HIRSAP Annual Meeting
Karlsruhe, November 2^{nd} - 3^{rd}

Spectral fitting of inclined showers and analysis

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PIERRE AUGER OBSERVATORY

HIRSAP Helmholtz International Research School for Astroparticle Physics

KIT Karlsruher Institut für Technologie

UNSAM UNIVERSIDAD NACIONAL DE SAN MARTÍN

- 2160 CoREAS p-simulations:

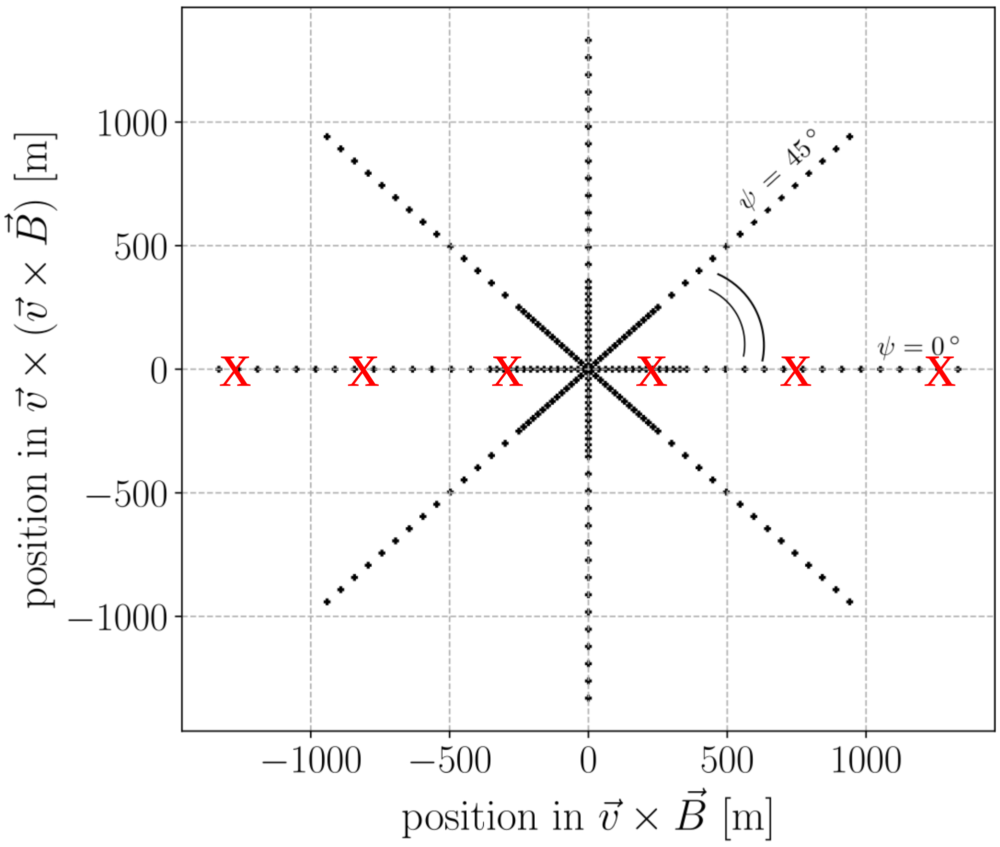
Log(E) = [18.4, 18.6, ..., 20.0, 20.2] eV
Azimuth = [0, 45, ..., 270, 315] deg
Zenith = [65.0, 67.5, ..., 82.5, 85.0] deg

- Simulated observer positions on a star-shaped grid

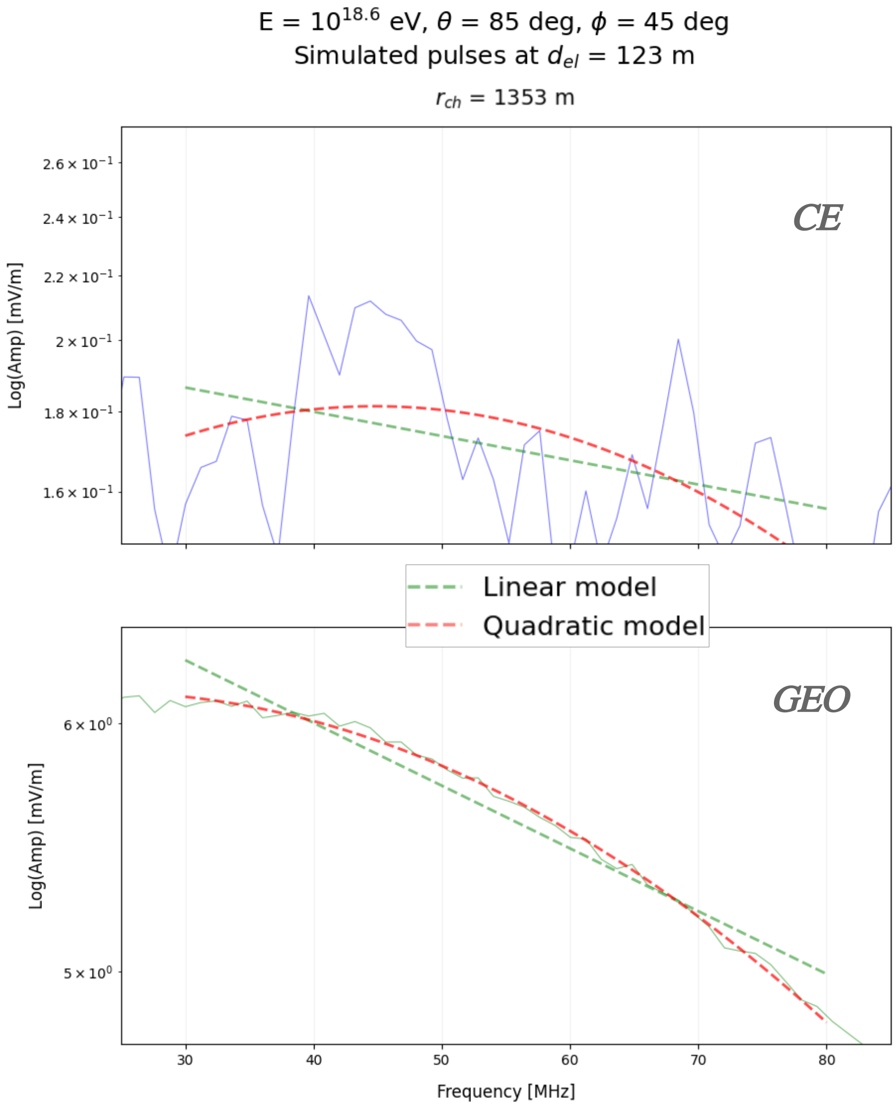
- **Geomagnetic (geo)** and **Charge-excess (ce)** field decomposition*:

$$E_{\vec{v}\times\vec{B}}\left(\vec{r},t\right) = E_{geo}\left(\vec{r},t\right) + cos\,\psi\, E_{ce}\left(\vec{r},t\right)$$
$$E_{\vec{v}\times(\vec{v}\times\vec{B})}\left(\vec{r},t\right) = sin\,\psi\, E_{ce}\left(\vec{r},t\right)$$

- Positions on the vxB-axis are excluded from the analysis



*From *"Simulation of radiation energy release in air showers", JCAP,* C. Glaser, M. Erdmann, J. R. Hörandel, T. Huege, J. Schulz

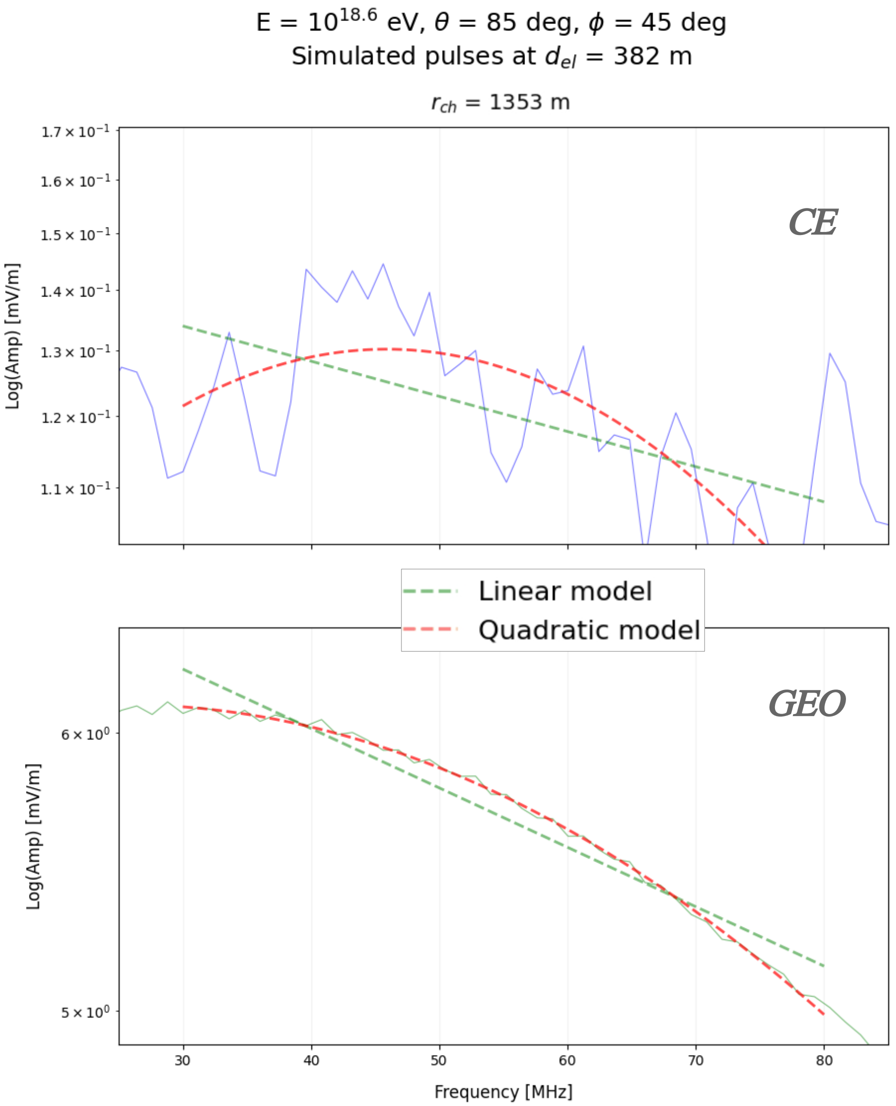


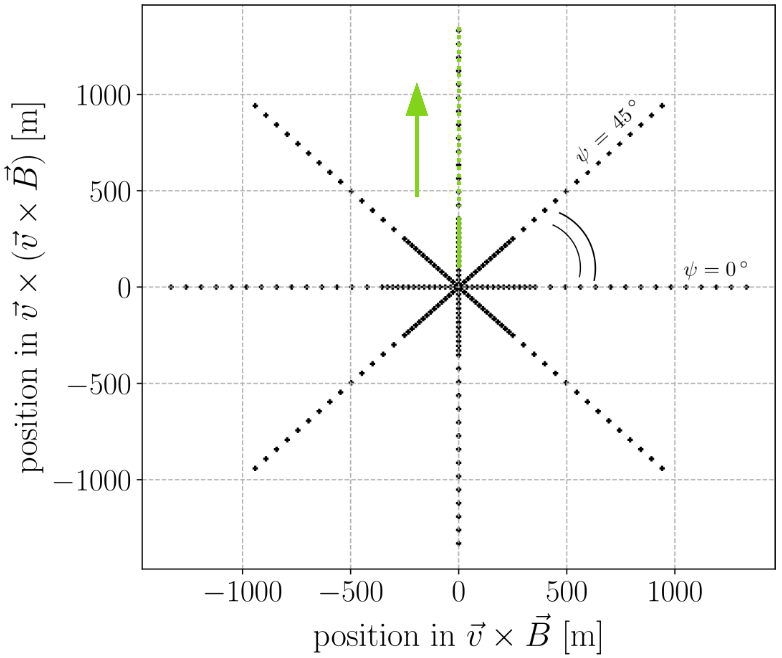
GEO and ***CE*** frequency spectra fitted separately in the 30-80 MHz band, comparing the models*:

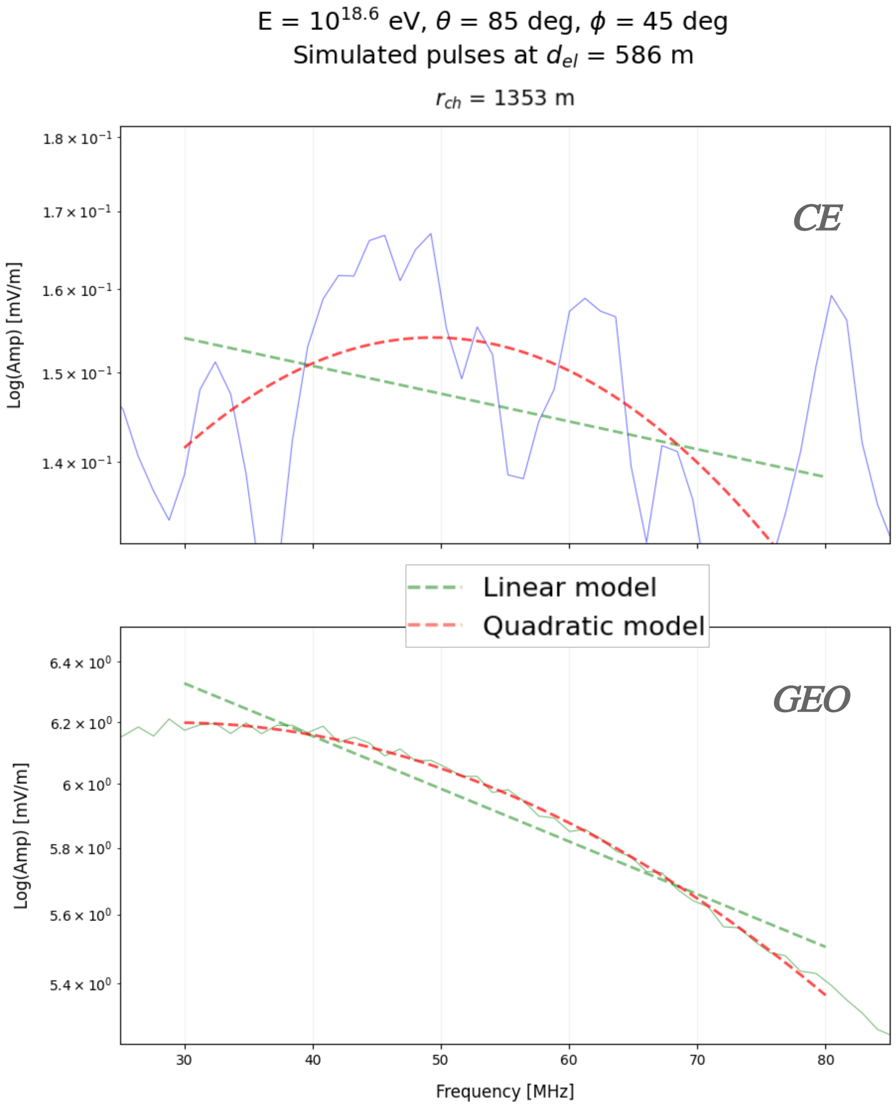
$$L = A \cdot 10^{(f-f_0)\,mf}$$

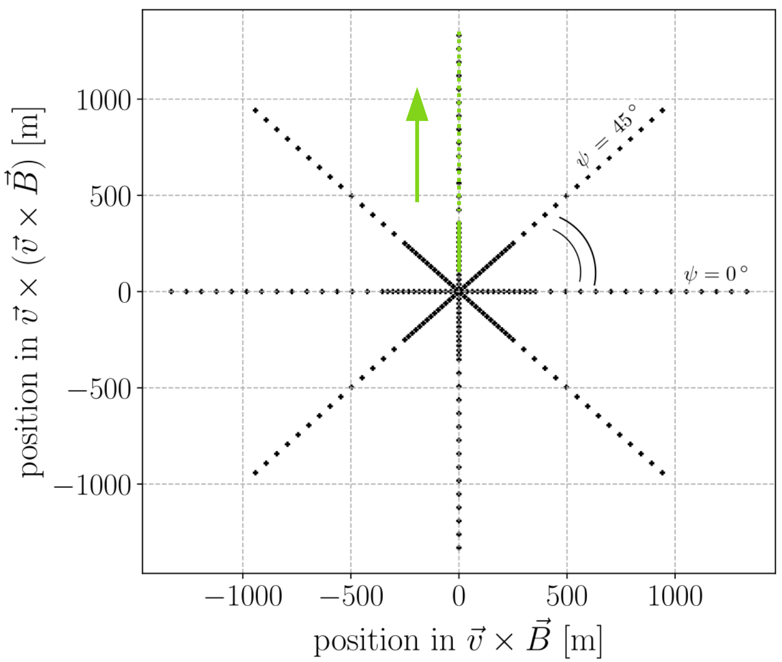
$$Q = A \cdot 10^{(f-f_0)\,mf + (f-f_0)^2\,mf_2}$$

*Models as described in: *"Reconstructing the cosmic-ray energy from the radio signal measured in one single station" (JCAP)* - C. Welling, C. Glaser, A. Nelles

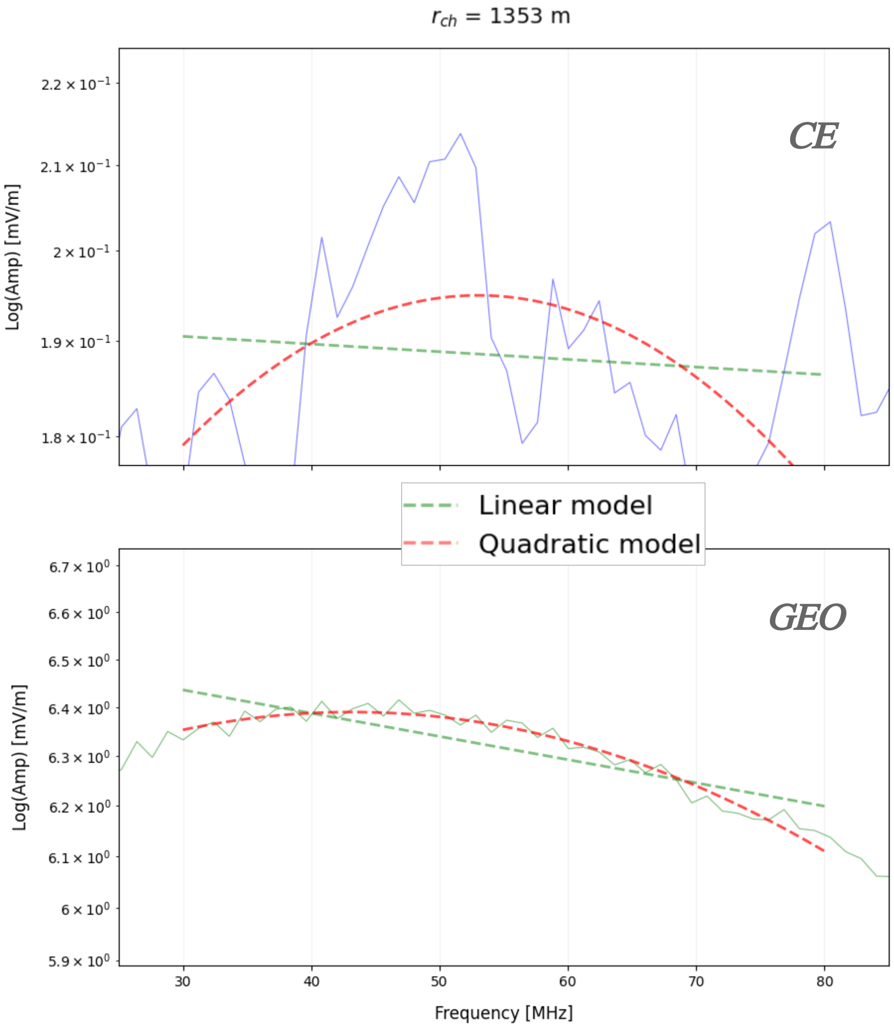

E = $10^{18.6}$ eV, θ = 85 deg, ϕ = 45 deg
Simulated pulses at d_{el} = 382 m
r_{ch} = 1353 m
CE
GEO
Linear model
Quadratic model
Log(Amp) [mV/m]
Frequency [MHz]

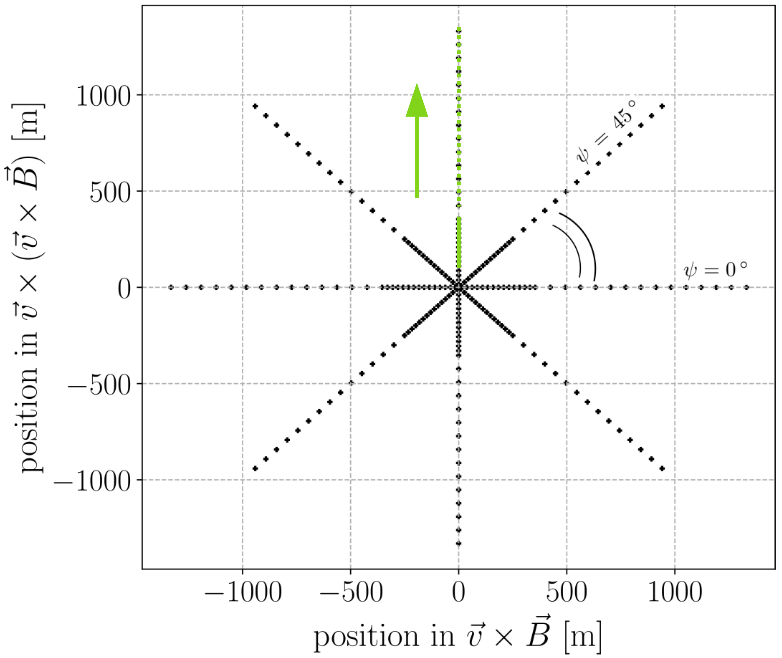

$\psi = 45°$
$\psi = 0°$
position in $\vec{v} \times (\vec{v} \times \vec{B})$ [m]
position in $\vec{v} \times \vec{B}$ [m]

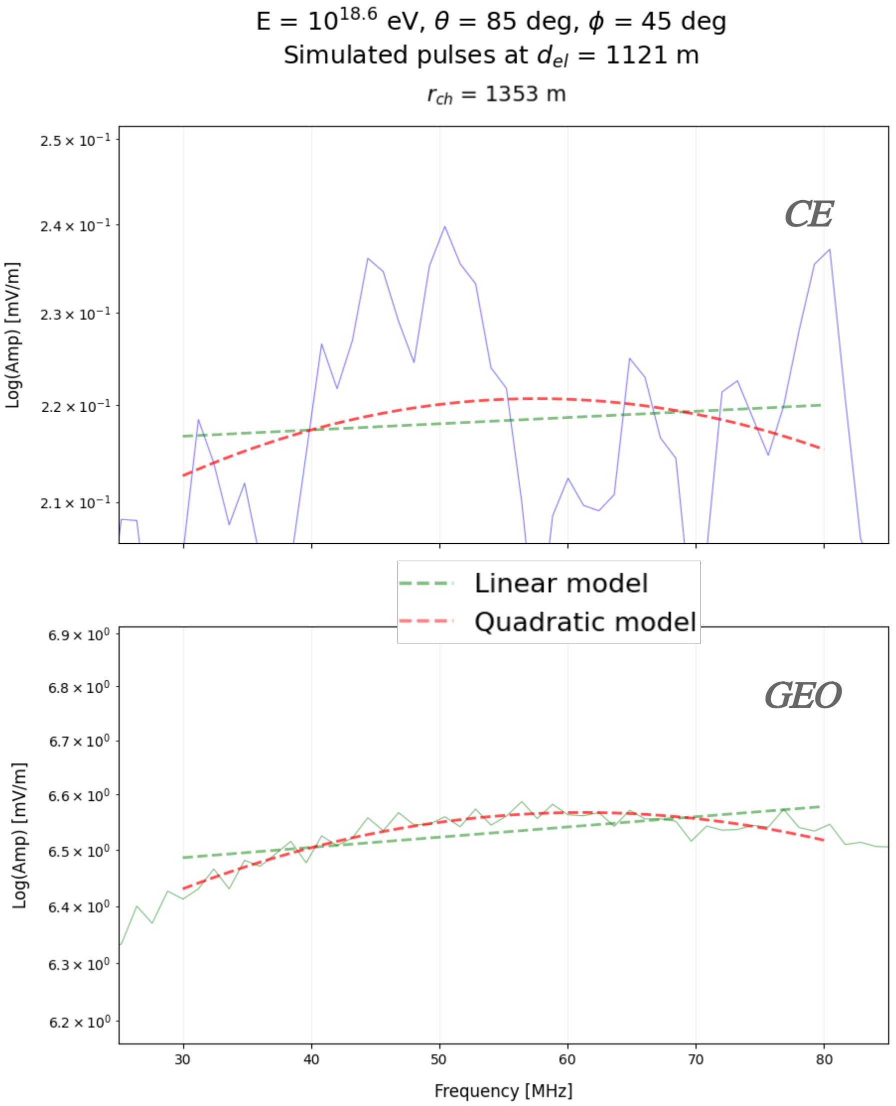

E = $10^{18.6}$ eV, θ = 85 deg, ϕ = 45 deg
Simulated pulses at d_{el} = 586 m
r_{ch} = 1353 m
CE
GEO
Linear model
Quadratic model
Log(Amp) [mV/m]
Log(Amp) [mV/m]
Frequency [MHz]
1.8×10^{-1}
1.7×10^{-1}
1.6×10^{-1}
1.5×10^{-1}
1.4×10^{-1}
6.4×10^{0}
6.2×10^{0}
6×10^{0}
5.8×10^{0}
5.6×10^{0}
5.4×10^{0}
30
40
50
60
70
80


$\psi = 45°$
$\psi = 0°$
position in $\vec{v} \times (\vec{v} \times \vec{B})$ [m]
position in $\vec{v} \times \vec{B}$ [m]
1000
500
0
−500
−1000
−1000
−500
0
500
1000

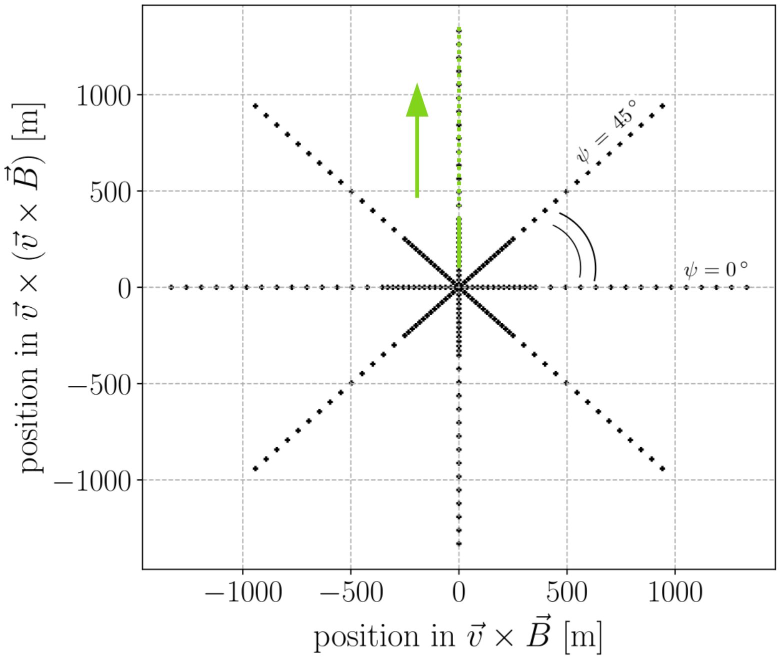
E = $10^{18.6}$ eV, θ = 85 deg, ϕ = 45 deg
Simulated pulses at d_{el} = 903 m



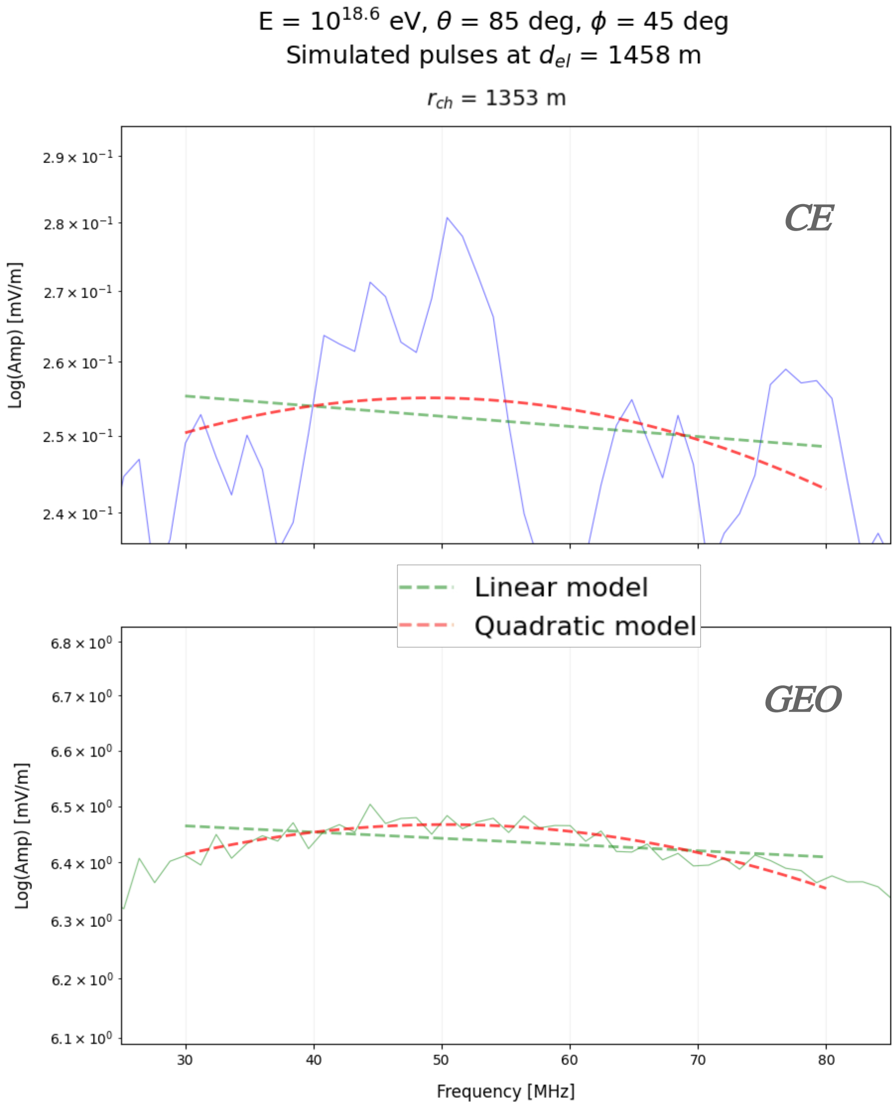


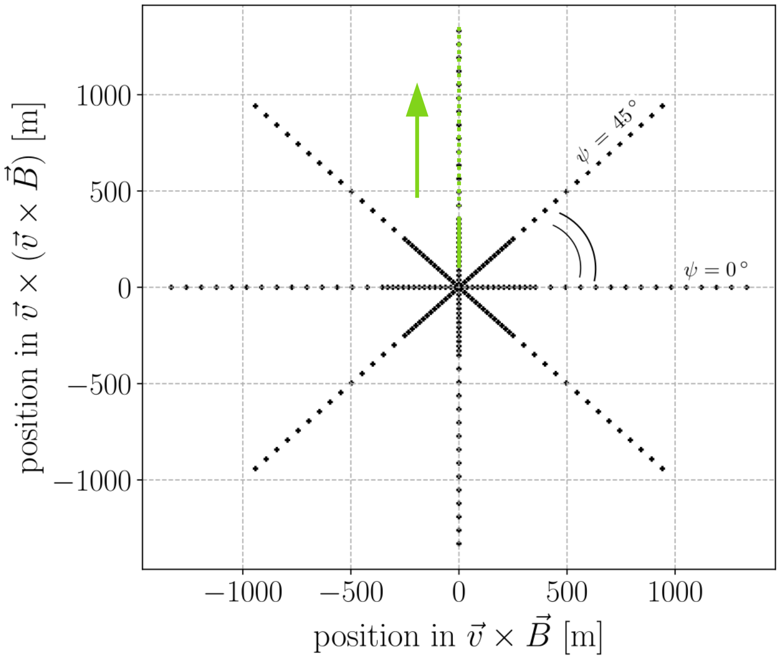


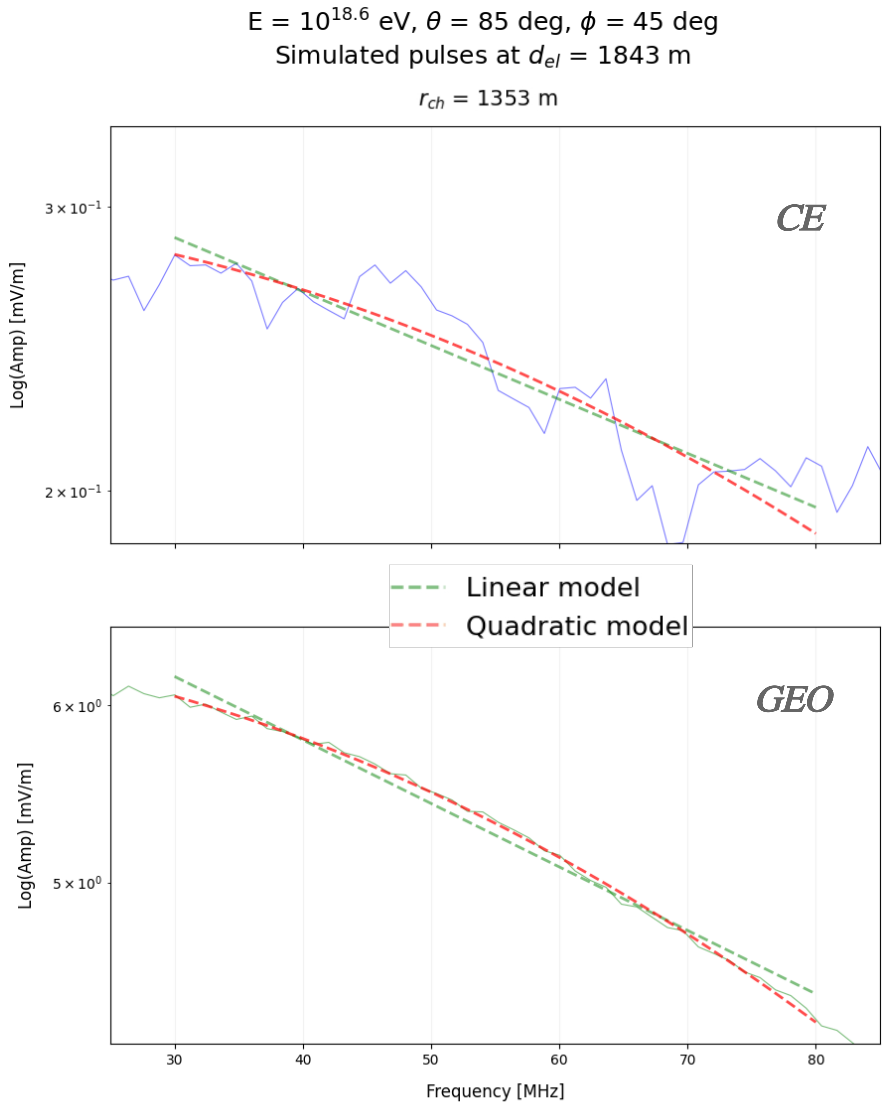
E = $10^{18.6}$ eV, θ = 85 deg, ϕ = 45 deg
Simulated pulses at d_{el} = 1121 m
r_{ch} = 1353 m
CE
GEO
Linear model
Quadratic model
Log(Amp) [mV/m]
Frequency [MHz]



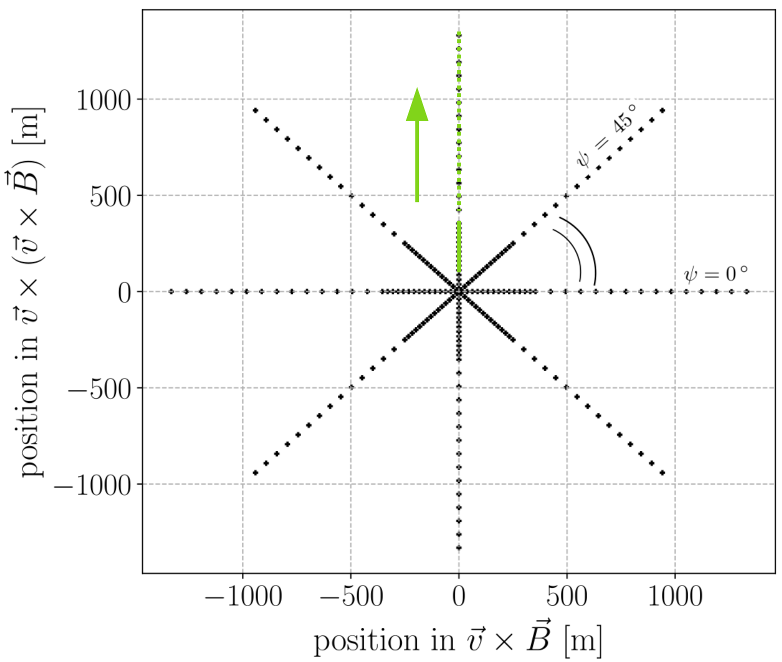
$\psi = 45°$
$\psi = 0°$
position in $\vec{v} \times (\vec{v} \times \vec{B})$ [m]
position in $\vec{v} \times \vec{B}$ [m]


E = $10^{18.6}$ eV, θ = 85 deg, ϕ = 45 deg
Simulated pulses at d_{el} = 1458 m
r_{ch} = 1353 m
CE
GEO
Linear model
Quadratic model
Log(Amp) [mV/m]
Frequency [MHz]

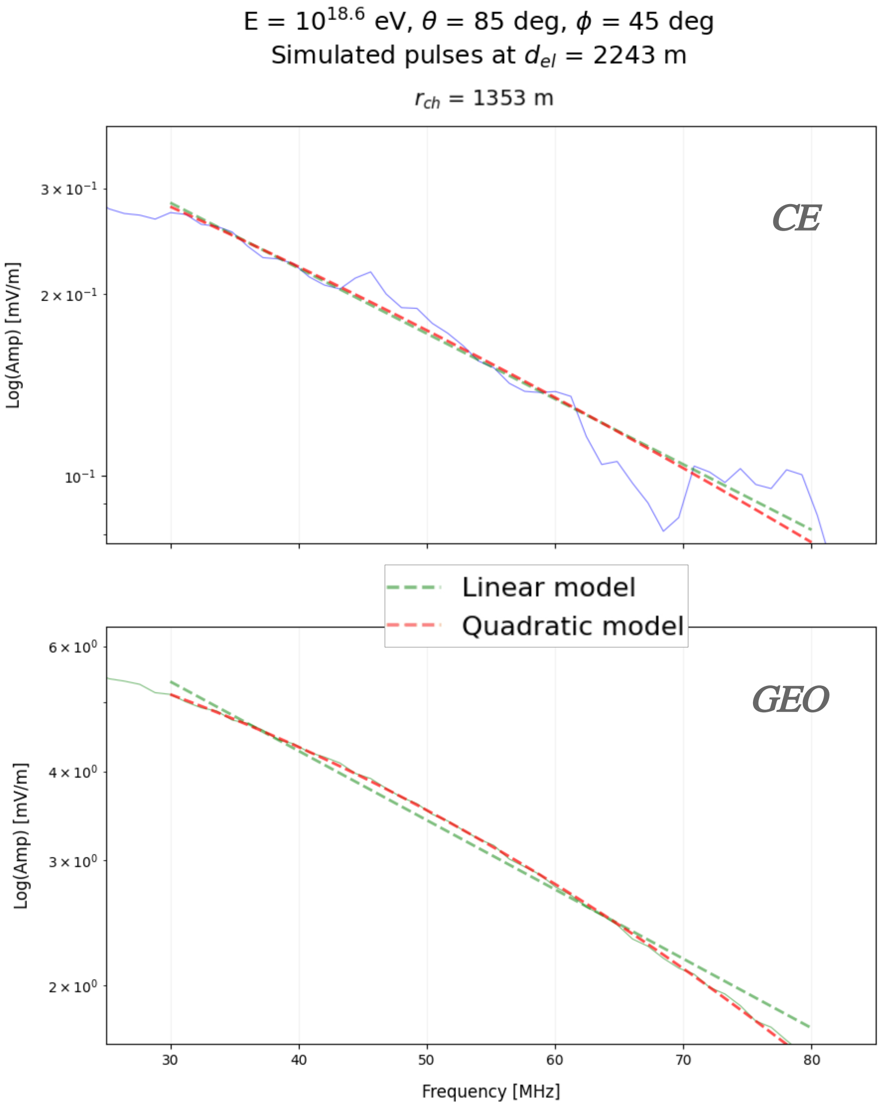

$\psi = 45°$
$\psi = 0°$
position in $\vec{v} \times \vec{B}$ [m]
position in $\vec{v} \times (\vec{v} \times \vec{B})$ [m]

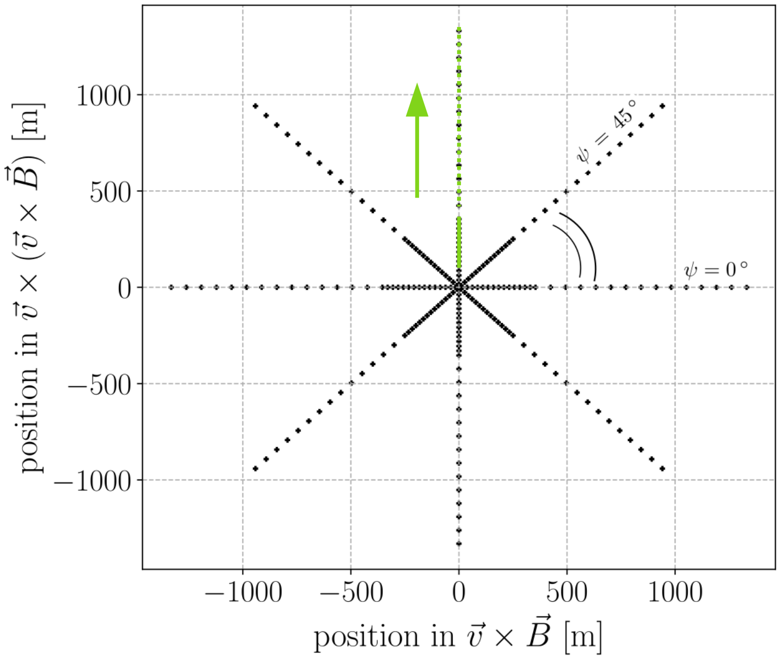


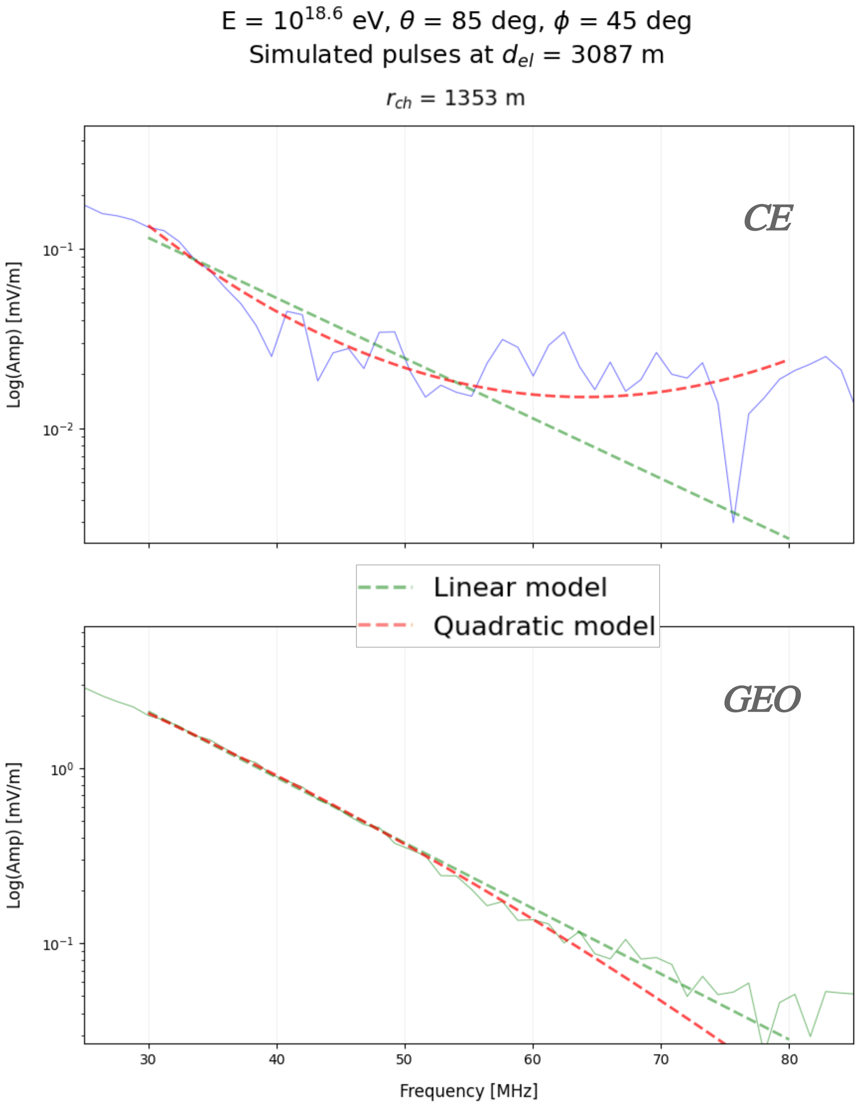
E = $10^{18.6}$ eV, θ = 85 deg, ϕ = 45 deg
Simulated pulses at d_{el} = 1843 m
r_{ch} = 1353 m
CE
GEO
Linear model
Quadratic model
Log(Amp) [mV/m]
Frequency [MHz]



$\psi = 45°$
$\psi = 0°$
position in $\vec{v} \times \vec{B}$ [m]
position in $\vec{v} \times (\vec{v} \times \vec{B})$ [m]


E = 10^18.6 eV, θ = 85 deg, ϕ = 45 deg
Simulated pulses at d_{el} = 2243 m
r_{ch} = 1353 m
CE
GEO
Linear model
Quadratic model
Log(Amp) [mV/m]
Frequency [MHz]

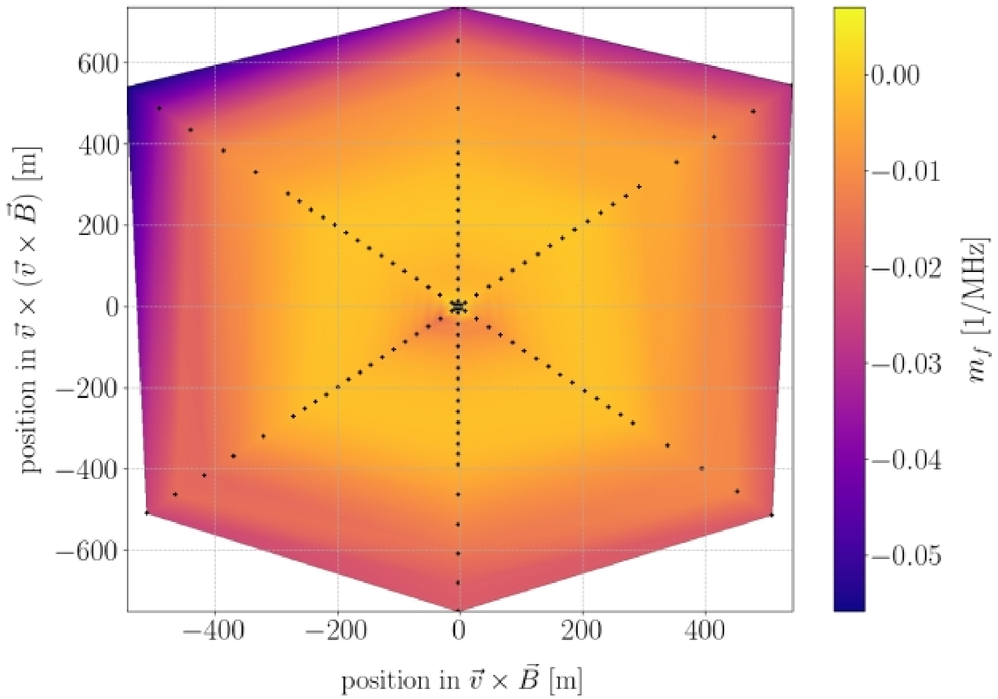

ψ = 45°
ψ = 0°
position in $\vec{v} \times (\vec{v} \times \vec{B})$ [m]
position in $\vec{v} \times \vec{B}$ [m]



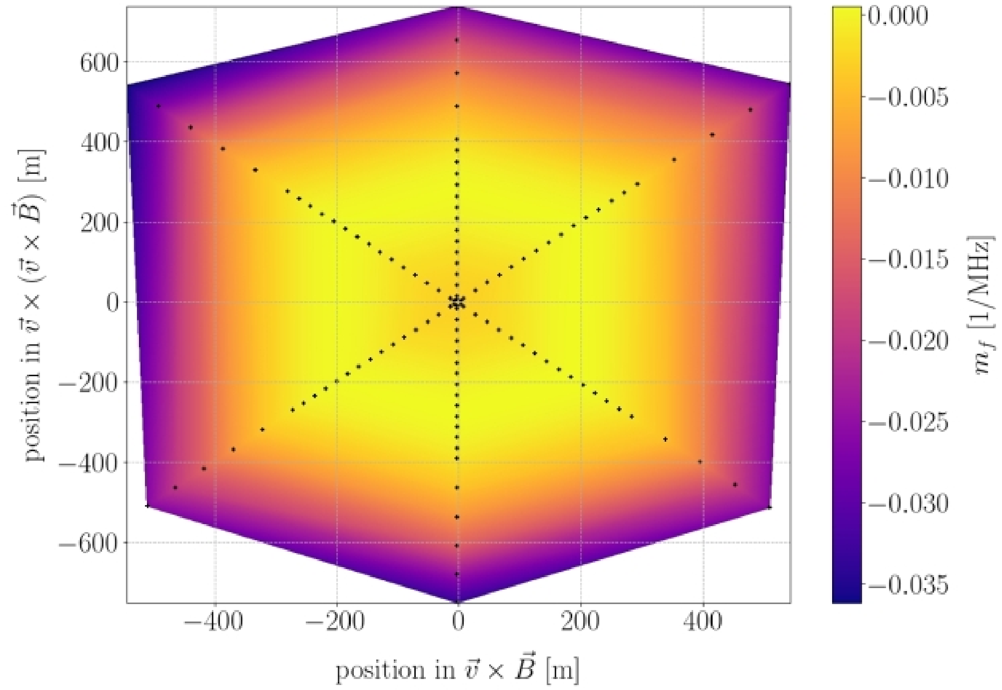
- ***CE:*** $L = A \cdot 10^{(f-f_0)\, mf}$
- ***GEO:*** $Q = A \cdot 10^{(f-f_0)\, mf + (f-f_0)^2\, mf_2}$
- Flattening of the spectrum around Cherenkov radius

Single event example:
Frequency slope mf footprint

CE



GEO



- Parameterization of the slope *mf* and the quadratic term *mf2*, as a function of *dmax*

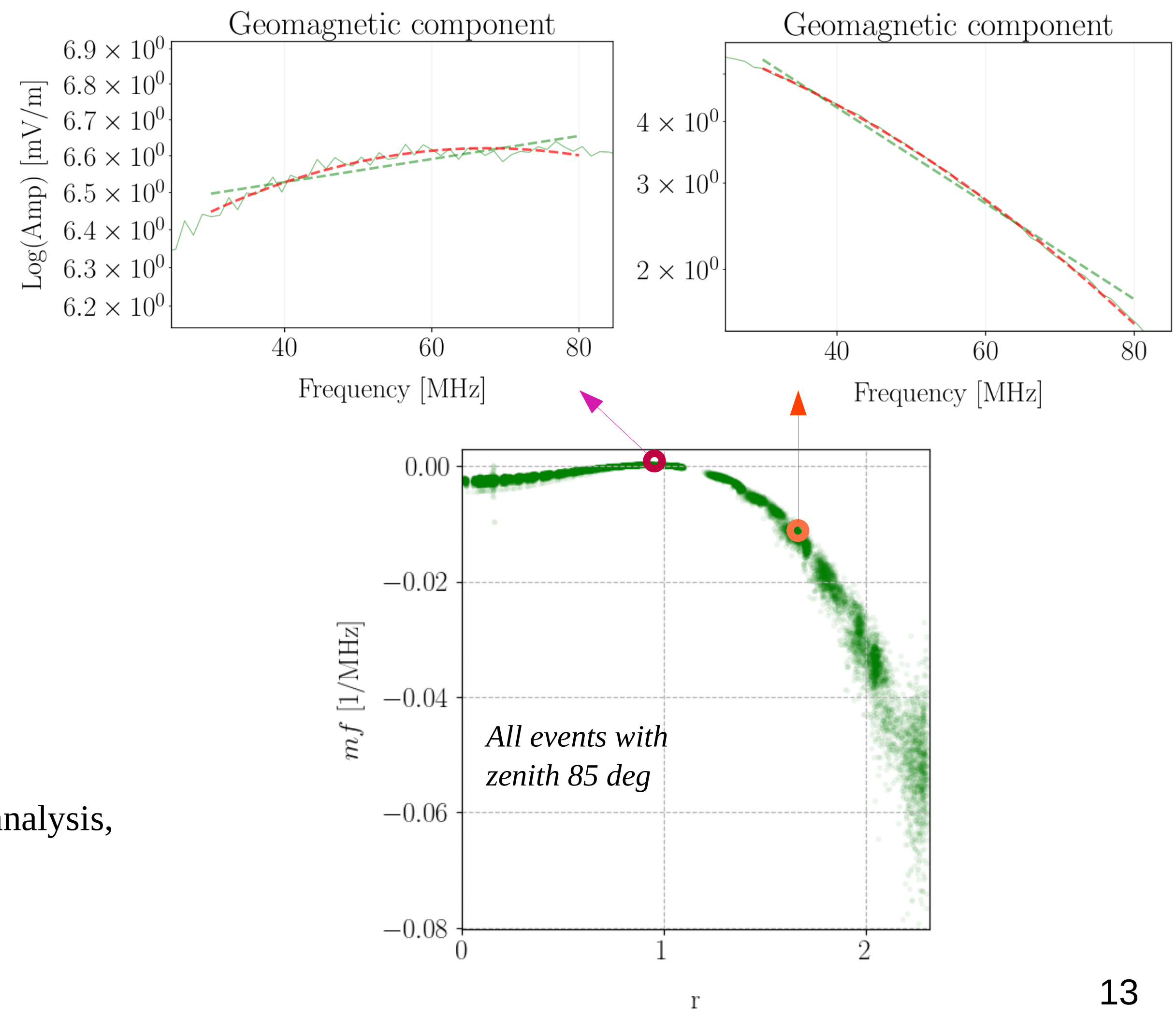
- Previous works about the spectral slope exploit second-order dependence of *Xmax* (F. Canfora, S. Jansen)

- This work can be used in reconstruction tools to better constrain the geometry (*e.g. the core*)

- Above about 2 Cherenkov radii, thinning becomes relevant

- Stations above r = 2 are excluded from the analysis, with $r = d_{el} / r_c$,
after applying (see backup slide):
  1. *core refraction displacement correction*
  2. *early-late (el) correction of the distance*

Example: slope lateral distribution for fixed zenith angle



GEOMAGNETIC COMPONENT

Quadratic Model

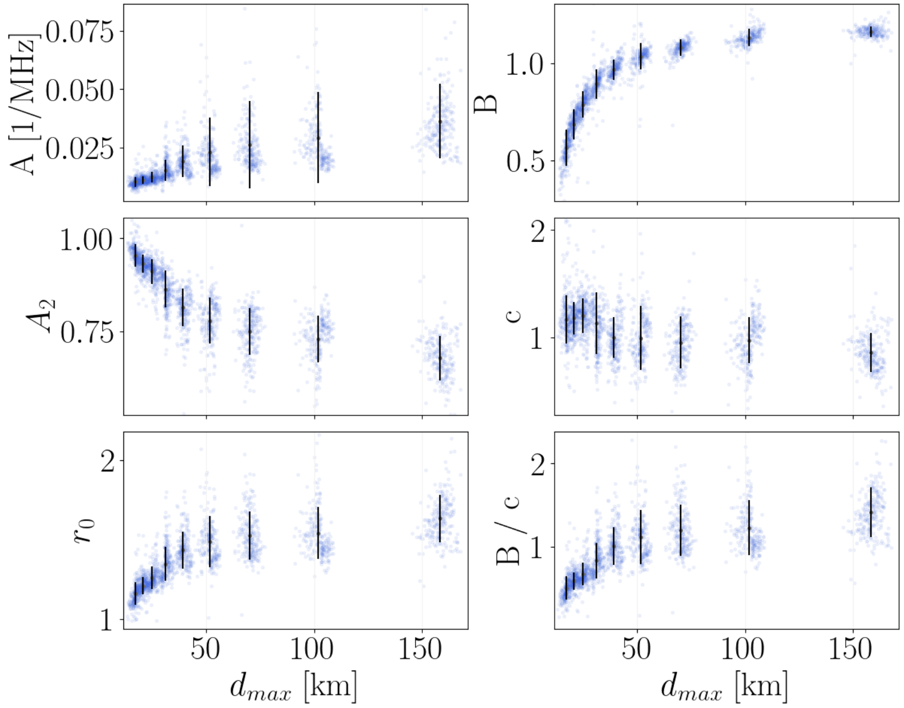
$$Q = A \cdot 10^{(f-f_0)\,mf + (f-f_0)^2\,mf_2}$$

- Function used to parameterize the slope:

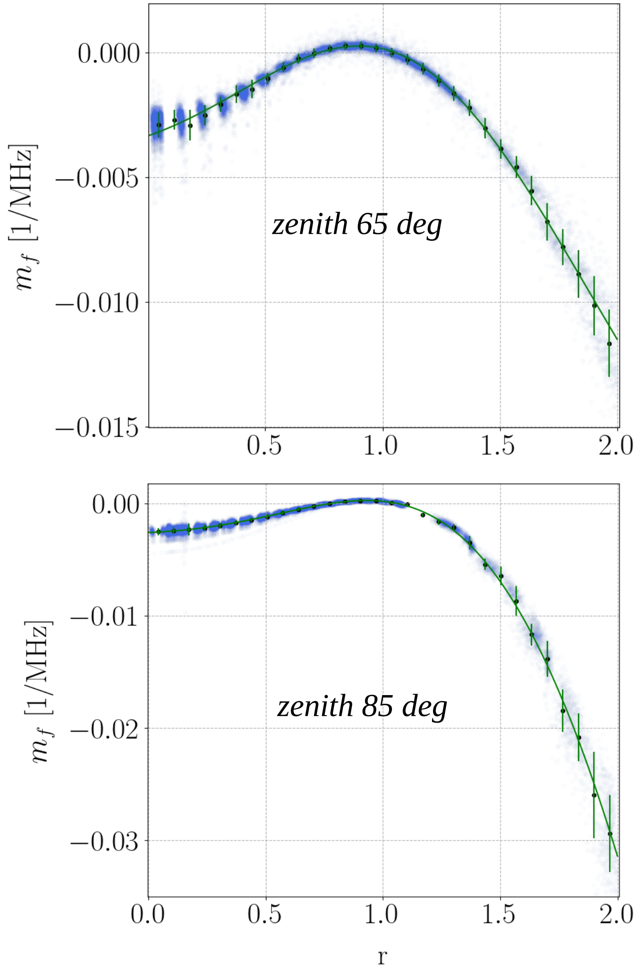
A * [– exp(B * (r – r0)) + A2 * exp(– c * (r – r0)** 2)]

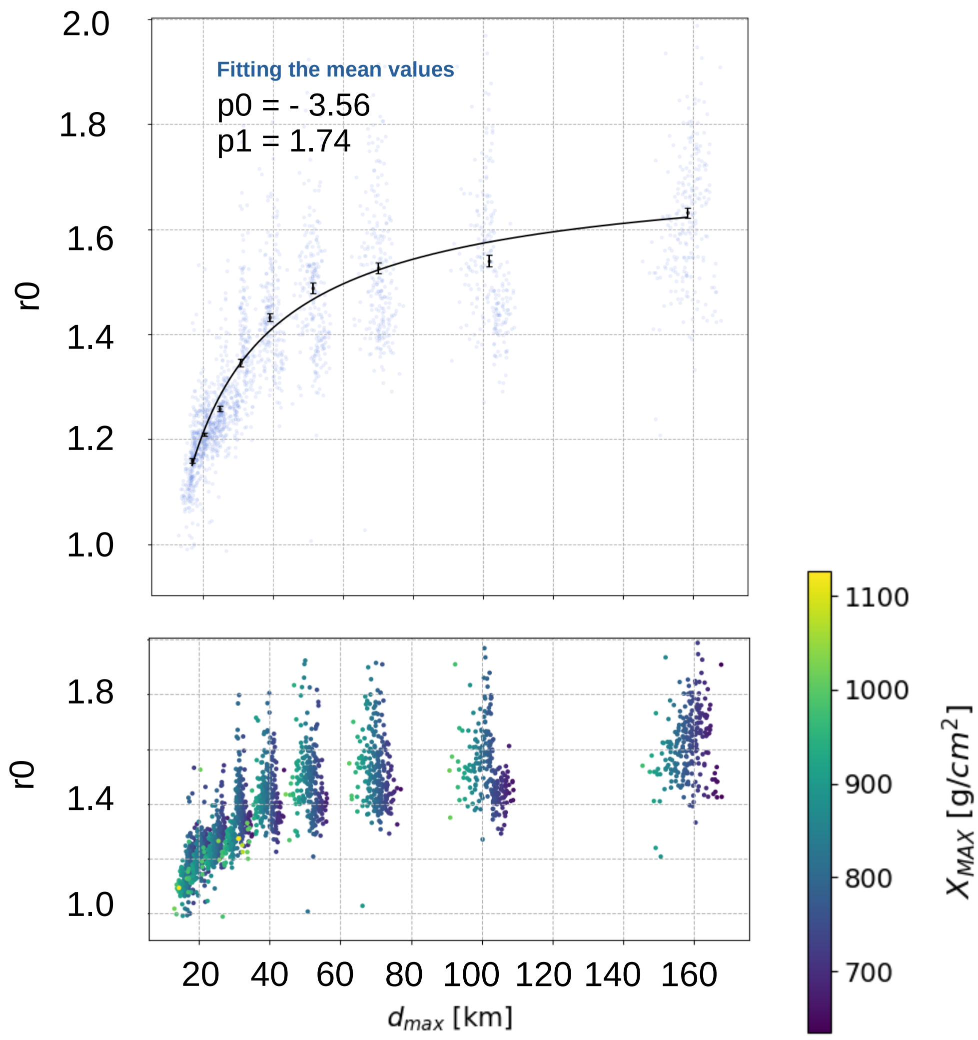
- 5 parameters, to be parameterized as a function of ***dmax***

Distributions from individual shower free fits



Free fit of mean values

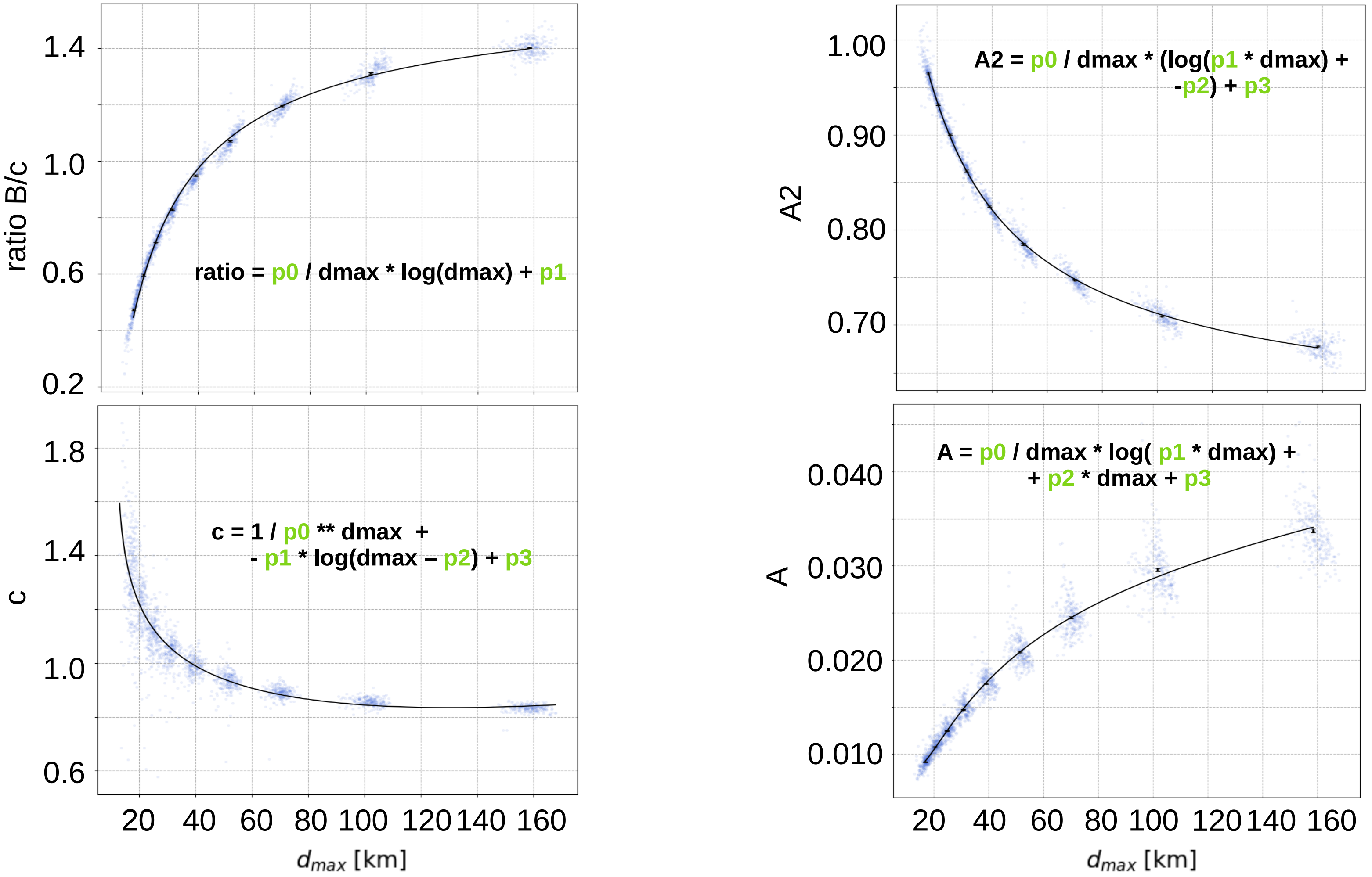




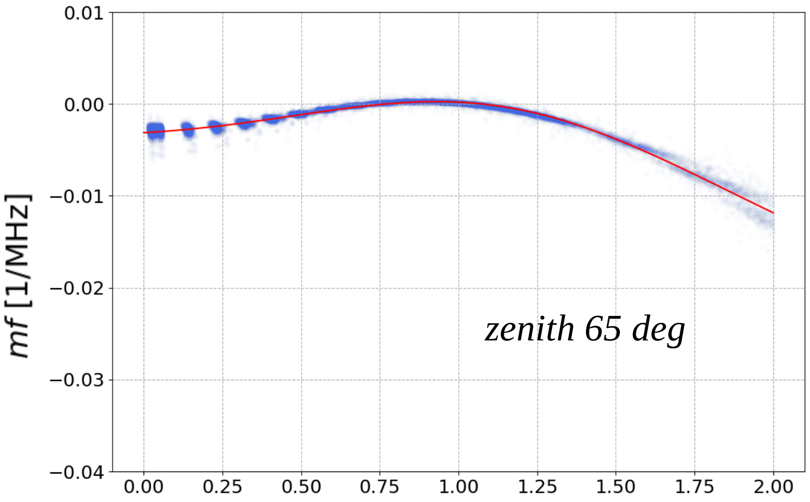
r0 = p0 / dmax * log(dmax) + p1

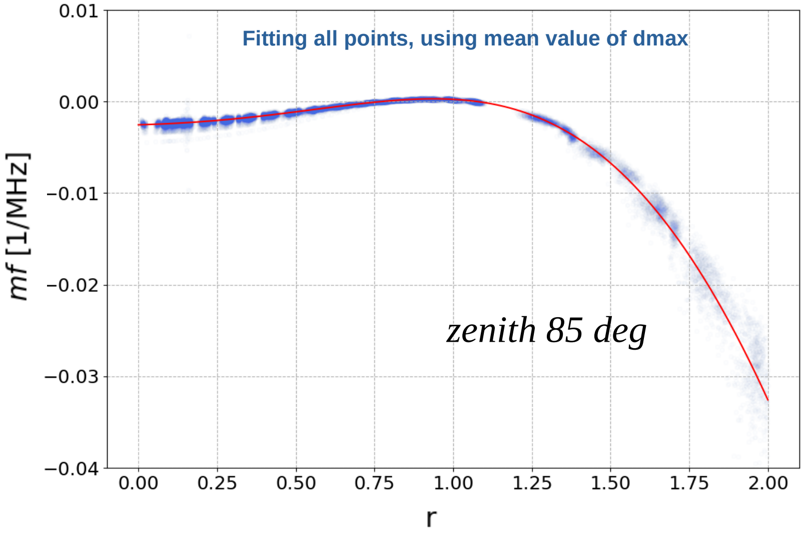
4 parameters left

A * [– exp(**B** * (r – **r0**)) +
+ **A2** * np.exp(– **c** * (r – **r0**)** 2)]

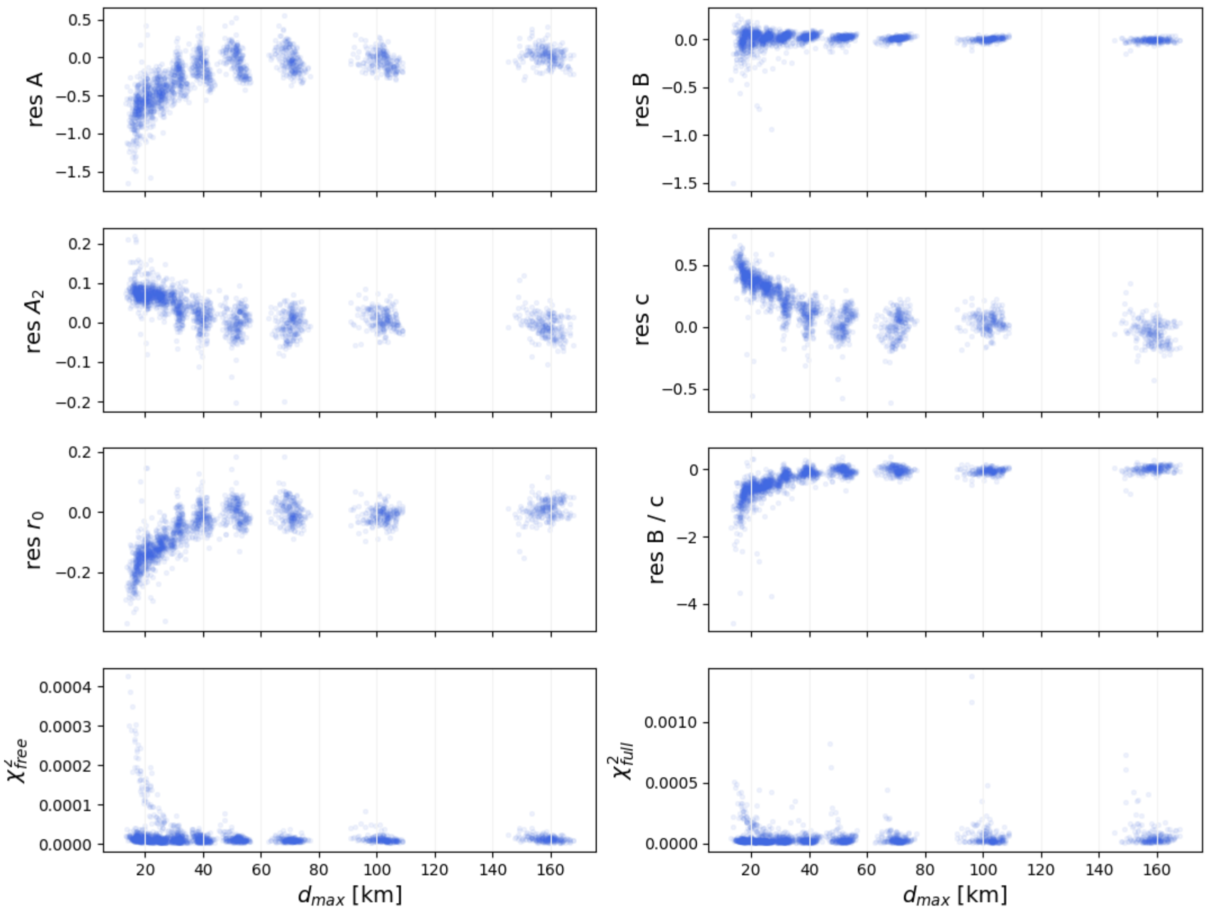

ratio = p0 / dmax * log(dmax) + p1
ratio B/c
1.4
1.0
0.6
0.2
A2 = p0 / dmax * (log(p1 * dmax) +
-p2) + p3
A2
1.00
0.90
0.80
0.70
c = 1 / p0 ** dmax +
- p1 * log(dmax – p2) + p3
c
1.8
1.4
1.0
0.6
A = p0 / dmax * log(p1 * dmax) +
+ p2 * dmax + p3
A
0.040
0.030
0.020
0.010
20 40 60 80 100 120 140 160
d_{max} [km]
20 40 60 80 100 120 140 160
d_{max} [km]

Full param. Function!



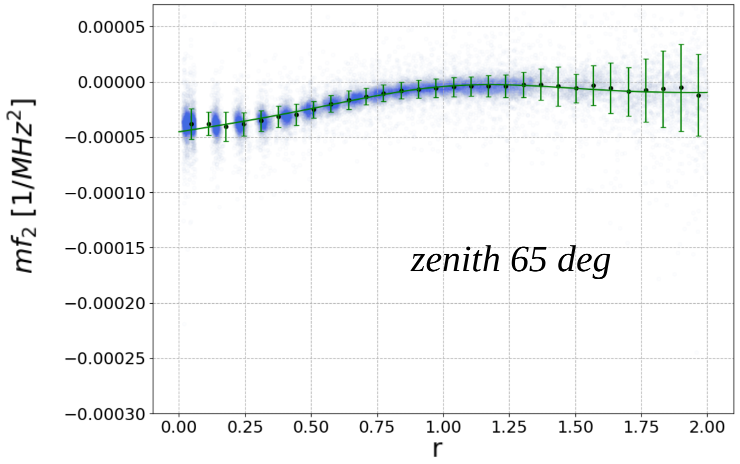


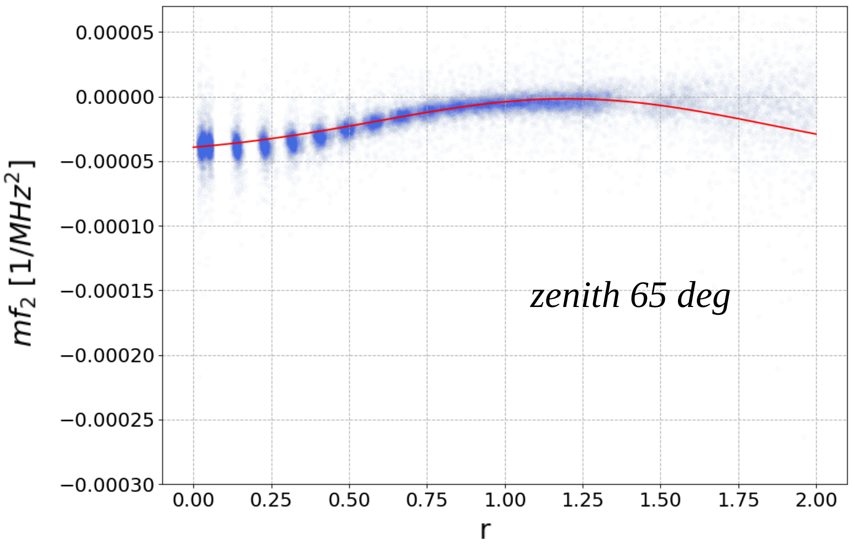
Residuals: free vs full param.

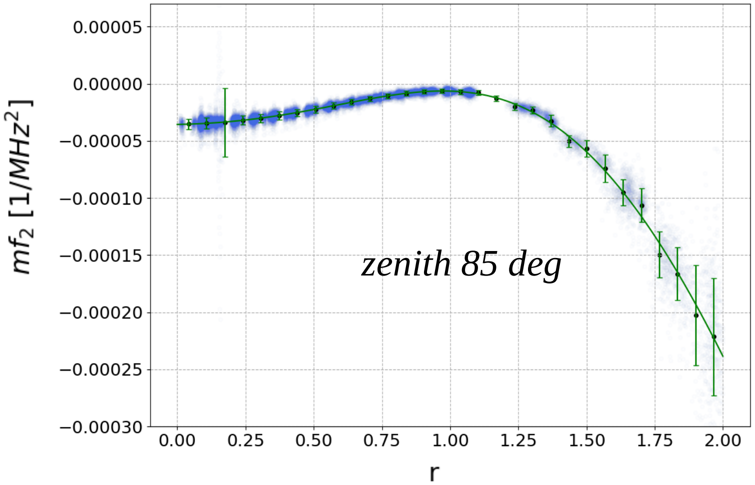


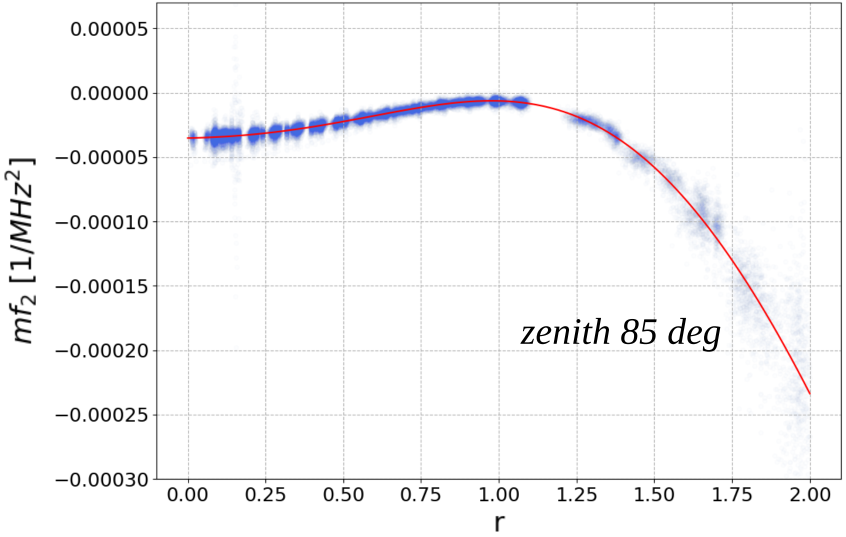
- The quadratic term ***mf2*** shows a similar trend as the slope
- Same function and similar procedure are used

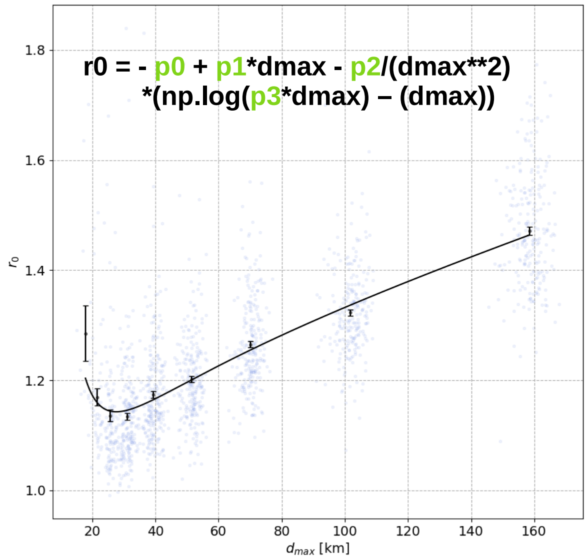
Full param. Function!

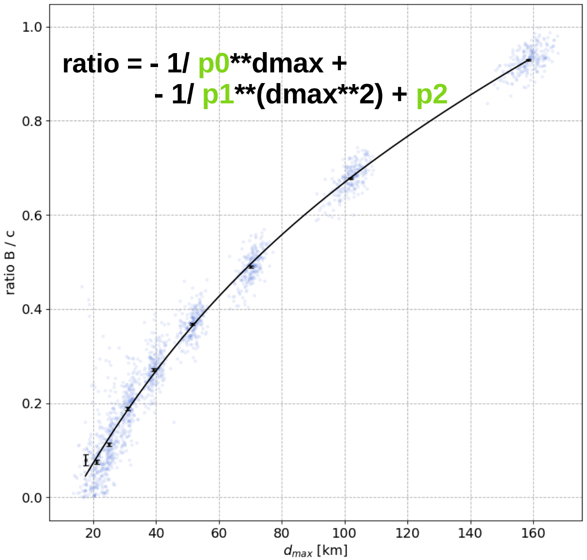


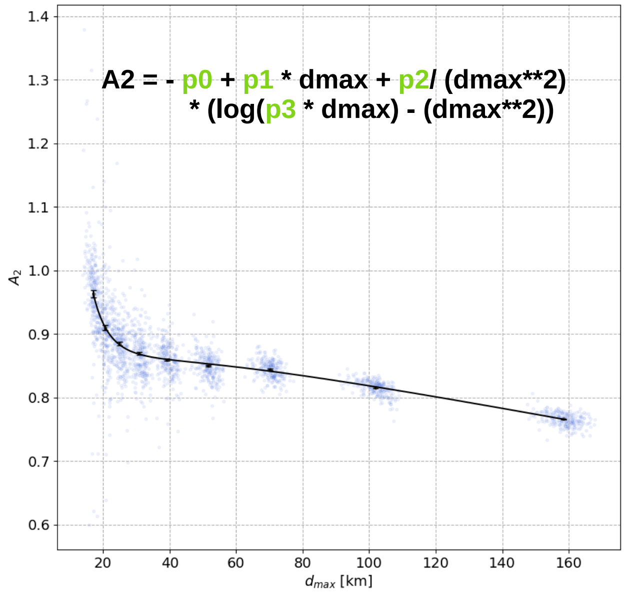


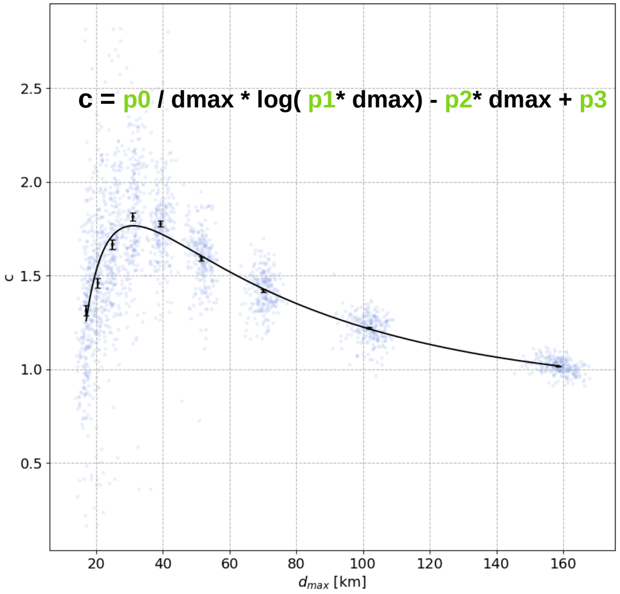


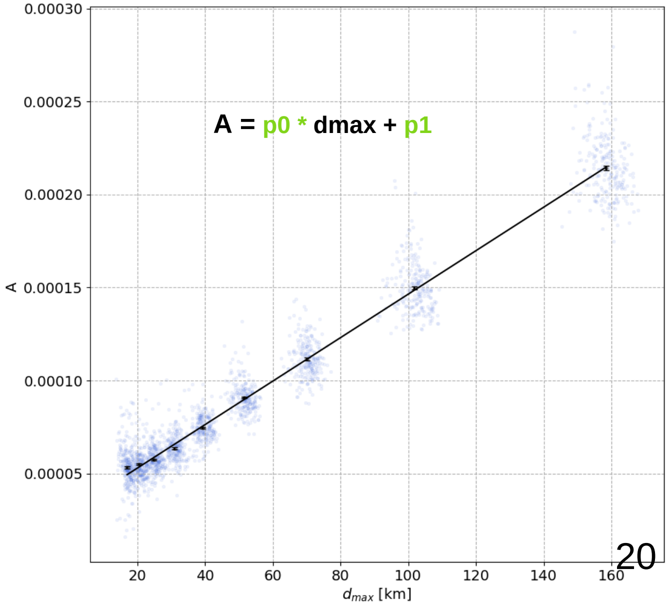



r0 = - p0 + p1*dmax - p2/(dmax**2)
*(np.log(p3*dmax) – (dmax))
r_0
d_{max} [km]


ratio = - 1/ p0**dmax +
- 1/ p1**(dmax**2) + p2
ratio B / c
d_{max} [km]


A2 = - p0 + p1 * dmax + p2/ (dmax**2)
* (log(p3 * dmax) - (dmax**2))
A_2
d_{max} [km]


c = p0 / dmax * log(p1* dmax) - p2* dmax + p3
c
d_{max} [km]


A = p0 * dmax + p1
A
d_{max} [km]

CHARGE-EXCESS COMPONENT

Linear Model

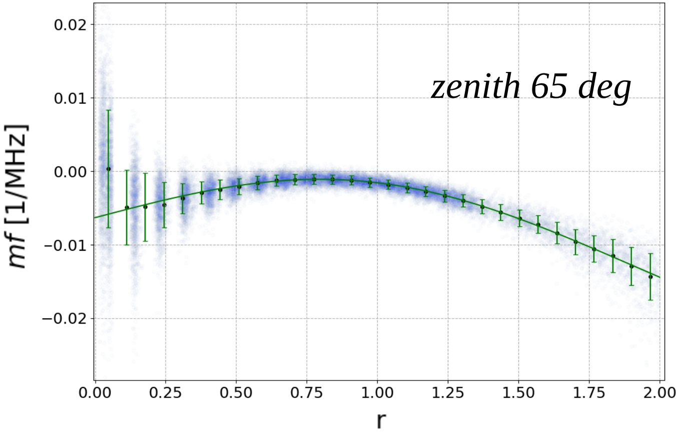
$$L = A \cdot 10^{(f - f_0) m f}$$

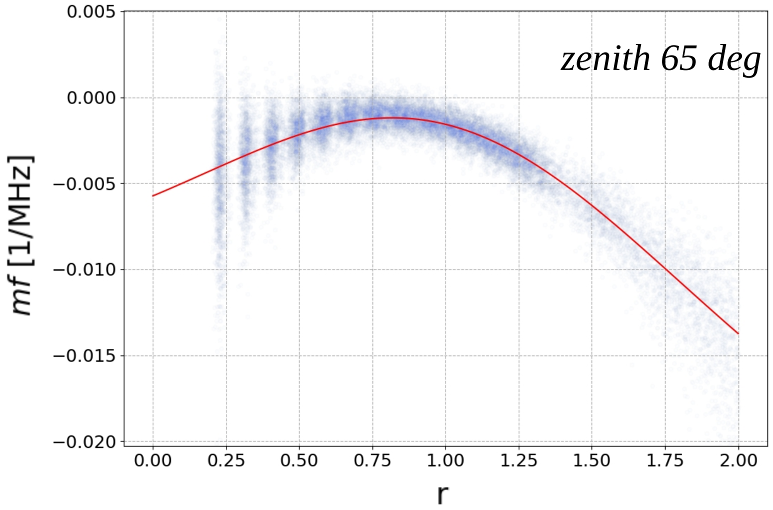
A * [– exp (B * (r – r0)) + A2 * exp (– c * (r – r0)** 2)]

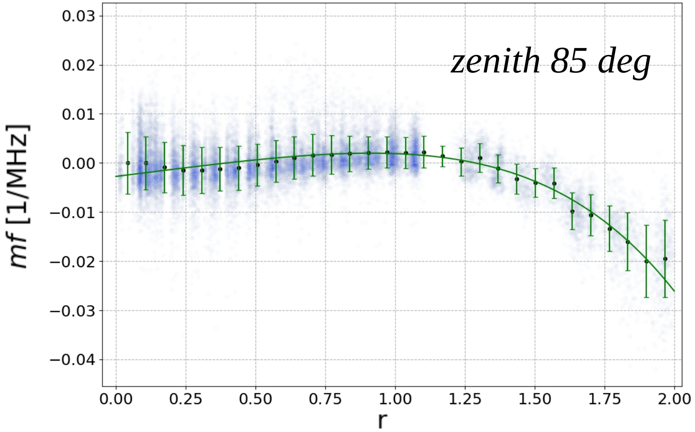
Full param. Function!

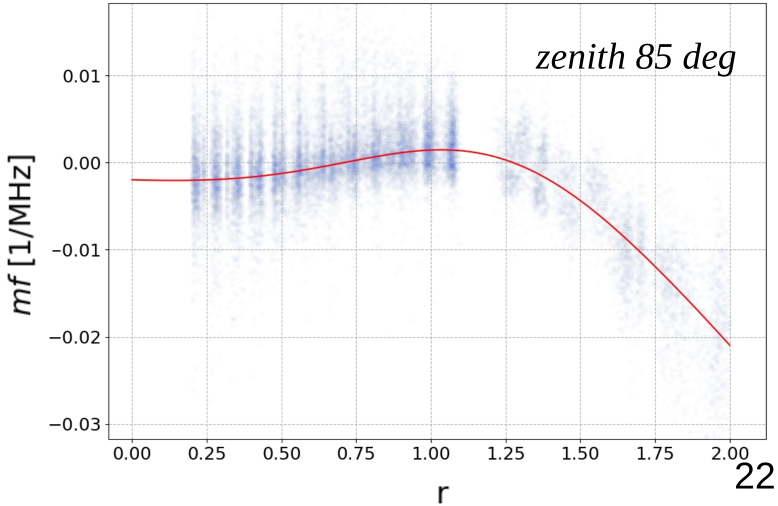
- Same function can be used to parameterize also the **CE slope**

Stations selected: 0.20 < r < 2.00









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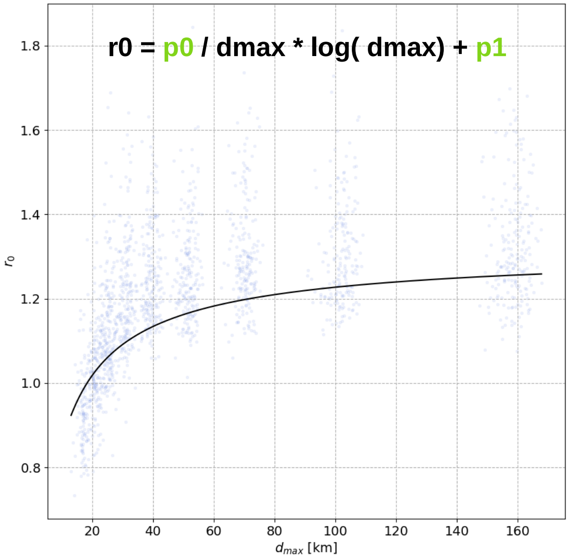
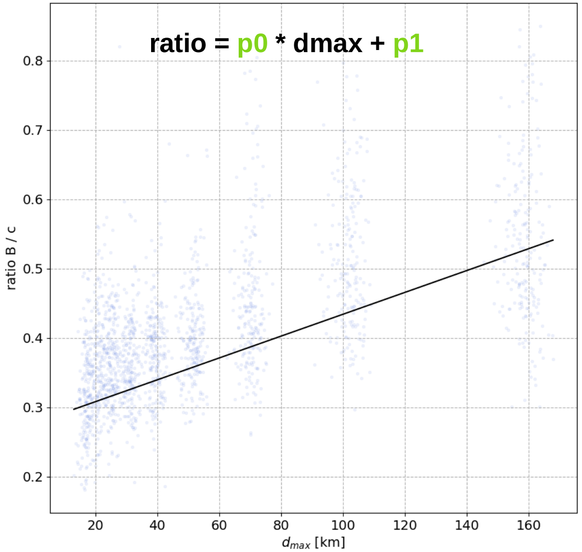
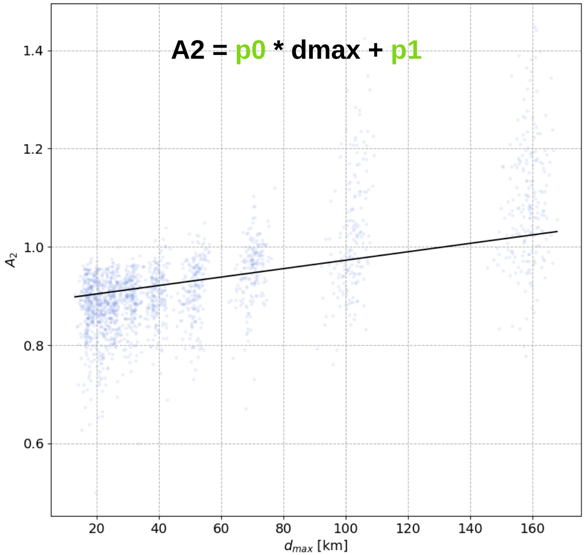
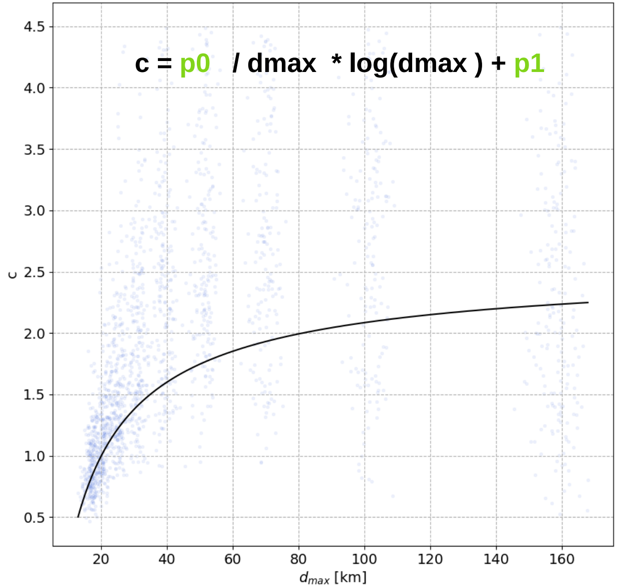
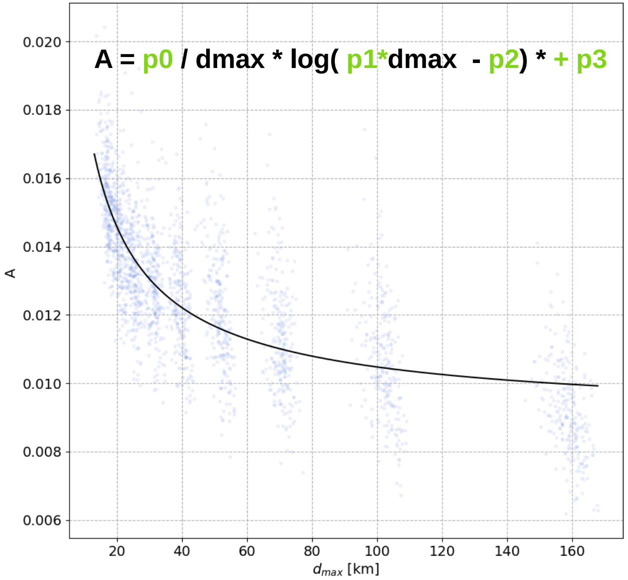

r0 = p0 / dmax * log(dmax) + p1
r_0
d_{max} [km]


ratio = p0 * dmax + p1
ratio B / c
d_{max} [km]


A2 = p0 * dmax + p1
A_2
d_{max} [km]


c = p0 / dmax * log(dmax) + p1
c
d_{max} [km]


A = p0 / dmax * log(p1*dmax - p2) * + p3
A
d_{max} [km]

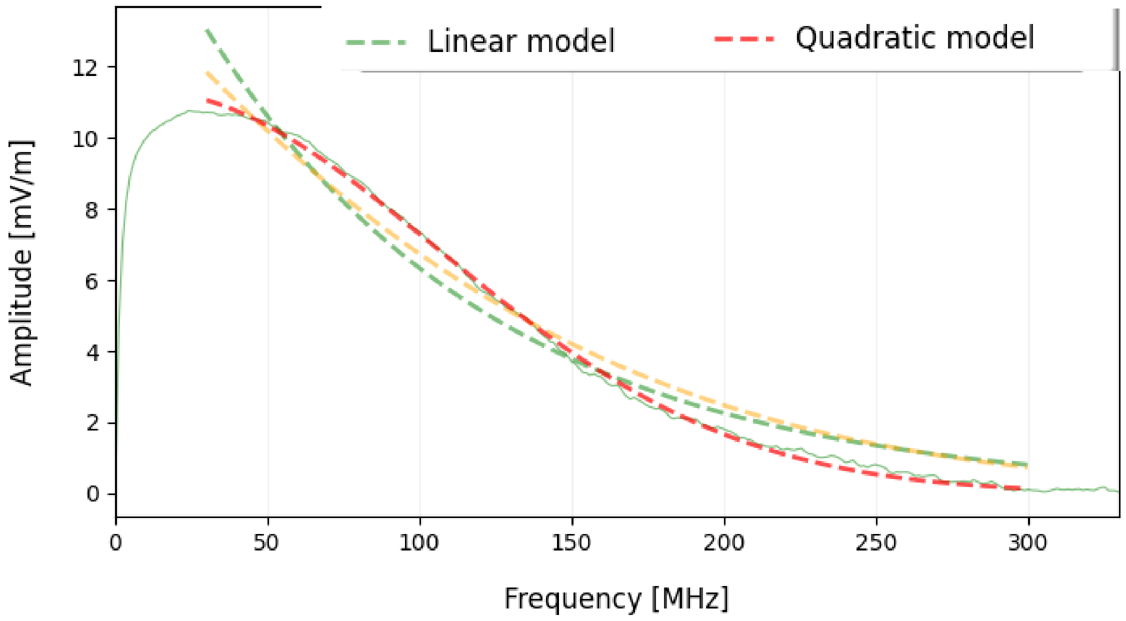
Conclusions

- The quadratic model describes better the geomagnetic spectrum
- Parameterizations for geomagnetic slope and quadratic term were found
- The charge-excess spectrum can be described by a linear model
- Parameterization for charge-excess slope was found
- Given zenith angle, xmax and antenna position, the slope and quadratic term can be analytically calculated and exploited in reconstruction algorithm to better constrain the geometry
- GAP note soon available

BACKUP SLIDES

- Wider frequencies regions:
  - linear model inadequate
  - quadratic model significant better

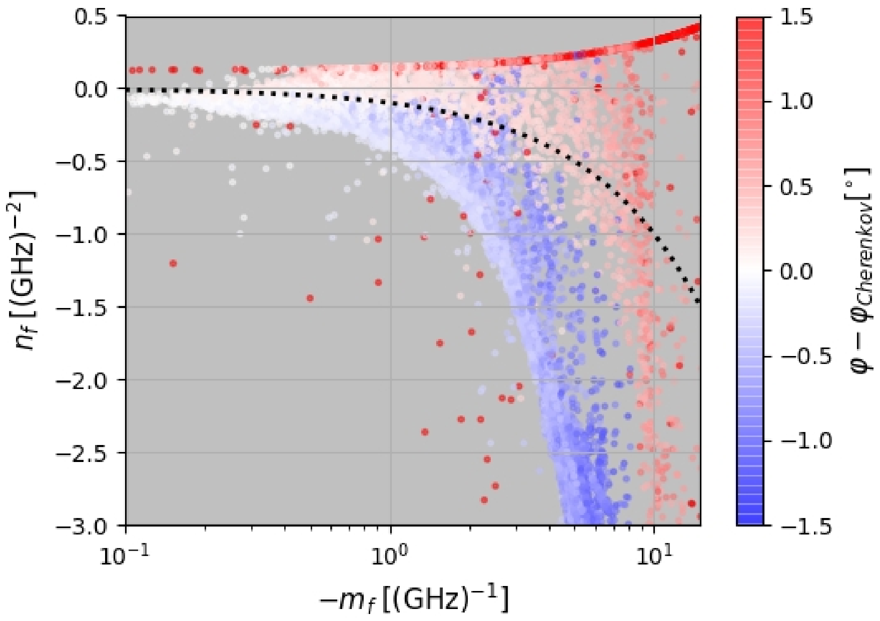
- An additional correction, not only quadratic, could be needed (see two curvatures)



“Reconstructing the cosmic-ray energy from the radio signal measured in one single station”, C. Welling, C. Glaser, A. Nelles

$$\begin{pmatrix} \mathcal{E}_\theta \\ \mathcal{E}_\phi \end{pmatrix} = \begin{pmatrix} A_\theta \\ A_\phi \end{pmatrix} 10^{f \cdot m_f + (f - 80\mathrm{MHz})^2 \cdot n_f} \exp(\Delta j)$$

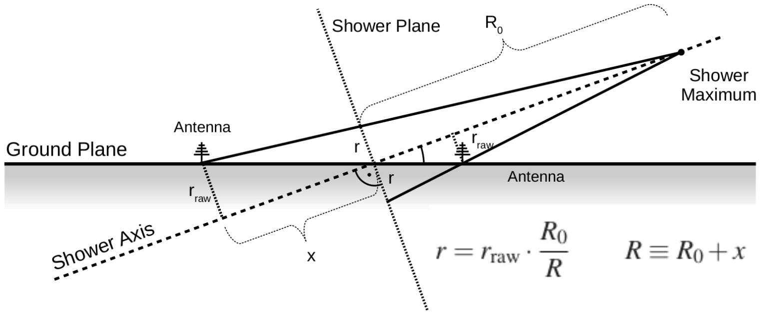
Quadratic correction of a broader frequency bandwidth (80 – 300 MHz – ARIANNA) shows interesting features



Positive quadratic correction identifies signals measured outside the Cherenkov ring, negative otherwise

- With larger distances, the scatter increases due to early-lateness ("*splitting*" feature)

  1. *core refraction displacement correction**
  2. *early-late (el) correction of the distance***



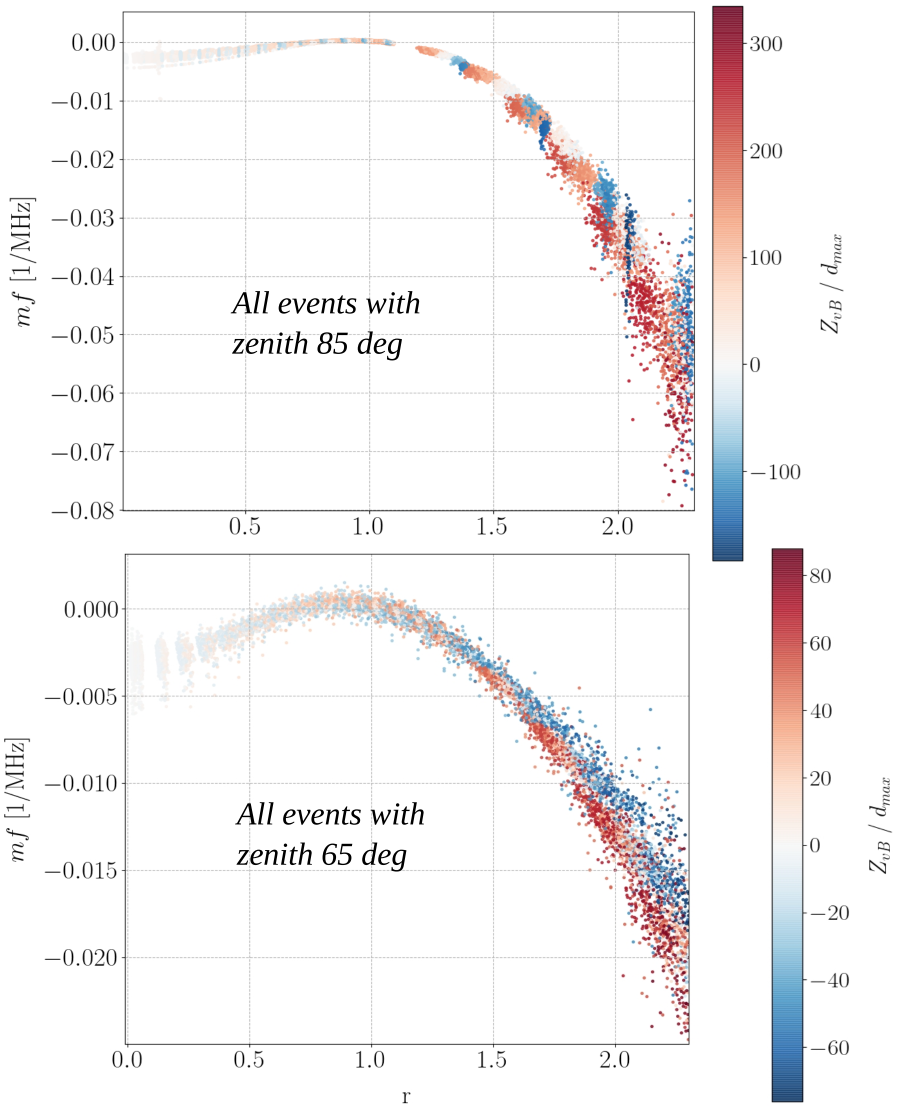
- Above about 2 Cherenkov radii, also thinning becomes relevant

- Stations above r = 2 are excluded from the analysis, with $r = d_{el} / r_c$

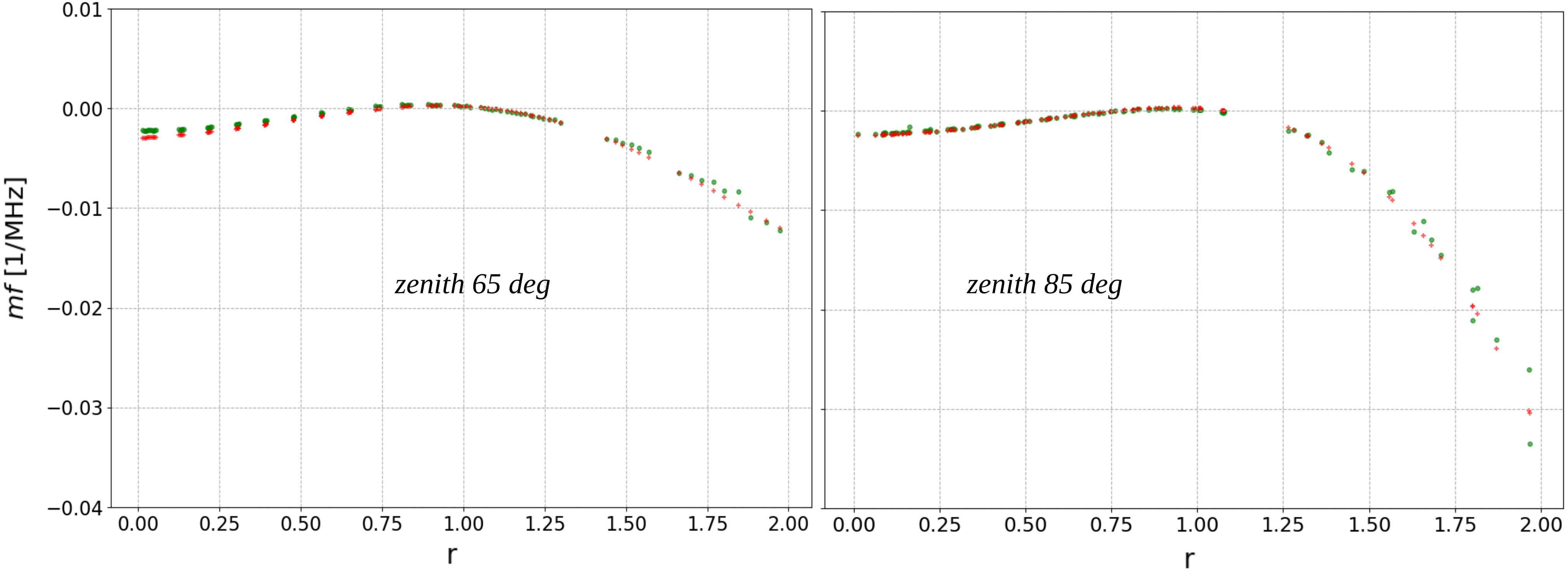
* *"Refractive displacement of the radio-emission footprint of inclined air showers simulated with CoREAS"* - F. Schlüter, M. Gottowik, T. Huege, J. Rautenberg

** *"A Rotationally Symmetric Lateral Distribution Function for Radio Emission from Inclined Air Showers"* - T. Huege, L. Brenk, F. Schlüter

Example: slope lateral distribution for fixed zenith angle



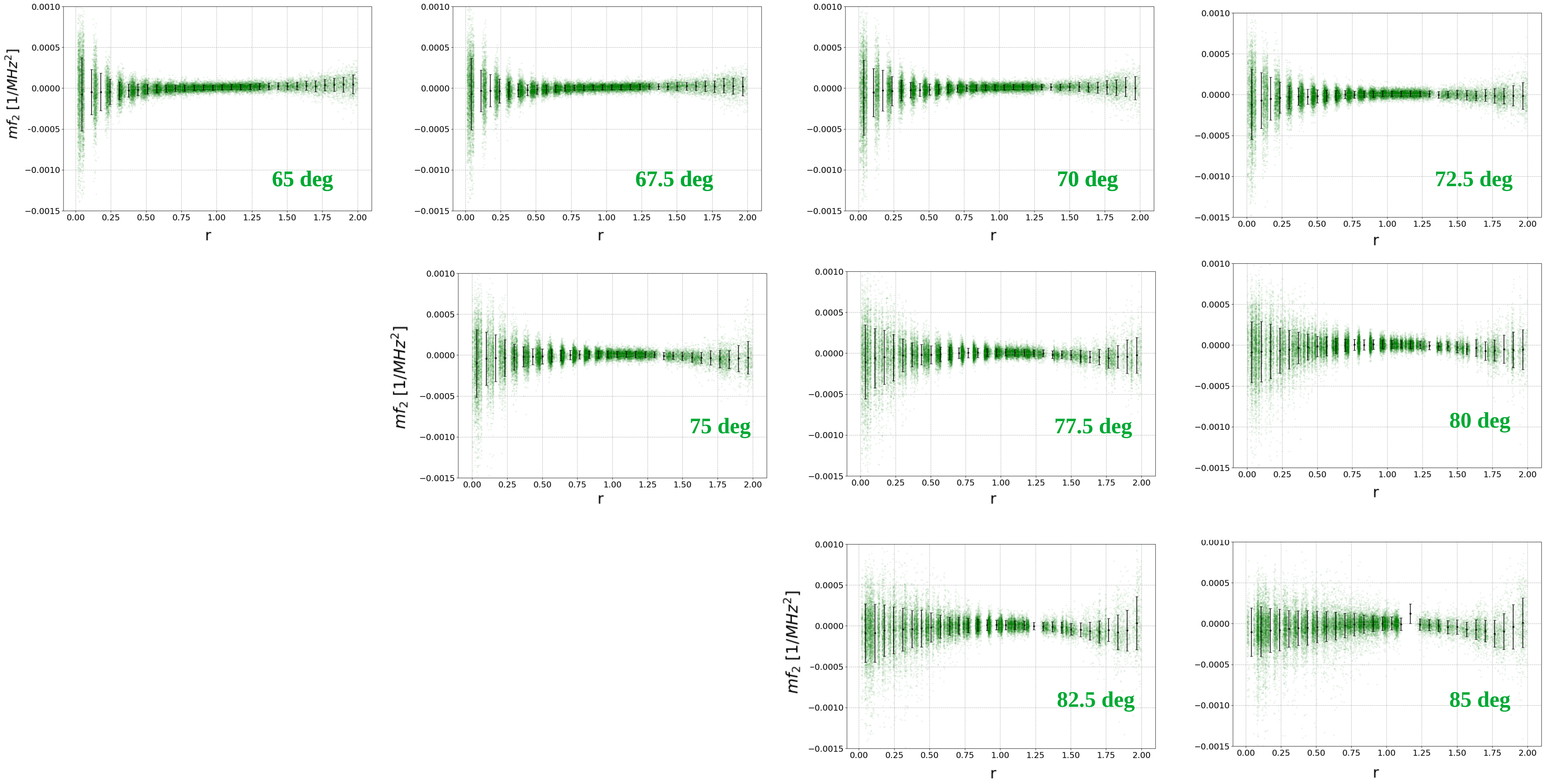
Geomagnetic slope:
single event examples

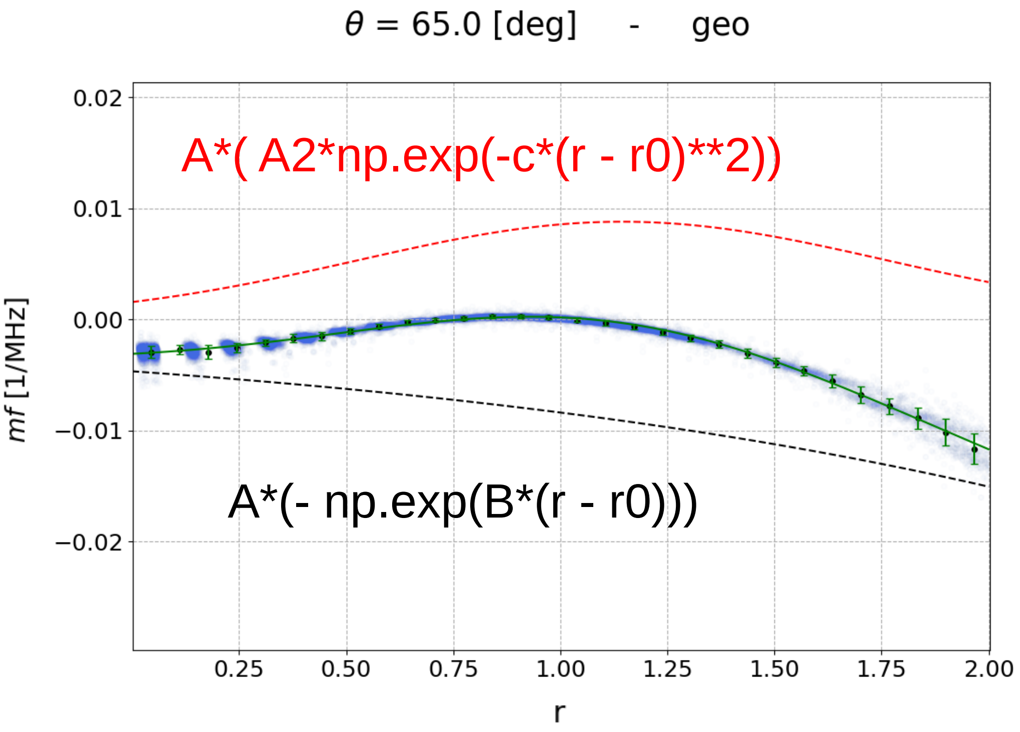


Red dots using full parameterization

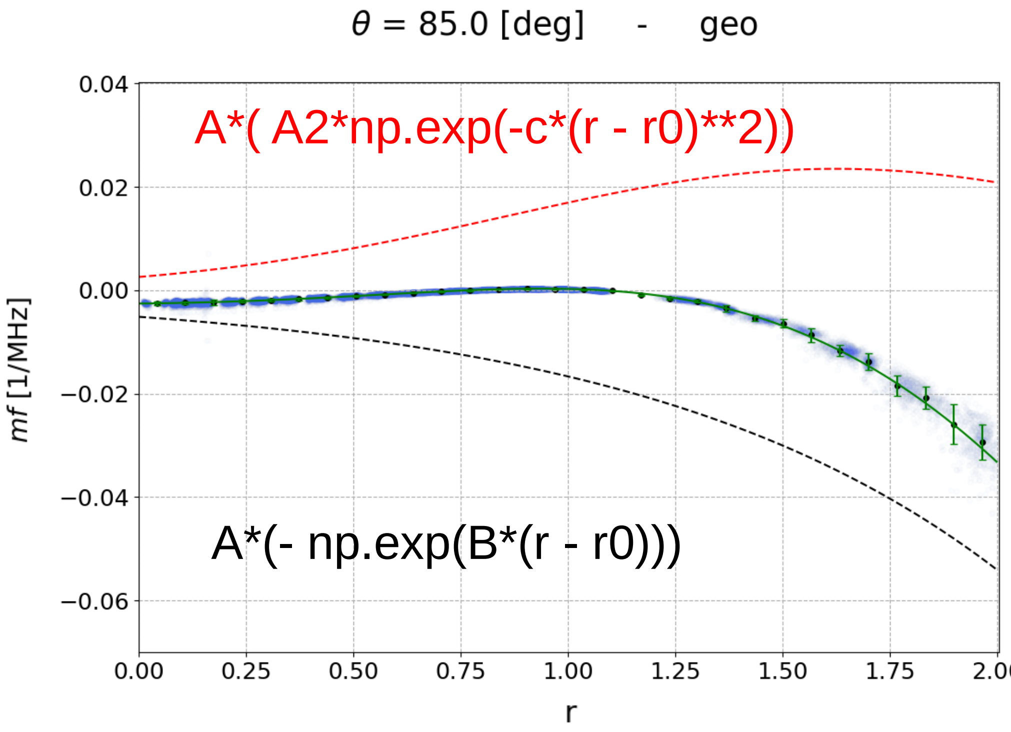
Green dots fitting the spectra

The CE quadratic term can just be set to zero





θ = 65.0 [deg] - geo
A*(A2*np.exp(-c*(r - r0)**2))
A*(- np.exp(B*(r - r0)))
mf [1/MHz]
r
0.02
0.01
0.00
−0.01
−0.02
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00



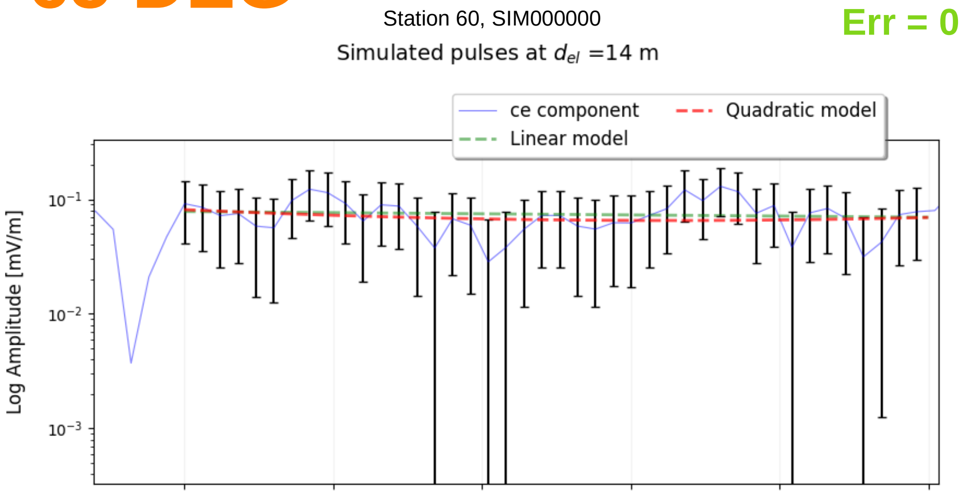
θ = 85.0 [deg] - geo
A*(A2*np.exp(-c*(r - r0)**2))
A*(- np.exp(B*(r - r0)))
mf [1/MHz]
r
0.04
0.02
0.00
−0.02
−0.04
−0.06
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

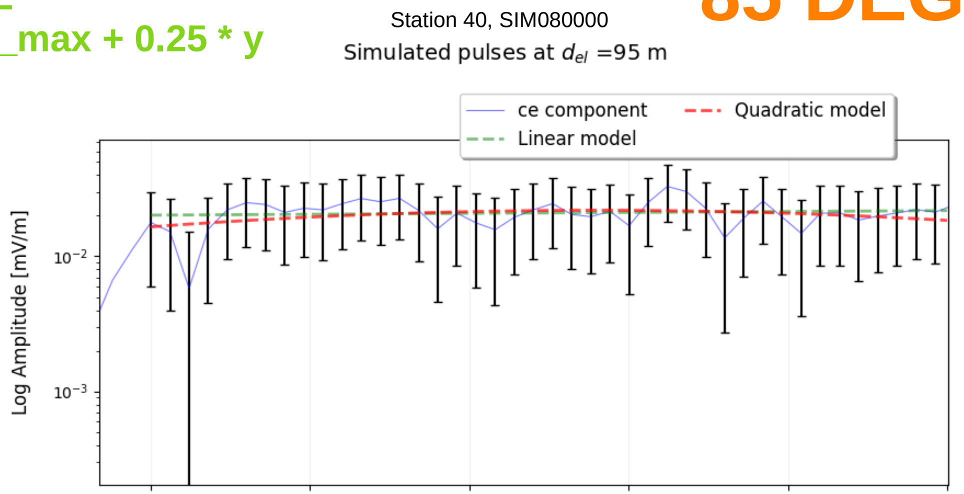
65 DEG

85 DEG

CE

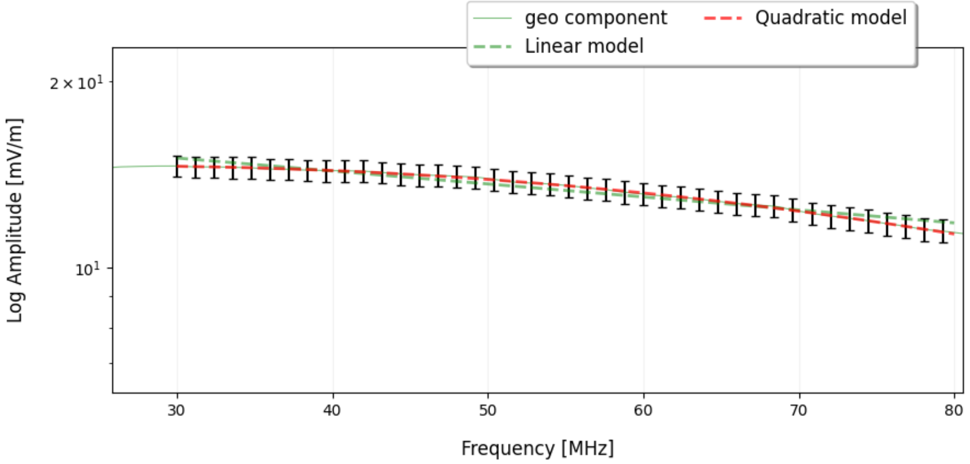
Err = 0.20 * y_max + 0.25 * y

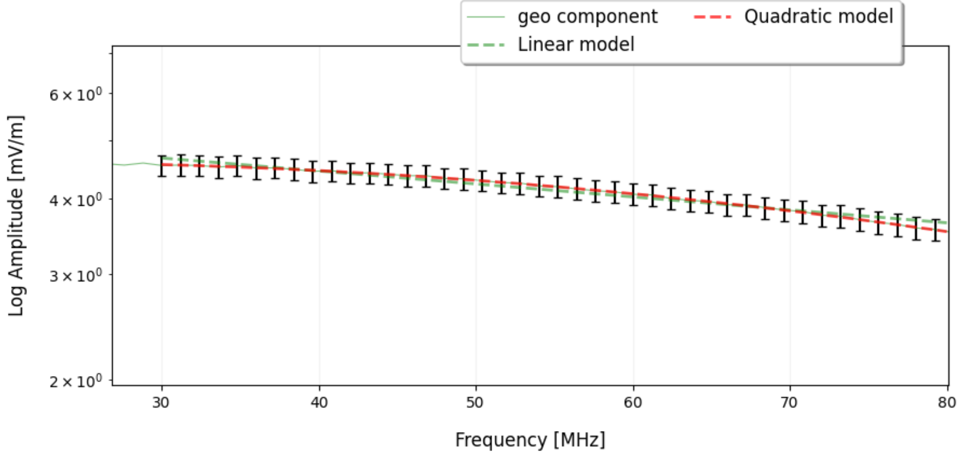




GEO

Err = 0.01 * y_max + 0.03 * y





QUAD. TERM: GEO component

A * [– exp(B * (r – r0)) + A2 * exp(– c * (r – r0)** 2)] → Fit having all 5 free parameters

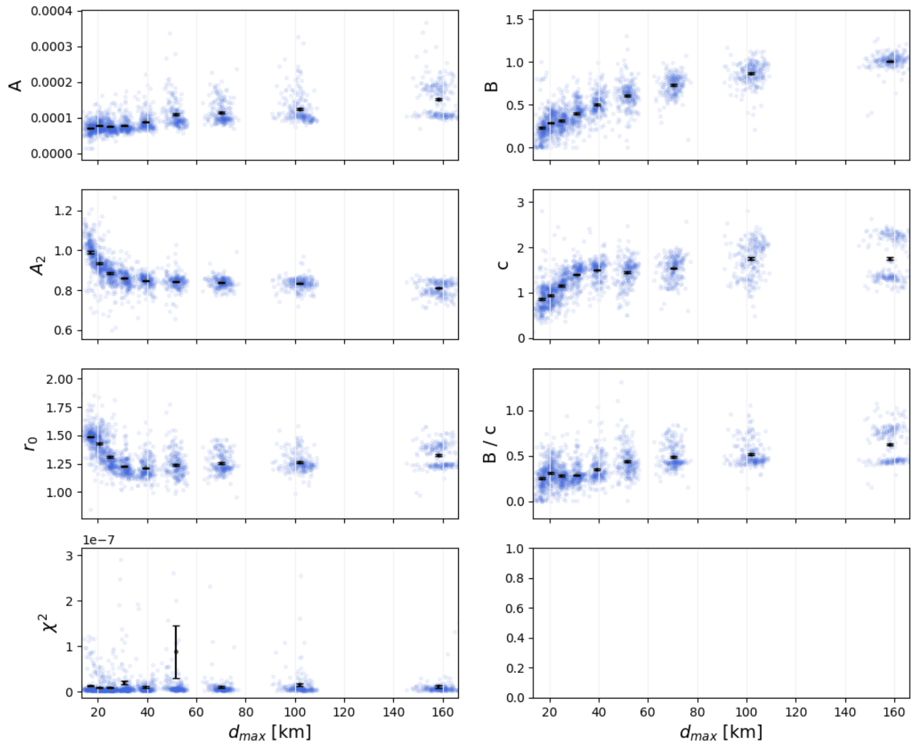
No statistical errors

Initial guesses: A , B , A2 , c , r0 = 0.000015, 0.8, 0.8, 1.35, 1.3

Fit bounds A , B , A2 , c , r0 , B/c:
(0, 0.0006), (0, 2.0), (0.6, 1.5), (0, 3.0), (0, 2.0), (0, 2.0)

Parameters distribution as a function of dmax

Number of total outliers: 22



65:	7(+2)
67.5:	3(+2)
70.0:	0
72.5:	3
75.0	0
77.5:	4
80.0	1
82.5:	0
85.0:	0

*Outliers:

A > 0.0004, A2 > 1.49,
c > 2.9, r0 > 1.9, B/c > 1.9

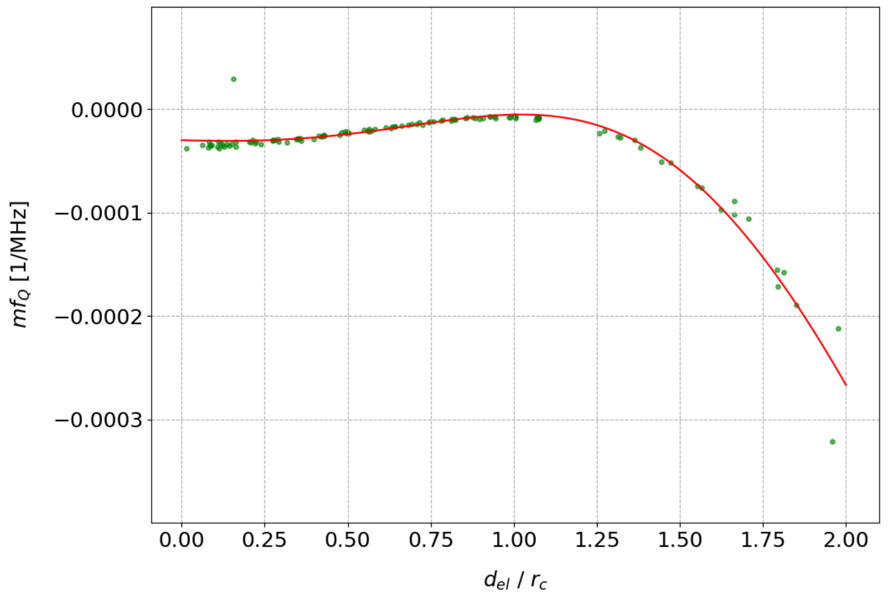
B < 0.001

- At higher zeniths, the parameters distribution shows a “splitting”, which is not related to any of the shower parameter

QUAD. TERM: GEO component & opposite examples

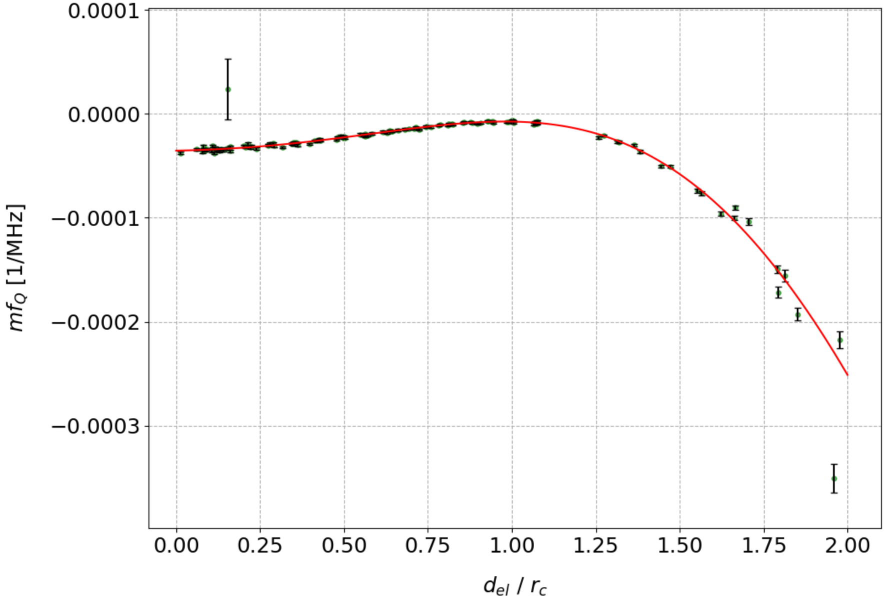
Shower having 85.0 deg zenith angle (SIM008700) → Fit having all 5 free parameters

No statistical errors free fit



A: 0.0001978
R0: 1.453906
A2: 0.758294
C: 1.3667073
B: 1.12834490

Err = 0.01 * y_max + 0.03 * y

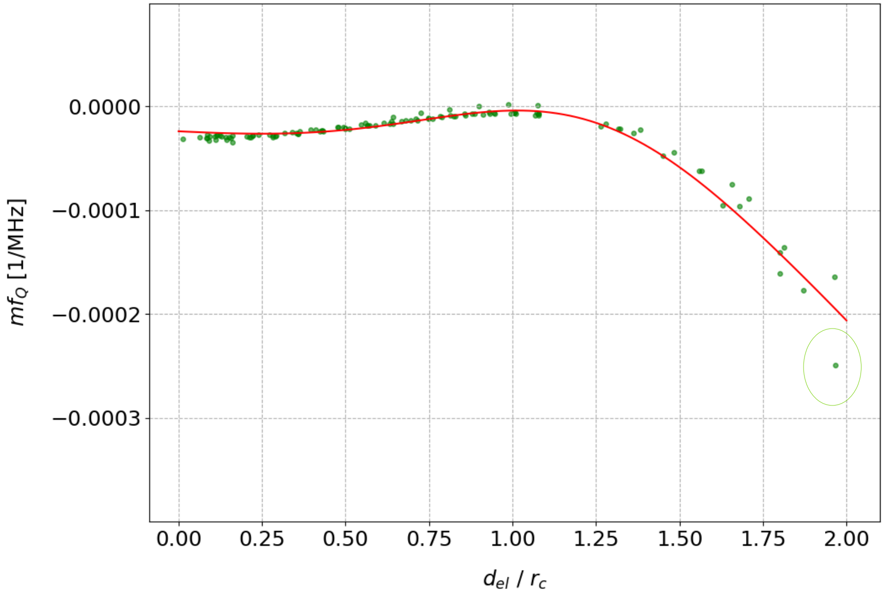


A: 0.0002983
R0: 1.612389
A2: 0.707670
C: 0.8200283
B: 0.9895646

QUAD. TERM: GEO component & opposite examples

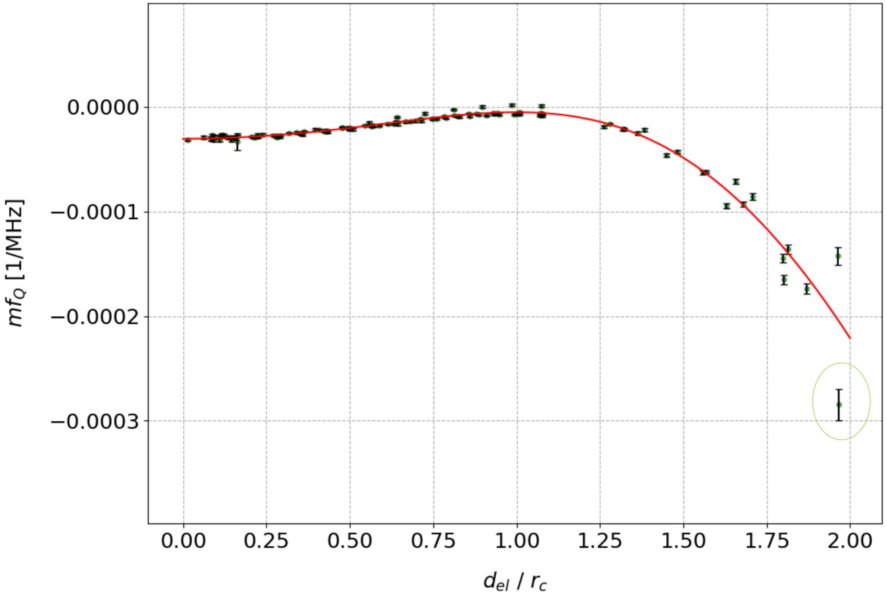
Shower having 85.0 deg zenith angle (SIM018102) → Fit having all 5 free parameters

No statistical errors free fit



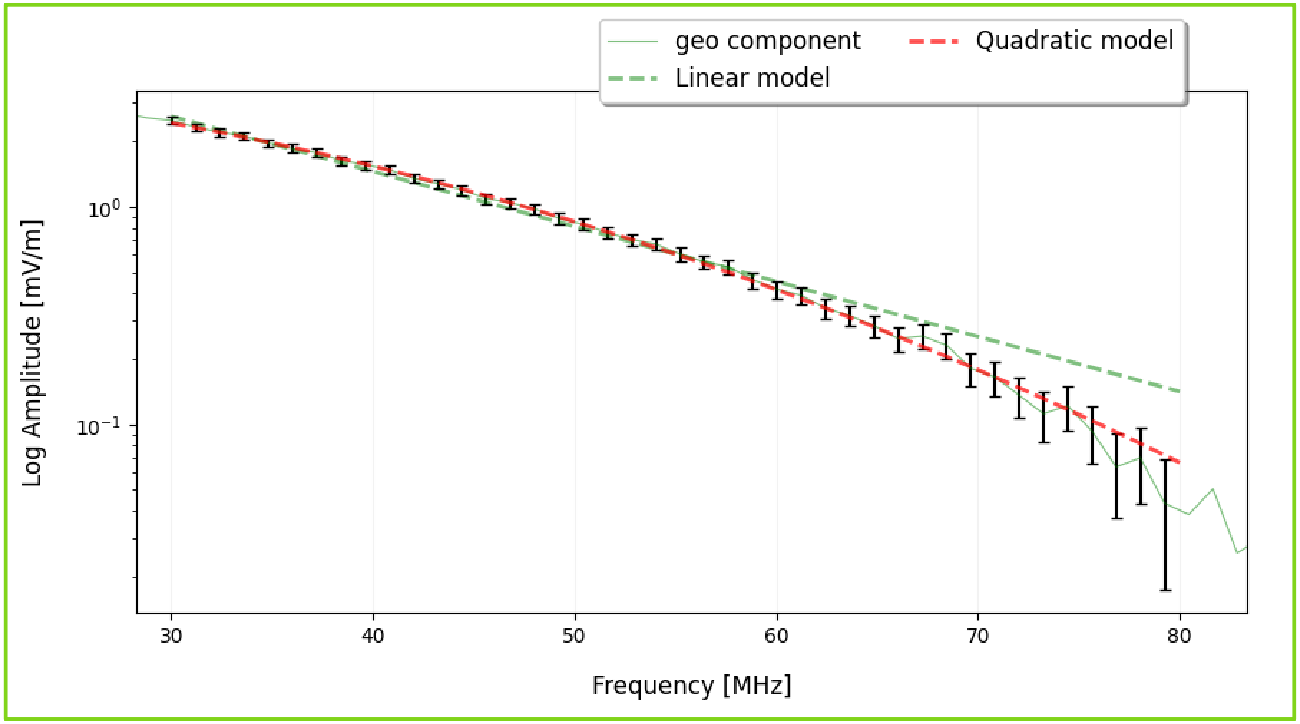
A: 0.0001022
R0: 1.2579461
A2: 0.8376234
C: 2.35153833
B: 1.089889801

Err = 0.01 * y_max + 0.03 * y



A: 0.0002440
R0: 1.5837290
A2: 0.7235978
C: 0.90286494
B: 1.01317462

QUAD. TERM: GEO component & opposite examples



Station 99 (SIM018102)
Corresponding to green circle of previous slide

QUAD. TERM: GEO component

A * [– exp(B * (r – r0)) + A2 * exp(– c * (r – r0)** 2)] → Fit having all 5 free parameters

Including uncertainties **Err = 0.001 * y_max + 0.001 * y**

Initial guesses: A , B , A2 , c , r0 = 0.000015, 0.8, 0.8, 1.35, 1.3

Fit bounds A , B , A2 , c , r0 , B/c:
(0, 0.0006), (0, 2.0), (0.6, 1.5), (0, 3.0), (0, 2.0), (0, 2.0)

65: 225(!)
67.5: 196(!)
70.0: 139(!)
72.5: 37
75.0 10
77.5: 4
80.0 1
82.5: 1
85.0: 12

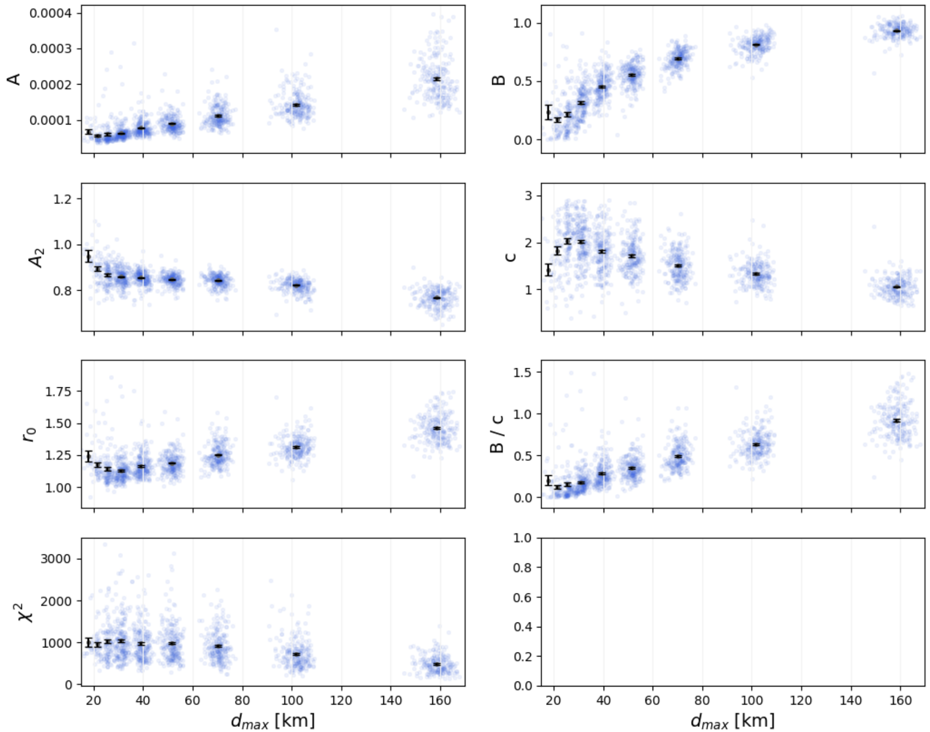
*Outliers:

A > 0.0004, A2 > 1.49,
c > 2.9, r0 > 1.9, B/c > 1.9

B < 0.001

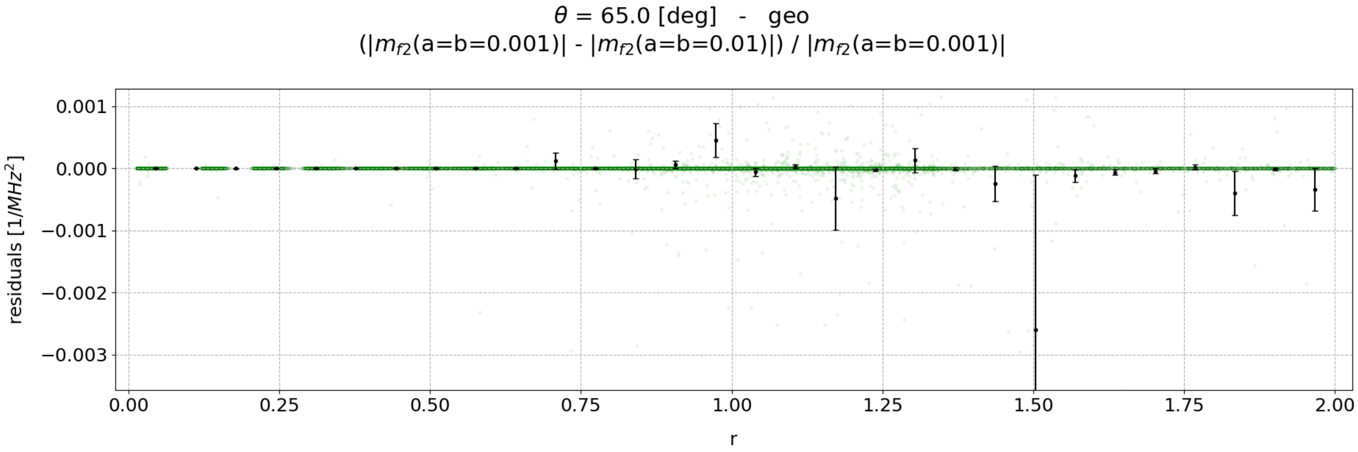
Parameters distribution as a function of dmax

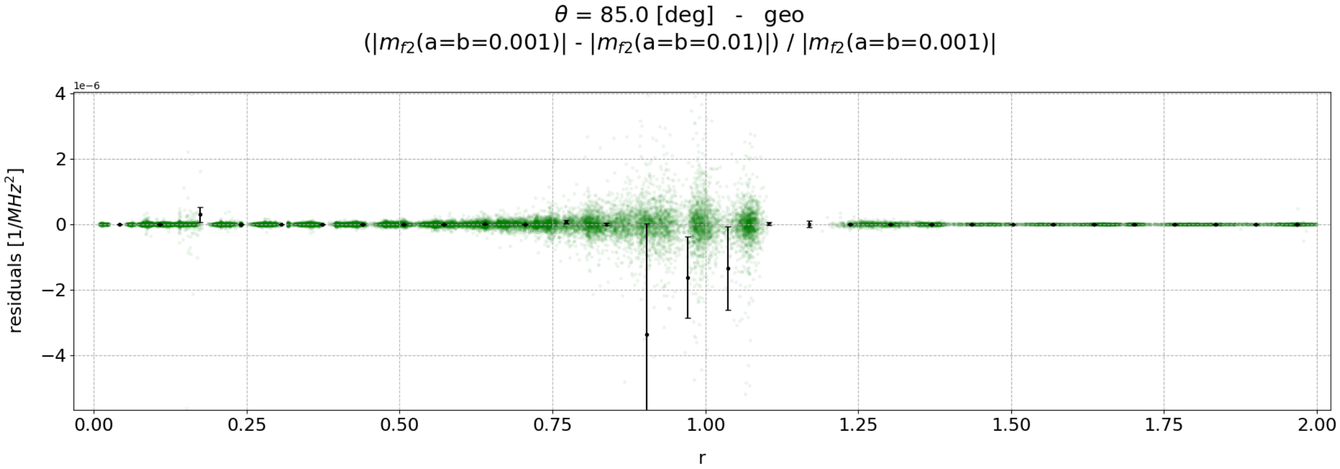
Number of total outliers: 625



- The "splitting seems to be gone, but further analysis on the uncertainties to be adopted are needed (backup)

QUAD. TERM: GEO component





Residuals using different values for the errors model

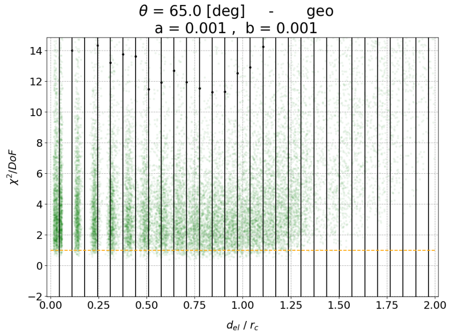
Spectra fit & errors scaling

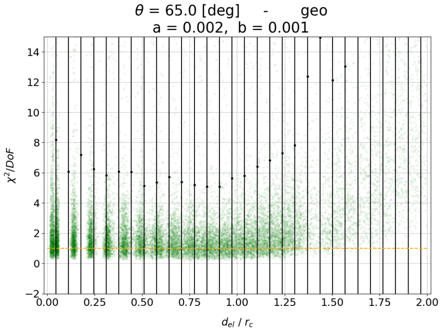
$$\chi^2 / \text{DoF} = 1$$

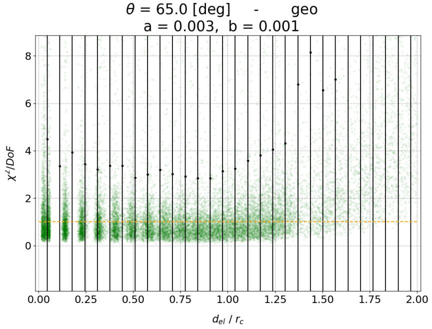
Reduced Chi2 distribution scaling the data errors

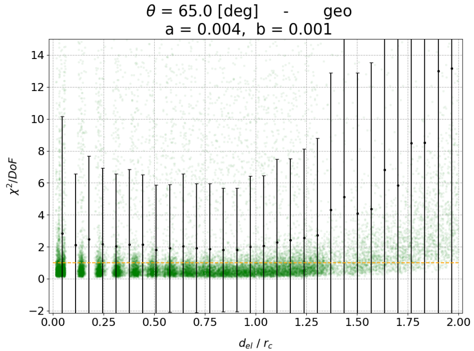
Err = a * y_max + b * y   with b = 0.001

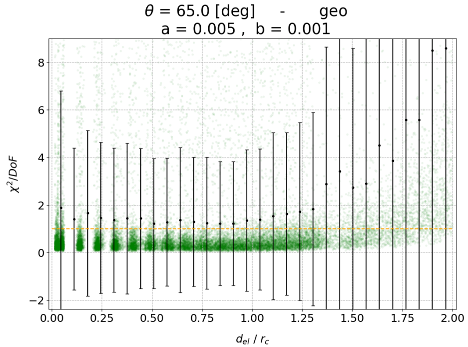
65 deg, Geo

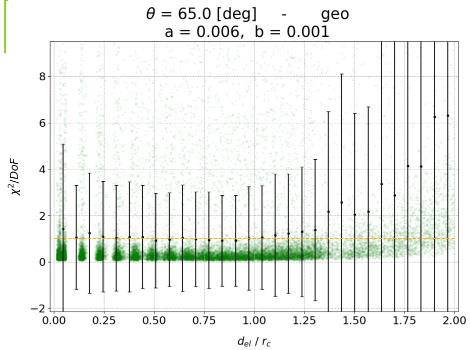


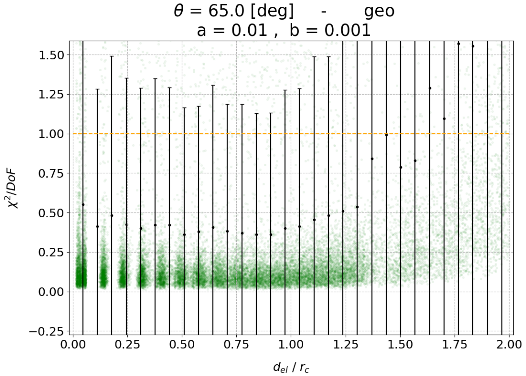






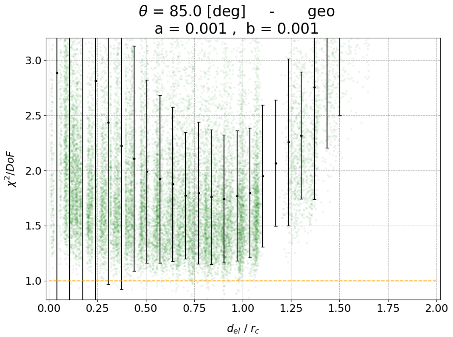


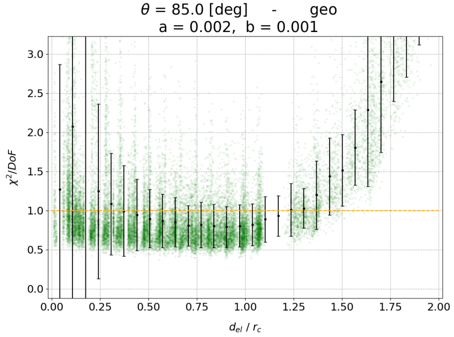


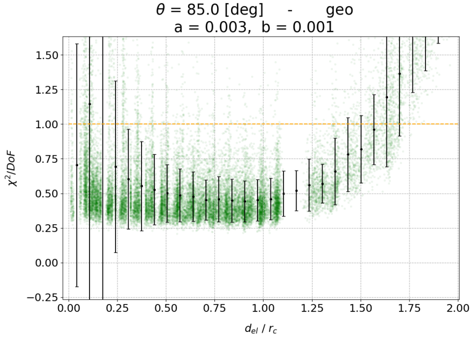


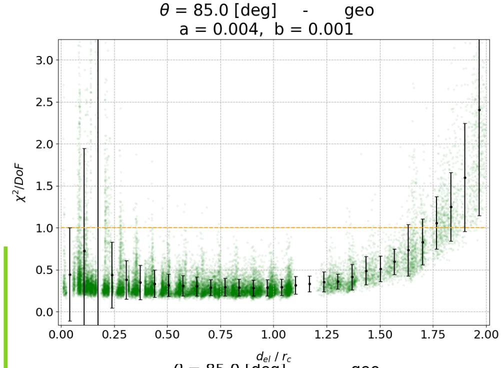
Reduced Chi2 distribution scaling the data errors

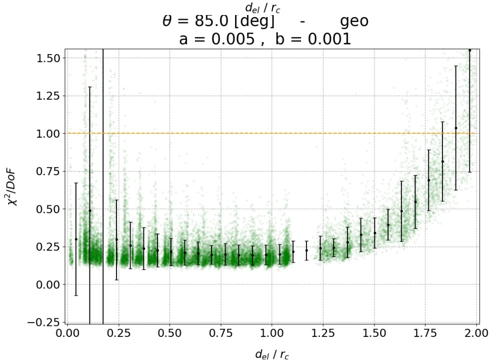
Err = a * y_max + b * y   with b = 0.001

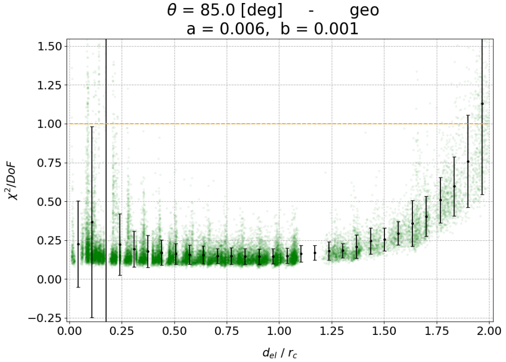




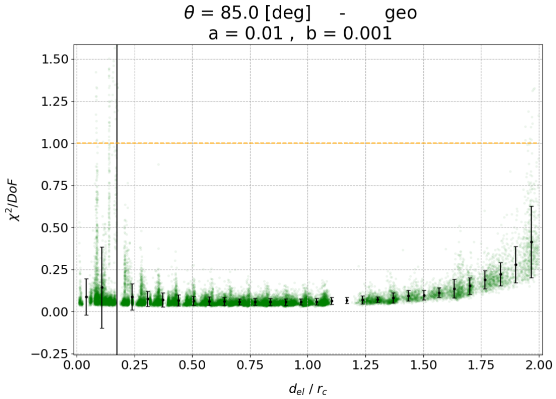








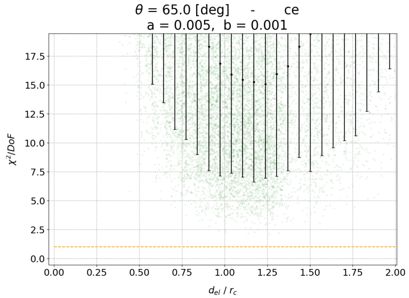
85 deg, Geo

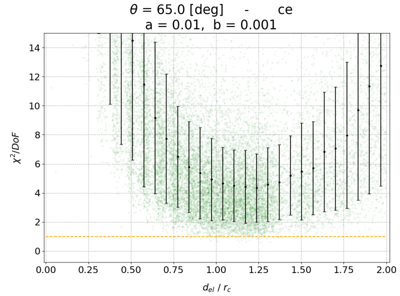


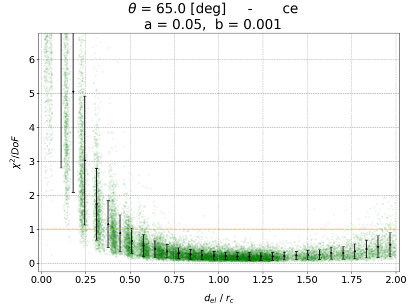
Reduced Chi2 distribution scaling the data errors

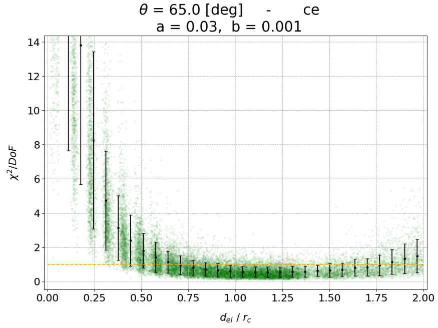
Err = a * y_max + b * y

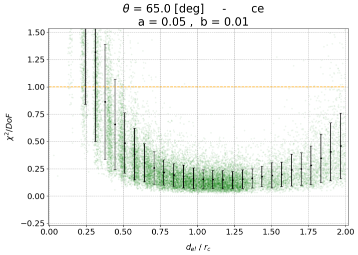
65 deg, CE

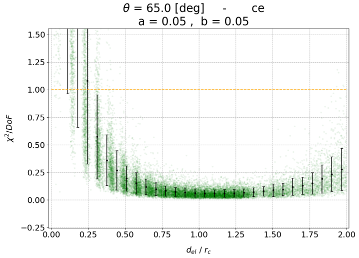


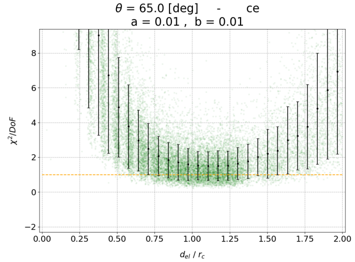


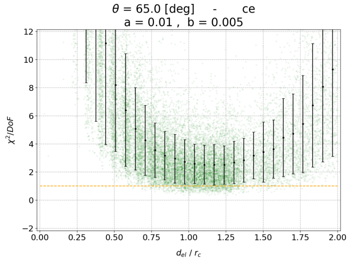


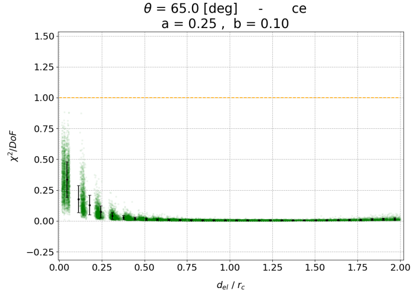


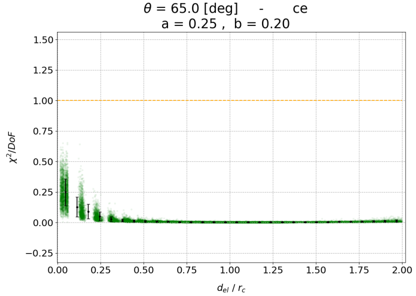


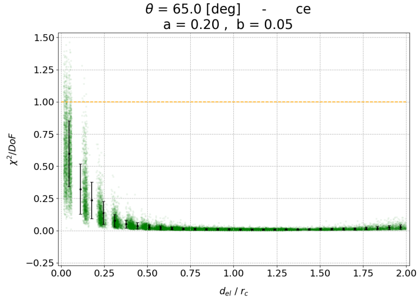












Reduced Chi2 distribution scaling the data errors

Err = a * y_max + b * y

85 deg, CE