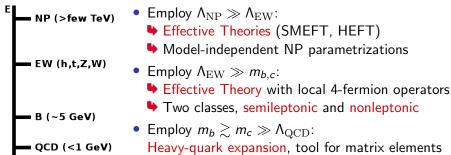




### $b \rightarrow c$ transitions in and beyond the SM

 $b \rightarrow c$  transitions...

- ...are an example of flavour-changing transitions
- ... proceed in the SM via the weak interaction
  - access to a fundamental SM parameter, V<sub>cb</sub>
- ... dominate lifetimes of singly-heavy groundstate B hadrons
- ... exhibit important hierarchies:

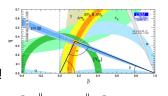


Tensions in  $b \to c \tau \nu$ ,  $b \to c \ell \nu$  and  $B_{d,s} \to D_{d,s}^{(*)}(\pi,K)$ 

# Importance of (semi-)leptonic hadron decays

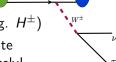
#### In the Standard Model:

- Tree-level,  $\sim |V_{ij}|^2 G_F^2 \, {\rm FF}^2$
- Determination of  $|V_{ij}|$  (6(+1)/9)
- Lepton-flavour universal W couplings!



### Beyond the Standard Model:

- Leptonic decays  $\sim m_I^2$ 
  - $\blacktriangleright$  large relative NP influence possible (e.g.  $H^{\pm}$ )
- NP in semi-leptonic decays small/moderate
  - ▶ Need to understand the SM very precisely!



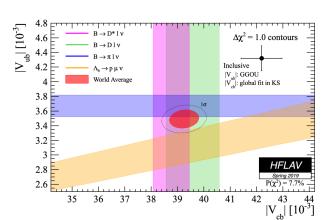
### Key advantages:

- Large rates
- Minimal hadronic input ⇒ systematically improvable
- Differential distributions ⇒ large set of observables

## Puzzling $V_{cb}$ results

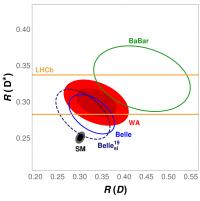
The  $V_{cb}$  puzzle has been around for 20+ years...

- ullet  $\sim 3\sigma$  between exclusive (mostly  $B o D^*\ell
  u$ ) and inclusive  $V_{cb}$
- Inclusive determination: includes  $\mathcal{O}(1/m_b^3, \alpha_s/m_b^2, \alpha_s^3)$ 
  - Excellent theoretical control,  $|V_{cb}| = (42.2 \pm 0.5)10^{-3}$  [Bordone+'21,Fael+'20,'21]
- Exclusive determinations:  $B \to D^{(*)} \ell \nu$ , using CLN ( $\to$  later)



### Lepton-non-Universality in $b \to c \tau \nu$

$$R(X) \equiv \frac{{\rm Br}(B \to X \tau \nu)}{{\rm Br}(B \to X \ell \nu)} \,, \quad \hat{R}(X) \equiv \frac{R(X)}{R(X)|_{\rm SM}} \,$$



contours: 68% CL filled: 95(68)% CL

R(D<sup>(\*)</sup>): BaBar, Belle, LHCb
 ⇒ average ~ 3 − 4σ from SM

More flavour  $b \to c \tau \nu$  observables:

- au-polarization (au au had) [1608.06391]
- $B_c o J/\psi au 
  u$  [1711.05623] : huge
- Differential rates from Belle, BaBar
- Total width of  $B_c$
- $b \to X_c \tau \nu$  by LEP
- D\* polarization (Belle)

Note: only 1 result  $\geq 3\sigma$  from SM

Form Factors (FFs) parametrize fundamental mismatch:

Experiment with hadrons

$$\left\langle D_q^{(*)}(p')|\bar{c}\gamma^{\mu}b|\bar{B}_q(p)\right\rangle = (p+p')^{\mu}f_+^q(q^2) + (p-p')^{\mu}f_-^q(q^2), \ q^2 = (p-p')^2$$

Most general matrix element parametrization, given symmetries: Lorentz symmetry plus P- and T-symmetry of QCD

 $f_{\pm}(q^2)$ : real, scalar functions of one kinematic variable

How to obtain these functions?

- Calculable w/ non-perturbative methods (Lattice, LCSR,...) Precision?
- ▶ Measurable e.g. in semileptonic transitions Normalization? Suppressed FFs? NP?

Form Factors (FFs) parametrize fundamental mismatch:

Experiment with hadrons

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Most general matrix element parametrization, given symmetries: Lorentz symmetry plus P- and T-symmetry of QCD  $f_{+}(q^2)$ : real, scalar functions of one kinematic variable

e functions?

n-perturbative methods (Lattice, LCSR,...)

in semileptonic transitions

Suppressed FFs? NP?

Form Factors (FFs) parametrize fundamental mismatch:

Experiment with hadrons

$$\left\langle D_q^{(*)}(p')|\bar{c}\gamma^{\mu}b|\bar{B}_q(p)\right\rangle = (p+p')^{\mu}f_+^q(q^2) + (p-p')^{\mu}f_-^q(q^2),\ q^2 = (p-p')^2$$

Most general matrix element parametrization, given symmetries: Lorentz symmetry plus P- and T-symmetry of QCD  $f_{+}(q^2)$ : real, scalar functions of one kinematic variable

e fur n-pe in se

methods (Lattice, LCSR,...)

transitions

Form Factors (FFs) parametrize fundamental mismatch:

Theory (e.g. SM) for partons (quarks) vs.

Experiment with hadrons

$$\left\langle D_q^{(*)}(p')|\bar{c}\gamma^{\mu}b|\bar{B}_q(p)\right\rangle = (p+p')^{\mu}f_+^q(q^2) + (p-p')^{\mu}f_-^q(q^2),\ q^2 = (p-p')^2$$

Most general matrix element parametrization, given symmetries: Lorentz symmetry plus P- and T-symmetry of QCD  $f_{+}(q^2)$ : real, scalar functions of one kinematic variable



# $q^2$ dependence

- $q^2$  range can be large, e.g.  $q^2 \in [0, 12] \text{ GeV}^2$  in  $B \to D$
- Calculations give usually one or few points
- $\blacktriangleright$  Knowledge of functional dependence on  $q^2$  cruical
- This is where discussions start...

### Give as much information as possible independent of this choice!

In the following: discuss BGL and HQE ( $\rightarrow$  CLN) parametrizations  $q^2$  dependence usually rewritten via conformal transformation:

$$z\left(t=q^{2},t_{0}
ight)=rac{\sqrt{t_{+}-t}-\sqrt{t_{+}-t_{0}}}{\sqrt{t_{+}-t}+\sqrt{t_{+}-t_{0}}}$$

$$t_{+} = (M_{B_q} + M_{D_q^{(*)}})^2$$
: pair-production threshold  $t_0 < t_{+}$ : free parameter for which  $z(t_0, t_0) = 0$ 

Usually  $|z| \ll 1$ , e.g.  $|z| \le 0.06$  for semileptonic  $B \to D$  decays

Good expansion parameter

### The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

- 1. Consider analytical structure, make poles and cuts explicit
- 2. Without poles or cuts, the rest can be Taylor-expanded in z
- 3. Apply QCD properties (unitarity, crossing symmetry)

  ▶ dispersion relation
- 4. Calculate partonic part perturbatively (+condensates)

Result:

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n [z(t, t_0)]^n.$$

- $a_n$ : real coefficients, the only unknowns
- P(t): Blaschke factor(s), information on poles below  $t_+$
- $\phi(t)$ : Outer function, chosen such that  $\sum_{n=0}^{\infty} a_n^2 \leq 1$
- Series in z with bounded coefficients (each  $|a_n| < 1$ )!
- Uncertainty related to truncation is calculable!

# $V_{cb} + R(D^*) \text{ w/ data} + \text{lattice} + \text{unitarity} \text{ [Gambino/MJ/Schacht'19]}$

Recent untagged analysis by Belle with 4 1D distributions [1809.03290]

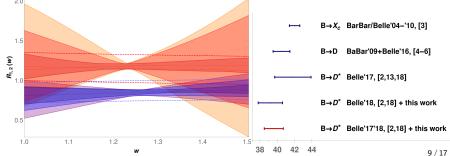
lacktriangle "Tension with the  $(V_{cb})$  value from the inclusive approach remains"

Analysis of 2017+2018 Belle data with BGL form factors:

- Datasets roughly compatible
- d'Agostini bias + syst. important
- All FFs to  $z^2$  to include uncertainties
  - ▶ 50% increased uncertainties

 $|V_{cb}^{D^*}| = 39.6_{-1.0}^{+1.1} \times 10^{-3}$  $R(D^*) = 0.254_{-0.006}^{+0.007}$ 

• 2018: no parametrization dependence



### **HQE** parametrization

HQE parametrization uses additional information compared to BGL

- ➡ Heavy-Quark Expansion (HQE)
  - $m_{b,c} \to \infty$ : all  $B \to D^{(*)}$  FFs given by 1 Isgur-Wise function
  - Systematic expansion in  $1/m_{b,c}$  and  $\alpha_s$
  - Higher orders in  $1/m_{b,c}$ : FFs remain related
    - Parameter reduction, necessary for NP analyses!

CLN parametrization [Caprini+'97]:

HQE to order  $1/m_{b,c}$ ,  $\alpha_s$  plus (approx.) constraints from unitarity [Bernlochner/Ligeti/Papucci/Robinson'17]: identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in  $B \to D$  and  $B \to D^*$ ) Dealt with by varying calculable ( $(01/m_{b,c})$ ) parameters, e.g.  $h_{A_1}(1)$ 

- ▶ Not a systematic expansion in  $1/m_{b,c}$  anymore!
- ▶ Related uncertainty remains  $\mathcal{O}[\Lambda^2/(2m_c)^2] \sim 5\%$ , insufficient

Solution: Include systematically  $1/m_c^2$  corrections [Bordone/MJ/vDyk'19,Bordone/Gubernari/MJ/vDyk'20] ,using [Falk/Neubert'92]

### Theory determination of $b \rightarrow c$ Form Factors

SM: BGL fit to data + FF normalization  $\rightarrow |V_{cb}|$ 

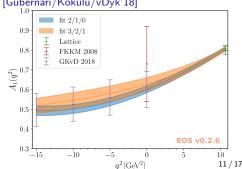
NP: can affect the  $q^2$ -dependence, introduces additional FFs

To determine general NP, FF shapes needed from theory

[MJ/Straub'18,Bordone/MJ/vDyk'19] used all available theory input:

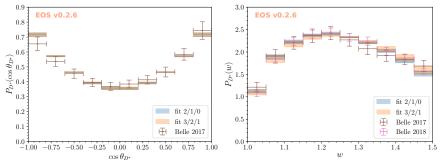
- Unitarity bounds (using results from [CLN, BGL] )
  - $\blacktriangleright$  non-trivial 1/m vs. z expansions
- LQCD for  $f_{+,0}(q^2)$  (B o D),  $h_{A_1}(q^2_{\max})$   $(B o D^*)$  [HPQCD'15,'17,Fermilab/MILC'14,'15]
- LCSR for all FFs (mod  $f_T$ ) [Gubernari/Kokulu/vDyk'18]
- QCDSR results for 1/m IW functions [Ligeti+'92'93]
- HQET expansion to  $\mathcal{O}(\alpha_s, 1/m_b, 1/m_c^2)$

FFs under control;  $R(D^*) = 0.247(6)$ [Bordone/MJ/vDyk'19]



# Robustness of the HQE expansion up to $1/m_c^2$ [Bordone/MJ/vDyk'19]

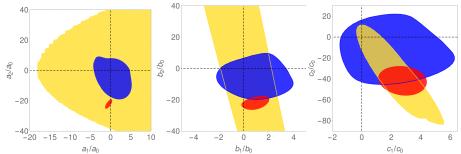
### Testing FFs by comparing to data and fits in BGL parametrization:



- Fits 3/2/1 and 2/1/0 are theory-only fits(!)
- k/I/m denotes orders in z at  $\mathcal{O}(1, 1/m_c, 1/m_c^2)$
- ullet w-distribution yields information on FF shape  $o V_{cb}$
- Angular distributions more strongly constrained by theory, only
- lacktriangle Predicted shapes perfectly confirmed by  $B o D^{(*)} \ell 
  u$  data
- $\triangleright$   $V_{cb}$  from Belle'17 compatible between HQE and BGL!

# Robustness of the HQE expansion up to $1/m_c^2$ [Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:



- B → D\* BGL coefficient ratios from:
  - 1. Data (Belle'17+'18) + weak unitarity (yellow)
  - 2. HQE theory fit 2/1/0 (red)
  - 3. HQE theory fit 3/2/1 (blue)
- Again compatibility of theory with data
- ▶2/1/0 underestimates the uncertainties massively
- ▶ For  $b_i, c_i$  (→  $f, \mathcal{F}_1$ ) data and theory complementary

# Including $ar{B}_s o D_s^{(*)}$ Form Factors [Bordone/Gubernari/MJ/vDyk'20]

Dispersion relation *sums* over hadronic intermediate states

- ▶ Includes  $B_s D_s^{(*)}$ , included via SU(3) + conservative breaking
- lacktriangle Explicit treatment can improve also  $ar{B} o D^{(*)} \ell 
  u$

Experimental progress in  $\bar{B}_s \to D_s^{(*)} \ell \nu$ :

2 new LHCb measurements [2001.03225, 2003.08453]

Improved theory determinations required, especially for NP

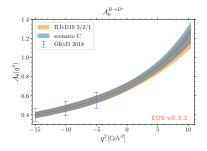
We extend our  $1/m_c^2$  analysis by including:

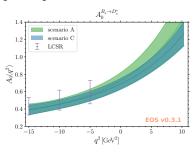
- Available lattice data:  $(2\ ar{B}_s o D_s\ \mathsf{FFs}\ (q^2\ \mathsf{dependent}),\ 1\ ar{B}_s o D^*\ \mathsf{FF}\ (\mathsf{only}\ q^2_{\mathrm{max}}))$
- Adaptation of existing QCDSR results [Ligeti/Neubert/Nir'93'94], including SU(3) breaking
- New LCSR results extending [Gubernari+'18] to  $B_s$ , including SU(3) breaking
- $\blacktriangleright$  Fully correlated fit to  $\bar{B} \to D^{(*)}, \bar{B}_s \to D_s^{(*)}$  FFs

# Including $\bar{B}_s \to D_s^{(*)}$ Form Factors, Results

#### We observe the following:

- Theory constraints fitted consistently in an HQE framework
- $\mathcal{O}(1/m_c^2)$  power corrections have  $\mathcal{O}(1)$  coefficients
- No indication of sizable SU(3) breaking
- Slight influence of strengthened unitarity bounds
- Improved determination of  $\bar{B}_s \to D_s^{(*)}$  FFs

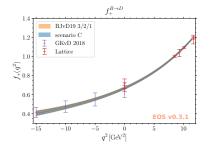


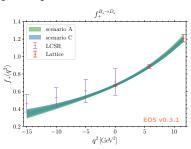


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Theory-only predictions:

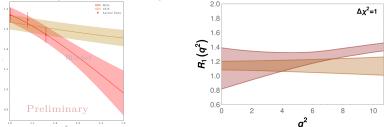
$$R(D) = 0.299(3)$$
  $R(D^*) = 0.247(5)$ 

$$R(D_s) = 0.297(3)$$
  $R(D_s^*) = 0.245(8)$ 

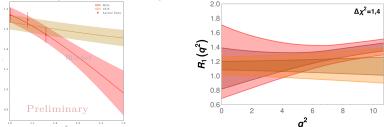
Theory+Experiment (Belle'17) predictions:

$$R(D) = 0.298(3)$$
  $R(D^*) = 0.250(3)$ 

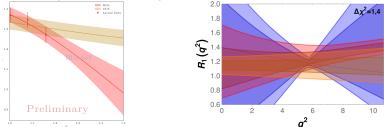
$$R(D_s) = 0.297(3)$$
  $R(D_s^*) = 0.247(8)$ 



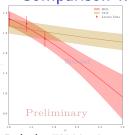
 $R_1(w)$ : FNAL slope surprising, compatible at  $1-2\sigma$ 

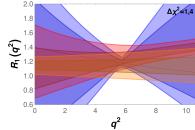


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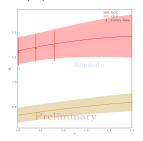


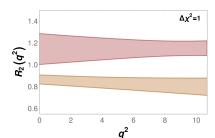
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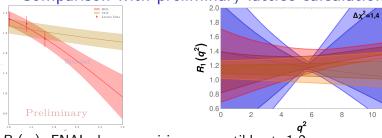


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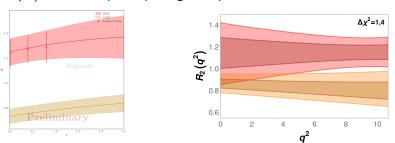




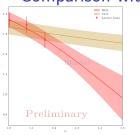
 $R_2(w=1)$ : Discrepancy FNAL (1.12  $\pm$  0.06) vs. (HQE fit, experiment)! HQE@1/ $m_c^2$ : 0.78 $^{+0.10}_{-0.06}$ , BGL: 0.81  $\pm$  0.11, HFLAV: 0.852  $\pm$  0.018

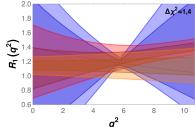


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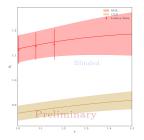


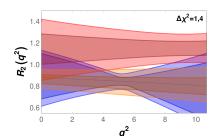
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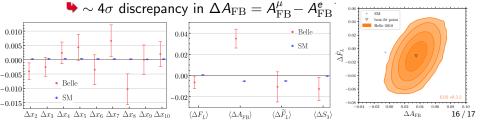
# Flavour universality in $B \to D^*(e, \mu)\nu$

[Bobeth/Bordone/Gubernari/MJ/vDyk'21]

So far: Belle'18 data used in SM fits, flavour-averaged

However: Bins 40 × 40 covariances given separately for  $\ell = e, \mu$  Belle'18:  $R_{e/\mu}(D^*) = 1.01 \pm 0.01 \pm 0.03$ 

- What can we learn about flavour-non-universality?  $\rightarrow$  2 issues:
  - 1.  $e \mu$  correlations not given  $\rightarrow$  constructable from Belle'18
- 2. 3 bins linearly dependent, but covariances not singular Two-step analysis:
  - Extract 2 × 4 angular observables for 2 × 30 angular bins
     Model-independent description including NP!
  - 2. Compare with SM predictions, using FFs@1/ $m_c^2$  [Bordone+'19]



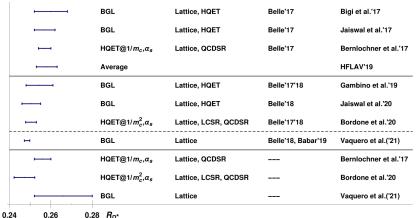
### Conclusions

Form factors essential ingredients in precision-flavour physics!

- $q^2$  dependence critical  $\rightarrow$  need FF-independent data
- ▶ Inclusion of higher-order (theory) uncertainties important
- BGL: model-independent, truncation uncertainty limited
- $lacktriangleright B o D^*$ : Reduced  $V_{cb}$  puzzle, somewhat lower  $R(D^*)$  prediction
- ullet Theory determinations for NP required o HQE to relate FFs
- $\mathcal{O}(1/m_c)$  not good enough for precision analyses
- First analysis at  $1/m_c^2$  provides all  $B \to D^{(*)}$  FFs
- $V_{cb}$  consistent w/ BGL
- First LQCD analyses in  $B \to D^*$  and  $B_s \to D_s^*$  @ finite  $q^2$
- Tension with experiment as well as other theory inputs
- LFU-violation in  $b \to c\ell\nu$  @ $\sim 4\sigma!$
- ► Experimental issues? NP?

Central lesson: experiment and theory need to work closely together!

## Overview over predictions for $R(D^*)$



Lattice  $B \to D^*$ :  $h_{A_1}(w=1)$  [FNAL/MILC'14,HPQCD'17]

Other lattice:  $f_{+,0}^{B\to D}(q^2)$  [MILC,HPQCD'15]

QCDSR: [Ligeti/Neubert/Nir'93,'94], LCSR: [Gubernari/Kokulu/vDyk'18]

Consistent SM predictions! Improvement expected from lattice FNAL/MILC('21) discussed in the following.

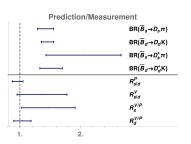
### A puzzle in non-leptonic $b \rightarrow c$ transitions

[Bordone/Gubernari/Huber/MJ/vDyk'20]

FFs also of central importance in non-leptonic decays:

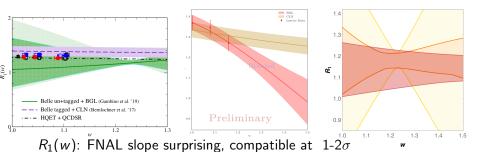
- Complicated in general,  $B o M_1 M_2$  dynamics
- Simplest cases:  $\bar{B}_d \to D_d^{(*)} \bar{K}$  and  $\bar{B}_s \to D_s^{(*)} \pi$  (5 diff. quarks)
  - lacktriangle Colour-allowed tree,  $1/m_b^0@\mathcal{O}(lpha_s^2)$  [Huber+'16] , factorizes at  $1/m_b$
  - lacktriangle Amplitudes dominantly  $\sim ar{B}_q 
    ightarrow D_q^{(*)}$  FFs
  - Used to determine  $f_s/f_d$  at hadron colliders [Fleischer+'11]

Updated and extended calculation: tension of  $4.4\sigma$  w.r.t. exp.!

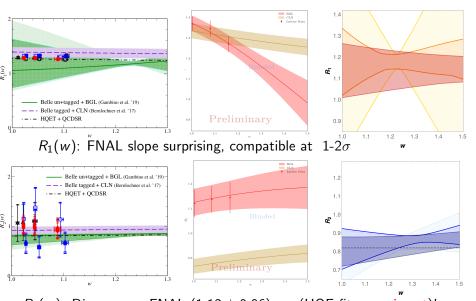


- Large effect,  $\sim -30\%$  for BRs
- Ratios of BRs ok
- QCDf uncertainty  $\mathcal{O}(1/m_b^2, \alpha_s^3)$
- Data consistent (too few abs. BRs)
- NP?  $\Delta_P \sim \Delta_V \sim -20\%$  possible
- ▶We will learn something important!

### Preliminary lattice calculations



# Preliminary lattice calculations



 $R_2(w)$ : Discrepancy FNAL (1.12  $\pm$  0.06) vs. (HQE fit, experiment)! HQE@1/ $m_c^2$ : 0.78 $^{+0.10}_{-0.06}$ , BGL: 0.81  $\pm$  0.11, HFLAV: 0.852  $\pm$  0.018

### Comments regarding systematics and fitting [MJ/Straub'18]

Present (and future!) precision renders small effects important:

- Form factor parametrization
- d'Agostini effect: assuming systematic uncertainties ~ (exp. cv) introduces bias
- e.g. 1-2 $\sigma$  shift in  $|V_{cb}|$  in Belle 2010 binned data
- Rounding in a fit with strong correlations and many bins:
  - lacktriangledown  $1\sigma$  between fit to Belle 2017 data from paper vs. HEPdata
- BR measurements and isospin violation [MJ 1510.03423] : Normalization depends on  $\Upsilon \to B^+B^-$  vs.  $B^0\bar{B}^0$  Taken into account, but simple HFLAV average problematic:
  - Potential large isospin violation in  $\Upsilon \to BB$  [Atwood/Marciano'90]
  - Measurements in  $r_{+0}^{
    m HFAG}$  assume isospin in exclusive decays
    - This is one thing we want to test!
  - Avoiding this assumption yields  $r_{+0} = 1.035 \pm 0.038$  (potentially subject to change, in contact with Belle members)
  - Relevant for all BR measurements at the %-level

### BR measurements and isospin violation [MJ 1510.03423]

Detail due to high precision and small NP

▶ Relevant for  $\sigma_{\rm BR}/{\rm BR} \sim \mathcal{O}(\%)$ 

Branching ratio measurements require normalization...

- B factories: depends on  $\Upsilon \to B^+B^-$  vs.  $B^0\bar{B}^0$
- LHCb: normalization mode, usually obtained from B factories

Assumptions entering this normalization:

- PDG: assumes  $r_{+0} \equiv \Gamma(\Upsilon \to B^+ B^-)/\Gamma(\Upsilon \to B^0 \bar{B}^0) \equiv 1$
- LHCb: assumes  $f_u \equiv f_d$ , uses  $r_{+0}^{\mathrm{HFAG}} = 1.058 \pm 0.024$

### Both approaches problematic:

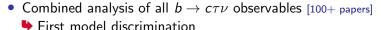
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### Generalities regarding this anomaly

- $\sim 15\%$  of a SM tree decay  $\sim V_{cb}$ : This is a huge effect!
  - Need contribution of  $\sim 5-10\%$  (w/ interference) or  $\gtrsim 40\%$  (w/o interference) of SM

#### What do we do about this?

- Check the SM prediction!
  - $[\rightarrow \mathsf{Bigi+}, \mathsf{Bordone+}, \mathsf{Gambino+}, \mathsf{Grinstein+}, \mathsf{Bernlochner+}]$
  - $\blacktriangleright \delta R(D^*)$  larger, anomaly remains



- Related indirect bounds (partly model-dependent)
  - $\blacktriangleright$  High  $p_T$  searches, lepton decays, LFV, EDMs, ...
- Analyze flavour structure of potential NP contributions
  - **\rightharpoonup** quark flavour structure, e.g.  $b \rightarrow u$
  - $\blacktriangleright$  lepton flavour structure, e.g.  $b \to c\ell (=e,\mu)\nu$

