Precision analyses of $b \rightarrow c$ transitions

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 $b \rightarrow c$ transitions in and beyond the SM $b \rightarrow c$ transitions...

- ... are an example of flavour-changing transitions
- ... proceed in the SM via the weak interaction
	- \bullet access to a fundamental SM parameter, V_{cb}
- \bullet ... dominate lifetimes of singly-heavy groundstate B hadrons
- . . . exhibit important hierarchies:

Importance of (semi-)leptonic hadron decays

In the Standard Model:

- Tree-level, $\sim |V_{ij}|^2 G_F^2 \, \mathrm{FF}^2$
- Determination of $|V_{ii}|$ $(6(+1)/9)$
- Lepton-flavour universal W couplings!

Beyond the Standard Model:

- Leptonic decays $\sim m_l^2$
	- large relative NP influence possible (e.g. $H^\pm)$
- NP in semi-leptonic decays small/moderate Reed to understand the SM very precisely!

Key advantages:

- Large rates
- Minimal hadronic input \Rightarrow systematically improvable
- Differential distributions \Rightarrow large set of observables

Puzzling V_{cb} results

The V_{cb} puzzle has been around for 20+ years...

- $\bullet \sim 3\sigma$ between exclusive (mostly $B \to D^*\ell \nu$) and inclusive V_{cb}
- Inclusive determination: includes $\mathcal{O}(1/m_b^3, \alpha_s/m_b^2, \alpha_s^3)$
	- \blacktriangleright Excellent theoretical control, $|V_{cb}| = (42.2 \pm 0.5)10^{-3}$

[Bordone+'21,Fael+'20,'21]

• Exclusive determinations: $B \to D^{(*)} \ell \nu$, using CLN (\to later)

Lepton-non-Universality in $b \rightarrow c \tau \nu$

$$
R(X) \equiv \frac{\text{Br}(B \to X\tau\nu)}{\text{Br}(B \to X\ell\nu)}, \quad \hat{R}(X) \equiv \frac{R(X)}{R(X)|_{\text{SM}}}
$$

• $R(D^{(*)})$: BaBar, Belle, LHCb \rightarrow average \sim 3 – 4 σ from SM

More flavour $b \to c\tau\nu$ observables:

- τ -polarization ($\tau \rightarrow$ had) [1608.06391]
- $B_c \rightarrow J/\psi \tau \nu$ [1711.05623] : huge
- Differential rates from Belle, BaBar
- Total width of B_c
- $b \to X_c \tau \nu$ by LEP
- D^* polarization (Belle)

Note: only 1 result $> 3\sigma$ from SM

Form Factors (FFs) parametrize fundamental mismatch:

Theory (e.g. SM) for partons (quarks) vs. Experiment with hadrons

 $\left\langle D_{q}^{(\ast)}(\rho')|\bar{c}\gamma^{\mu}b|\bar{B}_{q}(\rho)\right\rangle =(\rho + \rho')^{\mu}f_{+}^{q}(q^{2}) + (\rho - \rho')^{\mu}f_{-}^{q}(q^{2})\,,\,q^{2} = (\rho - \rho')^{2}$

Most general matrix element parametrization, given symmetries: Lorentz symmetry plus P- and T-symmetry of QCD $f_{\pm}(q^2)$: real, scalar functions of one kinematic variable

How to obtain these functions?

Calculable w/ non-perturbative methods (Lattice, LCSR,. . .) Precision?

► Measurable e.g. in semileptonic transitions Normalization? Suppressed FFs? NP?

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q^2 dependence

- q^2 range can be large, e.g. $q^2 \in [0, 12]~{\rm GeV}^2$ in $B \to D$
- Calculations give usually one or few points
- Knowledge of functional dependence on q^2 cruical
- This is where discussions start. . .

Give as much information as possible independent of this choice!

In the following: discuss BGL and HQE (\rightarrow CLN) parametrizations

 q^2 dependence usually rewritten via conformal transformation:

$$
z(t = q^2, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}
$$

 $t_+=(\mathit{M}_{\mathit{B}_{q}}+\mathit{M}_{\mathit{D}_{q}^{(\ast)}})^{2}$: pair-production threshold $t_0 < t_+$: free parameter for which $z(t_0, t_0) = 0$

Usually $|z| \ll 1$, e.g. $|z| \leq 0.06$ for semileptonic $B \to D$ decays Good expansion parameter

The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

- 1. Consider analytical structure, make poles and cuts explicit
- 2. Without poles or cuts, the rest can be Taylor-expanded in z
- 3. Apply QCD properties (unitarity, crossing symmetry) **↓** dispersion relation
- 4. Calculate partonic part perturbatively $(+$ condensates)

Result:

$$
F(t)=\frac{1}{P(t)\phi(t)}\sum_{n=0}^{\infty}a_n[z(t,t_0)]^n.
$$

- a_n : real coefficients, the only unknowns
- $P(t)$: Blaschke factor(s), information on poles below t_{+}
- $\phi(t)$: Outer function, chosen such that $\sum_{n=0}^{\infty} a_n^2 \leq 1$
- Series in z with bounded coefficients (each $|a_n| \leq 1$)!
- ♦ Uncertainty related to truncation is calculable!

 $V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

Recent untagged analysis by Belle with 4 1D distributions [1809.03290] \blacktriangleright "Tension with the (V_{cb}) value from the inclusive approach remains"

Analysis of 2017+2018 Belle data with BGL form factors:

- Datasets roughly compatible
- \bullet d'Agostini bias $+$ syst. important
- All FFs to z^2 to include uncertainties **► 50% increased uncertainties**

$$
|V_{cb}^{D^*}| = 39.6_{-1.0}^{+1.1} \times 10^{-3}
$$

$$
R(D^*) = 0.254_{-0.006}^{+0.007}
$$

• 2018: no parametrization dependence

HQE parametrization

HQE parametrization uses additional information compared to BGL **► Heavy-Quark Expansion (HQE)**

- $m_{b,c} \rightarrow \infty$: all $B \rightarrow D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
	- ♦ Parameter reduction, necessary for NP analyses!

CLN parametrization [Caprini+'97] : HQE to order $1/m_{b,c}, \alpha_s$ plus (approx.) constraints from unitarity [Bernlochner/Ligeti/Papucci/Robinson'17] : identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in $B \to D$ and $B \to D^*)$ Dealt with by varying calculable $(\mathcal{Q}1/m_{b,c})$ parameters, e.g. $\,h_{A_1}(1)\,$ \blacktriangleright Not a systematic expansion in $1/m_{b,c}$ anymore! Related uncertainty remains $\mathcal{O}[\Lambda^2/(2m_c)^2] \sim 5\%$, insufficient Solution: Include systematically $1/m_c^2$ corrections

[Bordone/MJ/vDyk'19,Bordone/Gubernari/MJ/vDyk'20] ,using [Falk/Neubert'92]

Theory determination of $b \rightarrow c$ Form Factors

SM: BGL fit to data + FF normalization $\rightarrow |V_{cb}|$

NP: can affect the q^2 -dependence, introduces additional FFs

► To determine general NP, FF shapes needed from theory

[MJ/Straub'18,Bordone/MJ/vDyk'19] used all available theory input:

- Unitarity bounds (using results from [CLN, BGL]) \rightarrow non-trivial $1/m$ vs. z expansions
- LQCD for $f_{+,0}(q^2)$ $(B \rightarrow D),$ $h_{A_1}(q^2_{\textrm{max}})$ $(B \rightarrow D^*)$ [HPQCD'15,'17,Fermilab/MILC'14,'15]
- LCSR for all FFs (mod f_T) [Gubernari/Kokulu/vDyk'18]
- QCDSR results for $1/m$ IW functions [Ligeti+'92'93]
- HQET expansion to $\mathcal{O}(\alpha_{\sf s},1/m_{\sf b},1/m_{\sf c}^2)$

FFs under control; $R(D^*) = 0.247(6)$ [Bordone/MJ/vDyk'19]

Robustness of the HQE expansion up to $1/m_c^2$ [Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:

• Fits $3/2/1$ and $2/1/0$ are theory-only fits $(!)$

- $k/l/m$ denotes orders in z at $\mathcal{O}(1, 1/m_c, 1/m_c^2)$
- w-distribution yields information on FF shape $\rightarrow V_{cb}$
- Angular distributions more strongly constrained by theory, only
- Predicted shapes perfectly confirmed by $B\to D^{(*)}\ell\nu$ data
- $\blacktriangleright V_{ch}$ from Belle'17 compatible between HQE and BGL!

Robustness of the HQE expansion up to $1/m_c^2$ [Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:

• $B \to D^*$ BGL coefficient ratios from:

- 1. Data (Belle'17+'18) + weak unitarity (yellow)
- 2. HQE theory fit 2/1/0 (red)
- 3. HQE theory fit 3/2/1 (blue)

► Again compatibility of theory with data

 \rightarrow 2/1/0 underestimates the uncertainties massively

For $b_i, c_i \, (\rightarrow f, \mathcal{F}_1)$ data and theory complementary

Including $\bar{B}_s \to D_s^{(*)}$ Form Factors [Bordone/Gubernari/MJ/vDyk'20]

Dispersion relation *sums* over hadronic intermediate states Includes $\mathit{B_sD_s^{(*)}}$, included via SU $(3)+$ conservative breaking Explicit treatment can improve also $\bar{B}\to D^{(*)}\ell\nu$

Experimental progress in $\bar{B}_s \to D_s^{(*)} \ell \nu$:

2 new LHCb measurements [2001.03225, 2003.08453]

Inproved theory determinations required, especially for NP \vert

We extend our $1/m_c^2$ analysis by including:

- Available lattice data: $(2 \bar{B}_s \rightarrow D_s$ FFs $(q^2$ dependent), $1 \bar{B}_s \rightarrow D^*$ FF $(\text{only } q^2_{\text{max}}))$
- Adaptation of existing QCDSR results [Ligeti/Neubert/Nir'93'94] , including SU(3) breaking
- New LCSR results extending [Gubernari+'18] to B_s , including SU(3) breaking
- Fully correlated fit to $\bar B\to D^{(*)}, \bar B_{\bar s}\to D^{(*)}_{\bar s}$ FFs

Including $\bar{B}_s \to D_s^{(*)}$ Form Factors, Results

We observe the following:

- Theory constraints fitted consistently in an HQE framework
- \bullet $\mathcal{O}(1/m_c^2)$ power corrections have $\mathcal{O}(1)$ coefficients
- No indication of sizable SU(3) breaking
- Slight influence of strengthened unitarity bounds
- Improved determination of $\bar B_\mathsf{s} \to D^{(*)}_\mathsf{s}$ FFs

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Theory-only predictions:

$$
R(D) = 0.299(3)
$$

\n
$$
R(Ds) = 0.297(3)
$$

\n
$$
R(Ds*) = 0.247(5)
$$

\n
$$
R(Ds*) = 0.245(8)
$$

Theory+Experiment (Belle'17) predictions:

 $R(D) = 0.298(3)$ $^{\ast})=0.250(3)$ $R(D_s) = 0.297(3)$ s^*) = 0.247(8)

 $-$ 1000 CLN Δx^2 =1 **A. Frederick** 1.4 12 1.2 $P_2(q^2)$ é $1\, \rm A$ 0.8 θ 0.6 Preliminary \mathfrak{p} 6 $\overline{8}$ 10 Ω 4 q^2

 $R_2(w = 1)$: Discrepancy FNAL (1.12 \pm 0.06) vs. (HQE fit, experiment)! $HQE@1/m_c^2$: 0.78 $_{-0.06}^{+0.10}$, BGL: 0.81 \pm 0.11, HFLAV: 0.852 \pm 0.018

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Flavour universality in $B \to D^*(e, \mu) \nu$

[Bobeth/Bordone/Gubernari/MJ/vDyk'21]

So far: Belle'18 data used in SM fits, flavour-averaged However: Bins 40 \times 40 covariances given separately for $\ell = e, \mu$ Belle' 18 : $R_{e/\mu}(D^*) = 1.01 \pm 0.01 \pm 0.03$

What can we learn about flavour-non-universality? \rightarrow 2 issues:

1. e – μ correlations not given \rightarrow constructable from Belle'18

2. 3 bins linearly dependent, but covariances not singular Two-step analysis:

1. Extract 2×4 angular observables for 2×30 angular bins

► Model-independent description including NP!

2. Compare with SM predictions, using FFs@ $1/m_c^2$ [Bordone+'19]

Conclusions

Form factors essential ingredients in precision-flavour physics!

- q^2 dependence critical \rightarrow need FF-independent data
- **►** Inclusion of higher-order (theory) uncertainties important
- BGL: model-independent, truncation uncertainty limited
- $B \to D^*$: Reduced V_{cb} puzzle, somewhat lower $R(D^*)$ prediction
- Theory determinations for NP required \rightarrow HQE to relate FFs
- $\mathcal{O}(1/m_c)$ not good enough for precision analyses
- First analysis at $1/m_c^2$ provides all $B \to D^{(*)}$ FFs
- V_{cb} consistent w/ BGL
- First LQCD analyses in $B \to D^*$ and $B_s \to D_s^*$ @ finite q^2
- Tension with experiment as well as other theory inputs
- LFU-violation in $b \to c \ell \nu \mathbb{Q} \sim 4\sigma!$
- **►** Experimental issues? NP?

Central lesson: experiment and theory need to work closely together!

Thank you $\&$ Happy birthday Alex! $17/17$

Overview over predictions for $R(D^*)$

 0.24 0.26

0.28 R_{D^*}

Lattice $B \to D^*$: $h_{A_1}(w=1)$ [FNAL/MILC'14,HPQCD'17] Other lattice: $f_{+,0}^{B\to D}(q^2)$ [MILC,HPQCD'15] QCDSR: [Ligeti/Neubert/Nir'93,'94] , LCSR: [Gubernari/Kokulu/vDyk'18]

Consistent SM predictions! Improvement expected from lattice FNAL/MILC('21) discussed in the following.

A puzzle in non-leptonic $b \rightarrow c$ transitions

[Bordone/Gubernari/Huber/MJ/vDyk'20] FFs also of central importance in non-leptonic decays:

- Complicated in general, $B \to M_1 M_2$ dynamics
- Simplest cases: $\bar{B}_d \to D_d^{(*)} \bar{K}$ and $\bar{B}_s \to D_s^{(*)} \pi$ (5 diff. quarks)
	- Colour-allowed tree, $1/m_b^0$ @ $O(\alpha_s^2)$ [Huber+'16], factorizes at $1/m_b$
	- Amplitudes dominantly $\sim \bar{B}_q \rightarrow D_q^{(*)}$ FFs
	- **↓** Used to determine f_s/f_d at hadron colliders [Fleischer+'11]

Updated and extended calculation: tension of 4.4σ w.r.t. exp.!

- Large effect, $\sim -30\%$ for BRs
- Ratios of BRs ok
- QCDf uncertainty $\mathcal{O}(1/m_b^2, \alpha_s^3)$
- Data consistent (too few abs. BRs)
- NP? $\Delta_P \sim \Delta_V \sim -20\%$ possible
- **► We will learn something important!**

Preliminary lattice calculations

Preliminary lattice calculations

 $R_2(w)$: Discrepancy FNAL (1.12 \pm 0.06) vs. (HQE fit, experiment)! $\mathsf{HQE@1/m_c^2}\text{: } 0.78_{-0.06}^{+0.10}\text{, } \mathsf{BGL}\text{: } 0.81\pm0.11\text{, } \mathsf{HFLAV}\text{: } 0.852\pm0.018$ $_{17/17}$

Comments regarding systematics and fitting [MJ/Straub'18]

Present (and future!) precision renders small effects important:

- Form factor parametrization
- d'Agostini effect:

assuming systematic uncertainties \sim (exp. cv) introduces bias e.g. 1-2 σ shift in $|V_{cb}|$ in Belle 2010 binned data

- Rounding in a fit with strong correlations and many bins: \rightarrow 1 σ between fit to Belle 2017 data from paper vs. HEPdata
- BR measurements and isospin violation [MJ 1510.03423] : Normalization depends on $\Upsilon \rightarrow B^+B^-$ vs. $B^0\bar{B}^0$ Taken into account, but simple HFLAV average problematic:
	- Potential large isospin violation in $\Upsilon \rightarrow BB$ [Atwood/Marciano'90]
	- Measurements in r_{+0}^{HFAG} assume isospin in exclusive decays \rightarrow This is one thing we want to test!
	- Avoiding this assumption yields $r_{+0} = 1.035 \pm 0.038$ (potentially subject to change, in contact with Belle members)
	- Relevant for all BR measurements at the %-level

BR measurements and isospin violation [MJ 1510.03423]

Detail due to high precision and small NP Relevant for $\sigma_{\rm BR}/\rm BR \sim \mathcal{O}(\%)$

Branching ratio measurements require normalization. . .

- B factories: depends on $\Upsilon \to B^+B^-$ vs. $B^0\bar{B}^0$
- LHCb: normalization mode, usually obtained from B factories Assumptions entering this normalization:
	- PDG: assumes $r_{+0} \equiv \Gamma(\Upsilon \to B^+B^-)/\Gamma(\Upsilon \to B^0 \bar{B}^0) \equiv 1$
	- LHCb: assumes $f_u \equiv f_d$, uses $r_{+0}^{\text{HFAG}} = 1.058 \pm 0.024$

Both approaches problematic:

- Potential large isospin violation in $\Upsilon \rightarrow BB$ [Atwood/Marciano'90]
- \bullet Measurements in $r_{+0}^{\rm HFAG}$ assume isospin in exclusive decays \rightarrow This is one thing we want to test!
- Avoiding this assumption yields $r_{+0} = 1.035 \pm 0.038$ (potentially subject to change, in contact with Belle members)

Generalities regarding this anomaly

 $\vert \sim 15\%$ of a SM tree decay $\sim V_{cb}$: This is a huge effect! Need contribution of \sim 5 − 10% (w/ interference) or $\geq 40\%$ (w/o interference) of SM

What do we do about this?

• Check the SM prediction!

[→ Bigi+,Bordone+,Gambino+,Grinstein+,Bernlochner+]

 $\delta R(D^*)$ larger, anomaly remains

- Combined analysis of all $b \to c\tau\nu$ observables [100+ papers] First model discrimination
- Related indirect bounds (partly model-dependent) \blacktriangleright High $p_{\mathcal{T}}$ searches, lepton decays, LFV, EDMs, ...
- Analyze flavour structure of potential NP contributions quark flavour structure, e.g. $b \rightarrow u$
	- **Le** lepton flavour structure, e.g. $b \rightarrow c\ell (= e, \mu)\nu$