Numerical Methods for Image Segmentation

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Numerical Methods for Image Segmentation

Outline

1. Active Contours Without Edges
2. GPU based diffusion for image segmentation
3. Results
4. Practical Session
5. Uncertainty Quantification
6. Inverse Image Segmentation
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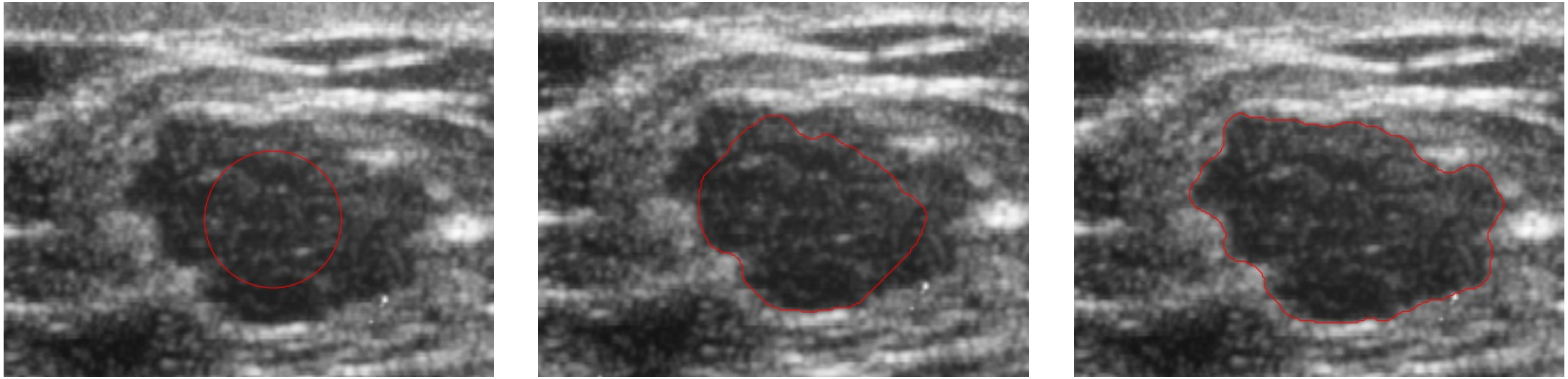
Active Contours Without Edges

Let the curve C in Ω be the boundary of an open subset $\omega \subset \Omega$, i.e. $C = \partial\omega$, $\omega = inside(C)$ and $\Omega \backslash \overline{\omega} = outside(C)$. Let $I(x, y)$ be the Image Data, $c_1 = average(inside)$ and $c_2 = average(outside)$, then we determine C so that it minimizes the functional

$$\begin{aligned} F(c_1, c_2, C) = {} & \alpha \cdot \text{length}(C) + \beta \cdot \text{Area}(inside(C)) \\ & + \lambda_1 \int_\omega |I(x, y) - c_1|^2 \, \mathrm{d}x\mathrm{d}y \\ & + \lambda_2 \int_{\Omega \backslash \overline{\omega}} |I(x, y) - c_2|^2 \, \mathrm{d}x\mathrm{d}y. \end{aligned}$$



Active Contours Without Edges

Level Set Method: C is represented by the zero level set of a Lipschitz function $\phi : \Omega \to \mathbb{R}$, such that

$$\begin{aligned} C = \partial\omega &= \{(x, y) \in \Omega : \phi(x, y) = 0\} \\ inside(C) &= \{(x, y) : \phi(x, y) > 0\} \\ outside(C) &= \{(x, y) : \phi(x, y) < 0\}. \end{aligned}$$

A curve that minimizes the Functional has to satisfy the Euler Lagrange equation

$$|\nabla\phi| \left[\alpha \operatorname{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \beta - \lambda_1(u_0 - c_1)^2 + \lambda_2(u_0 - c_2)^2\right] = 0.$$

Introducing an artificial time $t \geq 0$, we have to solve

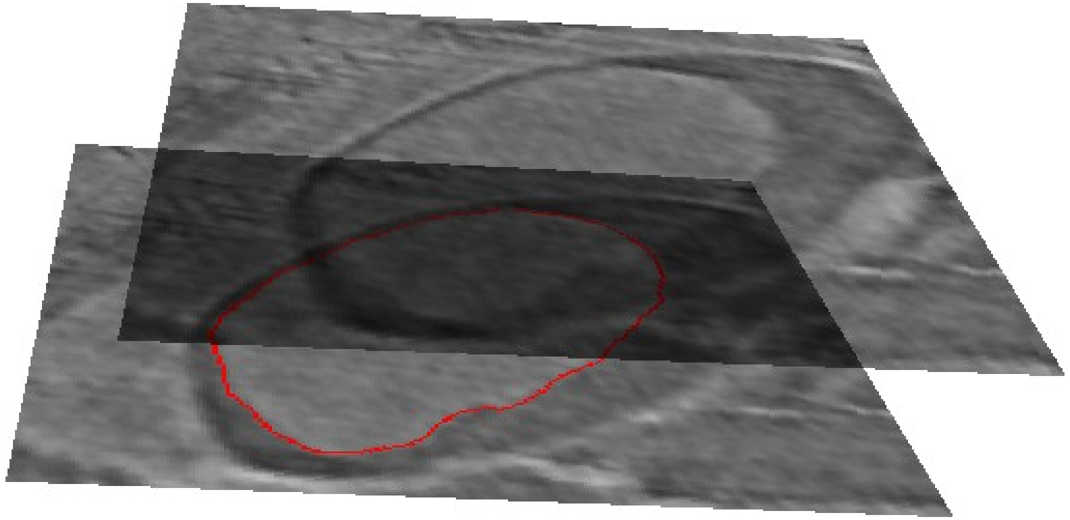
$$\frac{\partial\phi}{\partial t} = |\nabla\phi| \left[\alpha \operatorname{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \beta - \lambda_1(u_0 - c_1)^2 + \lambda_2(u_0 - c_2)^2\right].$$

Active Contours Without Edges

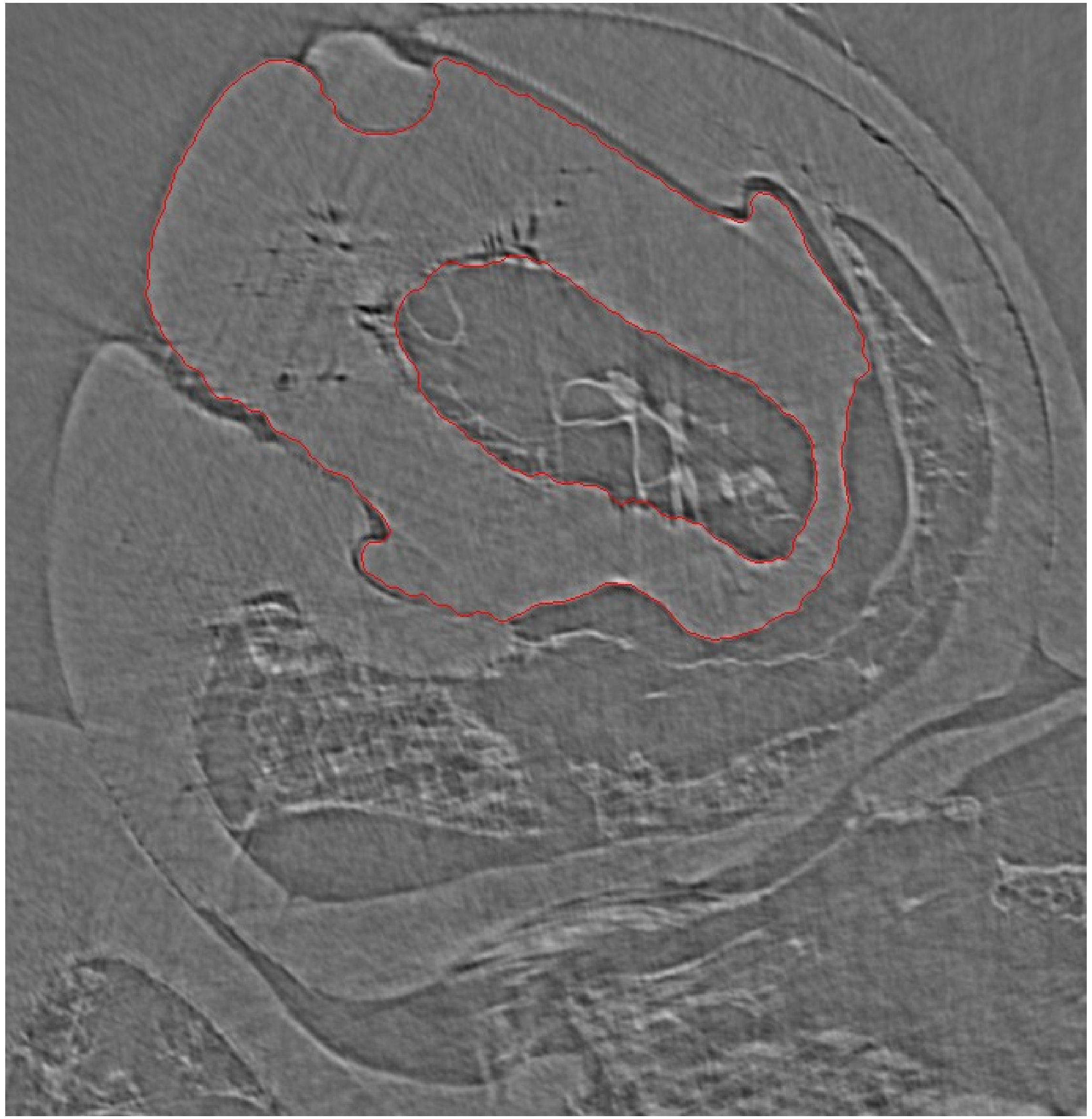
Practical session

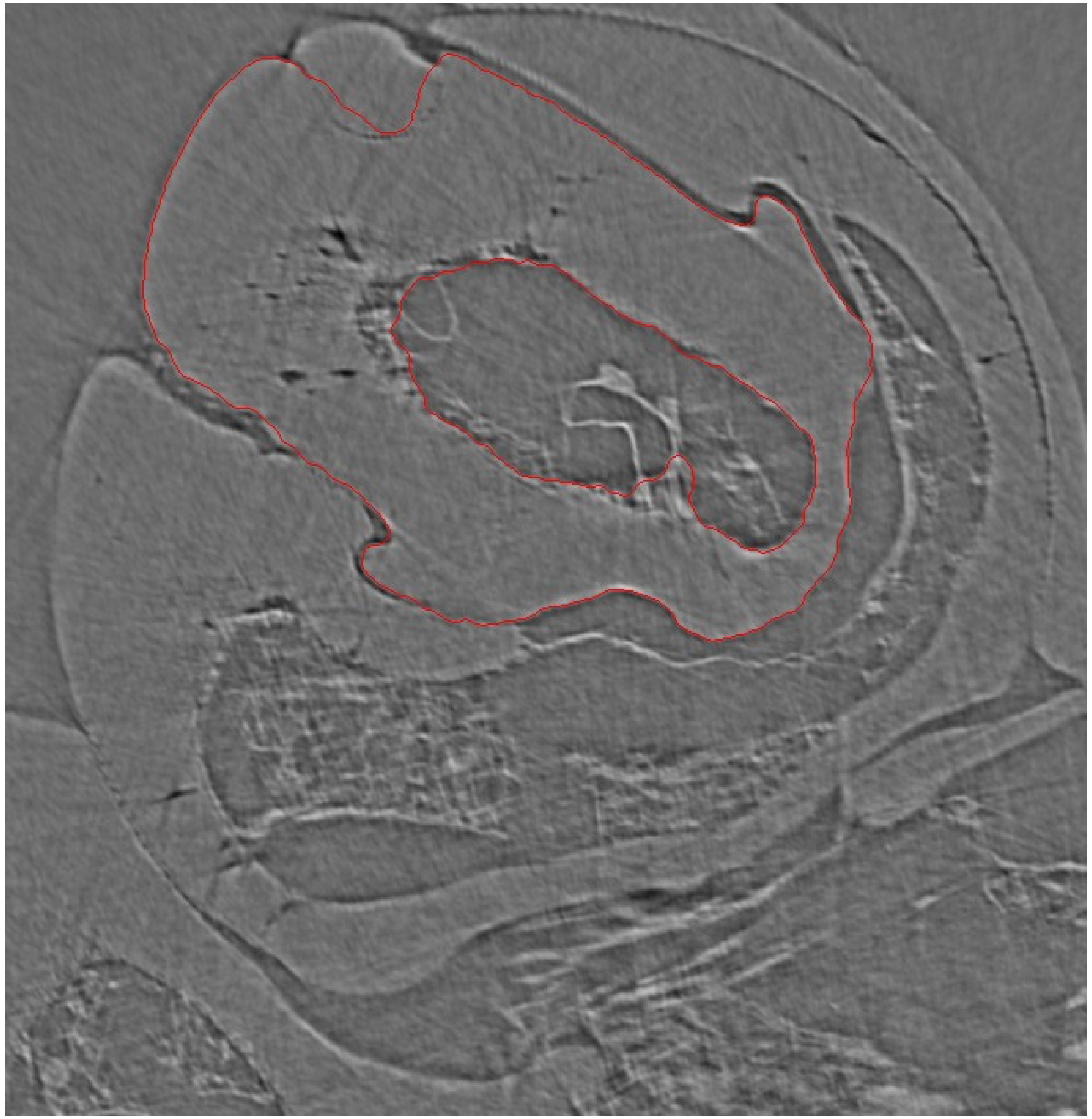
Active Contours Without Edges

- How to segment 3D data?
- Active contours slice by slice
- Use prior segmentation as initialization and move forward in z-direction



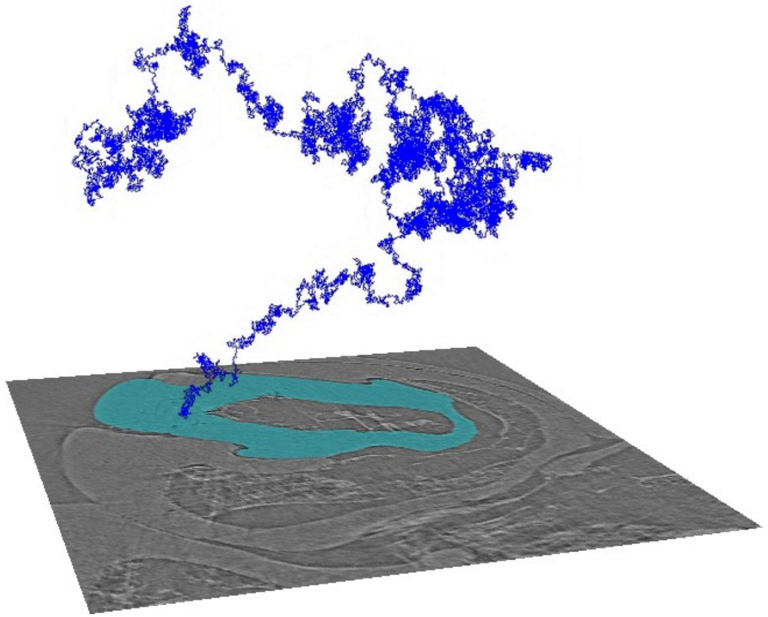
Active Contours Without Edges

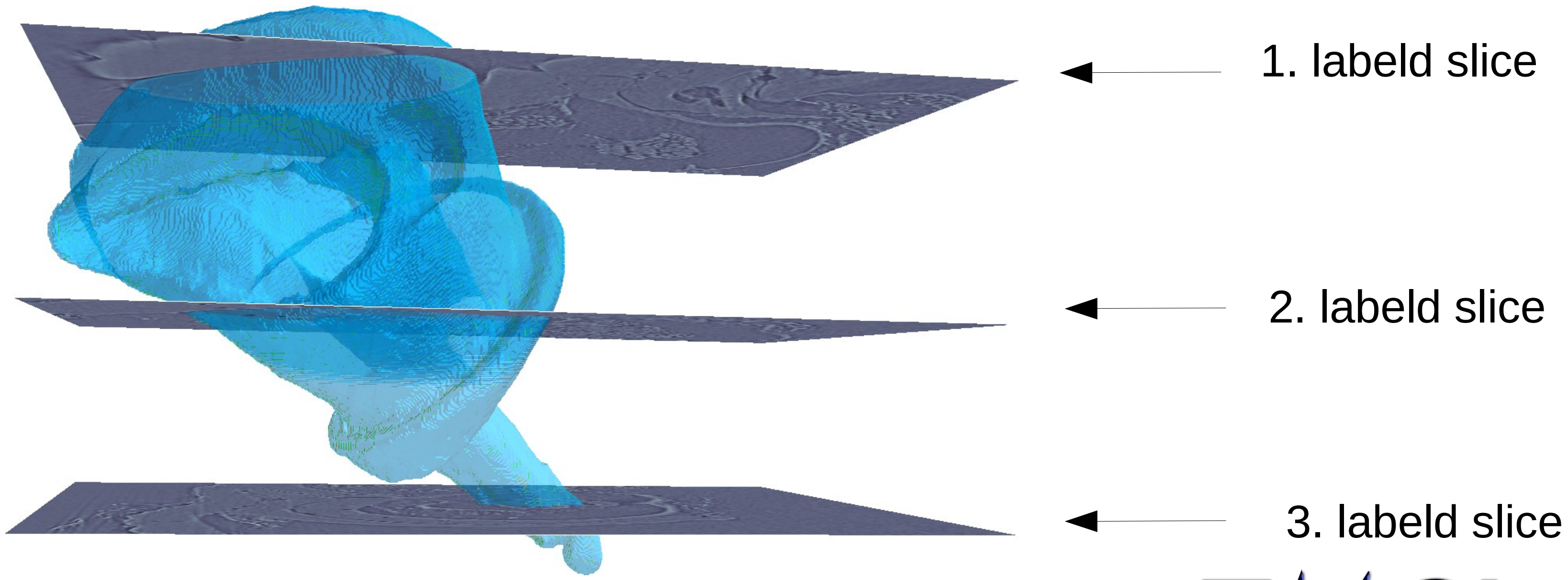




Multi-GPU based, parmeter-free diffusion method

- Segmentation by means of weighted random walks
- Several weighted random walks start in manually labeled slices
- An unlabeled voxel will be hit by random walks over time
- A voxel can be hit by random walks starting in the background or the labels
- Image Segmentation by assigning each voxel to the segment where the most random walks started from





Multi-GPU based, parmeter-free diffusion method

Let V be a set of vertices in $\mathbb{R}^n$. For $x, y \in V$ we write $x \sim y$ if x is adjacent to y. A weighted graph is a couple (V, P) where P is a non-negatove function on $V \times V$. Let $I(x)$ be the image data. Further, let x_0 be the starting point of a random walk and σ_{x_0} the standard deviation from x_0 in a local area, then we set

$$\mu_{x_0}(y) = \frac{1}{\sqrt{2\pi\sigma_{x_0}^2}} \exp\left(-\frac{(I(x_0) - I(y))^2}{2\sigma_{x_0}^2}\right) \text{ for all } y \in V$$

and

$$\mu(x) = \sum_{y \in V, y \sim x} \mu_{x_0}(y) \text{ for all } x \in V.$$

Multi-GPU based, parmeter-free diffusion method

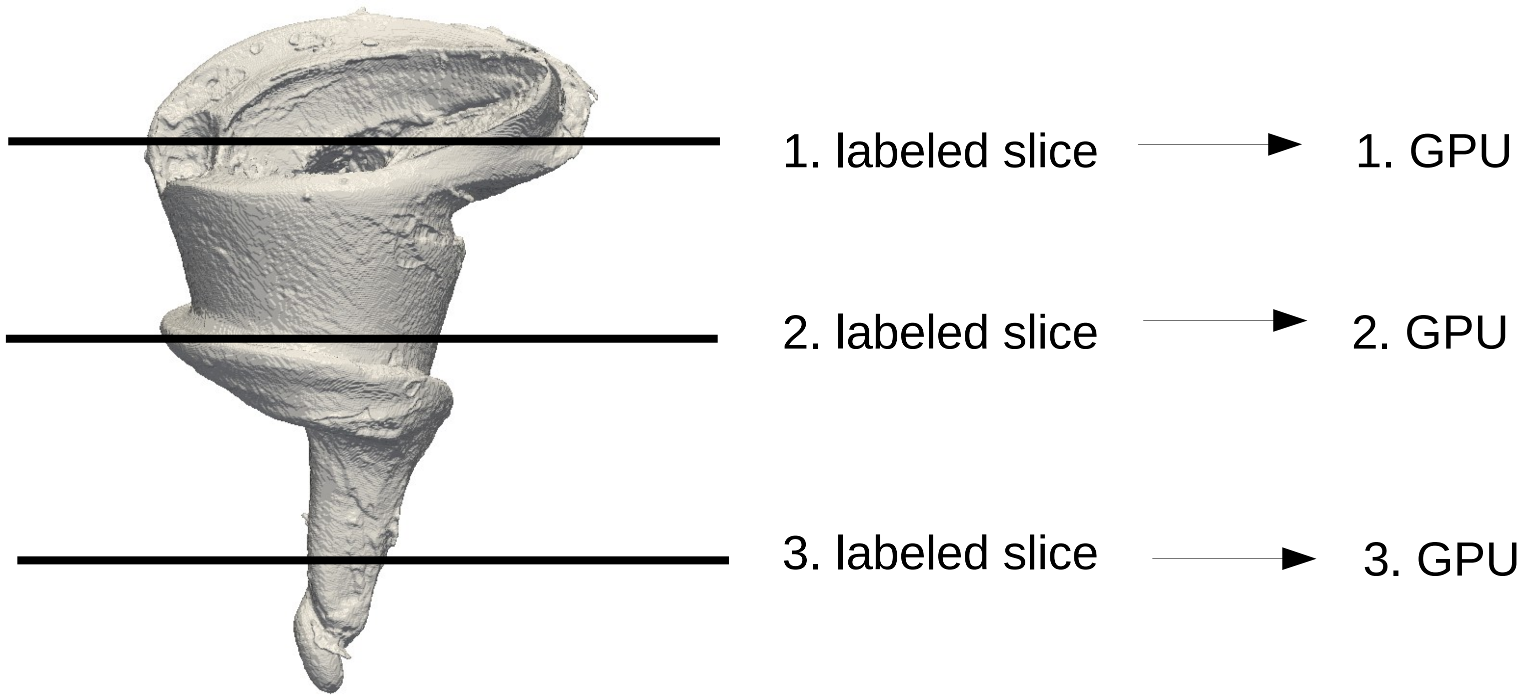
The function μ induces a Markov kernel

$$P_{x_0}(y, x) = \begin{cases} \frac{\mu_{x_0}(y)}{\mu(x)} & \text{if } y \sim x \\ 0 & \text{else,} \end{cases}$$

where $P_{x_0}(y, x)$ is the conditional probability of a random walk moving from x to y given its position x.

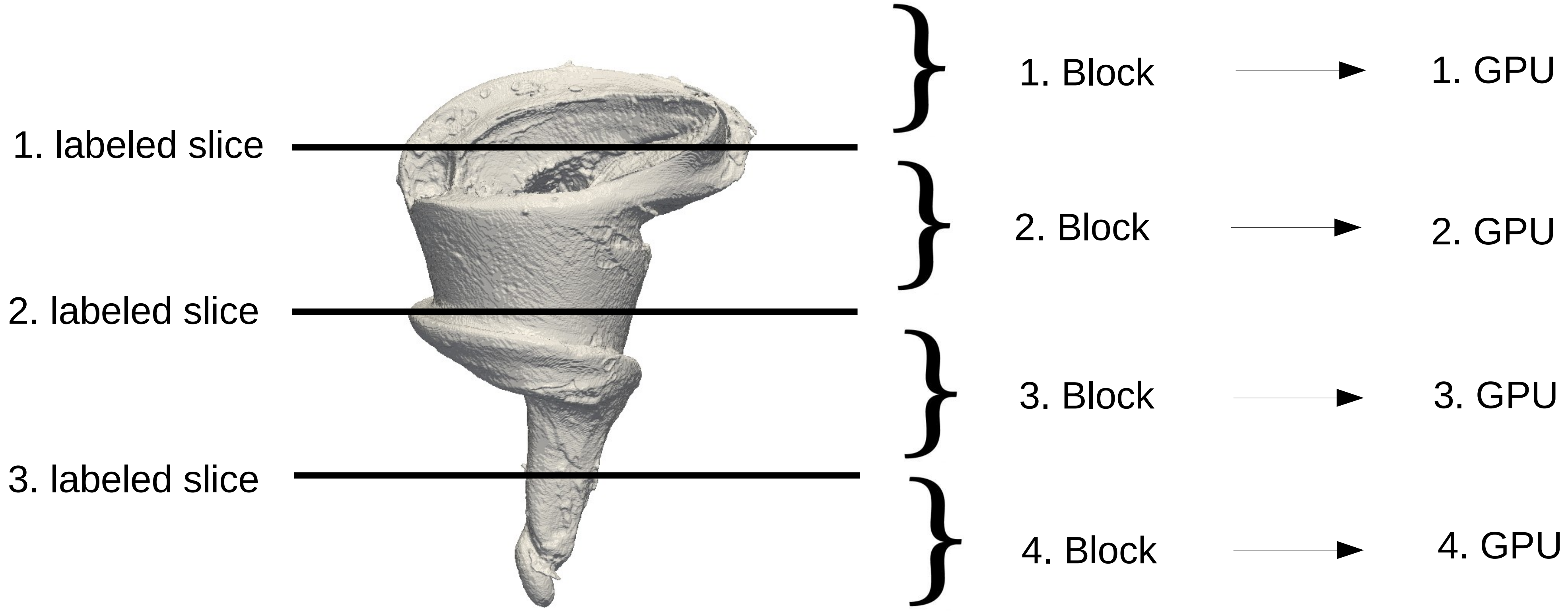
Multi-GPU diffusion (small images)

- Images < 125 MB
- Spread the labeled slices over several GPUs
- 1 GPU per labeled slice
- The GPU runs only the random walks belonging to this slice



Domain decomposition (large images)

- Split the image into blocks
- Each GPU runs all random walks for all labeled slices
- Each GPU considers only the hits in one block
- No storage of all hits necessary
- Argmax on the fly



Biomedical image segmentation app

A free online application for segmentation of tomographic images

Wasp Project – Thomas van de Kamp (ANKA @ KIT)

Practical session

Results

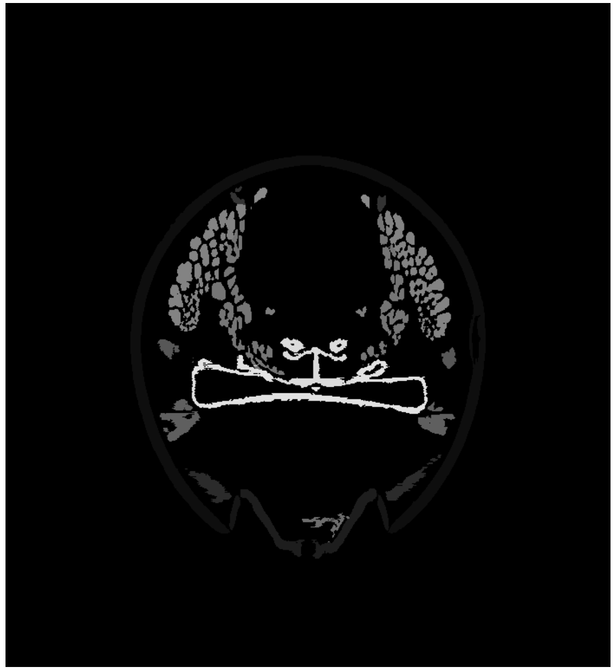
vs.

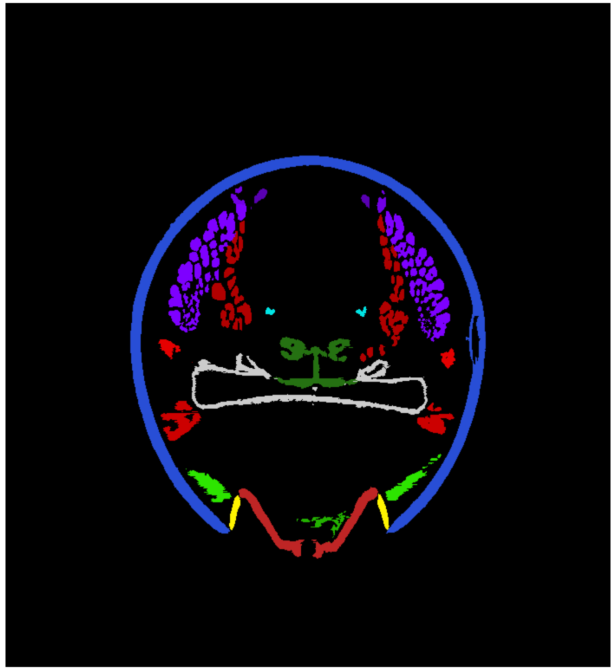
Biomedisa

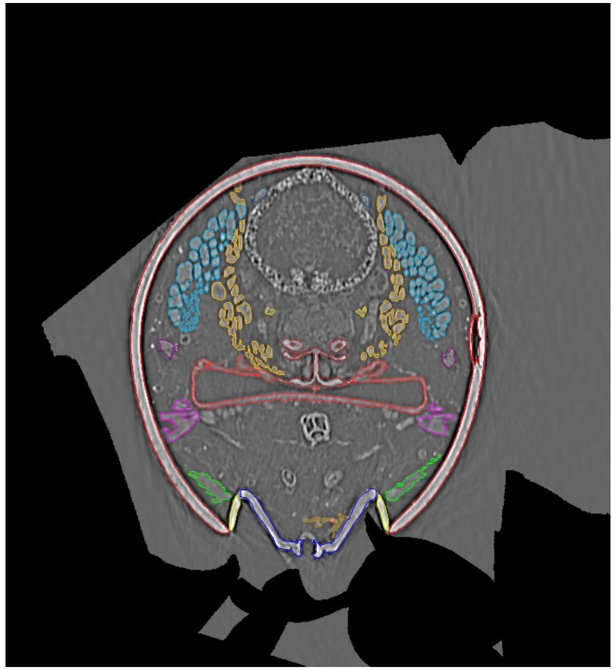
manual segmentation and
interpolation with Amira

Results

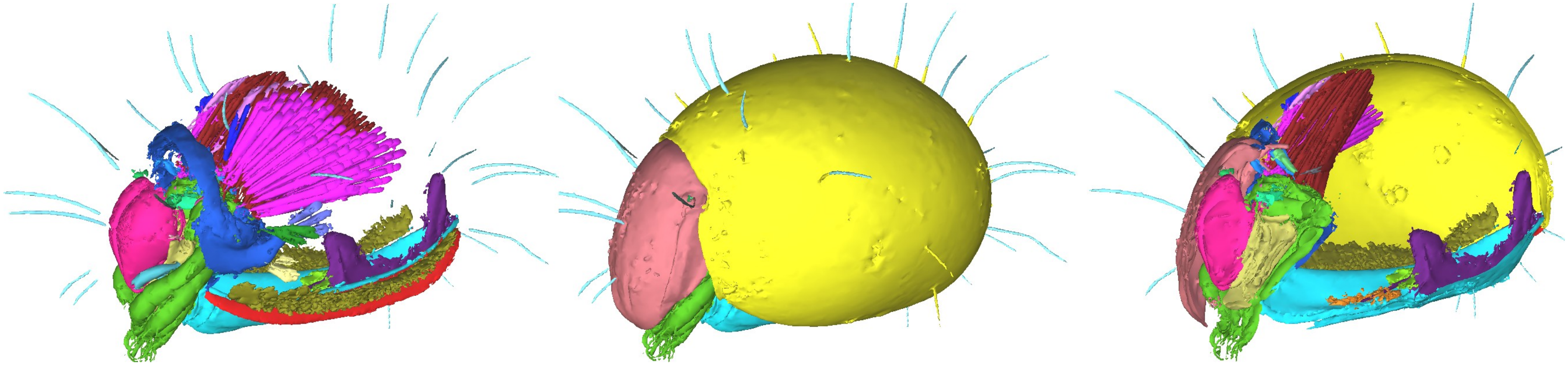
- Any number of labels
- Gray values or colors (ImageJ)







Results



Euphthiracarus reticulatus - Sebastian Schmelzle (TU Darmstadt)

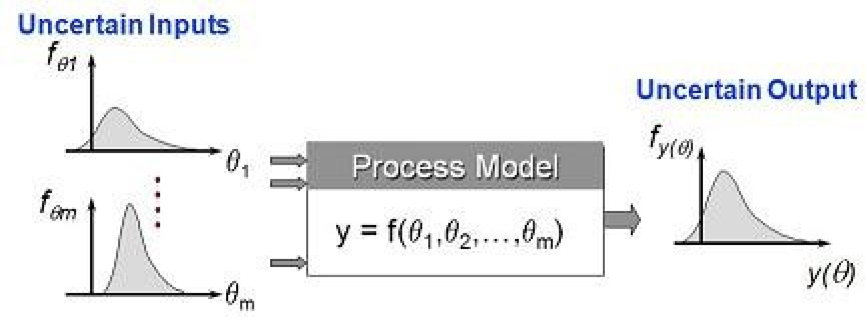
Results (7 GPUs)

Image	Size	No. of labeled slices	No. of segments	time	Dice coefficient
Wasp female	2553 x 992 x 1077	224	56	4 h 21 min	
Wasp male	3311 x 1223 x 1267	299	40	6 h 4 min	
Eucrib	1088 x 766 x 698	109	33	1 h 7 min	0.926
Eucrib	1088 x 766 x 698	55	33	35 min 0 sec	0.924
Eucrib	1088 x 766 x 698	22	33	27 min 55 sec	0.915
Trigonopterus	479 x 482 x 440	21	2	3 min 17 sec	
Hair	348 x 477 x 412	29	2	3 min 51 sec	
Brain Tumor	240 x 240 x 155	1	1	8 sec	

Uncertainty Quantification

„It tries to determine how likely certain outcomes are if some aspects of the system are not exactly known.“ (wiki)

- Uncertainty of parameters, boundary conditions, initialization
- None-intrusive (Monte Carlo) and intrusive methods (Polynomial Chaos Expansion)



Active contours (deterministic)

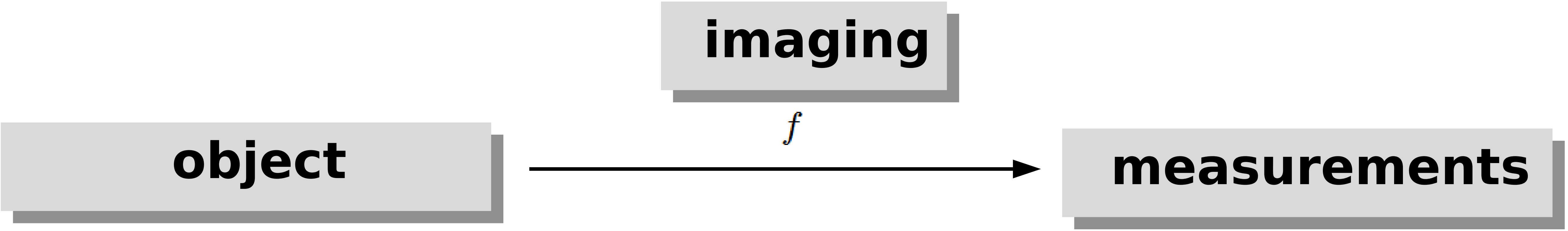
$$\frac{\partial \phi(x)}{\partial t} = |\nabla \phi(x)| \left(-\lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2\right)$$

Active contours (uncertain weights)

$$\frac{\partial \phi(x,\theta)}{\partial t} = |\nabla \phi(x,\theta)| \left(-\lambda_1(\theta) (u_0 - c_1)^2 + \lambda_2(\theta) (u_0 - c_2)^2\right)$$

Inverse Image Segmentation

Forward:



Inverse Image Segmentation

Which segmentation satisfies the measured data?

model

$$\textbf{segmentation} \quad \underset{f^{-1}}{\overset{f}{\rightleftarrows}} \quad \textbf{output}$$

Summary

1. Active Contours for image segmentation
2. Parameter-free, semi-automatic diffusion algorithm
3. Uncertainty Quantification
4. Inverse Segmentation

Thank you for your attention!