HIDDeν

Karlsruher Institut für Technologie

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# A self-Consistent Wave Description of Axion Miniclusters and their Survival in the Galaxy

*V.Dandoy, T.Schwetz and E.Todarello*

# Motivation

- QCD Axion is able to solve both the strong CP problem and the dark matter through the misalignment mechanism

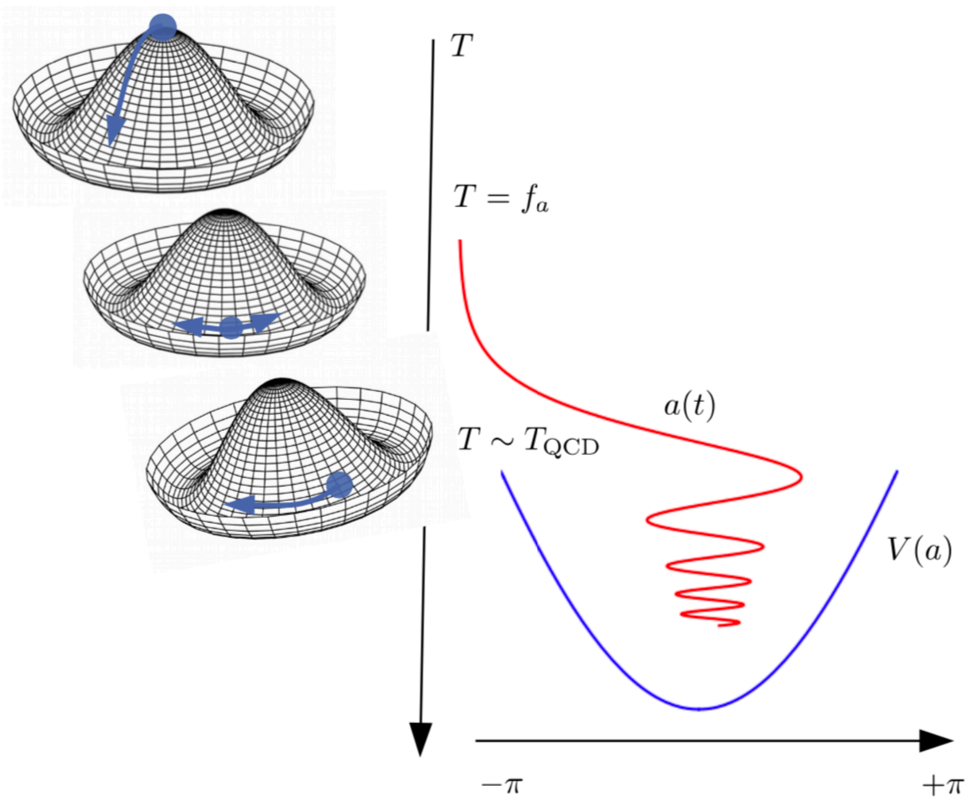



*Fig. 1: Axion Misalignment [1]*

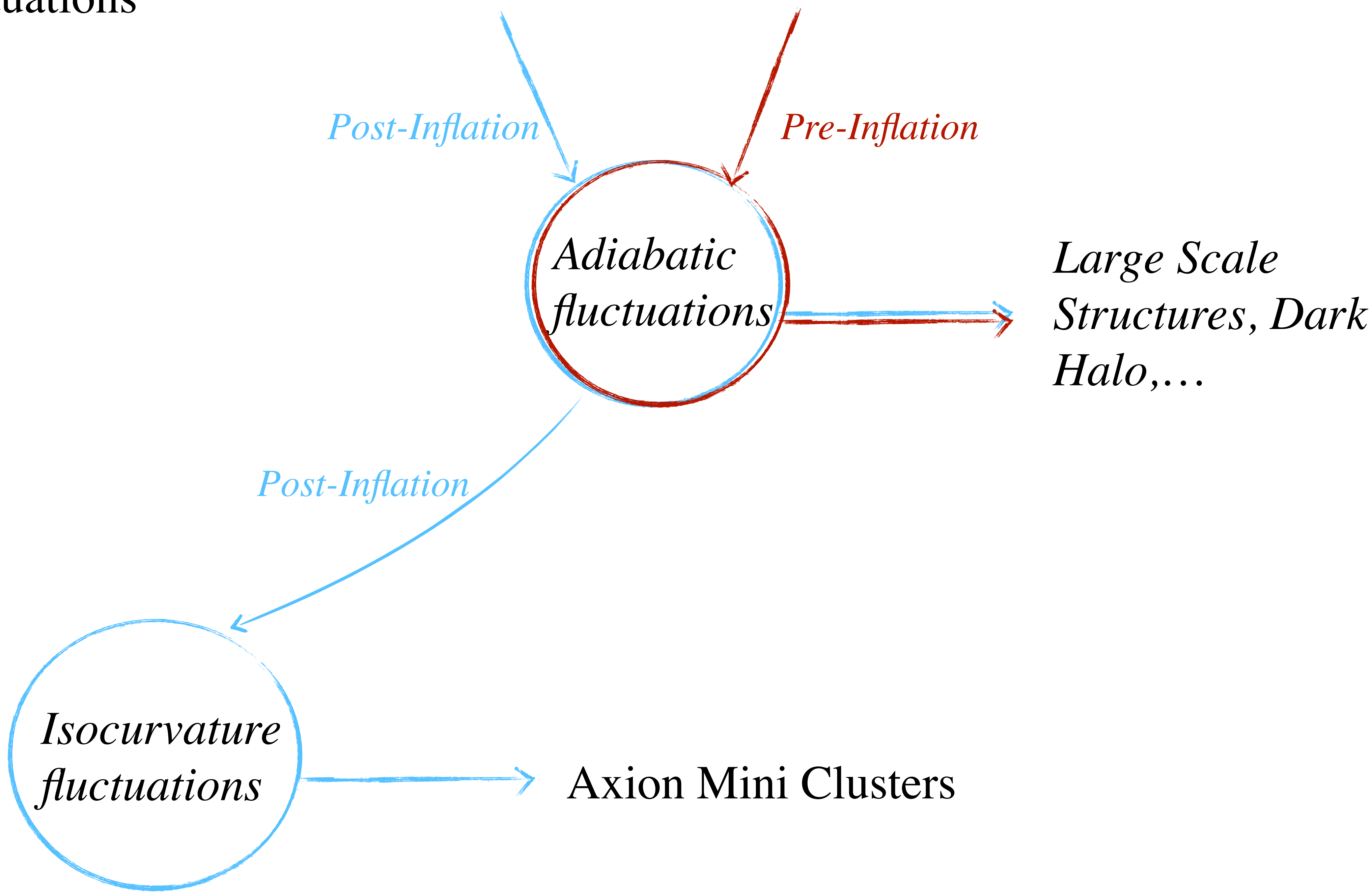
- U(1) Symmetry Breaking: Axion field reaches random initial value

- Cooling down of the Universe: potential for the axion + oscillation of the field

- Dark Matter energy density

## Motivation

- As expected the axion will also develop different types of density fluctuations



# Motivation

- *Post-Inflation*

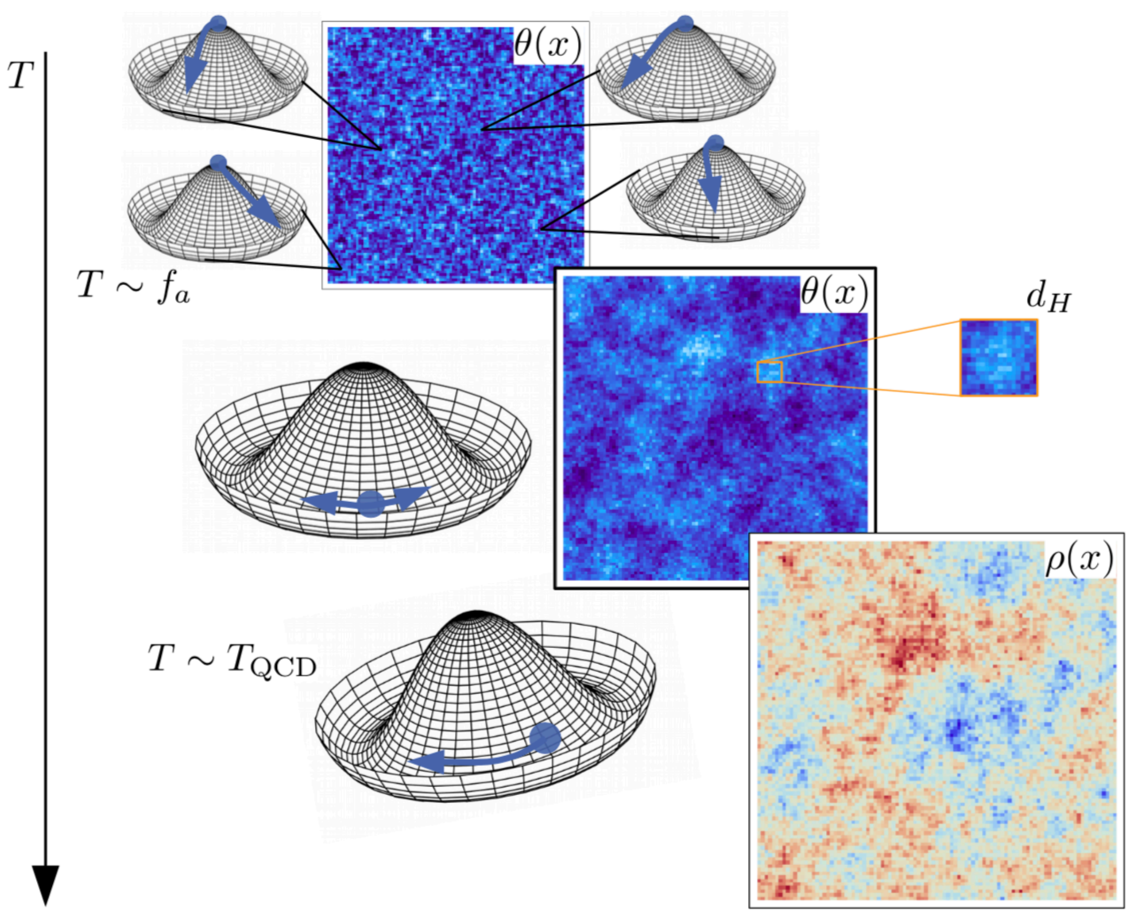



*Fig. 2: Post-Inflation Scenario [1]*

- Different values of the field in causal disconnected regions
- The initial energy density depends on the initial field value

Energy fluctuations

# Motivation

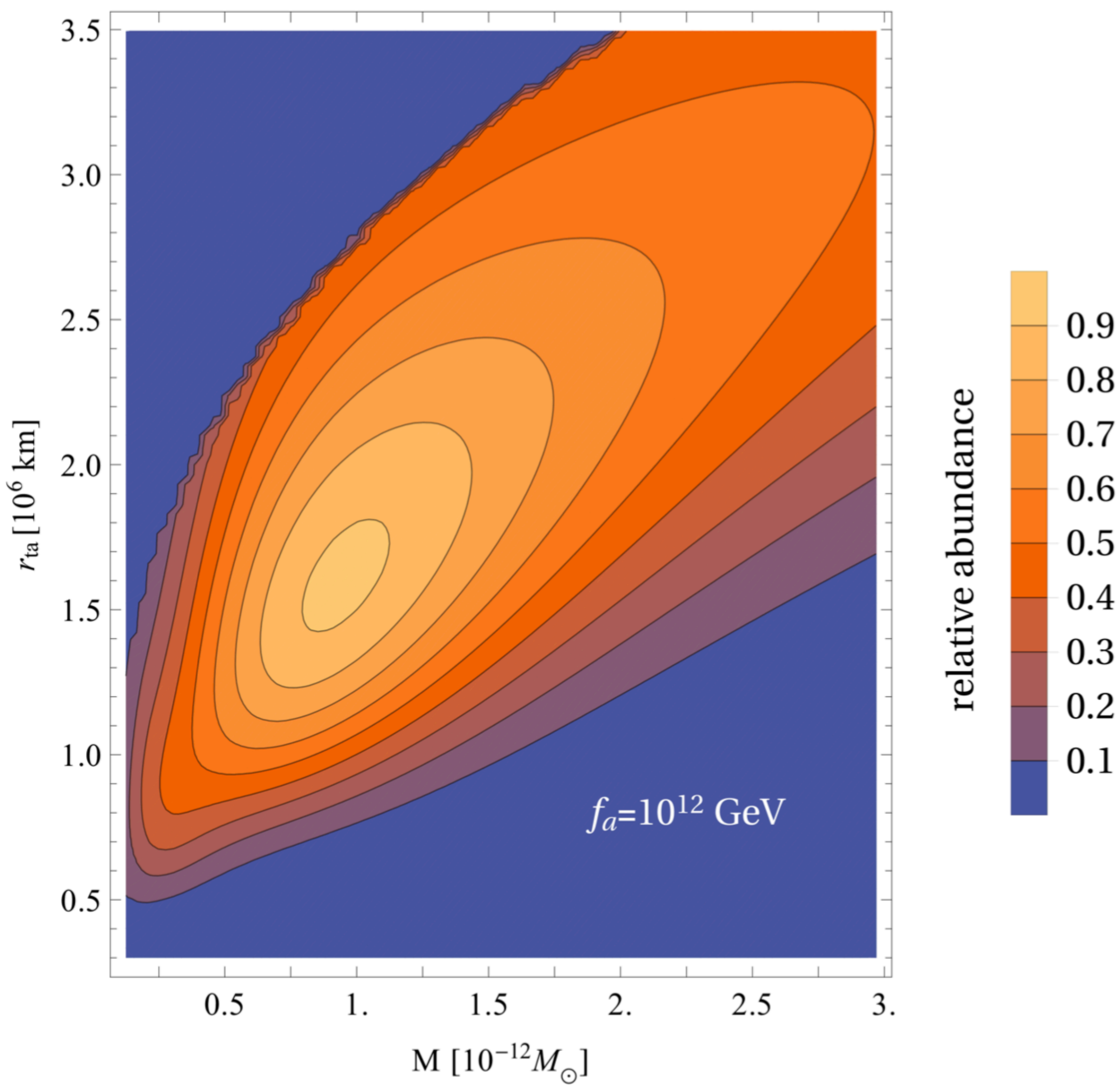
- *Post-Inflation and Miniclusters formation*

Energy fluctuations

→ Not interesting at cosmological scales

→ Large enough to collapse at high redshift: formation of miniclusters

*Fig. 3: Distribution of Miniclusters in mass and radius [1].*



# Motivation

- *How Do Axions Behave Inside Miniclusters?*

- Axions inside miniclusters have escape velocity $v_{\text{esc}} \ll 1$

→ Non-relativistic version of the Klein-Gordon equation: Schrodinger equation

- Occupation number is very high inside miniclusters

→ Classical interpretation of the field and mass density given by

$$\rho = m|\psi|^2$$

# Motivation

1. Axions can be described through a classical wave description

→ How to construct self-gravitating axion mini-cluster in that formalism?

2. Survival of the axion miniclusters

→ How do axion miniclusters interact with 'normal' stars?

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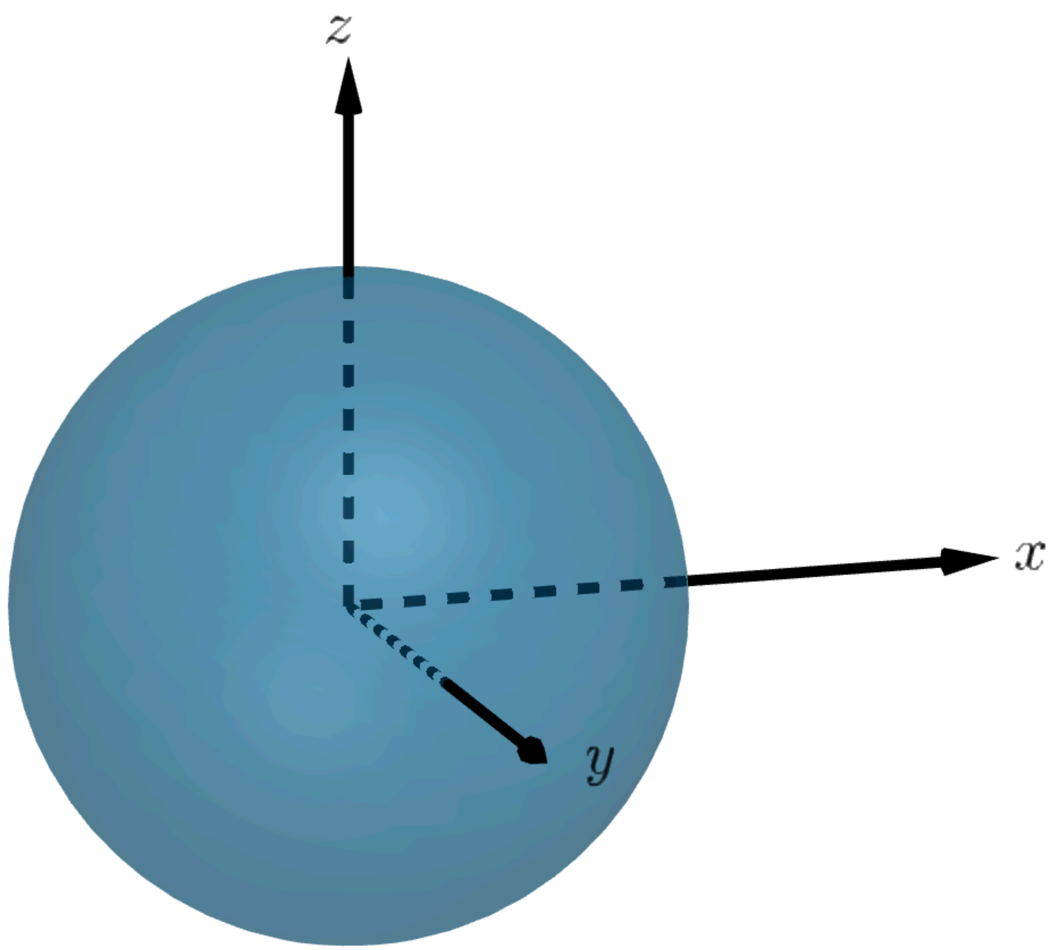
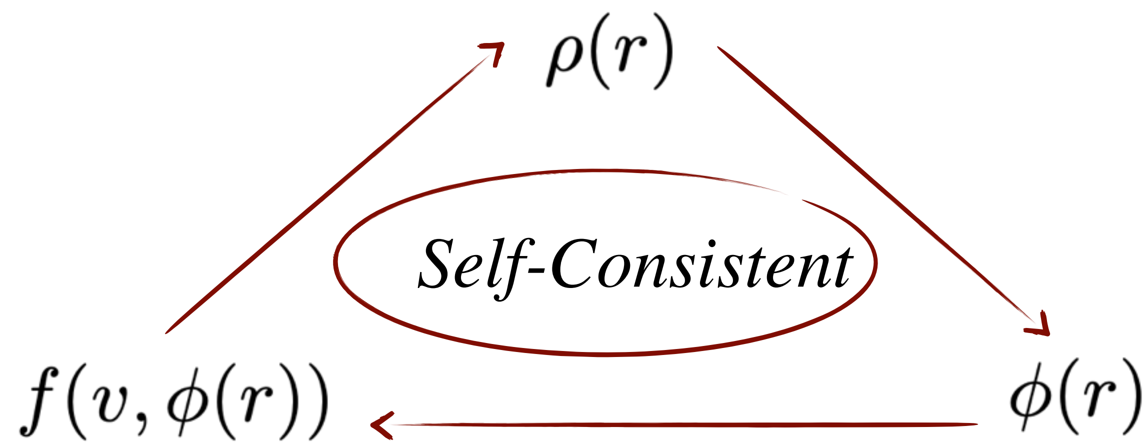
# Self-Consistent Description of Mini-Clusters

- *How Do I Describe a Self-Gravitating System with Particles?*




- The bound system is a solution of:

$$\rho(r) = m_a \int d^3v \, f(v, \phi(r))$$

$$\nabla^2 \phi(r) = 4\pi G \rho(r)$$

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*Ex:*

- Isothermal Sphere:

$$f(\mathcal{E}) = \frac{\rho_1}{(2\pi\sigma^2)^{3/2}}\, \mathrm{e}^{\mathcal{E}/\sigma^2}$$

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

$$\Phi(r) = 2\sigma^2 \ln(r) + \text{constant}$$

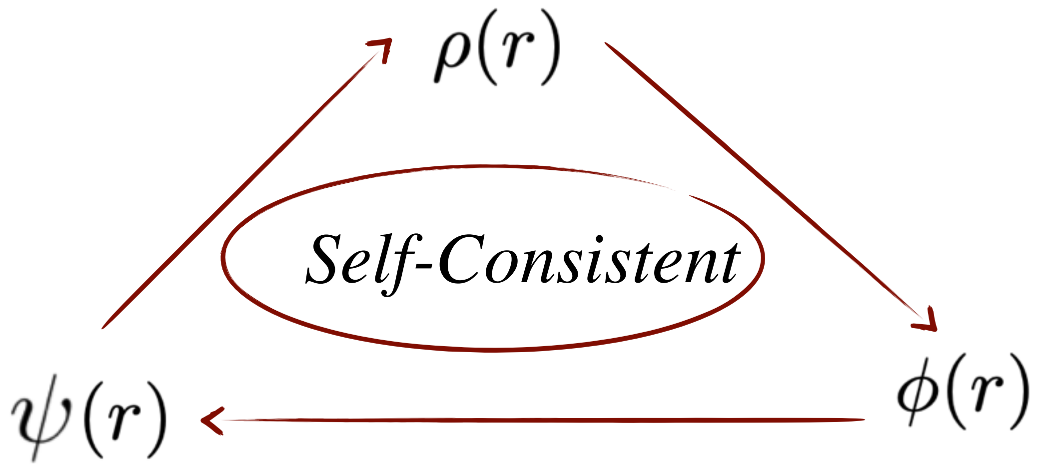
# Self-Consistent Description of Mini-Clusters

- *How Do I Describe a Self-Gravitating System with Waves?*

  - Non-relativistic axions in their own gravitational potential are described  through the Schrodinger-Poisson system

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m_a}\nabla^2\psi + m_a\phi\psi$$

$$\frac{\nabla^2\phi}{4\pi G} = m_a|\psi|^2, \quad m_a|\psi|^2 = \rho(r)$$




- How do I find a solution of that system?

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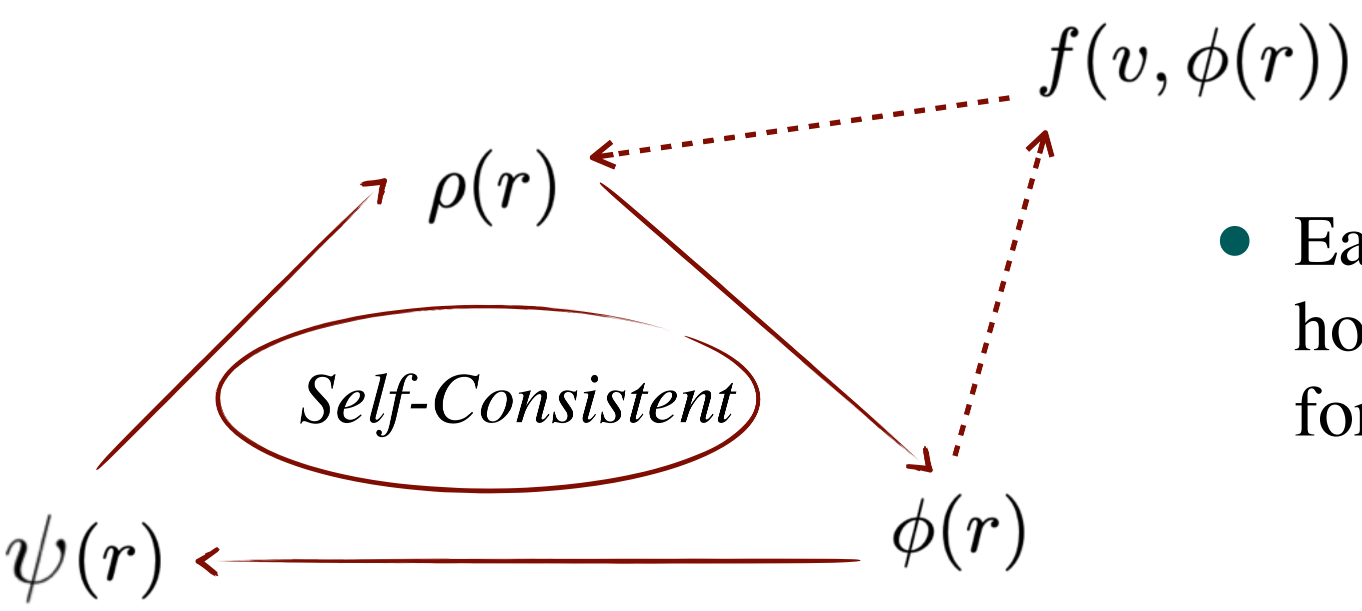


- Easiest way is to use our knowledge about how clusters are made in the particle formalism

- We first define the cluster we want to create and define the three self consistent functions

# Self-Consistent Description of Mini-Clusters

- *How Do I Describe a Self-Gravitating System with Waves?*

  - How do I find a solution of that system?

    ↓

  - We first define the cluster we want to create and define the three self consistent functions

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m_a}\nabla^2\psi + m_a\phi\psi$$

→ *Input gravitational potential*

*+ Constraint:* $m_a|\psi|^2 = \rho(r)$ → *Input density profile*

# Self-Consistent Description of Mini-Clusters

- Need to find the appropriate superposition of eigenfunctions such that the constraint is fulfilled

*WKB Approximation*

$$\psi(r,\theta,\phi) = \sum_{n,l,m} \frac{C_{n,l,m}}{\left(\frac{2m_a}{\hbar^2}(E_n - V_{eff}(r))\right)^{1/4} r} \sin\left[\int_r |p_r(r)| dr + \pi/4\right] Y_{lm}(\theta,\phi),$$

$$m_a |\psi|^2 = \rho(r)$$

- Fulfilled if each coefficient is given in term of the distribution function

$$C_{nlm} = 4\pi \sqrt{\frac{m_a}{\hbar} \mathcal{N}_{nlm}} \sqrt{f(E_n)\, dE\, dl\, dm}\, e^{i\phi_{nlm}}$$

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*Input distribution function*

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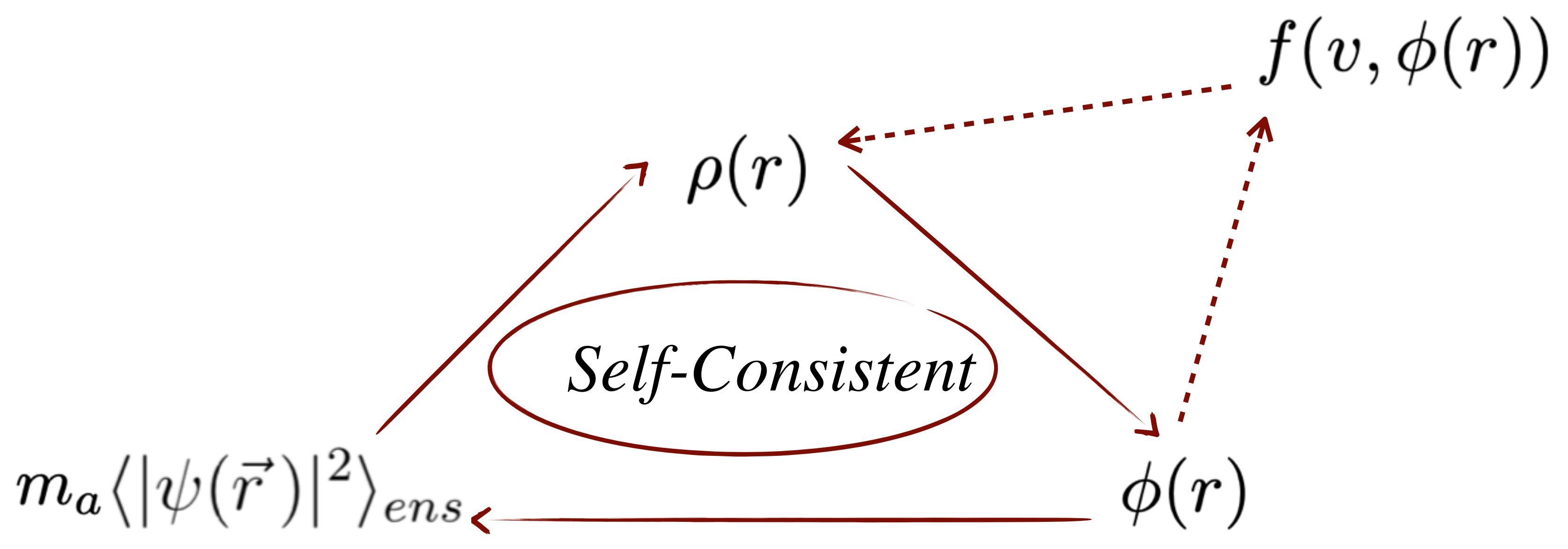
*Random Phase*

Does it correctly reproduce the density profile?



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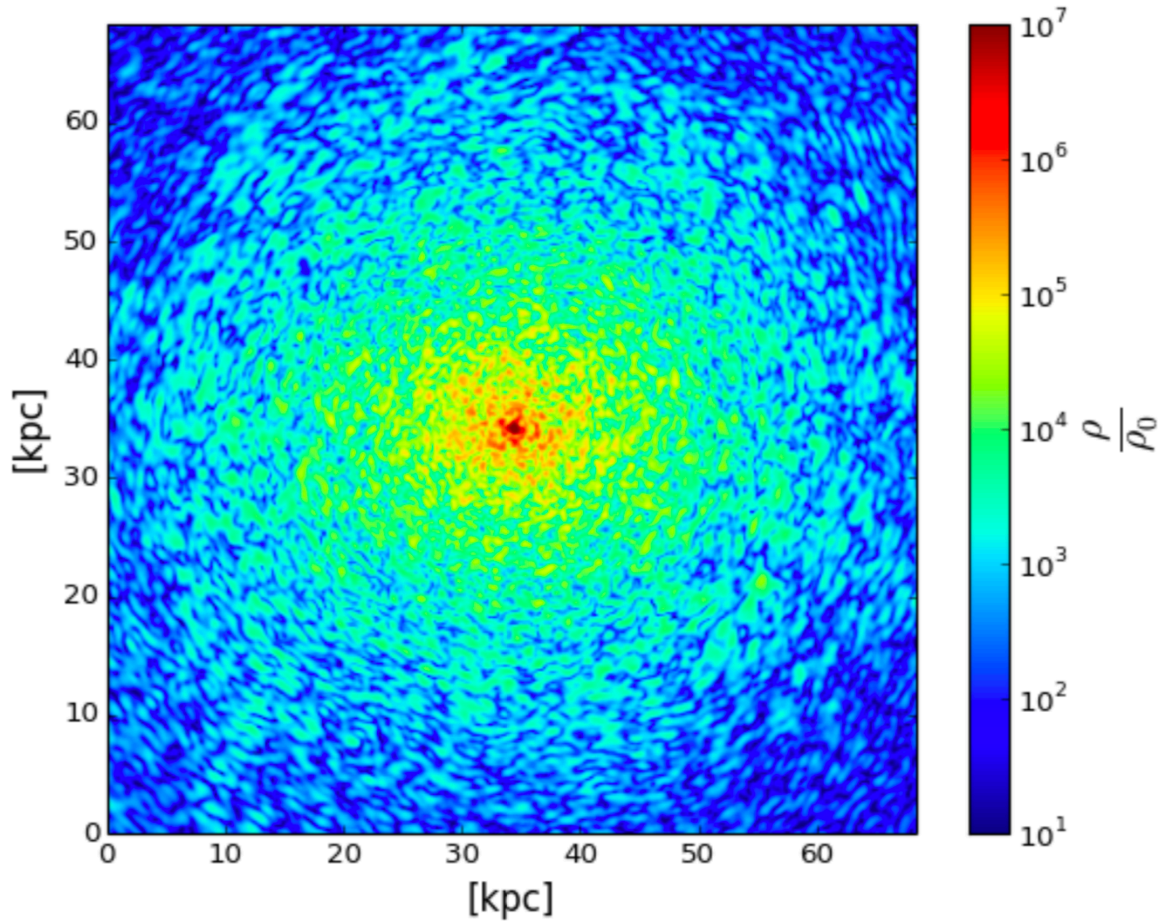



*Fig. 4: Wave numerical simulation dark halo[6].*

# Self-Consistent Description of Mini-Clusters

- For any minicluster characterized by a gravitational potential, density profile and distribution function, we are able to find a self-consistent solution for the axion field as

$$\psi(r,\theta,\phi) = \sum_{nlm} 4\pi \sqrt{\frac{m_a}{\hbar}\mathcal{N}_{nlm}} \sqrt{f(E_n)\, dE\, dl\, dm}\, e^{i\phi_{nlm}}\, R_{n,l}(r) Y_{lm}(\theta,\phi)$$

*Ex:*

$$\rho(r) = \frac{\pi}{4}\frac{M_c}{R_c^3}\mathrm{Sinc}(\pi r/R_c),$$

$$\phi(r) = -\frac{M_c G}{R_c}\mathrm{Sinc}(\pi r/R_c),$$

$$f(E) = \begin{cases} \frac{1}{m_a^4}\frac{1}{2^{7/2}\pi R_c^2 G}(-E/m_a)^{-1/2} & \text{if} \quad E \leq 0 \\ 0 & \text{if} \quad E > 0 \end{cases}$$

*Lane-Emden Profile*

# Survival in the Galaxy

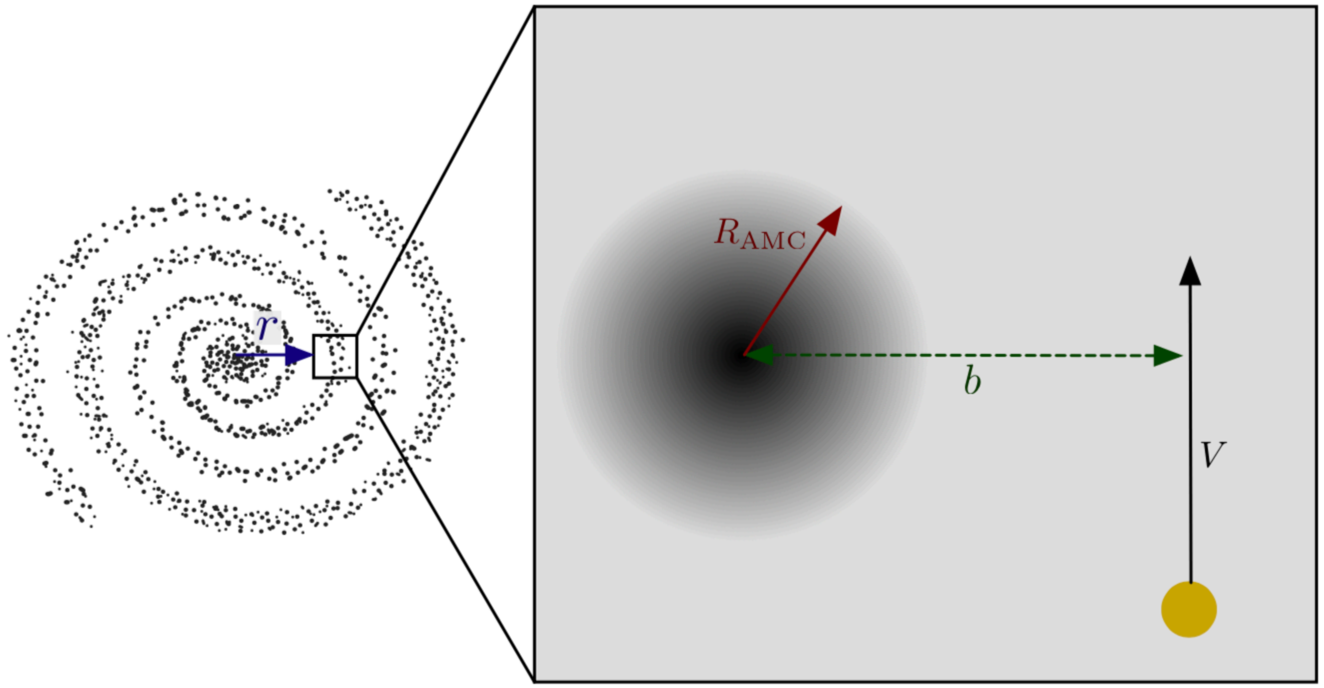



*Fig. 5: Interaction between a milky way star and a mini cluster [2].*

- Assuming that initially all the dark matter is bound inside mini-clusters
- At each tidal interaction with a star, the mini-cluster will lose some mass
- What is the today probability of finding a surviving mini-cluster at a given location?

→ Whether or not the miniclusters are still abundant in our galaxy will drastically alter the kind of axion signal observed on earth

# Survival in the Galaxy

- Recent studies have considered such interactions using ‘classical’ axion particles (see [2])

- What would be the difference if we consider now the axions as a field?

$$\psi(r,\theta,\phi) = \sum_{nlm} 4\pi \sqrt{\frac{m_a}{\hbar}\mathcal{N}_{nlm}} \sqrt{f(E_n)\, dE\, dl\, dm}\, e^{i\phi_{nlm}}\, R_{n,l}(r) Y_{lm}(\theta,\phi)$$

- The interaction will have to be treated in the quantum mechanics formalism: Tidal perturbation in the hamiltonian

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m_a}\nabla^2\psi + m_a\phi\psi \left( -\frac{GM_* m r^2}{(b^2+v^2t^2)^{3/2}} P_2(\cos\gamma(t)) \right)$$

# Survival in the Galaxy

*Single Encounter*

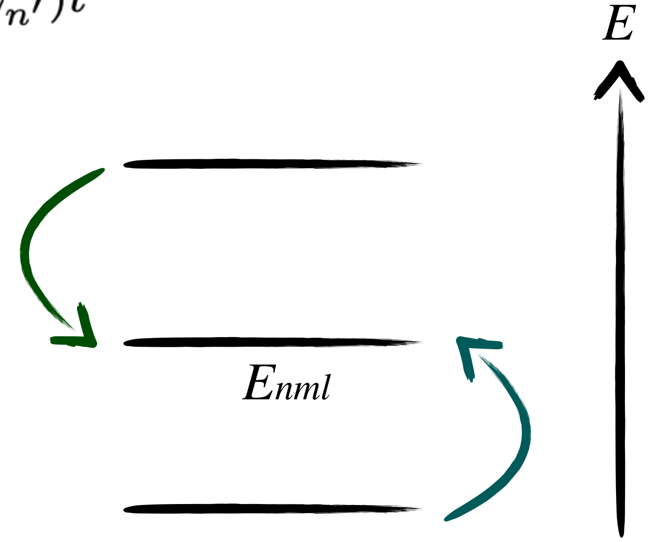
- The interaction will have to be treated in the quantum mechanics formalism: Tidal perturbation in the hamiltonian

$$H_1(t) = -\frac{GM_* m r^2}{(b^2 + v^2 t^2)^{3/2}} P_2(\cos\gamma(t))$$

→ Transition between energy levels / Modification of the coefficients

$$C^{(1)}_{nlm} = -i \sum_{n'l'm'} C^{(0)}_{n'l'm'} \int_{-\infty}^{\infty} dt \; \langle nlm | \, H_1(t) \, | n'l'm' \rangle \; e^{i(E_n - E_{n'})t}$$

*First order correction* — *Initial coefficient* — *Matrix Elements*




- Each energy level receives from the others

# Survival in the Galaxy



- These transitions will modify the initial distribution function

- Modification of the mass and energy of the system

$$\Delta M = 16\pi^2 m^2 \int_0^R dr \int_{m\phi(r)}^0 dE \int_0^{l_{\max}(E,r)} dl\, l \frac{\Delta f_{lost}(E,l)}{\sqrt{2m(E - V_l(r))}}$$

$$\Delta E = 16\pi^2 m \int_0^R dr \int_{m\phi(r)}^0 dE\, E \int_0^{l_{\max}(E,r)} dl\, l \frac{\Delta f(E,l)}{\sqrt{2m(E - V_l(r))}}$$

# Survival in the Galaxy



- We can assume that the time interval between the interactions is long enough to allow a re-virialization of the system

- We assume that the new profile is again on a Lane-Emden shape

- The radius is modified to return to a new stable configuration

$$E_{\rm tot} = -\frac{1}{8}\frac{GM_i^2}{R_i} \xrightarrow{\textit{Relaxation}} R_f = -\frac{1}{8}\frac{G(M_i + \Delta M)^2}{E_{\rm tot} + \Delta E}$$

# Conclusion and Outlook

- We have first develop a solution of the Schrodinger-Poisson system able to describe any self-consistent minicluster
- Using this formalism, we are able to perturb this system thanks to the quantum mechanical formalism and extract the lost mass and the variation of the radius when a star flies by
- Finally we will run a simulation over the initial mini-cluster population (reproducing the NFW dark halo profile) taking into account the number if interactions each of these clumps undergo.
- With this, we will be able to calculate the survival probability at a given location of the Milky Way

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