QED Corrections to QCDF

Gael Finauri

Status and Prospects of Non-leptonic B Meson Decays

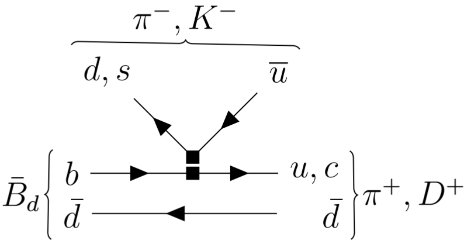
Siegen - 31 May 2022

based on
M. Beneke, P. Böer, J.-N. Toelstede, K. K. Vos 2008.10615
M. Beneke, P. Böer, GF, K. K. Vos 2107.03819

Motivation

QCDF known at $\mathcal{O}(\alpha_s^2)$ in two body non-leptonic B decays

[Beneke, Huber, Li *0911.3655*][Huber, Kränkl, Li *1606.02888*]



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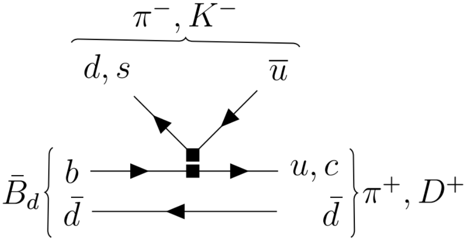

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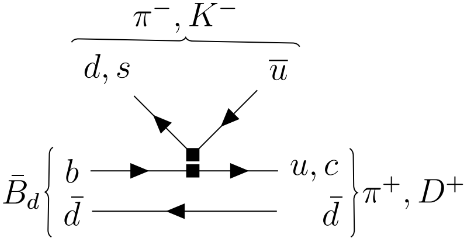

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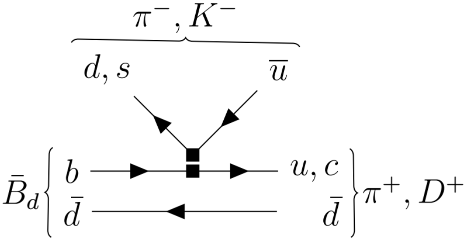

- Electrically charged mesons:
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- Conceptually interesting problem

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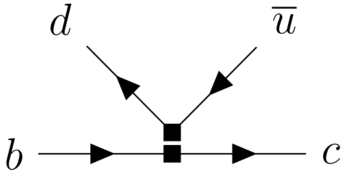

colour-allowed tree topology

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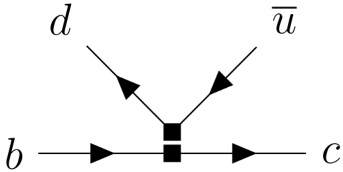

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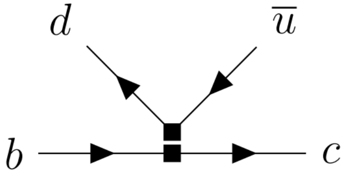

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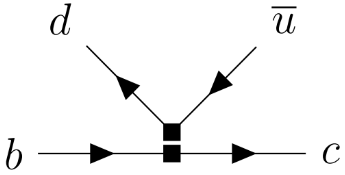

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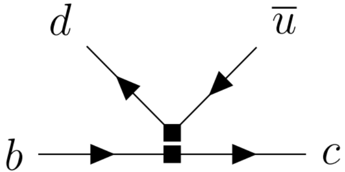

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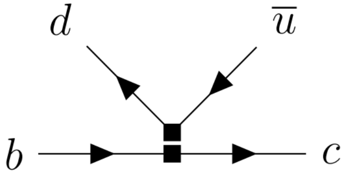

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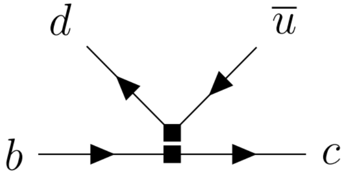

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Keri will tell you more about the pheno in $\bar{B} \to \pi K$

QCD Factorization in $\bar{B} \to DL$

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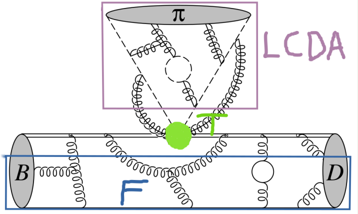

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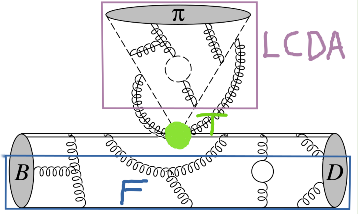

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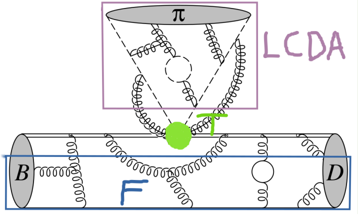

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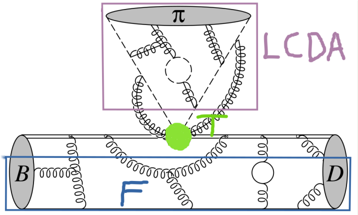

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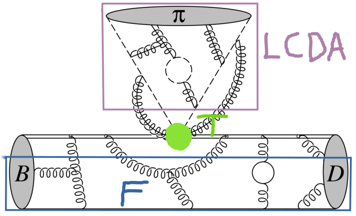

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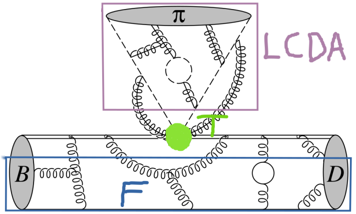

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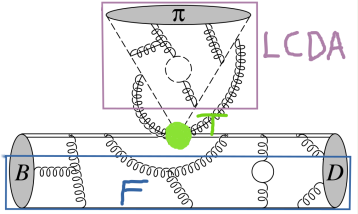

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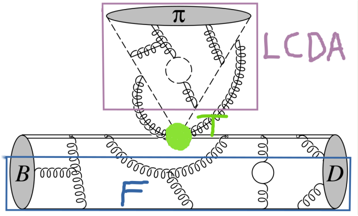

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may $\mathcal{O}(\alpha_{\rm em})$ corrections be of the same order of QCD uncertainties?

QED Factorization Formula

The factorization formula has the same form

$$\langle DL|Q_i|\bar{B}\rangle = 4if_L E_L E_D \, \zeta^{BD} \int_0^1 du \, H_i(u,z)\Phi_L(u) \qquad z \equiv \frac{m_c^2}{m_b^2} \sim \mathcal{O}(1)$$

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- Form factor: soft photons do not decouple from L if charged $\Rightarrow$

semi-leptonic factorization!

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such that in principle it can be determined from experiments

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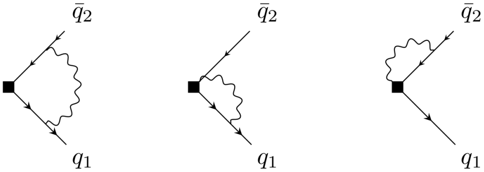

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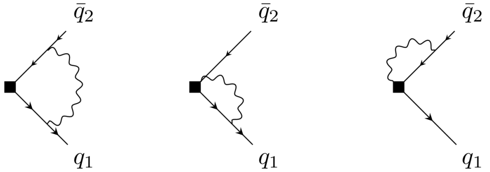

$$\Gamma(u,v;\mu) = -\frac{\alpha_s C_F + \alpha_{\rm em} Q_{q_1} Q_{q_2}}{\pi}\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\theta(v-u) + \left(1+\frac{1}{\bar v-\bar u}\right)\frac{\bar u}{\bar v}\theta(\bar v-\bar u)\right]_+^{(u)}$$
$$-\frac{\alpha_{\rm em}}{\pi}\delta(u-v)Q_M\left(Q_M\left(\ln\frac{\mu}{2E}+\frac{3}{4}\right) - Q_{q_1}\ln u + Q_{q_2}\ln\bar u\right)$$

LCDA Evolution

In addition to a simple rescaling of the QCD ERBL kernel, for charged mesons new local terms arise with interesting features:

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Hadronic Amplitude and QED Corrections on $a_1(DL)$

$$\mathcal{A}(\bar{B} \to DL) = A_{BD}^{QCD} \left(\frac{\hat{\mathcal{F}}^{BD}}{F_0^{BD}} \right) \sum_i \frac{C_i}{C_{\rm sl}} \int_0^1 du \, \frac{H_i(u,z)}{H_{\rm sl}} \frac{\Phi_L(u)}{Z_\ell}$$

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- $\delta a_1^{\rm L}(D\pi) = \delta a_1^{\rm L}(DK) = +0.0035^*$ [Beneke, Böer, Toelstede, Vos *2108.05589*]

*not included in *2107.03819*

Numbers for $|a_1(D^{(*)}L)|$

$\lvert a_1(D^{(*)+}L^-)\rvert$	LO	NLO	NNLO	+QED NLO
$\lvert a_1(D^+\pi^-)\rvert$	1.008	$1.032^{+0.024}_{-0.018}$	$1.064^{+0.019}_{-0.017}$	$1.059^{+0.019}_{-0.017}$
$\lvert a_1(D^{*+}\pi^-)\rvert$	1.008	$1.031^{+0.023}_{-0.018}$	$1.063^{+0.020}_{-0.017}$	$1.058^{+0.020}_{-0.017}$
$\lvert a_1(D^+K^-)\rvert$	1.008	$1.032^{+0.024}_{-0.018}$	$1.062^{+0.017}_{-0.016}$	$1.057^{+0.017}_{-0.016}$
$\lvert a_1(D^{*+}K^-)\rvert$	1.008	$1.031^{+0.023}_{-0.018}$	$1.061^{+0.017}_{-0.016}$	$1.056^{+0.017}_{-0.016}$

QED corrections turned out to be 1 order of magnitude smaller than QCD uncertainties

Ultrasoft Radiation

$$\Gamma[\bar{B} \to DL](\Delta E) \equiv \Gamma[\bar{B} \to DL + X_s]|_{E_{X_s} < \Delta E} = \Gamma^{(0)}(\bar{B} \to DL) U(DL)$$

in QED in order to have IR finite quantities we have to consider the soft-photon inclusive rate, which for $\Delta E \ll \Lambda_{\mathrm{QCD}}$ factorizes

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$$U(D^+K^-) = 0.960 \pm 0.001 \qquad U(D^{*+}K^-) = 0.962 \pm 0.001$$

$$U(D^+\pi^-) = 0.938 \pm 0.005 \qquad U(D^{*+}\pi^-) = 0.940 \pm 0.005$$

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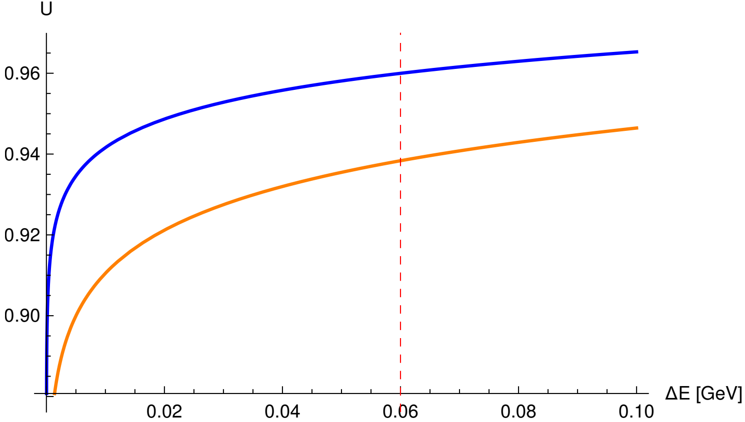
$\bar{B} \to D\pi$

$\bar{B} \to DK$



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In ratios of non-leptonic/semi-leptonic rates the $\hat{\mathcal{F}}^{BD^{(*)}}$ contribution drops

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- $X_L^{(*)}(= 1$ for $m_L \to 0)$ encodes the QCD form factor contribution

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$$R_L^{(*)} = R_L^{(*)}|_{\mathrm{QCD}}(1 + \delta_{\mathrm{QED}} + \delta_U)$$

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effects from kernels + WC + LCDA $< 1\%$

$$\delta_{\mathrm{QED}}(D\pi) = \delta_{\mathrm{QED}}(DK) = -0.85 \cdot 10^{-2} \qquad \delta_{\mathrm{QED}}(D^*\pi) = \delta_{\mathrm{QED}}(D^*K) = -0.84 \cdot 10^{-2}$$

Numbers for $R_L^{(*)}$

$R_L^{(*)}$	LO	QCD NNLO	$+\delta_{\rm QED}$	$+\delta_{\rm U}$ $(\delta_{\rm U}^{(0)})$
R_π	0.969 ± 0.021	$1.078^{+0.045}_{-0.042}$	$1.069^{+0.045}_{-0.041}$	$1.074^{+0.046}_{-0.043}$ $(1.003^{+0.042}_{-0.039})$
R_π^*	0.962 ± 0.021	$1.069^{+0.045}_{-0.041}$	$1.059^{+0.045}_{-0.041}$	$1.065^{+0.047}_{-0.042}$ $(0.996^{+0.043}_{-0.039})$
$R_K \cdot 10^2$	7.47 ± 0.07	$8.28^{+0.27}_{-0.26}$	$8.21^{+0.27}_{-0.26}$	$8.44^{+0.29}_{-0.28}$ $(7.88^{+0.26}_{-0.25})$
$R_K^* \cdot 10^2$	6.81 ± 0.16	$7.54^{+0.31}_{-0.29}$	$7.47^{+0.30}_{-0.29}$	$7.68^{+0.32}_{-0.30}$ $(7.19^{+0.29}_{-0.28})$

how ultrasoft effects are treated experimentally is crucial!

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Thank you!

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with the four-fermion operators

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so the goal is to compute these hadronic matrix elements: how?

factorization needed!

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We have included in the definition of the exponentiated ultrasoft function also the RG evolution factors from the non-radiative amplitude in order to have (at double-log accuracy) a **renormalization scale independent quantity**

$$U(D^{(*)}L) \equiv \left|e^{S_L(\mu_b,\mu_c)}\right|^2 e^{\frac{\alpha_{\mathrm{em}}}{4\pi} S_{\otimes}^{(1)}}$$

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$\Rightarrow$ strictly speaking U contains also some of the virtual corrections contributions... **difficult to compare with experiments!**

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Expanding in the light-meson mass (neglecting the lepton mass)

$$X_L = 1 + \frac{4m_L^2 m_B m_D}{(m_B^2 - m_D^2)^2} + \mathcal{O}\left(\frac{m_L^4}{m_B^4}\right)$$

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[Gubernari, Kokulu, van Dyk *1811.00983*]

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- for $\bar{B} \to D^* K$ large correction!