Overview of PQCD for non-leptonic decays

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Outlines

- Framework
  - kT factorization
  - Power counting
  - TMD wave functions
- Recent progress
  - Global analysis
  - Multi-body decays
  - Heavy baryon decays
- Summary

Framework

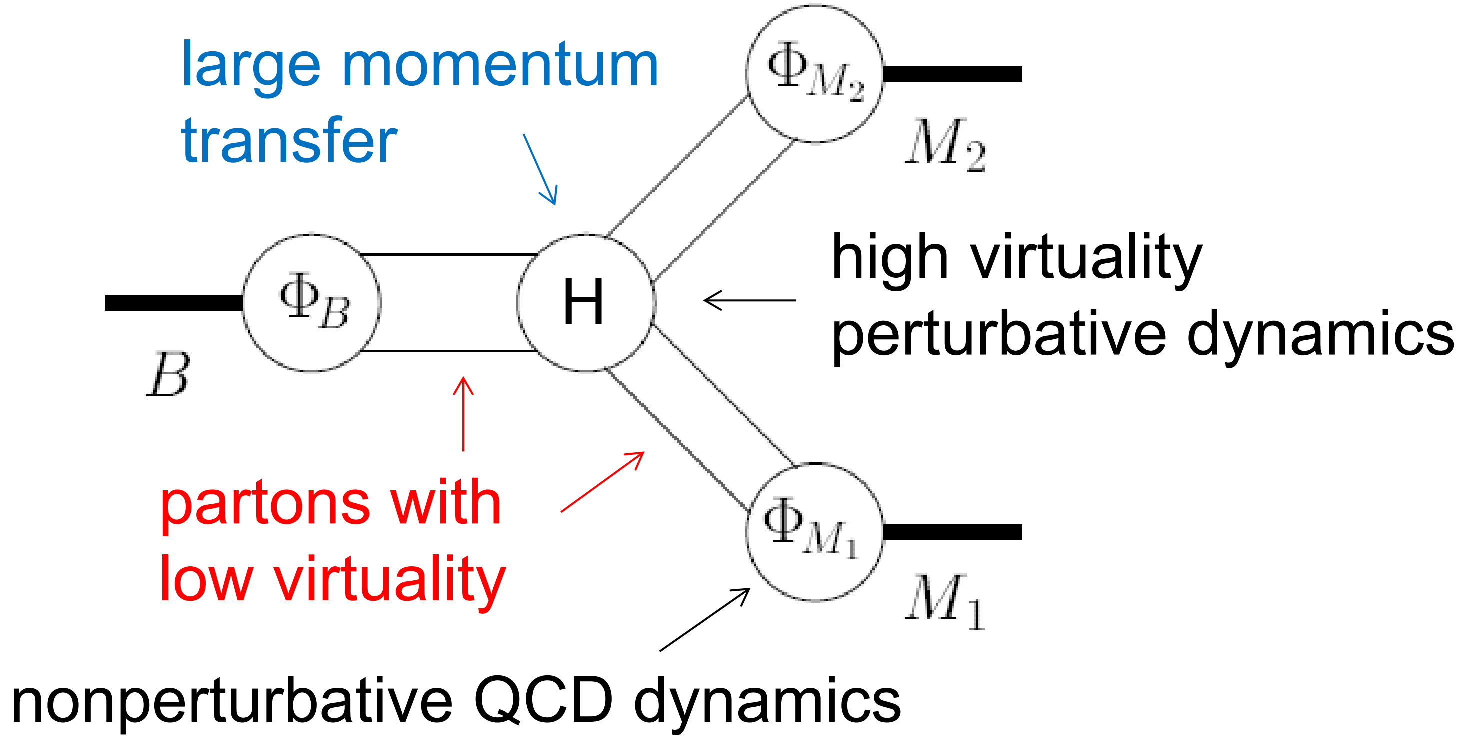
kT factorization

Power counting

TMD wave functions

Goal of factorization

- Factorize hadronic matrix element for B decay into hard kernel H and universal hadron wave function Φ to have predictive power



kT factorization

- Hadron momentum $P_1 = (P_1^+, 0, \mathbf{0}_T)$
- Parton momentum k with small k^2 has four components $k = (k^+, k^-, k_T)$

  ← $2k^+k^- - k_T^2$
- Convolution $\int d^4k \Phi(k^+, k^-, k_T) H(k^+, k^-, k_T)$
- Drop smallest k^- in H, integrate Φ over k^-
- Arrive at kT factorization

$$\int dk^+ d^2k_T \Phi(k^+, k_T) H(k^+, 0, k_T)$$

↑ transverse-momentum-dependent (TMD)

- $k^2 = -k_T^2$ in H, off-shell parton

Collinear factorization

- Can further drop smaller k_T, if no endpoint singularity

$k^+ \sim 0$

- Integrate Φ over k_T to get LCDA ϕ
- Arrive at collinear factorization

$$\int dk^+ \phi(k^+) H(k^+, 0, 0)$$

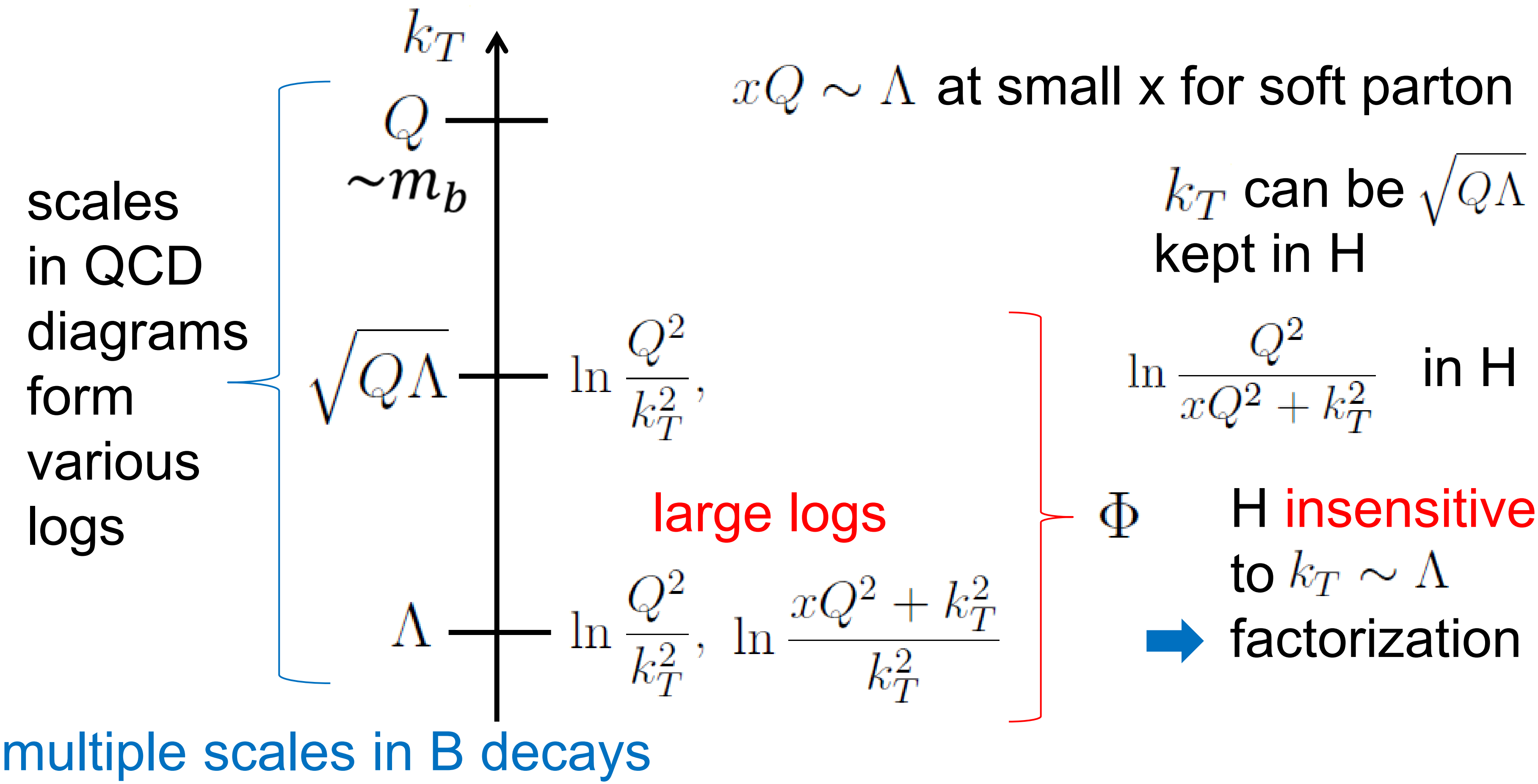
$k^2 = 0$
on-shell parton

- If singularity appears, stay at kT factorization
- Endpoint singularity appears in B decays (factorizable emission and annihilation)
- Collinear ➡ QCDF ➡ FF inputs, cutoffs

Beneke's talk

Power counting

- k_T is integration variable in kT factorization
- No fixed power counting for parton virtuality



Resummations

- Both Φ and H contain multiple scales, thus double logs in kT factorization
- kT resummation for Φ and threshold resummation for H
- kT resmmation organizes dynamics down to $\sqrt{Q\Lambda}$ into Sudakov factor S
- Factorization $\Phi(x,Q,b) = S(Q,b)\phi(x,b)$

$1/b \sim \Lambda$ ↓ intrinsic k_T dependence neglected

↑ impact parameter

↑ suppress large b (small kT)

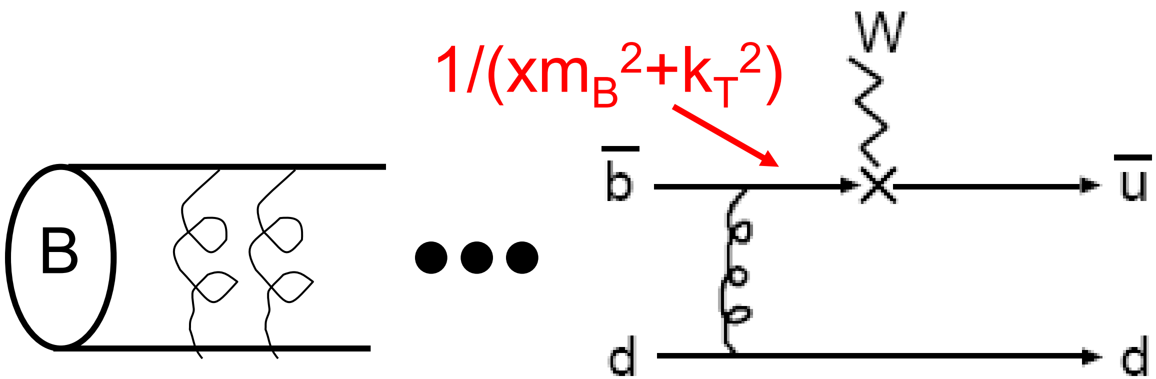
- Threshold gives jet function J, H -> J(x)*H,

↑ suppress small x region

Zhang, Li 2020

Smearing of endpoint singularity

- Parton k_T accumulated through Sudakov gluon emissions up to $\sqrt{Q\Lambda}$
- Similar to DGLAP evolution



$k_T \sim \Lambda$ $\qquad$ $k_T \sim \sqrt{Q\Lambda}$

$$\int_0^1 dx \frac{1}{x + k_T^2 / m_B^2}$$

is finite

- Subleading contributions under control, so smearing is effective

Gauge invariance

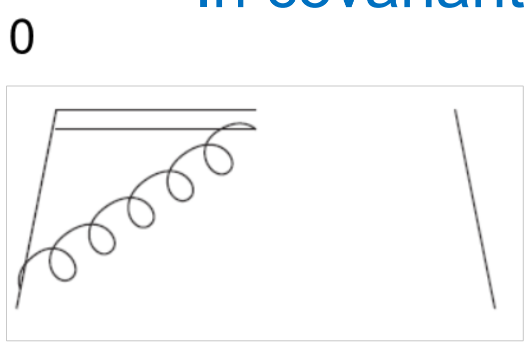
- Both QCD diagrams G and Φ depend on gauges, due to off-shell partons
- Here $\Phi \sim \langle 0|\ldots|q\bar{q}\rangle$, not gauge-invariant $\langle 0|\ldots|\text{hadron}\rangle$
- IR finite H can be extracted from quark process or hadron process
- $H=G/\Phi$ is gauge invariant
- related to exact cancellation of IR logs
- Proved using covariant gauge Nandi, Li 2007

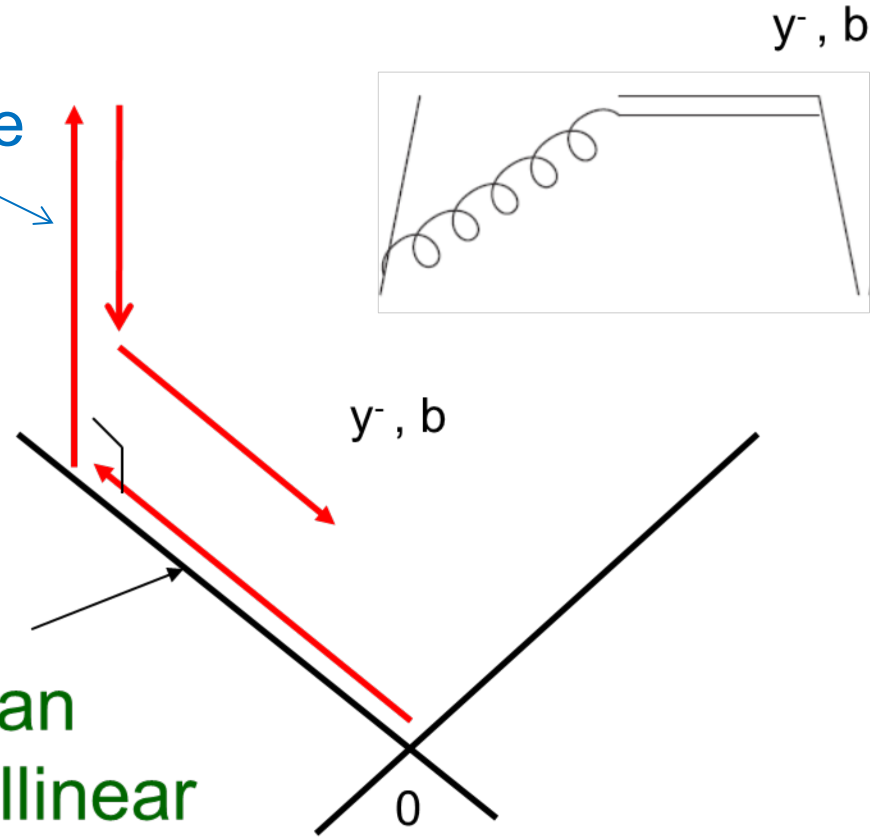
naïve TMD definition

Wilson links

loop momentum l flows through the hard kernel

no contribution
In covariant gauge





l does not flow through the other parts. Feynman rules the same as in collinear factctorization

Light-cone singularity

- Compute $H^{(1)} = G^{(1)} - \phi^{(1)} \otimes H^{(0)}$
- $1/(n_- \cdot l) = 1/l^+$ from Wilson lines in $\phi^{(1)}$ (vertex correction) gives light-cone singularity
- They cancel in collinear factorization

$$\phi^{(1)} \otimes H = \int \frac{dl^+}{l^+} [H(x) - H(x + l^+ / P^+)]$$

- Difference of $H^{(0)}$ removes singularity $l^+ \to 0$
- They exist in k_T factorization:

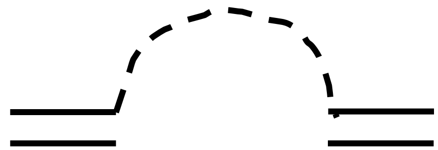
$$\int \frac{dl^+}{l^+} [H(x, k_T) - H(x + l^+ / P^+, k_T + l_T)]$$

- because $H(x, k_T) \neq H(x, k_T + l_T)$

Soft subtraction

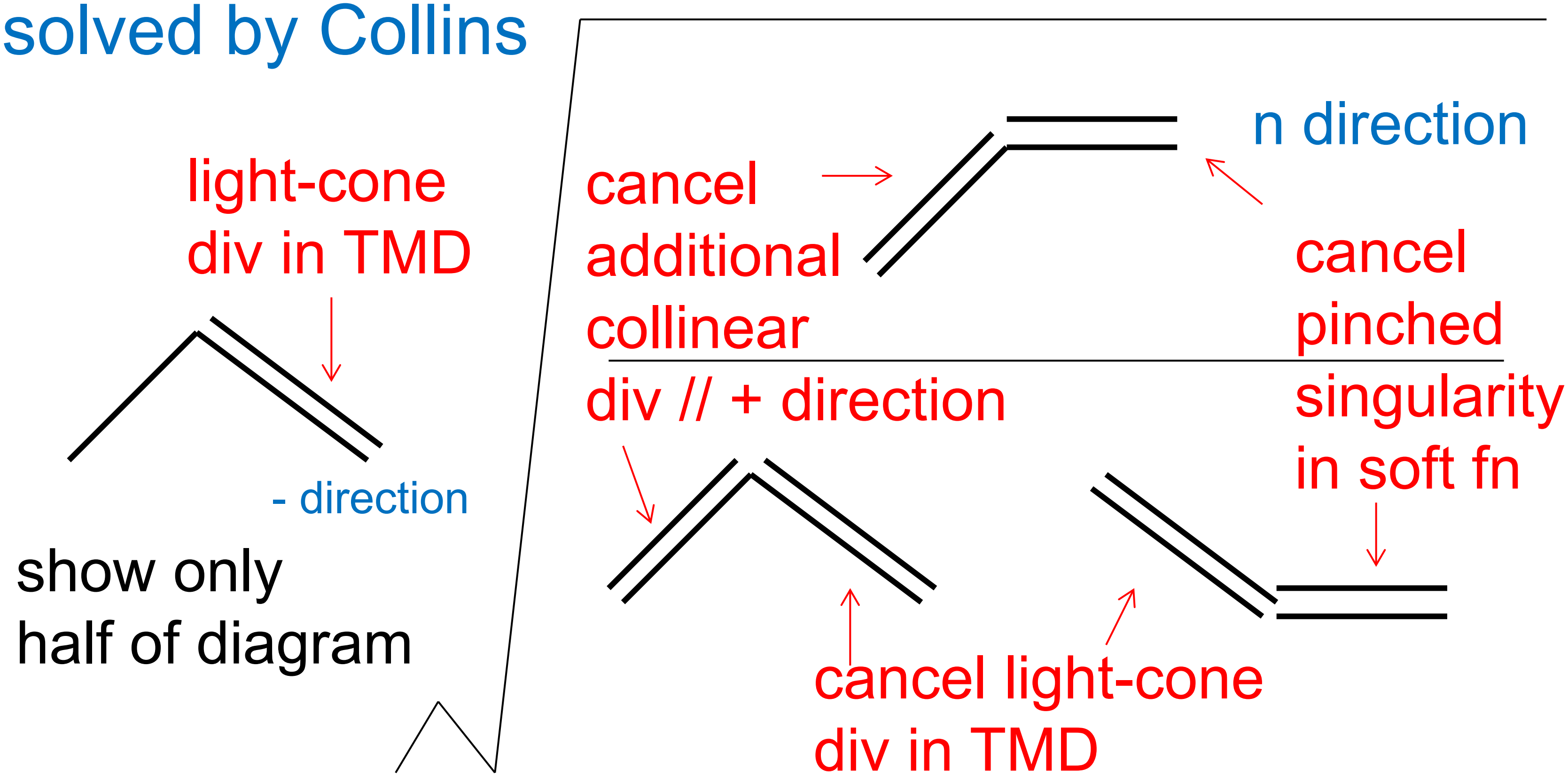
$n_{-} \to n,\, n^2 \neq 0$ introduces self-energy correction to Wilson lines

$1/(n \cdot l)^2$



generates power divergence or pinched singularity

solved by Collins

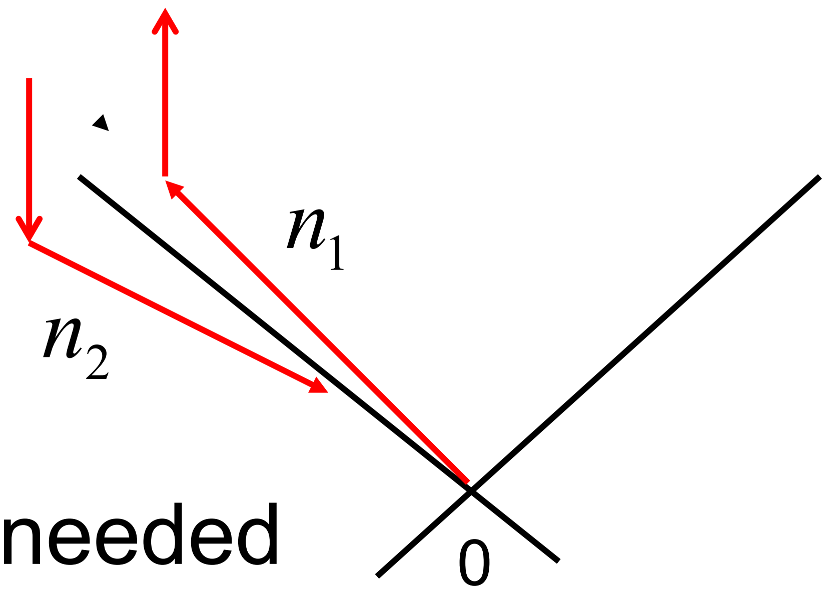


Non-dipolar Wilson lines

Wang, Li 2014

- Using dipolar Wilson lines, Collins' soft subtraction is the unique solution
- Choose orthogonal gauge vectors for off-light-cone Wilson lines
- Pinched singularity disappears
- soft subtraction is not needed

$$\frac{n_1 \cdot n_2 = 0}{(n_1 \cdot l + i\varepsilon)(n_2 \cdot l - i\varepsilon)}$$



Recent progress

Global analysis

Multi-body decays

Heavy baryon decays

Global analysis

Hua et al. 2021

- Consistent formalism with only universal inputs allows global analyses in PQCD

$$\phi_P(x) = \frac{f_P}{2\sqrt{2N_c}} 6x(1-x)\left[1 + a_1^f C_1^{3/2}(1-2x) + a_2^f C_2^{3/2}(1-2x) + a_4^f C_4^{3/2}(1-2x)\right]$$

- Gegenbauer coefficients from LO global fit

	a_1^π	a_2^π	a_4^π	a_{P2}^π	a_{T2}^π	$a_1^{\rho\parallel}$	$a_2^{\rho\parallel}$	
fit	–	0.644 ± 0.075	-0.41 ± 0.098	1.08 ± 0.15	-0.48 ± 0.33	0	0.16 ± 0.084	
	a_1^K	a_2^K	a_4^K	a_{P2}^K	a_{T2}^K	$a_1^{K^*\parallel}$	$a_2^{K^*\parallel}$	γ
fit	0.331 ± 0.082	0.28 ± 0.10	-0.398 ± 0.073	–	–	–	0.137 ± 0.029	$(75.2 \pm 2.9)^\circ$

- Compared with sum rule results

	a_1^π	a_2^π	a_4^π	$a_1^{\rho\parallel}$	$a_2^{\rho\parallel}$
QCD sum rule	–	0.25 ± 0.15	-0.015 ± 0.025	–	0.15 ± 0.07
	a_1^K	a_2^K	a_4^K	$a_1^{K^*\parallel}$	$a_2^{K^*\parallel}$
QCD sum rule	0.06 ± 0.03	0.25 ± 0.15	–	0.03 ± 0.02	0.11 ± 0.09

channel	data		fit	
	branching ratio(10^{-6})	A_{CP}(%)	branching ratio(10^{-6})	A_{CP}(%)
$B^0 \to \bar{K}^0 K^0$	1.21 ± 0.16	-60 ± 70	1.23 ± 0.08	0 ± 0
$B^0 \to \bar{K}^0 \pi^0$	9.90 ± 0.50	0 ± 13	8.98 ± 0.19	-4.02 ± 0.48
$B^0 \to K^- \pi^+$	19.6 ± 0.50	-8.3 ± 0.6	20.3 ± 0.36	-8.34 ± 0.36
$B^0 \to \pi^- \pi^+$	5.12 ± 0.19	32 ± 4	5.24 ± 0.17	23.2 ± 2.1
$B^0 \to \rho^0 \bar{K}^0$	3.40 ± 1.10	4 ± 20	3.06 ± 0.37	2.853 ± 0.068
$B^0 \to \pi^0 \bar{K}^{*0}$	3.30 ± 0.60	-15 ± 13	1.73 ± 0.10	-6.02 ± 0.6
$B^0 \to \pi^- \rho^+ / \pi^+ \rho^-$	23.0 ± 2.30	$13 \pm 6 / -8 \pm 8$	23.33 ± 0.8	$-24.3 \pm 1 / 8.1 \pm 1.1$
$B^- \to K^0 K^-$	1.31 ± 0.17	4 ± 14	1.47 ± 0.09	22.5 ± 2.7
$B^- \to \pi^0 K^-$	12.9 ± 0.50	3.7 ± 2.1	12.99 ± 0.23	-6.44 ± 0.6
$B^- \to \bar{K}^0 \pi^-$	23.7 ± 0.80	-1.7 ± 1.6	23.15 ± 0.42	-2.84 ± 0.24
$B^- \to \rho^- \pi^0$	10.9 ± 1.40	2 ± 11	8.73 ± 0.25	24.2 ± 2.3
$B^- \to \pi^0 K^{*-}$	6.80 ± 0.90	-39 ± 21	3.51 ± 0.19	-33.5 ± 1.7
$B^- \to K^- K^{*0}$	0.59 ± 0.08	12 ± 10	0.476 ± 0.022	22.5 ± 1.3
$B_s \to K^- K^+$	26.6 ± 2.20	-14 ± 11	24.8 ± 1.50	-8.1 ± 2.3
$B_s \to \pi^- \pi^+$	0.7 ± 0.1	–	0.798 ± 0.092	-1.62 ± 0.39
$B_s \to K^0 \bar{K}^0$	20.0 ± 6.00	0 ± 0	26.2 ± 1.60	0 ± 0
$B_s \to \pi^- K^+$	5.80 ± 0.70	22.1 ± 1.5	5.69 ± 0.64	22.1 ± 1.2
$B_s \to K^+ K^{*-} / K^- K^{*+}$	19.0 ± 5.0	–	15.28 ± 0.90	$-33.8 \pm 1.3 / 53.5 \pm 2.4$
$B_s \to K^0 \bar{K}^{*0} / \bar{K}^0 K^{*0}$	20.0 ± 6.00	–	15.06 ± 0.96	0 ± 0

large errors, weak constraint

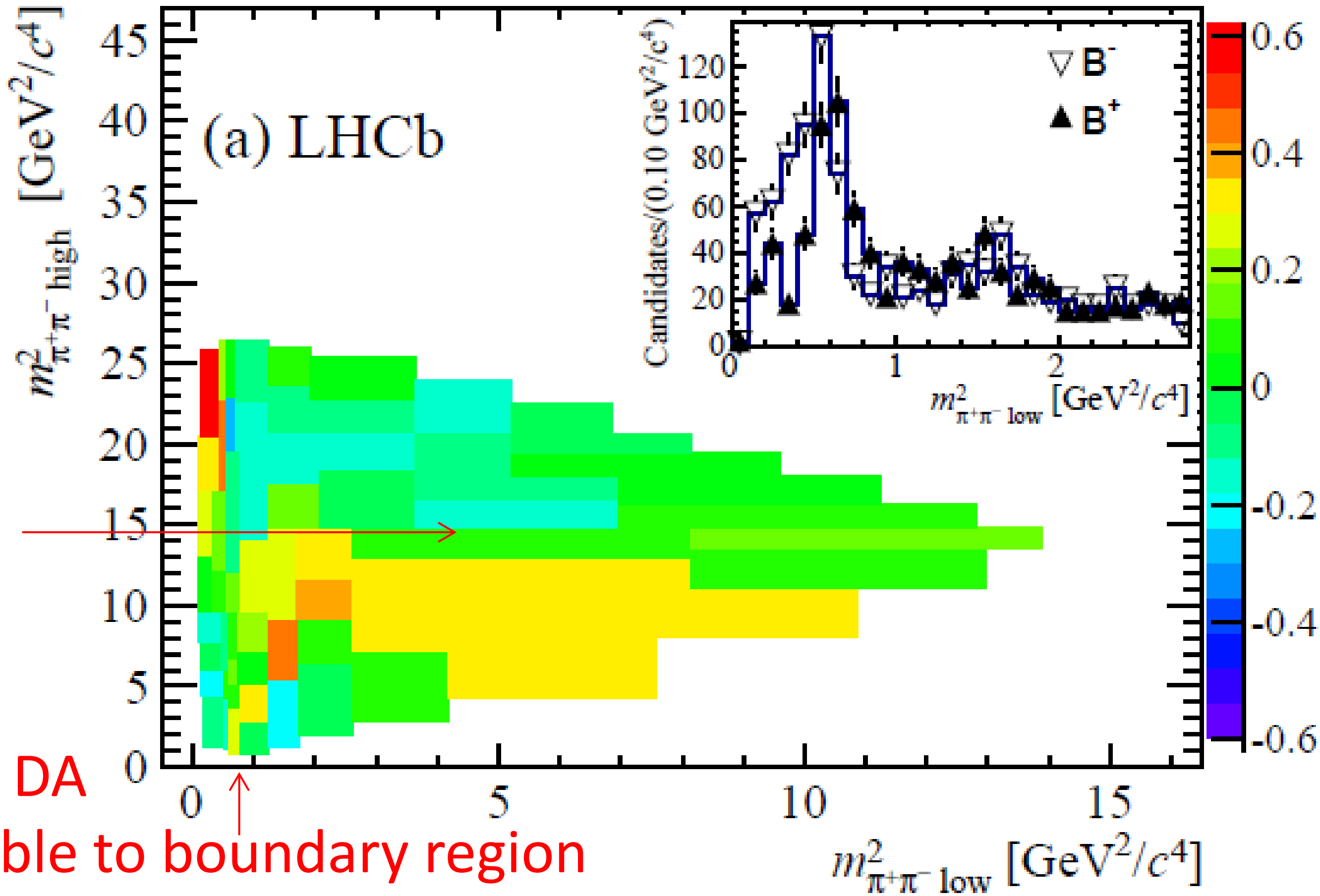
modes with puzzles not included

Three-body B decays

Virto, Magalhaes's talks

- LHCb has measured CP asymmetries in whole Dalitz plot for $B^- \to \pi^+\pi^-\pi^-$

genuine 3-body decay

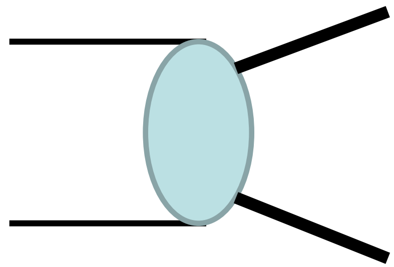


di-pion DA applicable to boundary region

Chen, Li, 2003

Two-hadron DA (TDA)

- Study CPV in localized boundary regions of Dalitz plot in three-body B decays
- Two-hadron DA collects collinear divergence appearing as two hadrons collimate roughly



quark momentum fraction x

p_1 p_2

$P = p_1 + p_2, \quad \varsigma = p_1^+ / P^+$

$\uparrow$ hadron momentum fraction

ω^2 invariant mass

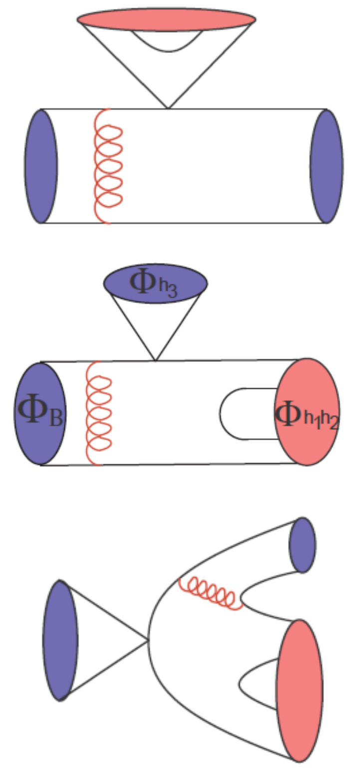
Gegenbauer Legendre

$$\Phi(x,\zeta,\omega^2) = \frac{6}{2\sqrt{2N_c}} x(1-x) \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(\omega^2) C_n^{3/2}(2x-1) P_l(2\zeta-1)$$

$\uparrow$ time-like form factor

Advantages of PQCD with TDA

- Calculation load reduced to that for 2-body, order 10 diagrams, not order 100
- Can calculate current-induced, transition, annihilation, nonfactorizable contributions
- Form factors can include both resonant and non-resonant contributions, and interference among final states ← time-like FF data
- Can calculate short-distance strong phases, so can predict direct CPV in localized regions



Regional CPV

- Factorization formula for decay amplitude

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}$$

- B meson and hadron DAs have been widely adopted in analysis of 2-body decay
- Calculate B+ and B- decays

$$A_{CP}^{reg}(B^\pm \to \pi^\pm \pi^+ \pi^-) = 0.52^{+0.12}_{-0.22}(\omega_B)^{+0.11}_{-0.09}(a_2^\pi)^{+0.03}_{-0.03}(m_0^\pi)$$

Wang et al, 2014

+-0.05 +-0.15 +-0.1

- Data $A_{CP}^{\rm region}(\pi^+\pi^-\pi^-) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007$

for $m^2_{\pi^+\pi^- \rm high} > 15\ \mathrm{GeV}^2$ and $m^2_{\pi^+\pi^- \rm low} < 0.4\ \mathrm{GeV}^2$

- Long-, short-distance phases equally important

Y. Li et al. 2021

Global analysis

Gegenbauer coefficients

different from V DA

	$a_{2\rho}^0$	$a_{2\rho}^s$	$a_{2\rho}^t$	$a_{2\phi}^0$	
fit	0.08 ± 0.13	-0.23 ± 0.24	-0.35 ± 0.06	-0.31 ± 0.19	
	$a_{1K^*}^0$ (Scenario I)	$a_{2K^*}^0$ (Scenario I)	$a_{1K^*}^0$ (Scenario II)	$a_{2K^*}^0$ (Scenario II)	$a_{4K^*}^0$ (Scenario II)
fit	0.31 ± 0.16	1.19 ± 0.10	0.57 ± 0.20	1.13 ± 0.32	-0.85 ± 0.16

B DA → Results ← Gegenbauer; scale

Modes		Results	Data
$B^+ \to K^+(\rho^0 \to)\pi\pi$	$\mathcal{B}(10^{-6})$	$2.91^{+0.68+0.77+1.43}_{-0.60-0.68-0.82}$	3.7 ± 0.5 †
	$\mathcal{A}_{CP}(\%)$	$53.5^{+0.4+4.5+11.9}_{-1.4-4.3-15.0}$	37 ± 10 †
$B^0 \to K^+(\rho^- \to)\pi\pi$	$\mathcal{B}(10^{-6})$	$8.48^{+2.20+1.63+3.87}_{-1.95-1.48-2.51}$	7.0 ± 0.9 †
	$\mathcal{A}_{CP}(\%)$	$33.0^{+1.1+5.2+8.9}_{-1.5-4.9-12.1}$	20 ± 11
$B_s^0 \to K^-(\rho^+ \to)\pi\pi$	$\mathcal{B}(10^{-6})$	$16.41^{+7.59+0.16+1.10}_{-5.30-0.15-1.31}$	–
	$\mathcal{A}_{CP}(\%)$	$19.4^{+3.6+3.3+3.1}_{-3.2-3.3-2.9}$	–
$B^+ \to K^0(\rho^+ \to)\pi\pi$	$\mathcal{B}(10^{-6})$	$7.86^{+2.07+1.51+3.68}_{-1.82-1.50-2.31}$	$7.3^{+1.0}_{-1.2}$ †
	$\mathcal{A}_{CP}(\%)$	$13.1^{+1.2+1.8+1.5}_{-0.5-2.5-3.6}$	-3 ± 15
$B^0 \to K^0(\rho^0 \to)\pi\pi$	$\mathcal{B}(10^{-6})$	$3.76^{+0.95+0.57+0.92}_{-0.81-0.52-0.81}$	3.4 ± 1.1 †
	$\mathcal{A}_{CP}(\%)$	$1.4^{+0.6+0.5+2.1}_{-0.5-0.6-3.1}$	-4 ± 20
$B_s^0 \to \bar{K}^0(\rho^0 \to)\pi\pi$	$\mathcal{B}(10^{-6})$	$0.17^{+0.04+0.02+0.01}_{-0.04-0.02-0.02}$	–
	$\mathcal{A}_{CP}(\%)$	$-51.0^{+1.1+11.7+26.6}_{-0.6-10.6-13.4}$	–

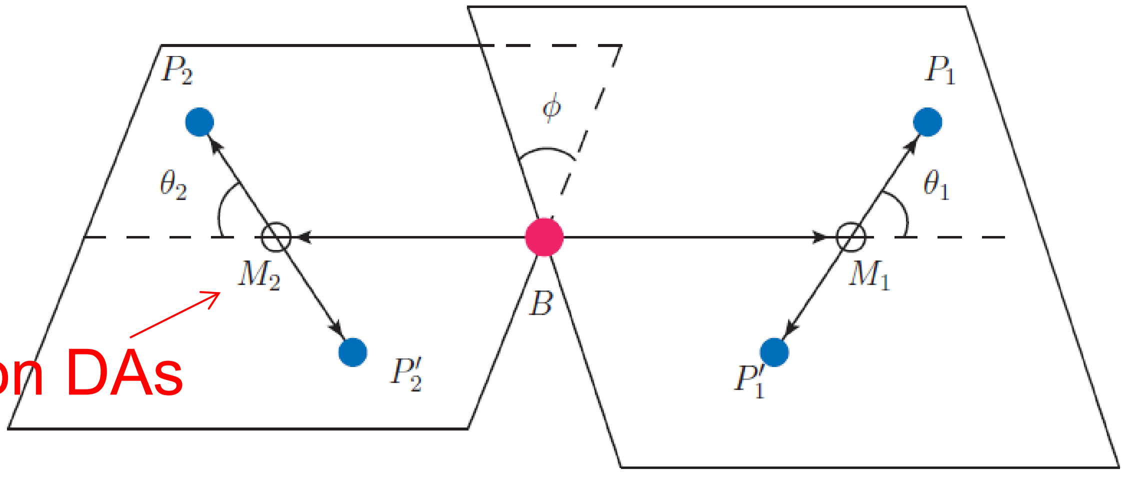
included in fit (†)

from our global fit

Four-body decays

Rui, Y. Li, Li, 2021

- Analyze angular distribution in four-body decays



- In addition to CPV, can predict triple product asymmetries (also need to know strong phase)

$$A_T^1 = \frac{\Gamma((2\zeta_1 - 1)(2\zeta_2 - 1)\sin\phi > 0) - \Gamma((2\zeta_1 - 1)(2\zeta_2 - 1)\sin\phi < 0)}{\Gamma((2\zeta_1 - 1)(2\zeta_2 - 1)\sin\phi > 0) + \Gamma((2\zeta_1 - 1)(2\zeta_2 - 1)\sin\phi < 0)}$$

Asymmetries	$B_s^0 \to (K^+\pi^-)(K^-\pi^+)$	$B_s^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^0 \to (K^-\pi^+)(K^+\pi^-)$	$B^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^+ \to (K^0\pi^+)(K^+\pi^-)$
A_T^1	$11.8^{+0.8}_{-1.1}$	$9.7^{+0.5}_{-0.6}$	$10.6^{+1.3}_{-1.7}$	~ 0	$8.5^{+0.9}_{-0.3}$

Heavy baryon decays

Han et al, 2022

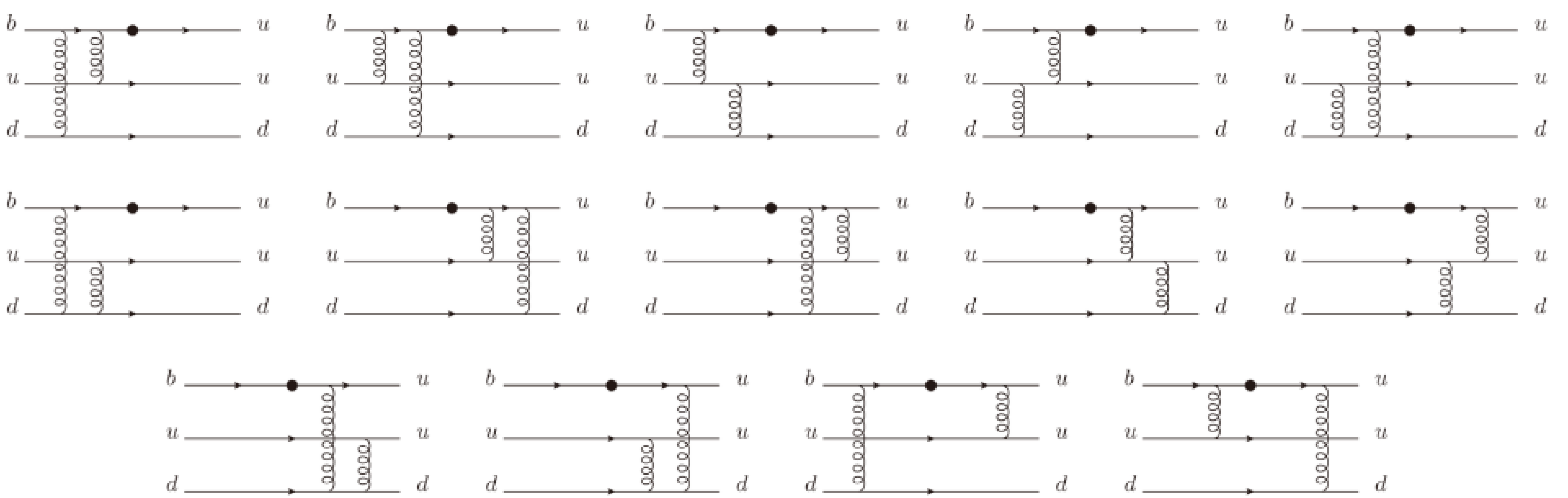
- Revisited transition form factors

$$\langle P(p', s')|\bar{u}\gamma_\mu(1-\gamma_5)b|\Lambda_b(p,s)\rangle = \overline{N}(p',s')(f_1\gamma_\mu - if_2\sigma_{\mu\nu}q^\nu + f_3q_\mu)\Lambda_b(p,s)$$
$$- \overline{N}(p',s')(g_1\gamma_\mu - ig_2\sigma_{\mu\nu}q^\nu + g_3q_\mu)\gamma_5\Lambda_b(p,s)$$

LCDAs up to twist 6

LCDAs up to twist 4

leading twist Das give small contribution



Numerical results

f1(0)	twist-3	twist-4	twist-5	twist-6	total
exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3^{+-}	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3^{-+}	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	0.25	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

	$f_1(0)$
NRQM [76]	0.043
heavy-LCSR [50]	$0.023^{+0.006}_{-0.005}$
light-LCSR-$\mathcal{A}$ [77]	$0.14^{+0.03}_{-0.03}$
light-LCSR-$\mathcal{P}$ [77]	$0.12^{+0.03}_{-0.04}$
QCD-light-LCSR [78]	0.018
HQET-light-LCSR [78]	-0.002
3-point QSR [49]	0.22
lattice [47]	0.22 ± 0.08
PQCD [32]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$
this work (exponential)	0.27 ± 0.12
this work (free parton)	0.24 ± 0.10

← previous leading-twist result

models for Λ_b DAs

consistent with other approaches, data indication; ready to study various exclusive heavy baryon decays and CPV systematically in PQCD

Summary

- Endpoint singularity in collinear factorization for B decays demands kT factorization
- No fixed power counting for kT
- Both TMD and H contain multiple scales; require kT and threshold resummatons
- Subleading contributions under control
- Prominent successes in phenomenology
- Global analyses of B decay data allowed
- Extended to multi-body decays with TDA