Factorization for Weak Annihilation B-meson Decays

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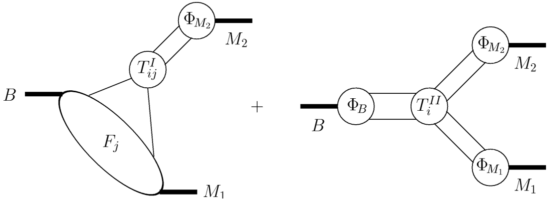
Status and Prospects of Non-Leptonic B-Meson Decays
Siegen University, June 1, 2022

Why hadronic B-meson decays?

- Important for the precision test of the CKM mechanism (CP violation).

$$\mathscr{A}(\bar{B} \to M_1 M_2) = \sum_i [\lambda_{\rm CKM} \times C \times \langle M_1 M_2 | \mathscr{O} | \bar{B} \rangle]_i \, .$$

- QCD factorization for exclusive heavy-hadron decays originally formulated in $\bar{B} \to M_1 M_2$ [BBNS].

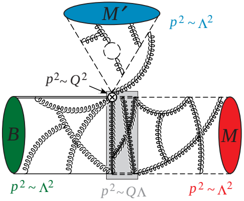


- Dedicated bottom-physics experiments $\to$ CP violation in B-meson decays (Nobel Prize 2008).
  - BaBar, Belle, Belle II, LHCb: A large number of exclusive decay channels.

- Explore the strong interaction dynamics governing heavy-hadron decay processes.
  - Perturbative factorization techniques (diagrammatic factorization, effective field theories).
  - Non-perturbative methods (lattice, QCDSR, LCSR, etc).
  - Soft and collinear fluctuations encoded in hadronic distribution amplitudes etc.

QCD factorization for hadronic B-meson decays

- Distinct energy scales appear in hadronic matrix elements.



- Three-scale problem at LP (?): m_b, $\sqrt{m_b \Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$.
- Separating physics at different energy scales.
- First-principles calculation @ leading power.

- General strategy of evaluating the hadronic matrix element $\langle M_1 M_2 | \mathcal{O} | \bar{B} \rangle$.
  - Λ / m_b and Λ / E expansion $\longrightarrow$ SCET + HQET.
  - Pioneer papers for hadronic B-meson decays in SCET [Chay, Kim, 2003, 2004; Bauer, Pirjol, Rothstein, Stewart, 2004; Beneke, Jäger, 2006, 2007; Beneke, Huber, Li, 2010].
  - Aim: express the desired matrix element in terms of simper dynamical quantities. Would it be possible to parameterize the IR physics only with the hadronic LCDAs?
  - Wide applications in the (almost) entire spectrum of heavy quark physics. $B \to \gamma \ell \nu$, $B \to \gamma\gamma$, $B \to \gamma \ell \ell$, $B \to M \ell \nu$, $B \to V \gamma$, $B \to V \ell \ell$.

QCD factorization for hadronic B-meson decays

- QCD $\to$ SCET$_\mathrm{I}$ matching (for a given tree operator Q) [Beneke, Jäger, 2006]:

$$\underbrace{(\bar{u}b)(\bar{d}u)}_{\text{only flavour structure}} \quad \to \quad [\bar{\chi}(tn_-)\,\chi(0)] \star \left(T^{\mathrm{I}} \star \underbrace{[\bar{\xi}(sn_+)\,h_v(0)]}_{\mathrm{A-type\ SCET_I\ form\ factor}} + H^{\mathrm{II}} \star [\bar{\xi}(s_1 n_+)\,\not{A}_\perp(s_2 n_+)\,h_v(0)] \right).$$

  - Factorization of the M_2 system from the $B \to M_1$ transition already at this step.
  - Strong phases of the $B \to M_1 M_2$ decay amplitudes arise from the hard coefficients only.
  - Factorization holds in the LP approximation.
  - Need the light-cone distribution amplitude of M_2 and the two SCET$_\mathrm{I}$ form factors.

$$\langle P(p')|(\bar{\chi} W_c) h_v|\bar{B}(v)\rangle = 2E\,\xi_P(E)\,.$$

    For the hadronic LCDAs and the A-type form factors $\to$ non-perturbative QCD methods.

  - Can redefine the first term in the bracket such that its matrix element corresponds to the relevant QCD form factor.

QCD factorization for hadronic B-meson decays

- $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$ matching for the B-type operator:

$$\langle M_1 | (\bar{\xi} W_c) \frac{\not{n}_+}{2} \left[W_c^\dagger i \not{D}_\perp W_c \right] (s n_+) (1+\gamma_5) h_v | \bar{B}(v) \rangle = -m_b m_B \int_0^1 d\tau \, e^{i m_B \tau s} \, \Xi_{M_1}(\tau) \, .$$

  - The same non-local form factor appears in the case of heavy-to-light form factor.
  - The explicit factorization formula [Beneke, Yang, 2006; Hill, Becher, Lee, Neubert, 2004]:

$$\Xi_{M_1}(\tau) = \frac{m_B}{4 m_b} \tilde{f}_B f_{M_1} \int_0^\infty d\omega \int_0^1 dv \, J_\parallel(\tau; v, \omega) \, \phi_B^+(\omega) \phi_{M_1}(v) \, .$$

    Can also construct the LCSR for $\Xi_{M_1}(\tau)$ [De Fazio, Feldmann, Hurth, 2006, 2008; Gao, Lü, Shen, YMW, Wei, 2020].

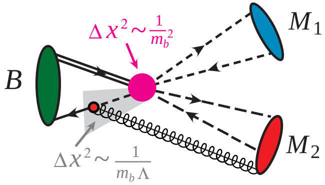
- The final factorized expression for the hadronic matrix element:

$$\langle M_1 M_2 | Q | \bar{B} \rangle = F_{B M_1}(0) \int_0^1 du \, T^{\mathrm{I}}(u) \, \phi_{M_2}(u) + \int_0^\infty d\omega \int_0^1 du \int_0^1 dv \, \underbrace{T^{\mathrm{II}}(\omega, u, v)}_{H^{\mathrm{II}} * J_\parallel} \, \phi_{M_1}(v) \, \phi_{M_2}(u) \, .$$

  - Consistent with the original BBNS factorization formula.
  - Rigorous at leading power in $1/m_b$ (for a review, see [Beneke, 2015]).

QCD factorization for hadronic B-meson decays

- Different sources of the subleading power contributions:
  - Chirally-enhanced twist-3 corrections (end-point divergences).
  - The scalar QCD penguin amplitude $r_\chi a_6$.
  - Higher Fock-state contribution [Arnesen, Rothstein, Stewart, 2007]:

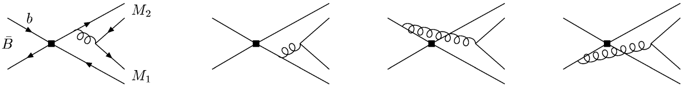


- ★ Actually unsuppressed in heavy quark expansion w.r.t. the two-particle contribution.
- ★ Calculable and free of the end-point divergence.
- ★ Numerically small w.r.t. the penguin annihilation contribution β_3^c.

  - Weak annihilation effect (end-point divergences):
    - ★ Phenomenologically relevant for realistic B-meson decays.
    - ★ Both the leading-twist and the twist-three contributions suffer from end-point divergences [BBNS, 2001].
    - ★ The negative helicity amplitude of $B \to VV$ even develops linear infrared divergence [Beneke, Yang, 2007].
    - ★ Making the theory prediction for CP asymmetries uncertain in QCD factorization.

Weak Annihilation Effect

- The LO Feynman diagrams [BBNS, 2001]:



$$A_1^i = \pi\alpha_s \int_0^1 dx \int_0^1 dy \left\{ \phi_{M_2}(x)\phi_{M_1}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2} \phi_{m_2}(x)\phi_{m_1}(y) \frac{2}{\bar{x}y} \right\},$$

$$A_3^f = \pi\alpha_s \int_0^1 dx \int_0^1 dy \left\{ r_\chi^{M_1} \phi_{M_2}(x)\phi_{m_1}(y) \frac{2(1+\bar{x})}{\bar{x}^2 y} + r_\chi^{M_2} \phi_{M_1}(y)\phi_{m_2}(x) \frac{2(1+y)}{\bar{x}y^2} \right\}.$$

The BBNS parametrization:

$$\int_0^1 dx \frac{\phi_M(x,\mu)}{\bar{x}^2} = \left(\lim_{u\to 1} \frac{\phi_M(u,\mu)}{\bar{u}} \right) \underbrace{\int_0^1 \frac{dx}{\bar{x}}}_{X_A^M = (1+\rho_A e^{i\varphi_A}) \ln \frac{m_B}{\Lambda_h}} + \underbrace{\int_0^1 \frac{dx}{\bar{x}} \left[\frac{\phi_M(x,\mu)}{\bar{x}} - \left(\lim_{u\to 1} \frac{\phi_M(u,\mu)}{\bar{u}} \right) \right]}_{\text{finite}}.$$

- Zero-bin subtraction [Arnesen, Ligeti, Rothstein, Stewart, 2008]:

$$\int_0^1 dx \frac{\phi_M(x,\mu)}{\bar{x}^2} = \int_0^1 dx \frac{\phi_M(x,\mu) + \bar{x}\phi_M'(1,\mu)}{\bar{x}^2} - \phi_M'(1,\mu) \ln\left(\frac{m_B}{\mu_-}\right).$$

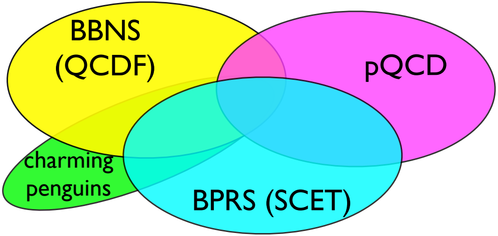
  - The factorization scale μ_- taken at the m_b scale. Cancellation of the μ_--scale dependence?
  - SCET definition of $\phi_M'(1,\mu)$? Weak annihilation is real? [Beneke, 2007]

Weak Annihilation Effect

- TMD factorization [Keum, Li, Sanda, 2001; Lü, Ukai, Yang, 2001]:

$$\frac{1}{x m_B^2 - k_T^2 + i\varepsilon} = \mathrm{P}\frac{1}{x m_B^2 - k_T^2} - i\pi\delta(x m_B^2 - k_T^2)\,.$$

  - Including the partonic transverse momentum generates the non-vanishing strong phase.
  - In the QCD factorization approach the TMD effect is part of the higher-twist contribution.
  - Earlier discussions on the distinct theory approaches in 2008 [From the talk by Stewart]:



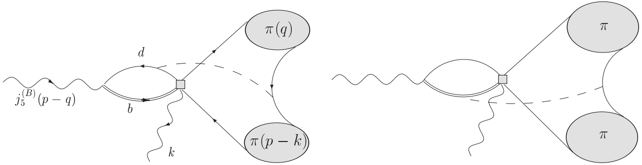
Keum, Li, Sanda (pQCD);
Lu et al.;
Beneke,Buchalla,Neubert, Sachrajda,(BBNS);
Chay, Kim;
Bauer, Pirjol, Rothstein, I.S. (BPRS)
Ciuchini et al (charming penguin),

See also the interesting talks by Beneke, Ciuchini, Li, Lü in the same workshop @ CERN.

  - Some insight from non-relativistic system [Bell, Feldmann, 2007] and [Beneke, Vernazza, 2008].

Weak Annihilation Effect

- The LCSR method [Khodjamirian, Mannel, Melcher, Melić, 2005]:



Non-trivial computation even at tree level.

- Introduce the unphysical four-momentum k flowing from the weak vertex to avoid a continuum of light "parasitic" contributions [Khodjamirian, 2001].
- Annihilation contributions are small and complex.
- There is no end-point divergence in the LCSR approach. Why is this so? *The power counting in QCDF implies that the momentum of the light quark in the B meson has to vanish. This is different in LCSR, since the B meson is effectively replaced by the spectral density of the heavy-light quark loop integrated over the duality interval, and thus the momentum of the light quark is non-vanishing.*
- The "modified" convolution integral for the annihilation effect:

$$A_1^i = \pi\alpha_s \int_0^\infty d\omega\, \phi_B^+(\omega,\mu) \int_0^1 du\, \phi_\pi(u) \int_0^1 dv\, \phi_\pi(v) \times \left\{ \frac{1}{\bar{u}v\,(\bar{u}-\omega/m_B)} + \frac{\bar{u}+\omega/m_B}{\bar{u}v\,[1-(u-\omega/m_B)(\bar{v}-\omega/m_B)]} \right\}.$$

Weak Annihilation Effect

- Open issues:
  - What are the implications of the comparison study of the annihilation effect?
    - ★ Different power counting schemes used in the LCSR and QCDF approaches?
    - ★ Missing ingredients in the framework of QCD factorization?
  - Do we really need to drop out the light-quark momentum of the B-meson in QCD factorization?
  - How to tackle the end-point divergence of the annihilation effect in QCD factorization?
  - Can we improve the treatment of the BBNS parametrization for the annihilation effect?
    - ★ Can the two non-perturbative parameters X_A^i and X_A^f be different? [Zhu, 2011]
    - ★ Flavour dependence of the annihilation parameters?
    - ★ Can the values of ρ_A in the quantity X_A be larger than one?
    - ★ Many more questions here ...
- The true story of revisiting the weak annihilation $B \to M_1 M_2$ decays (for us) is however different.

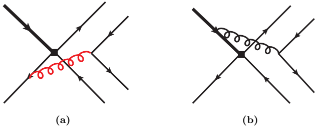
Pure annihilation $B \to M_1 M_2$ decays

- A clean environment to understand factorization properties of the weak annihilation effect.
  - No need to worry about other topological amplitudes.
  - Easier to identify the perturbative enhancement mechanism at NLO.
  - Suitable for understanding sources of the strong phases.

- Generic structure of the weak annihilation decay amplitude:

$$\bar{\mathscr{A}}(\bar{B}_q \to M_1 M_2) = -\langle M_1(p_1) M_2(p_2) | \mathscr{H}_{\rm eff} | \bar{B}_q(p_B) \rangle$$
$$= \mp i \frac{4 G_F}{\sqrt{2}} f_B f_{M_1} f_{M_2} \sum_{p=u,c} V_{pb} V_{pq}^* \, (\pi \alpha_s) \, \frac{C_F}{N_c^2} \times \left[\mathscr{T}^{p,(0)} + \left(\frac{\alpha_s}{4\pi} \right) \mathscr{T}^{p,(1)} + \mathscr{O}(\alpha_s^2) \right].$$

  - Adopt the CMM basis for the effective weak Hamiltonian.
  - The two relevant Feynman diagrams at LO.



The gluon emission from the final-state partons leads to the vanishing contribution in the limit of symmetric LCDAs and assuming SU(3) flavour symmetry [BBNS, 2001].

Weak annihilation amplitude at LO

- Parametrization for the transition amplitude:

$$\mathscr{T}^{p,(0)} = \delta_{pu}\, \mathscr{C}^{(1)}_{M_1 M_2}\, \mathscr{B}_1(M_1 M_2) + \mathscr{C}^{p,(4)}_{M_1 M_2}\, \mathscr{B}_4(M_1 M_2) + \mathscr{C}^{p,(4,\mathrm{EW})}_{M_1 M_2}\, \mathscr{B}_{4,\mathrm{EW}}(M_1 M_2)\,.$$

  - The prefactors $\mathscr{C}^{(p),(i,(\mathrm{EW}))}_{M_1 M_2}$ collect the CG coefficients from the flavour structures of the B-meson and $M_{1,2}$ as well as the electric-charge coefficients from P_{iQ}.
  - The gluon emission from the initial-state bottom quark (i.e., the diagram (b)) is calculable in QCD factorization [BBNS, 2001].
  - The yielding amplitude of the diagram (a) with an insertion of P_2^u:

$$\langle P_2^u \rangle^{(0)}_{(a)} = (\pi\, \alpha_s)\, \frac{C_F}{N_c^2} \int_0^\infty d\omega \int_0^1 dx \int_0^1 dy\, \mathrm{Tr}\Big\{ \mathscr{M}^B(v,\omega) \left[\gamma_{\perp\nu} \left(\not p_1 + \not q_2 - \not k\right) \gamma_\mu\, (1-\gamma_5)\right]$$
$$\mathscr{M}^{M_1}(p,y)\, [\gamma_\perp^\nu]\, \mathscr{M}^{M_2}(q,x)\, [\gamma^\mu\, (1-\gamma_5)] \Big\} \frac{1}{[(p_1+q_2-k)^2 + i\varepsilon][(p_1+q_2)^2 + i\varepsilon]}\,.$$

    - ★ The momentum-space projectors $\mathscr{M}^B$, $\mathscr{M}^{M_1}$ and $\mathscr{M}^{M_2}$ are well-known [Beneke, Feldmann, 2001; Beneke, 2002].
    - ★ The simplified quark and gluon propagators:

$$\frac{1}{y\, m_B^2\, [\bar{x} - \omega/m_B + i\varepsilon]}\, \frac{1}{y\bar{x}\, m_B^2 + i\varepsilon} \stackrel{?}{=} \frac{1}{[y\bar{x}\, m_B^2 + i\varepsilon]^2} + (\text{NLP effect})\,.$$

Weak annihilation amplitude at LO

- Identify the leading contributions of the convolution integral:

$$\mathcal{G}_{\mathcal{B}_1} \equiv \int_0^\infty d\omega\, \phi_B^+(\omega) \int_0^1 dx\, \phi_{M_2}(x) \int_0^1 dy\, \phi_{M_1}(y)\, \frac{1}{\bar{x} y\, (\bar{x} - \omega/m_B + i\varepsilon)}\,.$$

  - For the generic $\bar{x} \sim \mathcal{O}(1)$, $y \sim \mathcal{O}(1)$ and $\omega \sim \mathcal{O}(\Lambda)$,

$$\phi_B^+(\omega) \sim \mathcal{O}(1/\Lambda)\,,\ \phi_{M_1}(y) \sim \phi_{M_2}(x) \sim \mathcal{O}(1)\,,\ \frac{1}{\bar{x} y\, (\bar{x} - \omega/m_B + i\varepsilon)} \sim \mathcal{O}(1)\,,$$
$$\int d\omega \sim \Lambda\,, \qquad \int dx \sim \mathcal{O}(1)\,, \qquad \int dy \sim \mathcal{O}(1)\,.$$

  - For the non-generic $\bar{x} \sim \mathcal{O}(\Lambda/m_b)$ but $y \sim \mathcal{O}(1)$, $\omega \sim \mathcal{O}(\Lambda)$

$$\phi_B^+(\omega) \sim \mathcal{O}(1/\Lambda)\,,\ \phi_{M_1}(y) \sim \mathcal{O}(1)\,,\ \phi_{M_2}(x) \sim \mathcal{O}(\Lambda/m_b)\,,\ \frac{1}{\bar{x} y\, (\bar{x} - \omega/m_B + i\varepsilon)} \sim \mathcal{O}(m_b^2/\Lambda^2)\,,$$
$$\int d\omega \sim \Lambda\,, \qquad \int dx \sim \mathcal{O}(\Lambda/m_b)\,, \qquad \int dy \sim \mathcal{O}(1)\,.$$

  - Apparently both $\bar{x} \sim \mathcal{O}(1)$ and $\bar{x} \sim \mathcal{O}(\Lambda/m_b)$ can give rise to the leading-power contributions. Both the hard and hard-collinear gluon exchanges will be relevant at LP.
  - The hard-collinear contribution suffers from the phase-space suppression but receives the dynamical enhancement from the quark/gluon propagators.

Weak annihilation amplitude at LO

- Reproduce the BBNS result at twist-two when setting $\omega \to 0$ (i.e., keeping only $\bar{x} \sim \mathcal{O}(1)$):

$$\int_0^\infty d\omega\, \phi_B^+(\omega) \int_0^1 dx\, \phi_{M_2}(x) \int_0^1 dy\, \phi_{M_1}(y)\, \frac{1}{\bar{x}^2\, y}\,.$$

  The very appearance of the end-point divergence already indicates the missing LP contribution from the hard-collinear gluon exchange.

- The yielding factorization formula holds for both the hard and hard-collinear gluon exchanges.

$$\mathcal{G}_{\mathcal{B}_1} \equiv \int_0^\infty d\omega\, \phi_B^+(\omega) \int_0^1 dx\, \phi_{M_2}(x) \int_0^1 dy\, \phi_{M_1}(y)\, \frac{1}{\bar{x}\, y\, (\bar{x} - \omega/m_B + i\epsilon)}\,.$$

  - This observation is in analogy to the smooth interpolation of the A-type contribution to $\bar{B}_q \to \gamma \ell \bar{\ell}$ between the hard and hard-collinear q^2 [Beneke, Bobeth, Y.M.W., 2020].
  - Employing the Grozin-Neubert model for ϕ_B^+ and the asymptotic forms of $\phi_{M_{1,2}}$ leads to

$$\mathcal{G}_{\mathcal{B}_1} \approx 18 \left[\left(\underbrace{\ln(m_B/\lambda_B)}_{\text{from the hard}-\text{collinear gluon exchange}} + \gamma_E - 2 \right) - i\pi \right].$$

  - Do not aim at performing the QCD resummation for the perturbatively generated logarithms.
  - Phenomenologically the end-point divergence is regularized by the soft-quark momentum [Keum, Li, 2001; Khodjamirian, Mannel, Melcher, Melić, 2005].

Weak annihilation amplitude at LO

- Comparison with the BBNS result (Grozin-Neubert model of ϕ_B^+):

$$X_A^i \approx \left[1 + \frac{(\gamma_E - 1) - i\pi}{\ln \frac{m_B}{\lambda_B}}\right] \ln \frac{m_B}{\lambda_B} \Longleftrightarrow X^i_{A,\,\mathrm{BBNS}} = (1 + \rho_A\, e^{i\varphi_A}) \ln \frac{m_B}{\Lambda_h} \,.$$

  - Setting $\Lambda_h = \lambda_B$ and $\lambda_B = \{200,\, 350,\, 500\}\,\mathrm{MeV}$ leads to

$$(\rho_A, \varphi_A) = \{(0.97, -97^\circ),\, (1.17, -97^\circ),\, (1.34, -97^\circ)\}\,.$$

  - Support the BBNS ansatz of $X^i_{A,\mathrm{BBNS}}$ with $\rho_A \sim \mathscr{O}(1)$ and the non-trivial strong phase.
  - Interpretation of the strong phase from [BBNS, 2001]:

    In practice, the singularity will be smoothed out by soft physics related to the intrinsic transverse momentum and off-shellness of the partons, which unfortunately does not admit a perturbative treatment. In particular, the resulting contribution may be complex due to soft rescattering in higher orders.

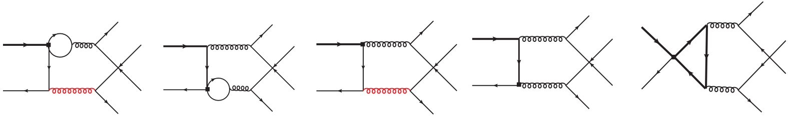
- Factorized expressions for the tree-level amplitudes:

$$\mathscr{B}_1 = \frac{1}{4}\left(C_2 - \frac{C_1}{2N_c}\right)\widehat{\mathscr{G}}_{\mathscr{B}_1}\,, \qquad \mathscr{B}_4 = \frac{1}{8}\left[(C_4 + 16\,C_6)\,\widehat{\mathscr{G}}_{\mathscr{B}_1} + (C_4 + 4\,C_6)\,\widehat{\mathscr{G}}_{\mathscr{B}_4}\right],$$

$$\mathscr{B}_{4,\mathrm{EW}} = \mathscr{B}_4\,(C_4 \to C_{4Q},\, C_6 \to C_{6Q})\,, \qquad \widehat{\mathscr{G}}_{\mathscr{B}_1} = \mathscr{G}_{\mathscr{B}_1} + \int_0^1 dx \int_0^1 dy\, \frac{\phi_{M_2}(x)\,\phi_{M_1}(y)}{y\,(1 - x\bar{y})}\,.$$

Weak annihilation amplitude at NLO

- Interesting NLO Feynman diagrams (plus symmetric diagrams by exchanging the two gluons):



(c) (d) (e) (f) (g)

- General analysis of the NLO amplitudes.
  - Matrix element of the B-meson decaying into two transversely polarized gluons:

$$\langle g^*(p_g,\alpha)\, g^*(\tilde{p}_g,\beta)|\mathscr{H}_{\rm eff}|\bar{B}_q\rangle = i\,\varepsilon^{\perp}_{\alpha\beta}\, F_V(p_g^2,\tilde{p}_g^2) + g^{\perp}_{\alpha\beta}\, F_A(p_g^2,\tilde{p}_g^2)\,.$$

    Symmetry relations of the two transition form factors:

$$F_V(p_g^2,\tilde{p}_g^2) = -F_V(\tilde{p}_g^2,p_g^2)\,, \qquad F_A(p_g^2,\tilde{p}_g^2) = F_A(\tilde{p}_g^2,p_g^2).$$

  - The matrix element for the two-hadron production:

$$\langle M_1(p)\, M_2(q)|g^*(p_g,\alpha)\, g^*(\tilde{p}_g,\beta)\rangle = \begin{cases} g^{\perp}_{\alpha\beta}\, \mathscr{S}_{\parallel}(M_1M_2) & \text{for } M_1M_2 = PP,\, V_LV_L\,, \\ i\,\varepsilon_{\alpha\beta pq}\, \mathscr{S}_{\perp}(M_1M_2) & \text{for } M_1M_2 = PV,\, VP\,. \end{cases}$$

    The transversity amplitudes $\mathscr{S}_{\parallel,\perp}$ proportional to the products $\phi_{M_2}(x)\,\phi_{M_1}(y)$.
    $\Longrightarrow$ Symmetric $\mathscr{S}_{\parallel,\perp}$ under the exchanges of $x \to \bar{x}$ and $y \to \bar{y}$ in the asymptotic limit.
  - The displayed NLO diagrams will NOT contribute to $B \to PV, VP$ decays.
  - Only the axial-vector form factor F_A will be relevant due to the symmetry properties.

Weak annihilation amplitude at NLO

- The triangle diagrams insensitive to the HQET B-meson distribution amplitudes.

$$\sum_{i=1}^{6} C_i \, \mathscr{I}_i^{p,(1)} = \left[(C_3 + 4\, C_5) + C_F \, (C_4 + 4\, C_6) \right] H_4 \, .$$

  - The resulting structure coincides with the one for the axil-vector form factor of $\bar{B}_q \to \gamma\gamma$ [Shen, YMW, Wei, 2020].
  - The current-current operators unfortunately generate the vanishing contribution.
    - ★ UV divergence does NOT appear in the QCD amplitude of the triangle diagram.
    - ★ The triangle diagram with an insertion of the vector current (the closed Fermion loop) will not contribute due to the Furry theorem.
    - ★ The triangle diagram with an insertion of the axial-vector current (the closed Fermion loop) will not contribute to F_A.
    - ★ Only the triangle diagram with an insertion of the penguin operator (the single Fermion line) will contribute to F_A.
  - The above mechanism dictating the disappearance of the $C_{1,2}$ term will NOT work, if the twist-2 mesonic DAs are no longer symmetric under the exchanges of $x \to \bar{x}$ and $y \to \bar{y}$ (e.g., the axial-vector mesons).

Weak annihilation amplitude at NLO

- The factorizable quark-loop effects (penguin contractions):

$$\sum_{i=1}^{6} C_i \, \mathscr{P}_i^{p,(1)} = \left(C_2 - \frac{C_1}{2N_c} \right) H_1(m_p) + \left[(C_3 + 16\, C_5) - \frac{1}{2N_c} (C_4 + 16\, C_6) \right] [H_1(m_b) + H_1(0)]$$
$$+ (C_4 + 10\, C_6)\, [H_1(m_b) + H_1(m_c) + 3\, H_1(0)] - \left[5\, C_4 - 8\, C_5 + 4 \left(\frac{1}{N_c} + 5 \right) C_6 \right] H_2 \, .$$

  - In agreement with the pattern of penguin contractions obtained in [BBNS, 2001].
  - The primitive kernels $H_{1,2}$ are again calculable in the QCD factorization framework.

$$H_{1,2} = \frac{1}{12} \int_0^\infty d\omega \, \phi_B^+(\omega) \int_0^1 dx \, \phi_{M_2}(x) \int_0^1 dy \, \phi_{M_1}(y)$$
$$\left[\left(\frac{h_{1,2}}{\bar{x} y} \left(\frac{1}{\bar{x} - \omega/m_B + i\varepsilon} + \frac{\bar{x}}{1 - x\bar{y}} \right) + \{x \leftrightarrow \bar{y}\} \right) + \{x \leftrightarrow y\} \right] .$$

  - The non-vanishing contribution from the current-current operators leads to the enhanced mechanism due to the large Wilson coefficients and/or the multiplication CKM factors.
  - The NLO contribution from the gluonic penguin operator can be expressed in a similar way.

Phenomenological implications

- The twist-two B-meson distribution amplitude in HQET [Beneke, Braun, Ji, Wei, 2018]:

$$\phi_B^+(\omega,\mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)}\, U\left(\beta-\alpha, 3-\alpha, \frac{\omega}{\omega_0}\right) \frac{\omega}{\omega_0^2} \exp\left(-\frac{\omega}{\omega_0}\right).$$

  - Such three-parameter ansatz is advantageous, since the resulting RG evolution can be done analytically in terms of ${}_2F_2$ functions.
  - An alternative parametrization of the twist-two B-meson DA in Laplace space [Galda, Neubert, Wang, 2022]:

$$\tilde{\phi}_B^+(\eta,\mu) = \int_0^\infty \frac{d\omega}{\omega} \left(\frac{\omega}{\bar{\omega}}\right)^\eta \phi_B^+(\omega,\mu) \stackrel{|\eta|\ll 1}{=} \frac{1}{\lambda_B(\mu)} \left[1 + \sum_{n\geq 1} \frac{\eta^n}{n!}\, \sigma_n^B(\mu)\right].$$

    Suitable for investigating $B \to \gamma \ell \nu$ in QCD factorization.
  - New parametrization of the momentum-space DA in terms of associated Laguerre polynomials [Feldmann, Lüghausen, van Dyk, 2022]:

$$\phi_B^+(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0} \sum_{k=0}^{K} \frac{a_k(\mu_0)}{1+k}\, L_k^{(1)}\left(\frac{2\omega}{\omega_0}\right).$$

  - The heavy-meson distribution amplitudes including QED interactions (soft functions) have been also investigated in [Beneke, Böer, Toelstede, Vos, 2022].

Phenomenological implications

- Numerical feature of the NLO QCD correction for $\bar{B}_s \to \pi^+ \pi^-$:

	$\nu = m_b/2$	$\nu = m_b$	$\nu = 2m_b$
$\mathscr{T}^{u,(0)}$	$2.03 - 17.7\,i$	$2.00 - 17.4\,i$	$1.97 - 17.2\,i$
$\mathscr{T}^{c,(0)}$	$-0.38 + 3.35\,i$	$-0.25 + 2.16\,i$	$-0.16 + 1.42\,i$
$\alpha_s/(4\pi)\ \mathscr{T}^{u,(1)}$	$-2.29 - 3.63\,i$	$-2.41 - 3.61\,i$	$-2.47 - 3.57\,i$
$\alpha_s/(4\pi)\ \mathscr{T}^{c,(1)}$	$-1.65 - 2.34\,i$	$-1.82 - 2.40\,i$	$-1.91 - 2.43\,i$

  - The NLO contribution results in the significant increase of the real part of $\mathscr{T}^{c,(0)}$ in magnitude and generates the considerable cancellation of its imaginary part.
  - The dominating perturbative correction arises from the charm-loop diagrams [(c), (d)].

- Theory predictions for CP violating observables:

	$\mathscr{A}_{\rm CP}^{\rm dir}$	$\mathscr{A}_{\rm CP}^{\rm mix}$
$\bar{B}_s \to \pi^+\pi^-,\ \pi^0\pi^0$	$-36.3^{+8.2}_{-1.3}$ (0.0 ± 0.0)	$-4.2^{+21.4}_{-9.0}$ $(35.9^{+15.6}_{-11.2})$
$\bar{B}_s \to \rho_L^+\rho_L^-,\ \rho_L^0\rho_L^0$	$-36.3^{+8.3}_{-1.8}$ (0.0 ± 0.0)	$-4.3^{+21.5}_{-9.0}$ $(35.9^{+15.6}_{-11.2})$
$\bar{B}_s \to \omega_L\,\omega_L$	$-36.3^{+8.3}_{-3.1}$ (0.0 ± 0.0)	$-3.8^{+21.8}_{-9.7}$ $(35.9^{+15.6}_{-11.2})$
$\bar{B}_s \to \rho_L\,\omega_L$	0.0 ± 0.0 (0.0 ± 0.0)	$-71.0^{+6.3}_{-5.4}$ $(-71.0^{+6.3}_{-5.4})$
$\bar{B}_d \to K^+K^-$	$39.0^{+3.2}_{-5.6}$ (0.0 ± 0.0)	$-2.2^{+19.1}_{-26.4}$ $(-47.0^{+15.7}_{-18.8})$
$\bar{B}_d \to K_L^{*+}K_L^{*-}$	$39.6^{+4.9}_{-6.7}$ (0.0 ± 0.0)	$-1.4^{+19.7}_{-26.9}$ $(-47.0^{+15.7}_{-18.8})$
$\bar{B}_d \to \phi_L\,\phi_L$	$38.3^{+11.4}_{-15.8}$ (0.0 ± 0.0)	$27.8^{+5.7}_{-25.9}$ (0.0 ± 0.0)

  - The NLO QCD correction provides an important source of the strong phases.

Conclusions

- The pure annihilation $\bar{B} \to M_1 M_2$ decay amplitudes are calculable in the factorization framework.
  - The unwanted end-point divergence can be eliminated when adding the missing hard-collinear gluon exchange on top of the hard gluon effect.
  - The new strategy can be straightforwardly applied to the analytical computation of the NLO diagrams under discussion.
  - The newly computed NLO correction yields the sizeable numerical impact on the strong phases of the weak annihilation amplitudes.

- Understanding QCD factorization for exclusive B-meson decays is rather challenging.
  - Systematic analysis for the weak annihilation amplitude in the SCET framework.
  - The off-shell gluon emission off the final-state quarks (relevant to $B \to \pi\pi, \pi K$).
  - Interesting progress on the end-point factorization in SCET.
    - ★ Novel end-point factorization relation for the NLP thrust distribution (a $\mathrm{SCET_I}$ problem) [Beneke, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 2022].
    - ★ The end-point factorization for the muon-electron scattering in the backward direction (resummation of the double-logarithmic corrections with the end-point refactorization condition) [Bell, Böer, Feldmann, 2022].
  - SCET factorization for heavy-to-light form factors.

- Very promising future for QCD aspects of heavy-quark physics!