B-anomalies in a twin Pati-Salam theory of flavour

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Based on [2209.00276] [hep-ph] and w.i.p in collaboration with Stephen F. King

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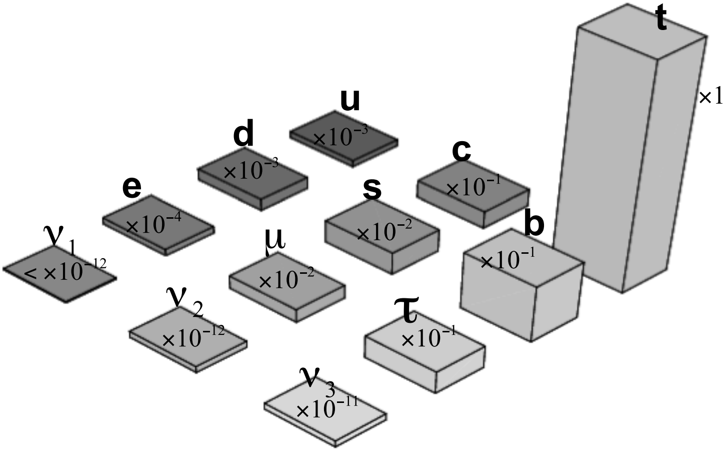
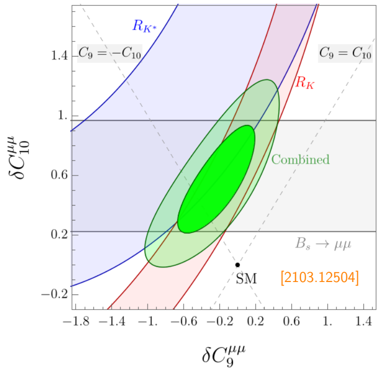
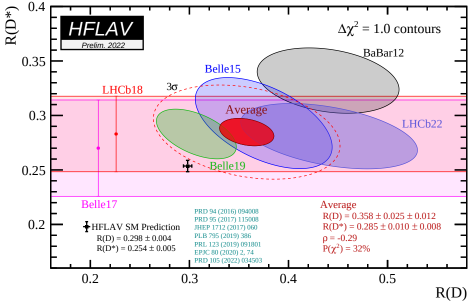


- Are we missing something?

- Hints of LFUV violation in B-mesons decays, both in neutral and charged currents.





- Left-handed NP preferred over SM at 4σ with clean observables only.
- Consistent picture of hierarchical anomalies involving 2nd-3rd family fermions.

U_1 vector leptoquark

- $U_1(3,1,2/3)$ provides a very good fit to all $b \to s\ell\ell$ and $b \to c\tau\nu$ data (see talk by Andreas Crivellin).
- U_1 is the only single leptoquark that can simultaneously address both LFUV anomalies. [Angelescu, Becirevic, Faroughy, Jaffredo, Sumensari, '21]

$$\mathcal{L}_{\mathrm{NP}} = \frac{1}{\Lambda^K_{\mathrm{NP}}} \left(\bar{s}_L \gamma_\mu b_L\right) \left(\bar{\mu}_L \gamma^\mu \mu_L\right) + \frac{1}{\Lambda^D_{\mathrm{NP}}} \left(\bar{c}_L \gamma_\mu b_L\right) \left(\bar{\tau}_L \gamma^\mu \nu_L\right)$$

- The different NP scales $\Lambda^K_{\mathrm{NP}} \approx (30\,\mathrm{TeV})^2$ and $\Lambda^D_{\mathrm{NP}} \approx (3\,\mathrm{TeV})^2$ point to a TeV scale U_1 with hierarchical couplings to second and third family fermions.
- Hierarchical pattern of anomalies and LQ couplings, connections with flavour hierarchies of the SM?

4321 and beyond

$\boxed{SU(4) \times SU(3)' \times SU(2)_L \times U(1)_{Y'}} \Rightarrow U_1,\, g',\, Z' +$ vector-like fermions

4321 gauge group proposed as the gauge origin of U_1, but two different approaches:

- The third family transforms under $SU(4)$ (the rest are singlets) $\Rightarrow$ natural low-energy limit of PS^3 multi-scale flavour model
  [Bordone, Cornella, Fuentes-Martin, Isidori, '17] (and more)
    - Predicts large U_1 couplings for LH and RH third family fermions $\Rightarrow$ is tightly constrained by high-p_T searches
      [Aebischer, Isidori, Pesut, Stefanek, Wilsch, '22]

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  - Predicts large U_1 couplings for LH and RH third family fermions
    $\Rightarrow$ is tightly constrained by high-p_T searches
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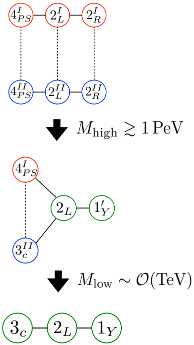
- Fermiophobic approach (all SM-like families are $SU(4)$ singlets)
  [Di Luzio, Greljo, Nardecchia, '17]
  $\Rightarrow$ predicts dominantly LH U_1 currents for B-anomalies, but is not a flavour model (ad-hoc flavour structure)
  - We will see that fermiophobic approaches can also be naturally connected with the SM flavour hierarchies

Twin Pati-Salam theory of flavour



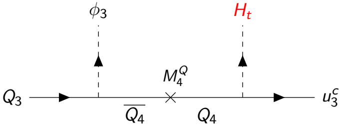
Field	$SU(4)^I_{PS}$	$SU(2)^I_L$	$SU(2)^I_R$	$SU(4)^{II}_{PS}$	$SU(2)^{II}_L$	$SU(2)^{II}_R$	Z_4	
$\psi_{1,2,3}$	**1**	**1**	**1**	**4**	**2**	**1**	$i, 1, 1$	
$\psi^c_{1,2,3}$	**1**	**1**	**1**	$\overline{\mathbf{4}}$	**1**	$\overline{\mathbf{2}}$	$i, -1, 1$	
$\psi_{4,5,6}$	**4**	**2**	**1**	**1**	**1**	**1**	$1, 1, i$	
$\overline{\psi}_{4,5,6}$	$\overline{\mathbf{4}}$	$\overline{\mathbf{2}}$	**1**	**1**	**1**	**1**	$1, 1, -i$	
$\psi^c_{4,5,6}$	$\overline{\mathbf{4}}$	**1**	$\overline{\mathbf{2}}$	**1**	**1**	**1**	$1, 1, i$	
$\overline{\psi^c}_{4,5,6}$	**4**	**1**	**2**	**1**	**1**	**1**	$1, 1, -i$	
ϕ	**4**	**2**	**1**	$\overline{\mathbf{4}}$	$\overline{\mathbf{2}}$	**1**	1	Break 4321 to SM and
$\overline{\phi}, \overline{\phi'}$	$\overline{\mathbf{4}}$	**1**	$\overline{\mathbf{2}}$	**4**	**1**	**2**	$1, -1$	mix VL-chiral fermions
H	$\overline{\mathbf{4}}$	$\overline{\mathbf{2}}$	**1**	**4**	**1**	**2**	1	Break EW symmetry
$\overline{H}$	**4**	**1**	**2**	$\overline{\mathbf{4}}$	$\overline{\mathbf{2}}$	**1**	1	
Ω_{15}	**15**	**1**	**1**	**1**	**1**	**1**	1	Splits VL quark-lepton masses

(plus extra scalars for high scale symmetry breaking)

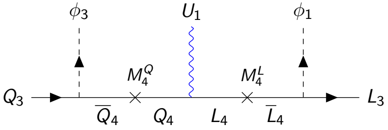
- High scale PS for chiral fermions: Quark-lepton unification, crucial universality constraints for a predictive framework
- Low energy PS for vector-like fermions: TeV scale U_1 coupled to vector-like fermions
- Full twin PS symmetry forbids SM-like Yukawa couplings (along with the choice of the scalar sector)
- Scalar sector generates VL-chiral fermion mixing $\Rightarrow$ 2nd-3rd family masses, mixings and B-anomalies
- Z_4 discrete shaping symmetry provides flavour structure and reduces the number of free parameters, protects from fine-tuning

Yukawa couplings & U_1 couplings

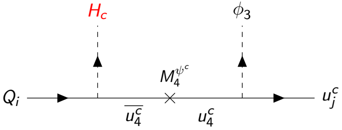
- SM-like Yukawa couplings for chiral fermions forbidden by twin PS symmetry.
- Only vector-like fermions coupled to U_1 (fermiophobic model).
- Effective Yukawa and U_1 couplings arise from mixing of VL and chiral fermions.



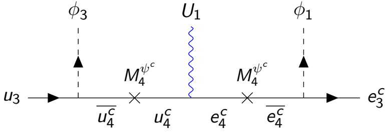
Third family masses, top mass requires $\frac{\langle \phi_3 \rangle}{M_4^Q} \approx 1$, the mixing is large



Eff. U_1 couplings to the third family EW doublet controlled by large mixing angles $s_{34}^{Q,L}$



$M_4^{\psi} \ll M_4^{\psi^c}$ required for second family masses and 2-3 CKM mixing, $\mu - \tau$ mixing also predicted



Eff. U_1 couplings to EW singlets are suppressed

- Personal Higgses $\langle H_t \rangle \approx v_{\rm SM}/\sqrt{2}$ and $\langle H_{b,c,s,\mu,\tau} \rangle \sim \mathcal{O}(\mathrm{GeV})$ break degeneracy of family masses (but only H_u and H_d are light and develop VEVs*).

GIM-like mechanism and FCNCs

- We need large flavour-violating U_1 couplings for the B-anomalies $\Rightarrow$ introduce $\Omega_{15}(\mathbf{15},1,1)_I$ to split VL quark-lepton masses

$$\left.\begin{array}{l}\begin{pmatrix} M_4^Q & 0 & 0 \\ 0 & M_5^Q & 0 \\ 0 & 0 & M_6^Q \end{pmatrix} = V_{45}^Q \begin{pmatrix} M_{44}^Q & M_{45}^\psi & 0 \\ M_{54}^\psi & M_{55}^Q & 0 \\ 0 & 0 & M_{66}^Q \end{pmatrix} V_{45}^{\bar{Q}\dagger} \,, \\ \begin{pmatrix} M_4^L & 0 & 0 \\ 0 & M_5^L & 0 \\ 0 & 0 & M_6^L \end{pmatrix} = V_{45}^L \begin{pmatrix} M_{44}^L & M_{45}^\psi & 0 \\ M_{54}^\psi & M_{55}^L & 0 \\ 0 & 0 & M_{66}^L \end{pmatrix} V_{45}^{\bar{L}\dagger} \,, \end{array}\right\} \Rightarrow \frac{g_4}{\sqrt{2}} Q_a^\dagger \gamma_\mu \begin{pmatrix} c_{\theta_{LQ}} & -s_{\theta_{LQ}} & 0 \\ s_{\theta_{LQ}} & c_{\theta_{LQ}} & 0 \\ 0 & 0 & 1 \end{pmatrix} L_b U_1^\mu + \text{h.c.}$$

  where $a, b = 4, 5, 6$ and $W_{LQ} = V_{45}^Q V_{45}^{L\dagger}$ is a CKM-like matrix in $SU(4)_I$ (VL quark-lepton) flavour space.
- Large flavour violating charged currents in VL flavour space, but no tree-level FCNCs. $s_{\theta_{LQ}} \approx 1/\sqrt{2}$ best benchmark for B-anomalies.

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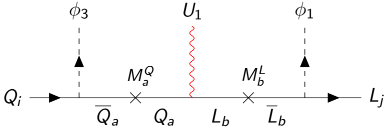
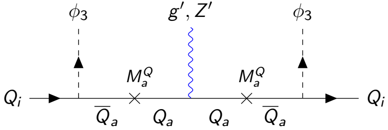
- Large flavour violating charged currents in VL flavour space, but no tree-level FCNCs. $s_{\theta_{LQ}} \approx 1/\sqrt{2}$ best benchmark for B-anomalies.

- Each VL family mixes with only one chiral family, i.e. we have only (3-4), (2-5) and (1-6) VL-chiral mixing, hence flavour-violating U_1 couplings and flavour diagonal g', Z' couplings for chiral fermions.



$i,j = 1,2,3$ and $a,b = 4,5,6$



$i = 1,2,3$, $a = 4,5,6$

GIM-like mechanism and FCNCs

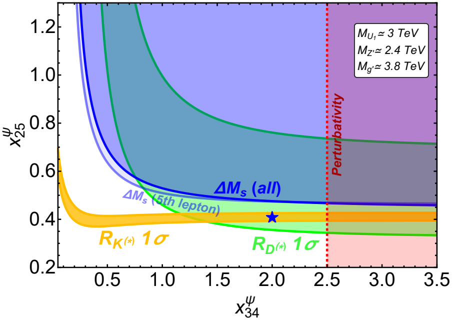
- In the large mixing angle formalism

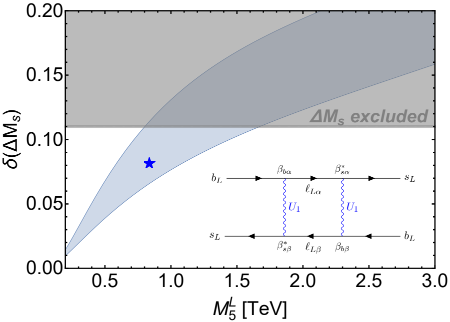
$$s_{34}^{Q,L}=\frac{x_{34}^{\psi}\left\langle\phi_{3,1}\right\rangle}{\sqrt{\left(x_{34}^{\psi}\left\langle\phi_{3,1}\right\rangle\right)^{2}+\left(M_{44}^{Q,L}\right)^{2}}}\,,\quad s_{25}^{Q,L}=\frac{x_{25}^{\psi}\left\langle\phi_{3,1}\right\rangle}{\sqrt{\left(x_{25}^{\psi}\left\langle\phi_{3,1}\right\rangle\right)^{2}+\left(M_{55}^{Q,L}\right)^{2}}}\,,\quad s_{16}^{Q,L}=\frac{x_{16}^{\psi}\left\langle\phi_{3,1}\right\rangle}{\sqrt{\left(x_{16}^{\psi}\left\langle\phi_{3,1}\right\rangle\right)^{2}+\left(M_{66}^{Q,L}\right)^{2}}}$$

- If $s_{25}^{Q,L}\approx s_{16}^{Q,L}$ then no tree level (1-2) FCNCs $\Rightarrow$ GIM-like mechanism in the (1-2) sector, down-alligned (2-3) sector protects from tree level B_s meson mixing.
- $s_{34}^{Q,L}\approx 1$ is motivated by the top mass and $R_{D^{(*)}}$, $s_{16}^{Q}\lesssim 0.2$ compatible with bounds from coloron production at LHC.
- We choose a suitable benchmark $\langle\phi_{3,1}\rangle\approx 0.6\,\mathrm{TeV},\ 0.3\,\mathrm{TeV}$ and $M_{44}^{Q,L}<M_{55}^{Q,L},M_{66}^{Q,L}\approx 1.2\,\mathrm{TeV}-\ 0.8\,\mathrm{TeV},$ and we explore the parameter space of x_{34}^{ψ} and x_{25}^{ψ}.

Observable	Experiment/constraint	Theory expr.
$\delta C_L^{\mu}\ (R_{K^{(*)}})$	$-0.40^{+0.08}_{-0.09}$[41]	(2.86), (3.58)
$g_{V_L}\ (R_{D^{(*)}})$	0.05 ± 0.02[14]	(2.87), (3.59)
$\delta(\Delta M_s)\ (B_s-\bar{B}_s)$	$\lesssim 0.11$ [54]	(2.96), (3.61)
$\mathcal{B}(\tau\to 3\mu)$	$<2.1\cdot 10^{-8}$ (90% CL)[67]	(3.71)
$\mathcal{B}(\tau\to\mu\gamma)$	$<5.0\cdot 10^{-8}$ (90% CL)[68]	(3.78)
$\mathcal{B}(B_s\to\tau^{\pm}\mu^{\mp})$	$<3.4\cdot 10^{-5}$ (90% CL)[69]	(3.82)
$\mathcal{B}(B^{+}\to K^{+}\tau^{\pm}\mu^{\mp})$	$<2.8\cdot 10^{-5}$ (90% CL)[70]	(3.84)
$\mathcal{B}(\tau\to\mu\phi)$	$<8.4\cdot 10^{-8}$ (90% CL)[71]	(3.85)
$(g_\tau/g_{e,\mu})_{\ell+\pi+K}$	1.0003 ± 0.0014[3]	(3.89)
$\mathcal{B}(B_s\to\tau^{+}\tau^{-})$	$<5.2\times 10^{-3}$ (90% CL)[72]	(3.92)
$\mathcal{B}(B\to K\tau^{+}\tau^{-})$	$<2.25\times 10^{-3}$ (90% CL)[73]	(3.93)
$\mathcal{B}\left(B\to K^{(*)}\nu\bar{\nu}\right)/\mathcal{B}\left(B\to K^{(*)}\nu\bar{\nu}\right)_{\mathrm{SM}}$	$<3.5\,(3.2)$ (90% CL)[74, 75]	(3.96)

$R_{K^{(*)}}$ and $R_{D^{(*)}}$, $B_s - \bar{B}_s$ mixing

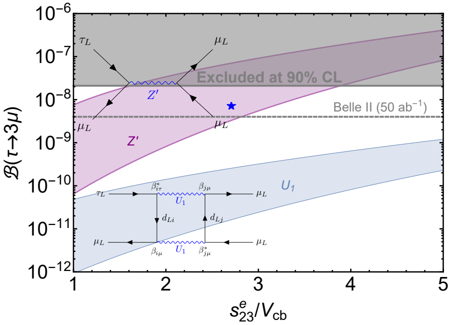


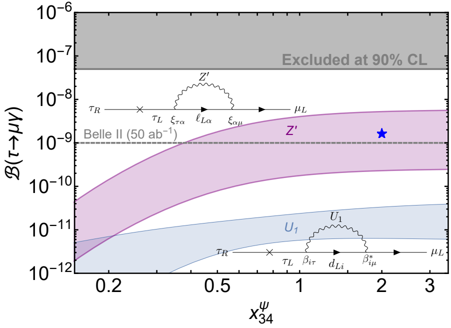


$$R_{D^{(*)}} \propto (x_{34}^{\psi})^3 x_{25}^{\psi}\,, \quad R_{K^{(*)}} \propto x_{34}^{\psi} (x_{25}^{\psi})^3$$

- R_D and R_{D^*} corrected in the same direction and with the same size.
- Light VL lepton required to relax ΔM_s, tantalizing 2.8σ excess at CMS searches of a VL lepton with these features [2208.09700] [hep-ex]

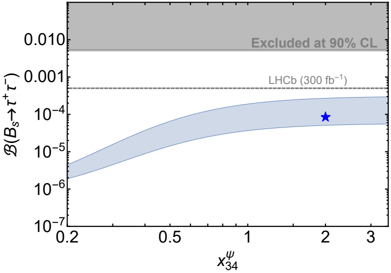
LFV processes



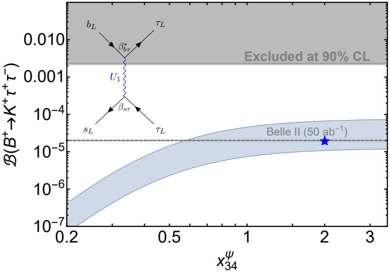


- General fermiophobic 4321 models predict only the U_1 signal.
- Z' signal is intrinsic to the twin PS model due to the $\mu-\tau$ mixing predicted.
- Signals also in $B_s\to\tau\mu$, $B\to K\tau\mu$, $\tau\to\mu\phi$.

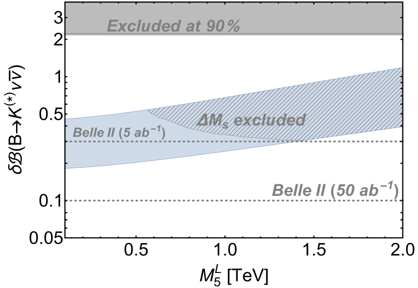
Rare decays



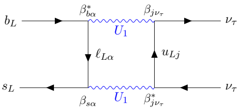
Excluded at 90% CL
LHCb (300 fb^{-1})
$\mathcal{B}(B_s\to\tau^+\tau^-)$
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0.2
0.5
1
2
3
x_{34}^{Ψ}



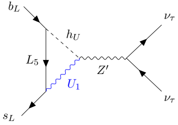
Excluded at 90% CL
Belle II (50 ab^{-1})
b_L
τ_L
$\beta^*_{b\tau}$
U_1
$\beta_{s\tau}$
s_L
τ_L
$\mathcal{B}(B^+\to K^+\tau^+\tau^-)$
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0.2
0.5
1
2
3
x_{34}^{Ψ}



Excluded at 90%
ΔM_s excluded
Belle II (5 ab^{-1})
Belle II (50 ab^{-1})
$\delta\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})$
3
2
1
0.5
0.1
0.05
0.5
1.0
1.5
2.0
M_5^L [TeV]

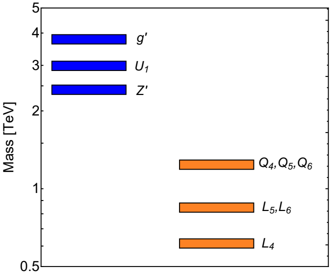


b_L
$\beta^*_{b\alpha}$
$\beta_{j\nu_\tau}$
ν_τ
U_1
$\ell_{L\alpha}$
u_{Lj}
s_L
$\beta_{s\alpha}$
U_1
$\beta^*_{j\nu_\tau}$
ν_τ



b_L
h_U
L_5
Z'
U_1
s_L
ν_τ
ν_τ

High-p_T signatures



Particle	Decay mode	$\mathcal{B}$(BP)	Γ/M
U_1	$Q_3L_5 + Q_5L_3$	~ 0.47	0.32
	Q_3L_3	~ 0.22	
	Q_5L_5	~ 0.24	
	$Q_iL_a + Q_aL_i$	~ 0.07	
g'	Q_3Q_3	~ 0.3	0.5
	Q_5Q_5	~ 0.3	
	Q_6Q_6	~ 0.3	
	$Q_1Q_6+Q_2Q_5+Q_3Q_4$	~ 0.1	
Z'	L_5L_5	~ 0.29	0.24
	L_6L_6	~ 0.29	
	L_3L_3	~ 0.27	
	$Q_3Q_3+Q_5Q_5+Q_6Q_6$	~ 0.09	
	$L_1L_6+L_2L_5+L_3L_4$	~ 0.06	

- Physical masses of VL fermions

$$\tilde{M}_a^Q = \sqrt{\left(x_{ia}^{\psi}\,\langle\phi_3\rangle\right)^2 + \left(M_a^Q\right)^2}\,, \quad \tilde{M}_a^L = \sqrt{\left(x_{ia}^{\psi}\,\langle\phi_1\rangle\right)^2 + \left(M_a^L\right)^2}$$

- The coloron $g' \to t\bar{t}$ bounds set the scale of the model as $M_{g'} \gtrsim 3.5\,\mathrm{TeV}$ [2103.16558].
- These lead to $M_{U_1} \approx 3\,\mathrm{TeV}$, beyond current limits but whithin projected sentivity for HL-LHC.
- Other than direct production, $pp \to \tau\tau$ is enhanced by U_1 exchange (most promising channel).
- Z' searches usually not competitive.

Take home messages

- Novel model building approach to connect the consistent picture of B-anomalies with the SM flavour hierarchies.
- Fermiophobic model with dominantly left-handed currents.
- U_1 couplings and Yukawa couplings arise via mixing with VL fermions $\Rightarrow$ B-anomalies and Yukawa couplings connected via the same physics
- GIM-like mechanism protects from the most dangerous FCNCs.
- Testable theory of flavour with predictions at low-energy observables and high-p_T searches.

Acknowledgements

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HIDDeN
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Backup: Gauge boson masses

$$M_{U_1} = \frac{1}{2} g_4 \sqrt{3 v_1^2 + 3 v_3^2 + \frac{4}{3} v_{15}^2}\,,$$

$$M_{g'} = \frac{\sqrt{3}}{\sqrt{2}} \sqrt{g_4^2 + g_3^2}\, v_3\,,$$

$$M_{Z'} = \frac{1}{2} \sqrt{\frac{3}{2}} \sqrt{g_4^2 + \frac{2}{3} g_1^2} \sqrt{3 v_1^2 + v_3^2}\,.$$

with

$$\langle \phi_3 \rangle = \begin{pmatrix} \frac{v_3}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{v_3}{\sqrt{2}} \\ 0 & 0 & 0 \end{pmatrix}, \quad \langle \phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \quad \langle \Omega_{15} \rangle = \frac{1}{2\sqrt{6}} \mathrm{diag}(1,1,1,-3)\, v_{15}$$

The choice $v_3 \gg v_1$ leads to $M_{g'} \approx \sqrt{2} M_{U_1}$ (phenomenologically motivated).

Backup: Personal Higgses

$$H(\bar{4},\bar{2},1;4,2,1) \to H_t(\bar{4},3,\bar{2},2/3),\ H_b(\bar{4},3,\bar{2},-1/3),\ H_\tau(\bar{4},1,\bar{2},-1),\ H_{\nu_\tau}(\bar{4},1,\bar{2},0)\,, \quad (1)$$

$$\overline{H}(\bar{4},\bar{2},1;\bar{4},1,\bar{2}) \to H_c(4,\bar{3},\bar{2},1/3),\ H_s(4,\bar{3},\bar{2},-2/3),\ H_\mu(\bar{4},1,\bar{2},0),\ H_{\nu_\mu}(\bar{4},1,\bar{2},1)\,, \quad (2)$$

- FCNCs in the Higgs basis? $\Rightarrow$ we assume that only one pair of Higgs doublets, H_u and H_d are light, given by linear combinations of the personal Higgs,

$$H_u = \widetilde{\alpha}_u H_t + \widetilde{\beta}_u H_c + \widetilde{\gamma}_u H_{\nu_\tau} + \widetilde{\delta}_u H_{\nu_\mu}\,,\ H_d = \widetilde{\alpha}_d H_b + \widetilde{\beta}_d H_s + \widetilde{\gamma}_d H_\tau + \widetilde{\delta}_d H_\mu \quad (3)$$

  where $\widetilde{\alpha}_{u,d}$, $\widetilde{\beta}_{u,d}$, $\widetilde{\gamma}_{u,d}$, $\widetilde{\delta}_{u,d}$ are complex elements of two unitary Higgs mixing matrices.
- The orthogonal linear combinations are assumed to be very heavy, well above the TeV scale in order to sufficiently suppress the FCNCs.
- We will further assume that only the light Higgs doublet states get VEVs in order to perform EW symmetry breaking, $\langle H_u \rangle = v_u$, $\langle H_d \rangle = v_d$, while the heavy linear combinations do not, i.e. we assume that in the Higgs basis the linear combinations which do not get VEVs are very heavy.
- The situation is familiar from $SO(10)$ models, where 6 Higgs doublets arise as H_{10}, H_{120}, $H_{\overline{126}}$, two from each, but below the $SO(10)$ breaking scale only two Higgs doublets are assumed to be light, similar to H_u and H_d above. We invert the unitary transformations

$$\begin{aligned} &H_t = \alpha_u H_u + \ldots\,, \quad H_b = \alpha_d H_d + \ldots\,, \quad H_\tau = \gamma_d H_d + \ldots\,, \quad H_{\nu_\tau} = \gamma_u H_u + \ldots\,, \\ &H_c = \beta_u H_u + \ldots\,, \quad H_s = \beta_d H_d + \ldots\,, \quad H_\mu = \delta_d H_d + \ldots\,, \quad H_{\nu_\mu} = \delta_u H_u + \ldots\,, \end{aligned} \quad (4)$$

  ignoring the heavy states indicated by dots. When the light Higgs H_u, H_d gain their VEVs, the personal Higgs in the original basis can be thought of as gaining VEVs $\langle H_t \rangle = \alpha_u v_u$, etc...

Backup: Mass matrix, block-diagonalisation

$$M^{\psi} = \left(\begin{array}{c|ccccccccc} & \psi_1^c & \psi_2^c & \psi_3^c & \psi_4^c & \psi_5^c & \psi_6^c & \overline{\psi_4} & \overline{\psi_5} & \overline{\psi_6} \\ \hline \psi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_{16}^{\psi}\phi \\ \psi_2 & 0 & 0 & 0 & y_{24}^{\psi}\overline{H} & y_{25}^{\psi}\overline{H} & 0 & 0 & x_{25}^{\psi}\phi & 0 \\ \psi_3 & 0 & 0 & 0 & y_{34}^{\psi}\overline{H} & y_{35}^{\psi}\overline{H} & 0 & x_{34}^{\psi}\phi & x_{35}^{\psi}\phi & 0 \\ \psi_4 & 0 & 0 & y_{43}^{\psi}H & 0 & 0 & 0 & \bar{M}_{44}^{\psi} & M_{45}^{\psi} & 0 \\ \psi_5 & 0 & 0 & y_{53}^{\psi}H & 0 & 0 & 0 & M_{54}^{\psi} & \bar{M}_{55}^{\psi} & 0 \\ \psi_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{M}_{66}^{\psi} \\ \overline{\psi_4^c} & 0 & x_{42}^{\psi^c}\overline{\phi'} & x_{43}^{\psi^c}\overline{\phi} & \bar{M}_{44}^{\psi^c} & M_{45}^{\psi^c} & 0 & 0 & 0 & 0 \\ \overline{\psi_5^c} & 0 & x_{52}^{\psi^c}\overline{\phi'} & x_{53}^{\psi^c}\overline{\phi} & M_{54}^{\psi^c} & \bar{M}_{55}^{\psi^c} & 0 & 0 & 0 & 0 \\ \overline{\psi_6^c} & x_{61}^{\psi^c}\overline{\phi} & 0 & 0 & 0 & 0 & \bar{M}_{66}^{\psi^c} & 0 & 0 & 0 \end{array} \right) ,$$

$$\begin{pmatrix} M_{44}^{Q} & 0 & 0 \\ 0 & M_{55}^{Q} & 0 \\ 0 & 0 & M_{66}^{Q} \end{pmatrix} = V_{45}^{Q} \begin{pmatrix} \bar{M}_{44}^{Q} & M_{45}^{\psi} & 0 \\ M_{54}^{\psi} & \bar{M}_{55}^{Q} & 0 \\ 0 & 0 & \bar{M}_{66}^{Q} \end{pmatrix} V_{45}^{\bar{Q}\dagger} , \qquad \bar{M}_{aa}^{Q} \equiv M_{aa}^{\psi} + \frac{\lambda_{15}^{aa}\langle\Omega_{15}\rangle}{2\sqrt{6}} ,$$

$$\begin{pmatrix} M_{44}^{L} & 0 & 0 \\ 0 & M_{55}^{L} & 0 \\ 0 & 0 & M_{66}^{L} \end{pmatrix} = V_{45}^{L} \begin{pmatrix} \bar{M}_{44}^{L} & M_{45}^{\psi} & 0 \\ M_{54}^{\psi} & \bar{M}_{55}^{L} & 0 \\ 0 & 0 & \bar{M}_{66}^{L} \end{pmatrix} V_{45}^{\bar{L}\dagger} , \qquad \bar{M}_{aa}^{L} \equiv M_{aa}^{\psi} - 3\frac{\lambda_{15}^{aa}\langle\Omega_{15}\rangle}{2\sqrt{6}} ,$$

Backup: Mass matrix, block-diagonalisation

Block-diagonalised via (effective mass matrices arise):

$$V_\psi = V_{16}^{\psi} V_{35}^{\psi} V_{25}^{\psi} V_{34}^{\psi} V_{45}^{\psi} V_{45}^{\overline{\psi^c}} \,,$$

$$V_{\psi^c} = V_{16}^{\psi^c} V_{35}^{\psi^c} V_{25}^{\psi^c} V_{34}^{\psi^c} V_{24}^{\psi^c} V_{45}^{\psi^c} V_{45}^{\bar{\psi}} \,.$$

$$M^{\psi'} = \left(\begin{array}{c|ccccccccc}
 & \psi_1^{c\prime} & \psi_2^{c\prime} & \psi_3^{c\prime} & \psi_4^{c\prime} & \psi_5^{c\prime} & \psi_6^{c\prime} & \overline{\psi_4'} & \overline{\psi_5'} & \overline{\psi_6'} \\
\hline
\psi_1' & & & & & & & 0 & 0 & 0 \\
\psi_2' & & & & & & & 0 & 0 & 0 \\
\psi_3' & & & & & & & 0 & 0 & 0 \\
\psi_4' & & & & \tilde{y}'^{\psi}_{\alpha\beta} & & & \tilde{M}_4^{\psi} & 0 & 0 \\
\psi_5' & & & & & & & 0 & \tilde{M}_5^{\psi} & 0 \\
\psi_6' & & & & & & & 0 & 0 & \tilde{M}_6^{\psi} \\
\overline{\psi_4^{c\prime}} & 0 & 0 & 0 & \tilde{M}_4^{\psi^c} & 0 & 0 & 0 & 0 & 0 \\
\overline{\psi_5^{c\prime}} & 0 & 0 & 0 & 0 & \tilde{M}_5^{\psi^c} & 0 & 0 & 0 & 0 \\
\overline{\psi_6^{c\prime}} & 0 & 0 & 0 & 0 & 0 & \tilde{M}_6^{\psi^c} & 0 & 0 & 0
\end{array}\right) \,,$$

Backup: VL-chiral mixing

$$s_{34}^{Q} = \frac{x_{34}^{\psi}\langle\phi_3\rangle}{\sqrt{\left(x_{34}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{44}^{Q}\right)^2}}\,, \qquad s_{34}^{L} = \frac{x_{34}^{\psi}\langle\phi_1\rangle}{\sqrt{\left(x_{34}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{44}^{L}\right)^2}}\,,$$

$$s_{25}^{Q} = \frac{x_{25}^{\psi}\langle\phi_3\rangle}{\sqrt{\left(x_{25}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{55}^{Q}\right)^2}}\,, \qquad s_{25}^{L} = \frac{x_{25}^{\psi}\langle\phi_1\rangle}{\sqrt{\left(x_{25}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{55}^{L}\right)^2}}\,,$$

$$s_{35}^{Q} = \frac{c_{34}^{Q}x_{35}^{\psi}\langle\phi_3\rangle}{\sqrt{\left(c_{34}^{Q}x_{35}^{\psi}\langle\phi_3\rangle\right)^2 + \left(x_{25}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{55}^{Q}\right)^2}}\,, \qquad s_{35}^{L} = \frac{c_{34}^{L}x_{35}^{\psi}\langle\phi_1\rangle}{\sqrt{\left(c_{34}^{L}x_{35}^{\psi}\langle\phi_1\rangle\right)^2 + \left(x_{25}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{55}^{L}\right)^2}}\,,$$

$$s_{16}^{Q} = \frac{x_{16}^{\psi}\langle\phi_3\rangle}{\sqrt{\left(x_{16}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{66}^{Q}\right)^2}}\,, \qquad s_{16}^{L} = \frac{x_{16}^{\psi}\langle\phi_1\rangle}{\sqrt{\left(x_{16}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{66}^{L}\right)^2}}\,,$$

$$\tilde{M}_{4}^{Q} = \sqrt{\left(x_{34}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{44}^{Q}\right)^2}\,, \qquad \tilde{M}_{4}^{L} = \sqrt{\left(x_{34}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{44}^{L}\right)^2}\,,$$

$$\tilde{M}_{5}^{Q} = \sqrt{\left(x_{25}^{\psi}\langle\phi_3\rangle\right)^2 + \left(x_{35}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{55}^{Q}\right)^2}\,, \qquad \tilde{M}_{5}^{L} = \sqrt{\left(x_{25}^{\psi}\langle\phi_1\rangle\right)^2 + \left(x_{35}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{55}^{L}\right)^2}\,,$$

$$\tilde{M}_{6}^{Q} = \sqrt{\left(x_{16}^{\psi}\langle\phi_3\rangle\right)^2 + \left(M_{66}^{Q}\right)^2}\,, \qquad \tilde{M}_{6}^{L} = \sqrt{\left(x_{16}^{\psi}\langle\phi_1\rangle\right)^2 + \left(M_{66}^{L}\right)^2}\,.$$

Backup: Effective Yukawa couplings (mass matrices)

- Zeroes enforced by Z_4:

$$M^u_{\text{eff}} = \begin{pmatrix} & u_1^{c\,\prime} & u_2^{c\,\prime} & u_3^{c\,\prime} \\ \hline Q_1' | & 0 & 0 & 0 \\ Q_2' | & 0 & 0 & s^Q_{25} y^\psi_{53} \\ Q_3' | & 0 & 0 & s^Q_{34} y^\psi_{43} \end{pmatrix} \langle H_t \rangle + \begin{pmatrix} & u_1^{c\,\prime} & u_2^{c\,\prime} & u_3^{c\,\prime} \\ \hline Q_1' | & 0 & 0 & 0 \\ Q_2' | & 0 & c^Q_{25} s^{q^c}_{24} y^\psi_{24} & c^Q_{25} s^{q^c}_{34} y^\psi_{24} \\ Q_3' | & 0 & c^Q_{34} s^{q^c}_{24} y^\psi_{34} & c^Q_{34} s^{q^c}_{34} y^\psi_{34} \end{pmatrix} \langle H_c \rangle + \text{h.c.}\,,$$

$$M^d_{\text{eff}} = \begin{pmatrix} & d_1^{c\,\prime} & d_2^{c\,\prime} & d_3^{c\,\prime} \\ \hline Q_1' | & 0 & 0 & 0 \\ Q_2' | & 0 & 0 & s^Q_{25} y^\psi_{53} \\ Q_3' | & 0 & 0 & s^Q_{34} y^\psi_{43} \end{pmatrix} \langle H_b \rangle + \begin{pmatrix} & d_1^{c\,\prime} & d_2^{c\,\prime} & d_3^{c\,\prime} \\ \hline Q_1' | & 0 & 0 & 0 \\ Q_2' | & 0 & c^Q_{25} s^{q^c}_{24} y^\psi_{24} & c^Q_{25} s^{q^c}_{34} y^\psi_{24} \\ Q_3' | & 0 & c^Q_{34} s^{q^c}_{24} y^\psi_{34} & c^Q_{34} s^{q^c}_{34} y^\psi_{34} \end{pmatrix} \langle H_s \rangle + \text{h.c.}\,,$$

$$M^e_{\text{eff}} = \begin{pmatrix} & e_1^{c\,\prime} & e_2^{c\,\prime} & e_3^{c\,\prime} \\ \hline L_1' | & 0 & 0 & 0 \\ L_2' | & 0 & 0 & s^L_{25} y^\psi_{53} \\ L_3' | & 0 & 0 & s^L_{34} y^\psi_{43} \end{pmatrix} \langle H_\tau \rangle + \begin{pmatrix} & e_1^{c\,\prime} & e_2^{c\,\prime} & u_3^{c\,\prime} \\ \hline L_1' | & 0 & 0 & 0 \\ L_2' | & 0 & c^L_{25} s^{e^c}_{24} y^\psi_{24} & c^L_{25} s^{e^c}_{34} y^\psi_{24} \\ L_3' | & 0 & c^L_{34} s^{e^c}_{24} y^\psi_{34} & c^L_{34} s^{e^c}_{34} y^\psi_{34} \end{pmatrix} \langle H_\mu \rangle + \text{h.c.}\,,$$

- CKM down alligned if

$$s^Q_{25} y^\psi_{53} \langle H_b \rangle + c^Q_{25} s^{q^c}_{34} y^\psi_{24} \langle H_s \rangle \approx 0 \Rightarrow y^\psi_{53} \approx (-)\mathcal{O}(0.1 - 0.5) \Rightarrow \theta^d_{23} \approx 0$$

$$\Rightarrow \begin{cases} \theta^u_{23} \approx \dfrac{-s^Q_{25} \left| y^\psi_{53} \right| \langle H_t \rangle + c^Q_{25} s^{q^c}_{34} y^\psi_{24} \langle H_c \rangle}{s^Q_{34} y^\psi_{43} \langle H_t \rangle} \approx s^Q_{25} \left| y^\psi_{53} \right| \approx \mathcal{O}(V_{cb})\,, \\ \theta^e_{23} \approx \dfrac{-s^L_{25} \left| y^\psi_{53} \right| \langle H_\tau \rangle + c^L_{25} s^{e^c}_{34} y^\psi_{24} \langle H_\mu \rangle}{s^L_{34} y^\psi_{43} \langle H_\tau \rangle} \approx \mathcal{O}(V_{cb} - 4V_{cb}) \end{cases}$$

In good approximation, the mass of the top quark is given by the (3,3) entry in M^u_{eff}, i.e.

$$m_t \approx s^Q_{34} y^\psi_{43} \langle H_t \rangle = s^Q_{34} y^\psi_{43} \alpha_u \frac{1}{\sqrt{1+\tan^{-2}\beta}} \frac{v_{\mathrm{SM}}}{\sqrt{2}} \,,$$

where $v_{\mathrm{SM}} = 246\,\mathrm{GeV}$ and we have applied $\langle H_t \rangle = \alpha_u v_u$, where

$$v_u = \frac{v_{\mathrm{SM}}}{\sqrt{2}} \sin\beta = \frac{1}{\sqrt{1+\tan^{-2}\beta}} \frac{v_{\mathrm{SM}}}{\sqrt{2}} \,,$$

as in usual 2HDM. If we consider $\tan\beta \approx 10$ and $\alpha_u \approx 1$[1], then we obtain

$$m_t \approx s^Q_{34} y^\psi_{43} \frac{v_{\mathrm{SM}}}{\sqrt{2}} \equiv y_t \frac{v_{\mathrm{SM}}}{\sqrt{2}} \,.$$

[1] This choice preserves $\langle H_t \rangle$ at the EW scale, larger values would break the decoupling approximation that we have assumed during the diagonalisation of the full mass matrix.

Backup: 1st family masses

- Add one VL family split across both PS groups, take advantage of scalars performing high scale breaking, Z_4 still provides flavour structure

Field	$SU(4)^I_{PS}$	$SU(2)^I_L$	$SU(2)^I_R$	$SU(4)^{II}_{PS}$	$SU(2)^{II}_L$	$SU(2)^{II}_R$	Z_4
ψ_7	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{1}$	1
$\overline{\psi_7}$	$\mathbf{1}$	$\overline{\mathbf{2}}$	$\mathbf{1}$	$\overline{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{1}$	1
ψ_7^c	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\overline{\mathbf{4}}$	$\overline{\mathbf{2}}$	$\overline{\mathbf{2}}$	1
$\overline{\psi_7^c}$	$\mathbf{1}$	$\overline{\mathbf{2}}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{2}$	$\mathbf{2}$	1
h	$\mathbf{1}$	$\overline{\mathbf{2}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	α^3
H'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$	1
$\overline{H}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{4}}$	$\mathbf{1}$	$\overline{\mathbf{2}}$	1
Φ, Φ'	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{2}}$	$\mathbf{1}$	1, α^2

$$M^\psi = \left(\begin{array}{c|ccccc} & \psi_1^c & \psi_2^c & \psi_3^c & \psi_7^c & \overline{\psi_7} \\ \hline \psi_1 & 0 & 0 & 0 & y_{17}^\psi h & 0 \\ \psi_2 & 0 & 0 & 0 & 0 & x_{27}^\psi \Phi \\ \psi_3 & 0 & 0 & 0 & 0 & x_{37}^\psi \Phi \\ \psi_7 & y_{71}^\psi h & 0 & 0 & 0 & M_7^\psi \\ \overline{\psi_7^c} & 0 & x_{72}^{\psi^c} \Phi' & x_{73}^{\psi^c} \Phi & M_7^{\psi^c} & 0 \end{array}\right)$$

$$M_{\mathrm{eff}} = \left(\begin{array}{c|ccc} & \psi_1^{c\prime} & \psi_2^{c\prime} & \psi_3^{c\prime} \\ \hline \psi_1' & 0 & 0 & 0 \\ \psi_2' & y_{71}^\psi x_{27}^\psi & 0 & 0 \\ \psi_3' & y_{71}^\psi x_{37}^\psi & 0 & 0 \end{array}\right) \frac{\langle \Phi \rangle}{M_7^\psi} \langle h \rangle + \left(\begin{array}{c|ccc} & \psi_1^{c\prime} & \psi_2^{c\prime} & \psi_3^{c\prime} \\ \hline \psi_1' & 0 & y_{17}^\psi x_{27}^{\psi^c} & y_{17}^\psi x_{37}^{\psi^c} \\ \psi_2' & 0 & 0 & 0 \\ \psi_3' & 0 & 0 & 0 \end{array}\right) \frac{\langle \Phi \rangle}{M_7^{\psi^c}} \langle h \rangle + \mathrm{h.c.}$$

- VL masses splitted via Ω_{15}. Texture zero and up-alligned structure in the (1-2) CKM sector.

Backup: Neutrino masses

Single right-handed neutrino dominance

$$M_\nu^M = \begin{pmatrix} \tilde{\xi}^2 & \tilde{\xi}^5 & \tilde{\xi}^4 \\ \tilde{\xi}^5 & \tilde{\xi}^2 & \tilde{\xi} \\ \tilde{\xi}^4 & \tilde{\xi} & 1 \end{pmatrix} \frac{\langle H' \rangle \langle H' \rangle}{\Lambda} \simeq \begin{pmatrix} M_1^M & 0 & 0 \\ 0 & M_2^M & \tilde{\xi} \\ 0 & \tilde{\xi} & M_3^M \end{pmatrix} , \tag{5}$$

$$M_1^M \simeq M_2^M \simeq \tilde{\xi}^2 M_3^M , \tag{6}$$

$$M_3^M = \frac{\langle H' \rangle \langle H' \rangle}{\Lambda} . \tag{7}$$

$$M_\nu^D = \begin{pmatrix} 0 & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix} . \tag{8}$$

Now we apply the seesaw formula:

$$m_\nu = M_\nu^D \left(M_\nu^M\right)^{-1} \left(M_\nu^D\right)^T . \tag{9}$$

If we neglect the off-diagonal $\tilde{\xi}$ terms

$$m_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & e^2 & ef \\ 0 & ef & f^2 \end{pmatrix} \frac{1}{M_1^M} + \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix} \frac{1}{M_2^M} + \begin{pmatrix} a'^2 & a'b' & a'c' \\ a'b' & b'^2 & b'c' \\ a'c' & b'c' & c'^2 \end{pmatrix} \frac{1}{M_3^M} . \tag{10}$$

Backup: Benchmark

Benchmark (BP)				Output			
g_4	3.5	λ_{15}^{44}	-0.5	s_{34}^{Q}	0.978	$M_{g'}$	3782.9 GeV
$g_{3,2,1}$	1, 0.65, 0.36	λ_{15}^{55}, λ_{15}^{66}	2.5, 1.1	s_{34}^{L}	0.977	$M_{Z'}$	2414.3 GeV
x_{34}^{ψ}	2	$x_{42}^{\psi^c}$	0.4	$s_{25}^{Q} = s_{16}^{Q}$	0.1986	s_{23}^{u}	0.042556
$x_{25}^{\psi} = x_{16}^{\psi}$	0.41	$x_{43}^{\psi^c}$	1	$s_{25}^{L} = s_{16}^{L}$	0.1455	s_{23}^{d}	0.001497
M_{44}^{ψ}	320 GeV	$M_{44}^{\psi^c}$	5 TeV	$s_{\theta_{LQ}}$	0.7097	s_{23}^{e}	-0.111
M_{55}^{ψ}	780 GeV	$y_{53,43,34,24}^{\psi}$	-0.3, 1, 1, 1	$\widetilde{M}_4^Q$	1226.8 GeV	V_{cb}	0.04106
M_{66}^{ψ}	1120 GeV	$\langle H_t \rangle$	177.2 GeV	$\widetilde{M}_5^Q$	1238.7 GeV	m_t	172.91 GeV
M_{45}^{ψ}	-700 GeV	$\langle H_c \rangle$	26.8 GeV	$\widetilde{M}_4^L$	614.04 GeV	m_c	1.270 GeV
M_{54}^{ψ}	50 GeV	$\langle H_b \rangle$	4.25 GeV	$\widetilde{M}_5^L$	845.26 GeV	m_b	4.180 GeV
$\langle \phi_3 \rangle$	0.6 TeV	$\langle H_s \rangle$	2.1 GeV	$\widetilde{M}_6^Q$	1234.6 GeV	m_s	0.0987 GeV
$\langle \phi_1 \rangle$	0.3 TeV	$\langle H_\tau \rangle$	1.75 GeV	$\widetilde{M}_6^L$	859.4 GeV	m_τ	1.7765 GeV
$\langle \Omega_{15} \rangle$	0.4 TeV	$\langle H_\mu \rangle$	4.58 GeV	M_{U_1}	2987.1 GeV	m_μ	105.65 MeV

Table 1: Input and output parameters for the benchmark point (BP).

Backup: $R_{K^{(*)}}$ and $R_{D^{(*)}}$

$$\mathcal{L}_{\rm eff} \supset C^{U_1}_{bs\mu\mu}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{\mu}_L\gamma^\mu\mu_L\right) + C^{U_1}_{bc\tau\nu}\left(\bar{c}_L\gamma_\mu b_L\right)\left(\bar{\tau}_L\gamma^\mu\nu_{\tau L}\right) + \text{h.c.}\,,$$

$$C^{U_1}_{bs\mu\mu} = -\frac{g_4^2}{2M_{U_1}^2}\beta^*_{b\mu}\beta_{s\mu} = \frac{g_4^2}{2M_{U_1}^2}c_{\theta_{LQ}}s_{\theta_{LQ}}s^Q_{25}s^Q_{34}\left(s^L_{25}\right)^2\,,$$

$$C^{U_1}_{bc\tau\nu_\tau} = -\frac{g_4^2}{2M_{U_1}^2}\beta^*_{b\tau}\beta_{c\nu_\tau} = -\frac{g_4^2}{2M_{U_1}^2}c_{\theta_{LQ}}s_{\theta_{LQ}}s^Q_{25}s^Q_{34}\left(s^L_{34}\right)^2\,,$$

in order to fit

$$C^{U_1}_{bs\mu\mu} = \frac{4G_F}{\sqrt{2}}V_{tb}V^*_{ts}\frac{\alpha}{4\pi}2\delta C^\mu_L\,, \qquad \delta C^\mu_L = -0.40^{+0.08}_{-0.09}\,,$$

$$C^{U_1}_{cb\tau\nu_\tau} = -2\sqrt{2}G_F V_{cb}g_{V_L}\,, \qquad g_{V_L} = 0.07 \pm 0.02\,,$$

at the matching scale $\mu \sim m_b$, provided that small corrections over the Wilson coefficients above due to RGE from the U_1 scale are at the percent level and can be safely neglected. In particular, δC^μ_L provides a good fit of R_K, R_{K^*} and $B_s \to \mu\bar{\mu}$ data, while g_{V_L} provides a good fit of R_D and R_{D^*}, imposing $\mathcal{B}(B_c \to \bar{\tau}\nu) \lesssim 30\,\%$.

$$\delta(\Delta M_s) \equiv \frac{\Delta M_s - \Delta M_s^{\rm SM}}{\Delta M_s^{\rm SM}} = \left|1 + \frac{C_{bs}^{\rm NP}}{C_{bs}^{\rm SM}}\right| - 1 = \frac{C_{bs}^{\rm NP}}{C_{bs}^{\rm SM}} \lesssim 0.11 \tag{11}$$

$$C_{bs}^{\rm NP-loop} = \frac{g_4^4}{(8\pi M_{U_1})^2} \sum_{\alpha,\beta} \left(\beta_{s\alpha}^* \beta_{b\alpha}\right) \left(\beta_{s\beta}^* \beta_{b\beta}\right) F(x_\alpha, x_\beta) \tag{12}$$

where $\alpha, \beta = \mu, \tau, E_4, E_5$ run for all charged leptons, including the vector-like partners, and $x_\alpha = (m_\alpha / M_U)^2$. We have generalised the loop function in [Fuentes-Martin et al, 2009.11296] to the case of more than one VL families,

$$F(x_\alpha, x_\beta) = \left(1 + \frac{x_\alpha x_\beta}{4}\right) B(x_\alpha, x_\beta)\,, \tag{13}$$

where

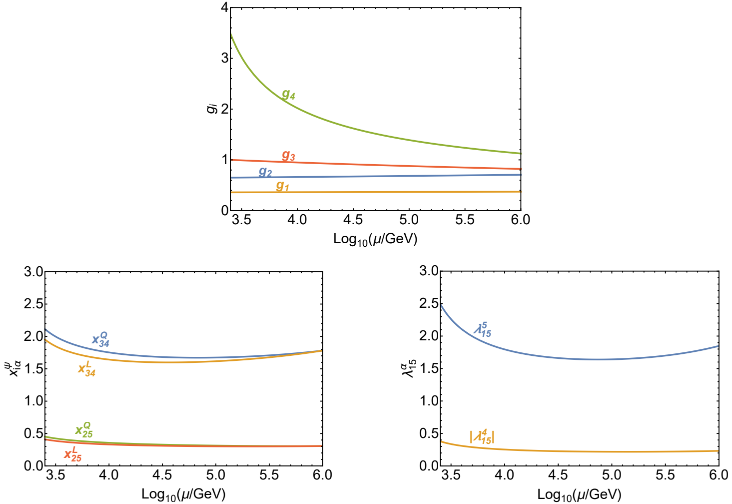
$$B(x_\alpha, x_\beta) = \frac{1}{(1 - x_\alpha)(1 - x_\beta)} + \frac{x_\alpha^2 \log x_\alpha}{(x_\beta - x_\alpha)(1 - x_\alpha^2)} + \frac{x_\beta^2 \log x_\beta}{(x_\alpha - x_\beta)\left(1 - x_\beta^2\right)}\,. \tag{14}$$

The product of couplings $\beta_{s\alpha}^* \beta_{b\alpha}$ has the fundamental property

$$\sum_\alpha \beta_{s\alpha}^* \beta_{b\alpha} = 0\,. \tag{15}$$

Backup: Perturbativity

The low-energy 4321 theory must remain perturbative until the high scale of the twin Pati-Salam symmetry.



Backup: Gauge bosons couplings

$$\mathcal{L}^{\text{gauge}}_{U_1} = \frac{g_4}{\sqrt{2}} Q_i^{\dagger\prime} \gamma_\mu \begin{pmatrix} s^Q_{16} s^L_{16} \epsilon & 0 & 0 \\ 0 & c_{\theta_{LQ}} s^Q_{25} s^L_{25} & s_{\theta_{LQ}} s^Q_{25} s^L_{34} \\ 0 & -s_{\theta_{LQ}} s^Q_{34} s^L_{25} & c_{\theta_{LQ}} s^Q_{34} s^L_{34} \end{pmatrix} L_j' U_1^\mu + \text{h.c.}\,,$$

$$\mathcal{L}^{\text{gauge}}_{g'} = \frac{g_4 g_s}{g_3} Q_i^{\dagger\prime} \gamma^\mu T^a \begin{pmatrix} \left(s^Q_{16}\right)^2 - \left(c^Q_{16}\right)^2 \frac{g_3^2}{g_4^2} & 0 & 0 \\ 0 & \left(s^Q_{25}\right)^2 - \left(c^Q_{25}\right)^2 \frac{g_3^2}{g_4^2} & 0 \\ 0 & 0 & \left(s^Q_{34}\right)^2 - \left(c^Q_{34}\right)^2 \frac{g_3^2}{g_4^2} \end{pmatrix} Q_j' g^{a\prime}_\mu \,.$$

$$\mathcal{L}^{\text{gauge}}_{Z',\ell} = -\frac{\sqrt{3}}{\sqrt{2}} \frac{g_4 g_Y}{g_1} L_i^{\dagger\prime} \gamma^\mu \begin{pmatrix} \frac{1}{2}\left(s^L_{16}\right)^2 - \left(c^L_{16}\right)^2 \frac{g_1^2}{3g_4^2} & 0 & 0 \\ 0 & \frac{1}{2}\left(s^L_{25}\right)^2 - \left(c^L_{25}\right)^2 \frac{g_1^2}{3g_4^2} & 0 \\ 0 & 0 & \frac{1}{2}\left(s^L_{34}\right)^2 - \left(c^L_{34}\right)^2 \frac{g_1^2}{3g_4^2} \end{pmatrix} L_j' Z'_\mu \,.$$

Backup: $B \to K\nu\bar{\nu}$

$$\mathcal{L}_{b\to s\nu\nu} = -C_\nu^{\tau\tau}\,(\bar{s}_L\gamma_\mu b_L)\,(\bar{\nu}_{L\tau}\gamma^\mu \nu_{L\tau})\;,\;\; C_\nu^{\tau\tau} = C_{\nu,\mathrm{NP}}^{\tau\tau} + C_{\nu,\mathrm{SM}}\,.$$

We parameterise corrections to the SM branching fraction as

$$\delta\mathcal{B}(B\to K^{(*)}\nu\bar{\nu}) = \frac{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})_{\mathrm{SM}}} - 1 \approx \frac{1}{3}\left|\frac{C_{\nu\nu}^{\mathrm{NP}} - C_{\nu\nu}^{\mathrm{SM}}}{C_{\nu\nu}^{\mathrm{SM}}}\right|^2 - \frac{1}{3}\,.$$

We split the NP effects into Z'-mediated and U_1-mediated contributions as follows

$$C_{\nu,\mathrm{NP}}^{\tau\tau} = C_{\nu,Z'}^{\tau\tau} + C_{\nu,U}^{\tau\tau}\,.$$

The U_1 contribution at NLO accuracy reads

$$C_{\nu,U}^{\tau\tau} \approx C_{\nu,U}^{\mathrm{RGE}} + \frac{g_4^4}{32\pi^2 M_{U_1}^2}\sum_{\alpha,j}\left(\beta_{s\alpha}^*\beta_{b\alpha}\right)\left(\beta_{j\nu_\tau}\right)^2 F(x_\alpha, x_j)\,,$$

where the second term arises from the semileptonic box diagram and the first term encodes the RGE-induced contribution from the tree-level leptoquark-mediated operator $(\bar{s}_L\gamma_\mu b_L)\,(\bar{\tau}_L\gamma^\mu\tau_L)$,

$$C_{\nu,U}^{\mathrm{RGE}} = -0.047\,\frac{g_4^2}{2M_{U_1}^2}\,\beta_{b\tau}\beta_{s\tau}\,.$$

$$C_{\nu,Z'}^{\tau\tau} \approx \frac{3g_4^2}{2M_{Z'}^2}\left[\xi_{bs}\xi_{\nu_\tau\nu_\tau}\left(1+\frac{3}{2}\,\frac{g_4^2}{16\pi^2}\,\xi_{\nu_\tau\nu_\tau}^2\right) + \frac{g_4^2}{16\pi^2}\,\beta_{sE_5}^*\beta_{bE_5}\,G_{\Delta Q=1}(x_{E_5}, x_{Z'}, x_R)\right]\,,$$

where $x_{E_5} \equiv (M_5^L)^2/M_U^2$, $x_{Z'} \equiv M_{Z'}^2/M_U^2$ and $x_R \equiv M_R^2/M_U^2$ with M_R being a scale associated to the radial mode $h_U(3,1,2/3)$ arising from $\phi_{3,1}$. The loop function [Fuentes-Martin et al, 2009.11296]

$$G_{\Delta Q=1}(x_1, x_2, x_3) \approx \frac{5}{4}x_1 + \frac{x_1}{2}\left(x_2 - \frac{3}{2}\right)\left(\ln x_3 - \frac{5}{2}\right)\,,$$

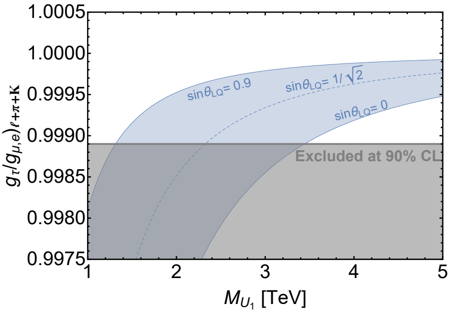
Backup: Tests of universality in leptonic τ decays

$$\left(\frac{g_\tau}{g_\mu}\right)_\ell = 1 + \frac{9}{12} C_{Z'} \left(|\xi_{\tau e}|^2 - |\xi_{\mu e}|^2\right) - \eta C_U \left(|\beta_{b\tau}|^2 - |\beta_{b\mu}|^2\right) , \tag{16}$$

$$\left(\frac{g_\tau}{g_e}\right)_\ell = 1 + \frac{9}{12} C_{Z'} \left(|\xi_{\tau \mu}|^2 - |\xi_{\mu e}|^2\right) - \eta C_U \left(|\beta_{b\tau}|^2 - |\beta_{be}|^2\right) , \tag{17}$$

where $\eta = 0.079$ parameterises the running from $\Lambda = 2\,\mathrm{TeV}$. Due to the hierarchy in leptoquark couplings, we find $\beta_{b\tau} \gg \beta_{b\mu}$ and $\beta_{be} \approx 0$, hence in good approximation both ratios receive the same contribution proportional to $\beta_{b\tau}$, so we can approximate

$$\left(\frac{g_\tau}{g_{\mu,e}}\right)_{\ell+\pi+K} \approx 1 - \eta C_U |\beta_{b\tau}|^2 , \tag{18}$$



Backup: Useful definitions

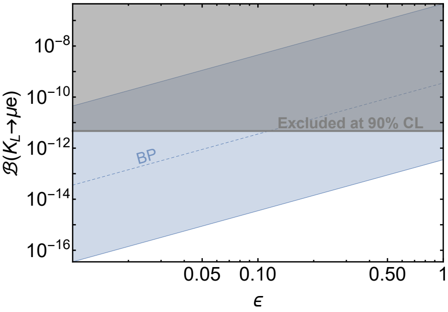
$$C_U = \frac{g_4^2 v_{\rm SM}^2}{4 M_{U_1}^2}\,, \quad C_{Z'} = \frac{3}{4}\frac{g_4^2 g_Y^2}{g_1^2}\frac{v_{\rm SM}^2}{M_{Z'}^2}\,.$$

Backup: $K_L \to \mu e$

The LFV process $K_L \to \mu e$ sets a strong constraint over all models featuring a vector leptoquark U_1 with first and second family couplings,

$$\mathcal{B}\left(K_L \to \mu e\right) = \frac{\tau_{K_L} G_F^2 f_K^2 m_\mu^2 m_K}{8\pi} \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 C_U^2 \left|\beta_{de}\beta_{s\mu}^*\right|^2 .$$

The first family coupling β_{de} can be diluted via mixing with vector-like fermions, which we parameterised via the effective parameter ϵ, so that $\beta_{se} \approx s_{16}^Q s_{16}^L \epsilon$.



Backup: $K_L \to \mu e$ (cont.)

field	Z_2
$\bar{\psi}_6,\ \psi_6$	1, 1
$\bar{\psi}'_6,\ \psi'_6$	-1, -1
χ	-1

$$\mathcal{L}_{\rm mix} = x_{66}\chi\bar{\psi}_6\psi'_6 + x'_{66}\chi^*\bar{\psi}'_6\psi_6 \ + \text{h.c.} \tag{19}$$

$$\mathcal{L}_{\rm mass} = (M^{\psi}_{66} + \lambda^{66}_{15}T_{15}\Omega_{15})\bar{\psi}_6\psi_6 + (M^{\psi}_{66'} + \lambda^{66'}_{15}T_{15}\Omega_{15})\bar{\psi}'_6\psi'_6 + \text{h.c.} \tag{20}$$

Then for LQ couplings

$$\mathcal{L}_{U_1} = \frac{g_4}{\sqrt{2}}\begin{pmatrix} Q^\dagger_6 & Q^{\dagger\prime}_6 \end{pmatrix}\gamma_\mu V^Q_{66'}\,\text{diag}(1,1)V^{L\dagger}_{66'}\begin{pmatrix} L_6 \\ L'_6 \end{pmatrix} U^\mu_1 + \text{h.c.} \tag{21}$$

If we define

$$V^Q_{66'}, V^{L\dagger}_{66'} \equiv \begin{pmatrix} \cos\theta_6 & \sin\theta_6 \\ -\sin\theta_6 & \cos\theta_6 \end{pmatrix}, \tag{22}$$

then the $Q^\dagger_6 L_6 U_1$ coupling receives a suppression via $\cos\theta_6$ as

$$\beta_{de} = s^Q_{16}s^L_{16}\cos\theta_6\ . \tag{23}$$

which is identified with the suppression parameter ϵ,

$$\epsilon \equiv \cos\theta_6\,. \tag{24}$$