More on Discrete Goldstone Bosons

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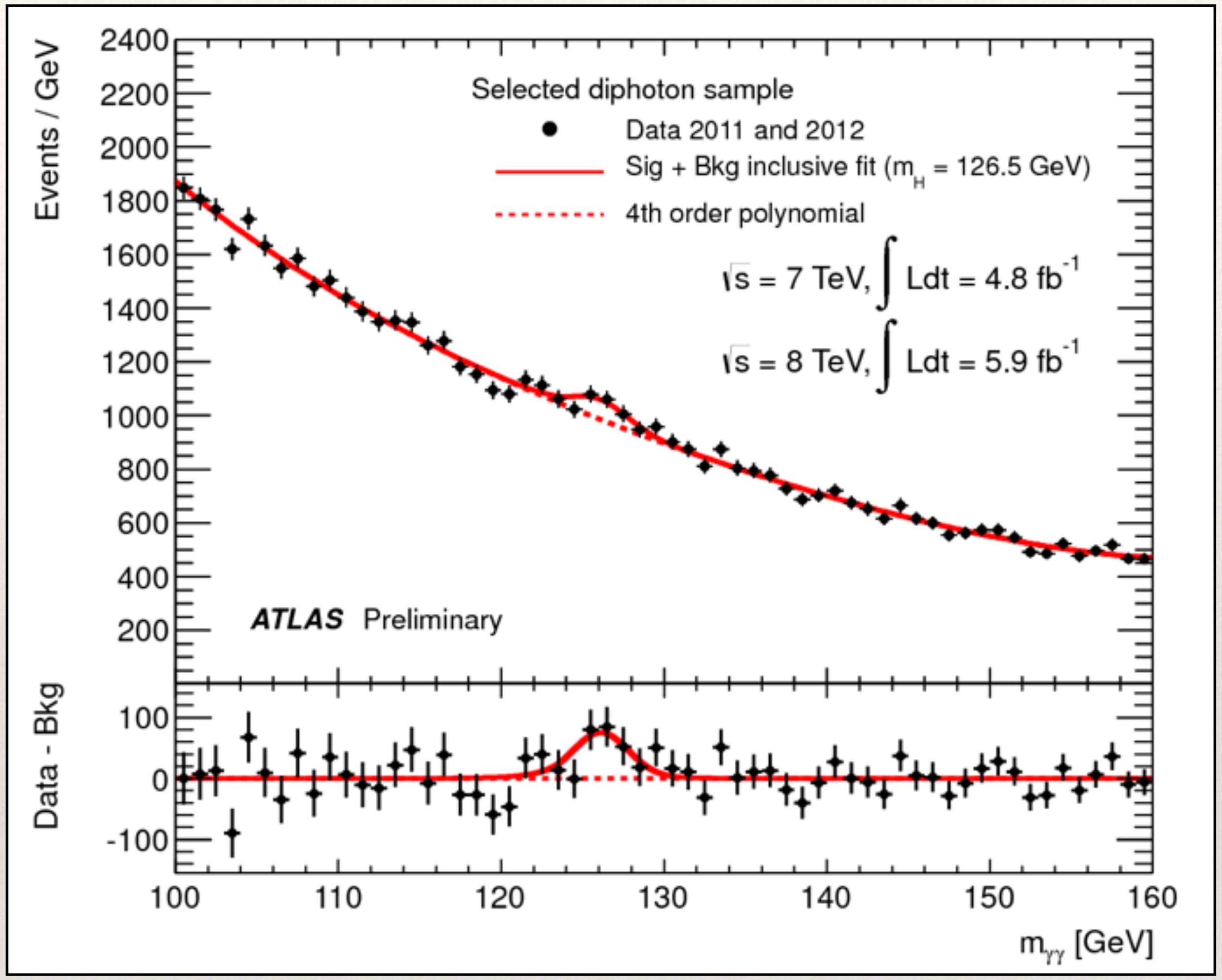
DISCRETE 2022
6-11th November 2022

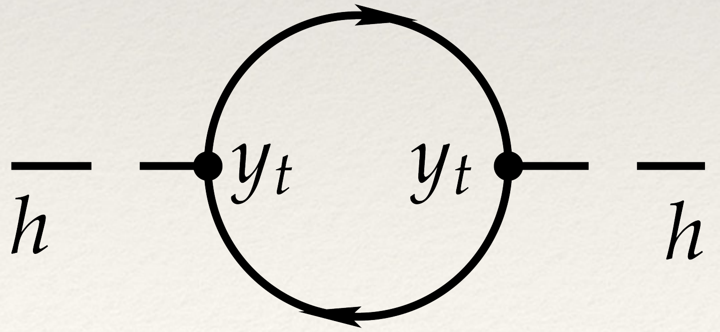
Work in collaboration with
Belén Gavela, Pablo Quílez and Rachel Houtz
(arXiv:2205.09131)

A “light” scalar in the spectrum

- $m_h = 125\,\mathrm{GeV}$, smaller than our expectations if $\exists$ NP.



- This is as m_h is sensitive quadratically to NP scales:



$$\rightarrow \quad \delta m_h^2 \sim y_t^2 \Lambda_{\mathrm{NP}}^2$$

How do we model light scalars?

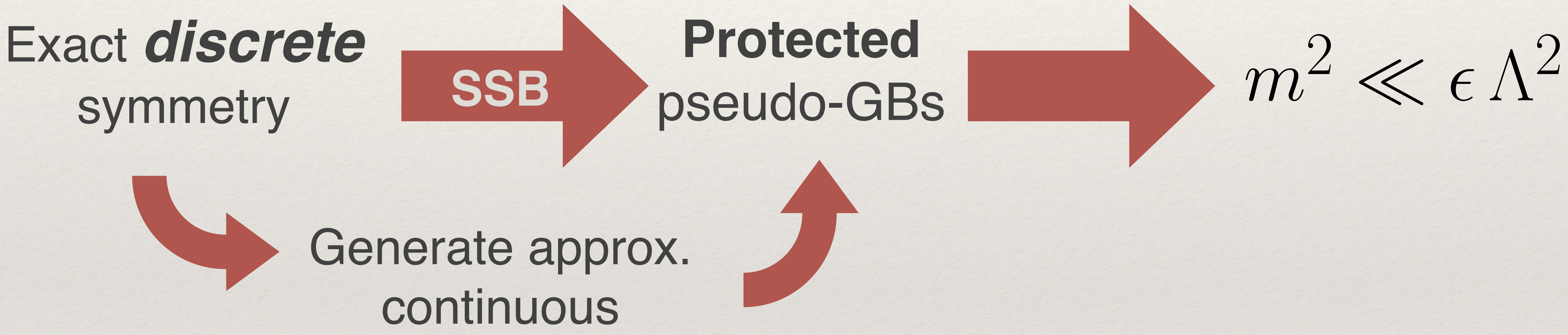
Exact *continuous* symmetry → **SSB** → Goldstone Bosons → **Explicit breaking** → $m^2 \sim \epsilon\,\Lambda^2$

Well-known examples:

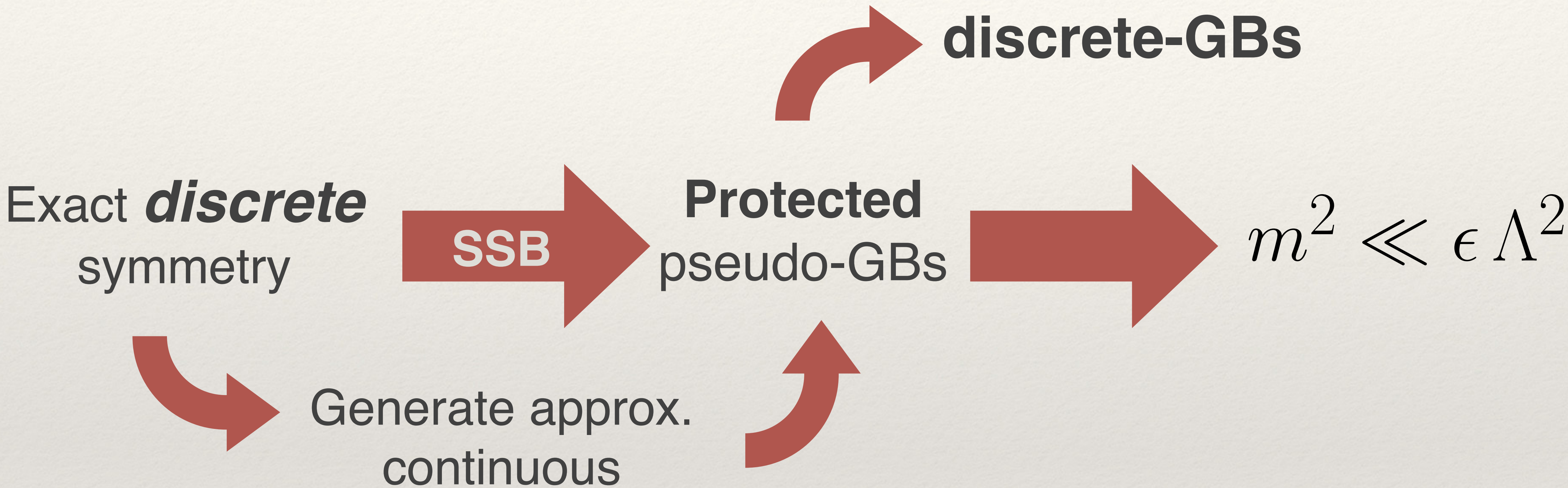
- Pions! $m_\pi \sim \epsilon\,\Lambda_{QCD}$
- The QCD axion $m_a f_a \sim \epsilon\, m_\pi f_\pi$
- Composite Higgs models… $m_h^2 \sim \epsilon\,\Lambda^2$ **Kaplan, Georgi (1984)** **Dugan, Georgi, Kaplan (1985)**

But can we get it better?

What about **discrete** symmetries?



What about **discrete** symmetries?



What about **discrete** symmetries?

Continuous symmetry → **Explicit breaking** → Exact discrete symmetry

Operators in the potential, very tightly constraint

Non-linearly realised discrete symmetries

$\Phi = (\phi_1, \ldots, \phi_N)^T$ in an N-dimensional irrep. of a discrete group D.

Non-linearity $\Rightarrow$ $$\Phi^T \Phi = \phi_1^2 + \phi_2^2 + \cdots + \phi_N^2 = f^2$$

$\Longrightarrow \Lambda_{\mathrm{UV}}^2$ contributions to the potential drop out

How?

A closer look into a interesting case

The EFT of a triplet of A_5

- A generic triplet of A_5: $\Phi \equiv (\phi_1, \phi_2, \phi_3)$ ⟶ Three d.o.f.s

- The *Molien formalism*, $\mathcal{F}_{A_5}(\mathbf{1},\mathbf{3};\lambda) = \dfrac{1+\lambda^{15}}{(1-\lambda^2)(1-\lambda^6)(1-\lambda^{10})}$

- $A_4 \subset A_5$ And its primary invariants are $\begin{cases} \mathcal{I}_2 = \phi_i\phi_i = \phi_1^2 + \phi_2^2 + \phi_3^2 \\ \mathcal{I}_3 = \prod\limits_{i<j<k} \phi_i\phi_j\phi_k = \phi_1\phi_2\phi_3 \\ \mathcal{I}_4 = \sum\limits_i \phi_i^4 = \phi_1^4 + \phi_2^4 + \phi_3^4 \end{cases}$

Also, the non-polynomial combination of order 6:

$$4\mathcal{I}_6^2 = 2\mathcal{I}_4^3 - 5\mathcal{I}_4^2\mathcal{I}_2^2 + 4\mathcal{I}_4\mathcal{I}_2^4 - 36\mathcal{I}_4\mathcal{I}_3^2\mathcal{I}_2 - \mathcal{I}_2^6 + 20\mathcal{I}_3^2\mathcal{I}_2^3 - 108\mathcal{I}_3^4$$

$\Longrightarrow$ can write A_5 invs. In terms of A_4 invs.

The EFT of a triplet of A_5

- A generic triplet of A_5: $\Phi \equiv (\phi_1, \phi_2, \phi_3)$ ⟶ Three d.o.f.s

- The *Molien formalism*, $\mathcal{F}_{A_5}(\mathbf{1},\mathbf{3};\lambda) = \dfrac{1+\lambda^{15}}{(1-\lambda^2)(1-\lambda^6)(1-\lambda^{10})}$

$\Longrightarrow$ three ***primary*** invariants:

$$\mathcal{I}_2^{(3,A_5)} = \mathcal{I}_2 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

⟵ SO(3) invariant

$$\mathcal{I}_6^{(3,A_5)} = 22\mathcal{I}_3^2 + \mathcal{I}_2\mathcal{I}_4 - 2\sqrt{5}\mathcal{I}_6$$

$$\mathcal{I}_{10}^{(3,A_5)} = \mathcal{I}_2^2\mathcal{I}_4 + 38\mathcal{I}_3^2\mathcal{I}_4 - \frac{7}{11}\mathcal{I}_2^3\mathcal{I}_4 - \frac{128}{11\sqrt{5}}\mathcal{I}_2^2\mathcal{I}_6 + \frac{6}{\sqrt{5}}\mathcal{I}_4\mathcal{I}_6$$

} SO(3) breaking, A_5 preserving

The EFT of a triplet of A_5

- Non-linearity $\Longrightarrow \mathcal{I}_2 = \phi_1^2 + \phi_2^2 + \phi_3^2 = f^2$ → ***non-dynamical!!***

- The most general A_5-symmetric potential, $V_{\rm dGB}(\Phi) = f(\mathcal{I}_6, \mathcal{I}_{10})$, …

$$V_{\rm dGB} = f^2\Lambda^2 \sum_{a,b}^{\infty} \left(\frac{\mathcal{I}_6}{f^6}\right)^a \left(\frac{\mathcal{I}_{10}}{f^{10}}\right)^b + g\,(\mathcal{I}_6, \mathcal{I}_{10}), \qquad \Lambda \leq 4\pi f$$

… contains no dimension 2 terms

$\Longrightarrow$ no $\Lambda_{\rm NP}^2$ at this level.

Quadratically divergent 1 -loop corrections all of the form $\propto \Lambda_{\rm UV}^2\,\mathcal{I}_2$

The EFT of a triplet of A_5

- Non-linearity $\Longrightarrow \mathcal{I}_2 = \phi_1^2 + \phi_2^2 + \phi_3^2 = f^2$ → ***non-dynamical!!***

- The most general A_5-symmetric potential, $V_{\rm dGB}(\Phi) = f(\mathcal{I}_6, \mathcal{I}_{10})$, …

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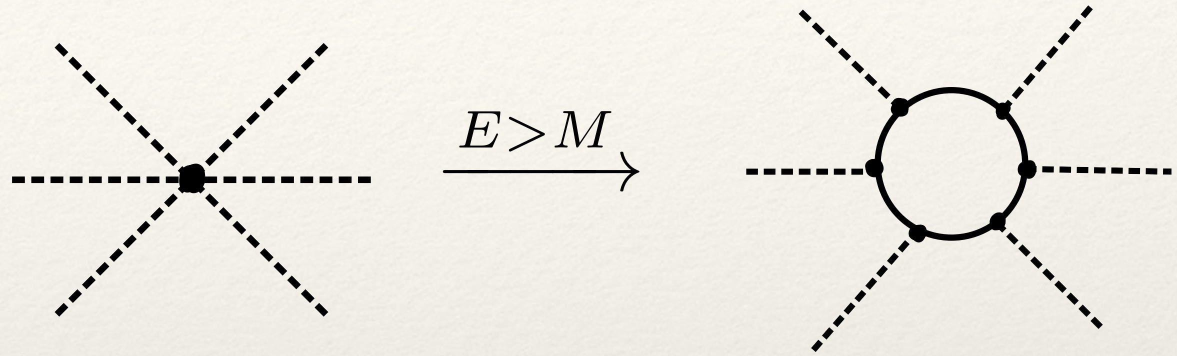
… contains no dimension 2 terms

More suppression features?

Additional $\hat{c}_n$ suppression

- $SO(3)$- breaking interactions mediated by physics above scale $M \gg \Lambda$.



- Integrating out → $\mathcal{L} = \frac{M^4}{16\pi^2} \sum_n \left(y \frac{\Phi}{M} \right)^n = \Lambda^2 f^2 \sum_n y^n \left(\frac{\Lambda}{M} \right)^{n-4} \left(\frac{\Phi}{\Lambda} \right)^n$

$\Longrightarrow \quad \hat{c}_n \sim y^n \left(\frac{\Lambda}{M} \right)^{n-4}$ suppressed so long as:
- $y < 1$
- $M > \Lambda$

$\Longrightarrow \quad m^2_{dGB} \sim y^6 \left(\frac{\Lambda}{M} \right)^2$ for the triplet of A_5

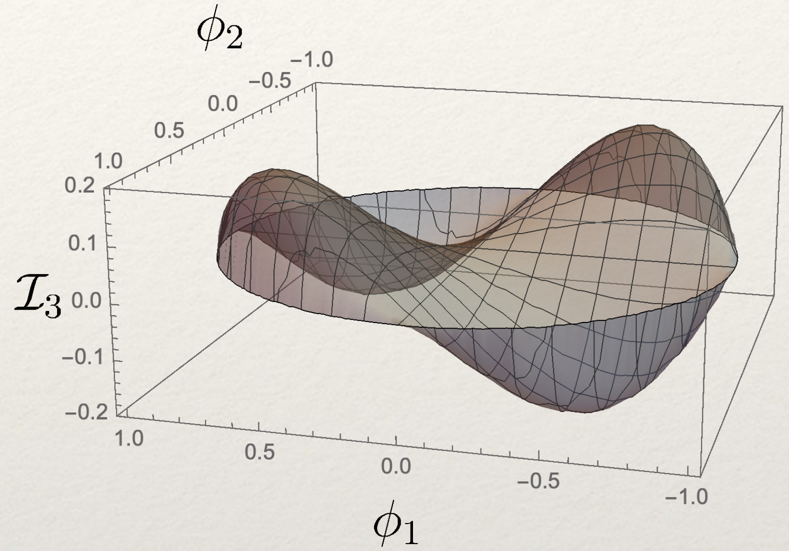
Classifying the extrema of $V_{\rm dGB}(\Phi) = f(\mathcal{I}_6, \mathcal{I}_{10}; \hat{c}_n)$

- Critical points occur when:

$$\frac{\partial V}{\partial \phi_j} = \sum_i \frac{\partial V}{\partial I_i}\frac{\partial I_i}{\partial \phi_j} = 0$$

1) Depends on the $\hat{c}_n$: **model-dependent**

2) Depends on the $\mathcal{I}_n$: **model-independent**

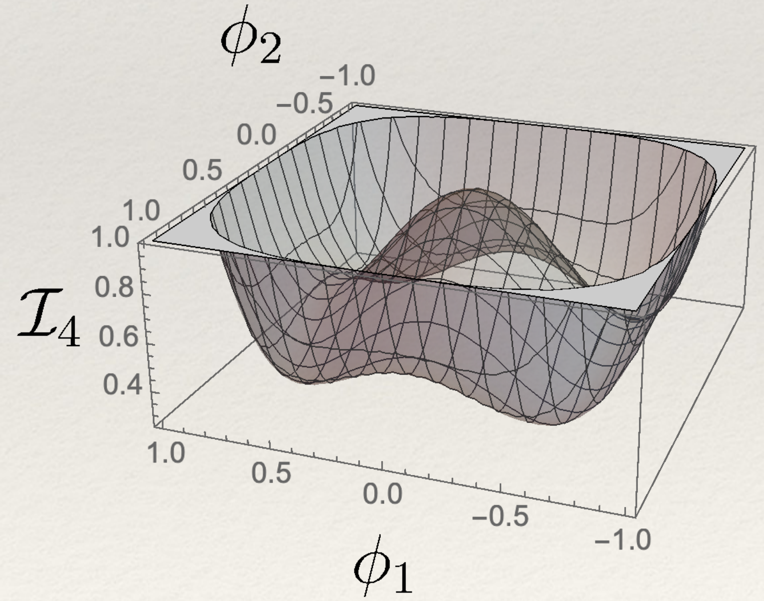


Maximally natural extrema: Extrema of the invariants themselves

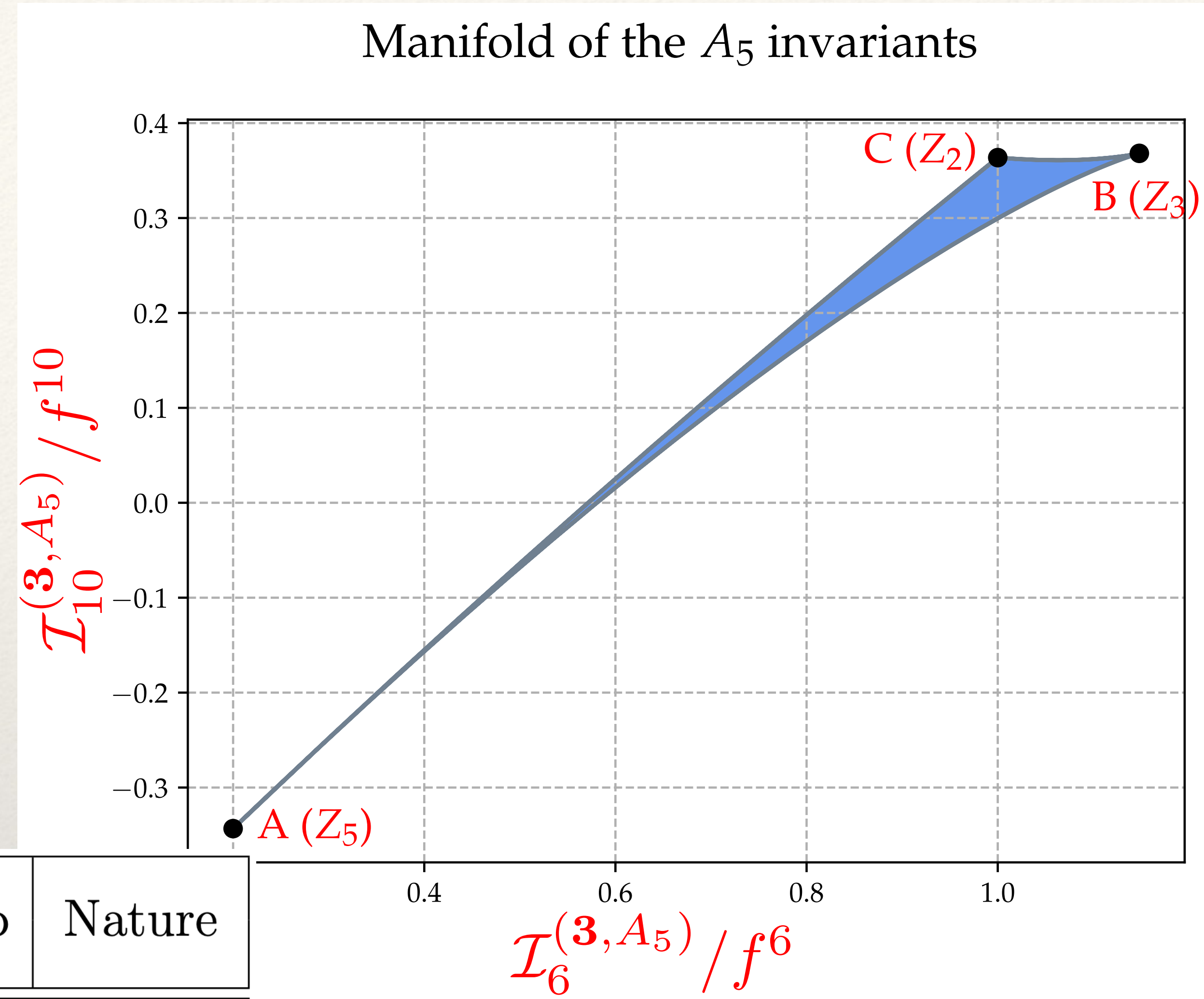
$$\det J = 0 \ , \qquad J_{ij} = \frac{\partial \mathcal{I}_i}{\partial \phi_j} = 0$$

➡ Only depend on the discrete symmetry

➡ Preserved symmetry at the extrema



The Natural Minima for a $\mathbf{3}$ of A_5



Maximally Natural Extrema:
Extrema of the invariants themselves

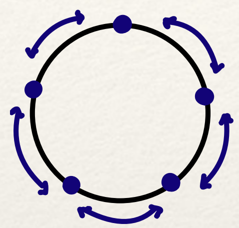
$$\det J = 0\ , \qquad J_{ij} = \frac{\partial \mathcal{I}_i}{\partial \phi_j}$$

Point	$\mathcal{I}_6$	$\mathcal{I}_{10}$	ϕ_1	ϕ_2 (Representatives)	ϕ_3	Little group	Nature
A	$\frac{1}{5}$	$-\frac{472}{1375}$	$\pm\sqrt{\frac{5-\sqrt{5}}{10}}$	0	$\mp\sqrt{\frac{5+\sqrt{5}}{10}}$	Z_5	Minima
B	$\frac{31}{27}$	$\frac{328}{891}$	$\frac{1}{\sqrt{3}}$ $\sqrt{\frac{3+\sqrt{5}}{6}}$	$\frac{1}{\sqrt{3}}$ 0	$\frac{1}{\sqrt{3}}$ $\sqrt{\frac{3-\sqrt{5}}{6}}$	Z_3	Minima
C	1	$\frac{4}{11}$	0 $\frac{(-1+\sqrt{5})}{4}$	0 $-\frac{1}{2}$	± 1 $\frac{(1+\sqrt{5})}{4}$	Z_2	Saddles

The Natural Minima for a $\mathbf{3}$ of A_5

At low energies, around A, Z_5 **-symmetric structure:**

$A_5 \rightarrow Z_5$



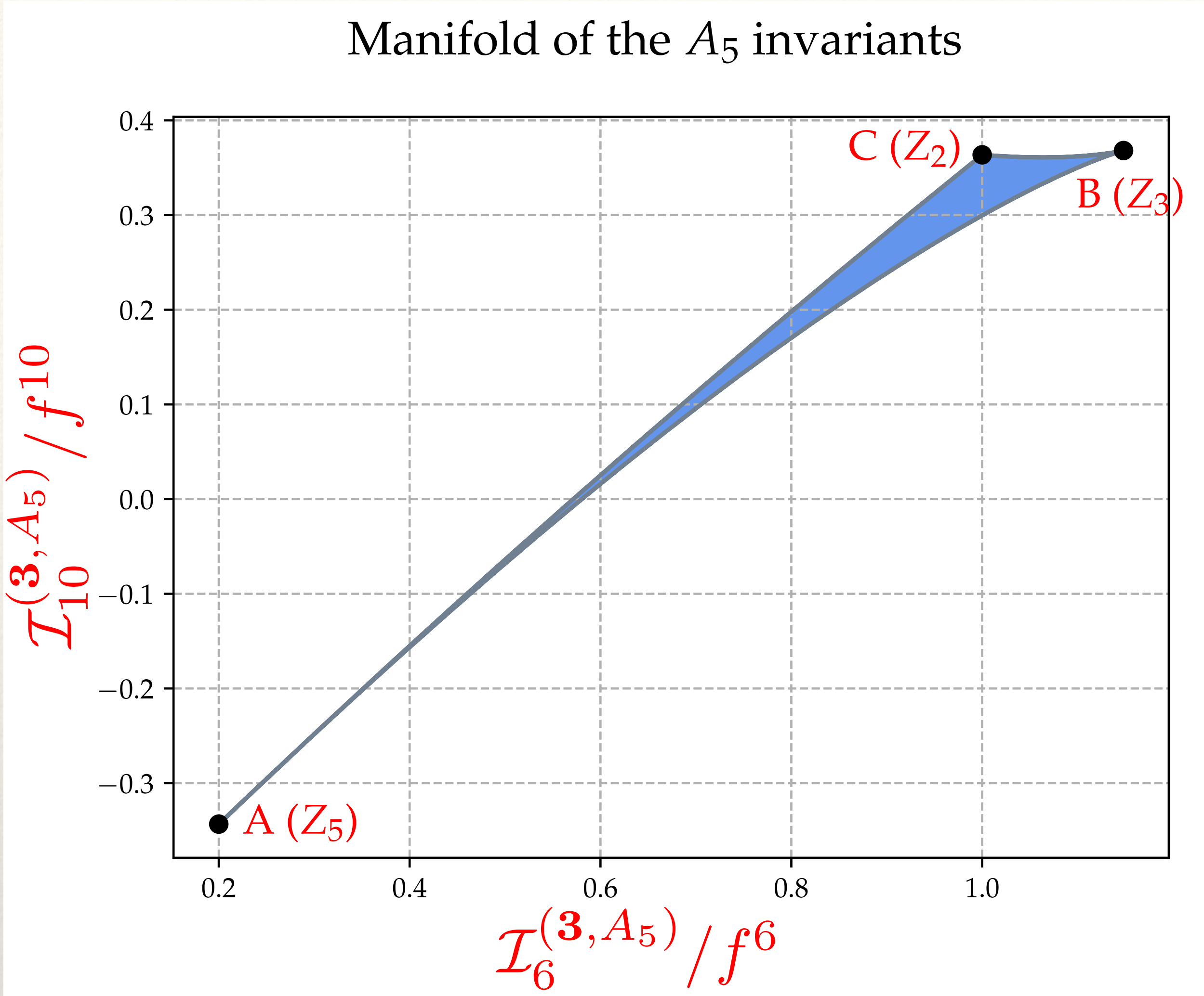
- Primary invariants, $\mathbf{2}$ of Z_5

$$\mathcal{I}_2^{(\mathbf{2},Z_5)} = \pi_1^2 + \pi_2^2$$

$$\mathcal{I}_5^{(\mathbf{2},Z_5)} = \pi_1^5 - 10\pi_1^3\pi_2^2 + 5\pi_1\pi_2^4$$

- Dominant invariant after SSB:

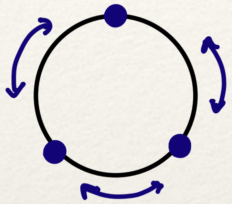
$$\mathcal{I}_6^{(3,A_5)} = \frac{32}{5} f^4 \left[\frac{f^2}{32} + \left(\pi_1^2 + \pi_2^2\right) - \frac{31}{12 f^2} \left(\pi_1^2 + \pi_2^2\right)^2 - \frac{1}{4 f^3} \left(\pi_1^5 - 10\pi_1^3\pi_2^2 + 5\pi_1\pi_2^4\right) \right]$$

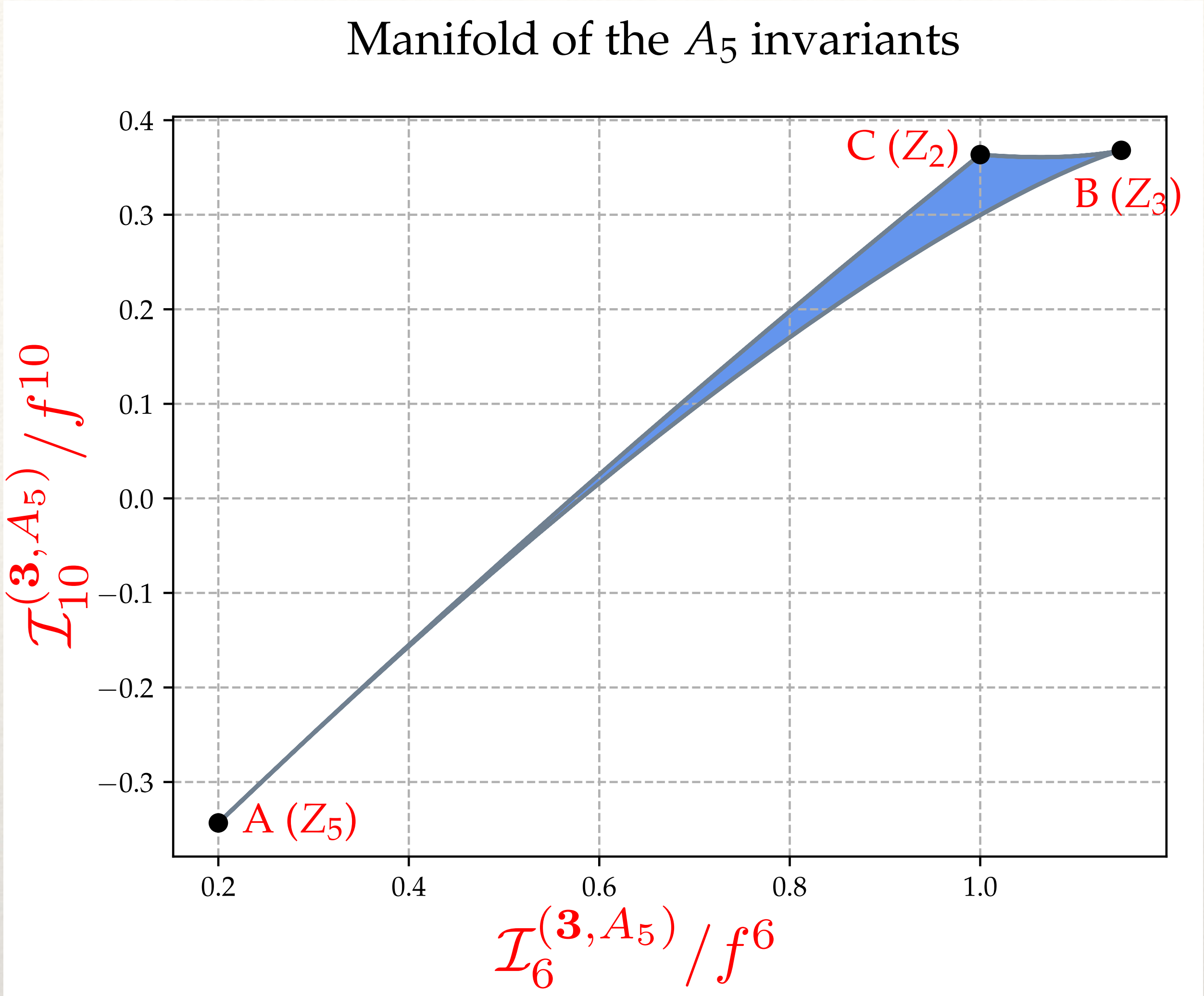


The Natural Minima for a $\mathbf{3}$ of A_5

At low energies, around B, Z_3**-symmetric structure:**

$A_5 \to Z_3$





- Primary invariants, $\mathbf{2}$ of Z_3

$$\mathcal{I}_2^{(2,Z_3)} = \pi_1^2 + \pi_2^2$$

$$\mathcal{I}_3^{(2,Z_3)} = \pi_1^3 - 3\pi_1\pi_2^2$$

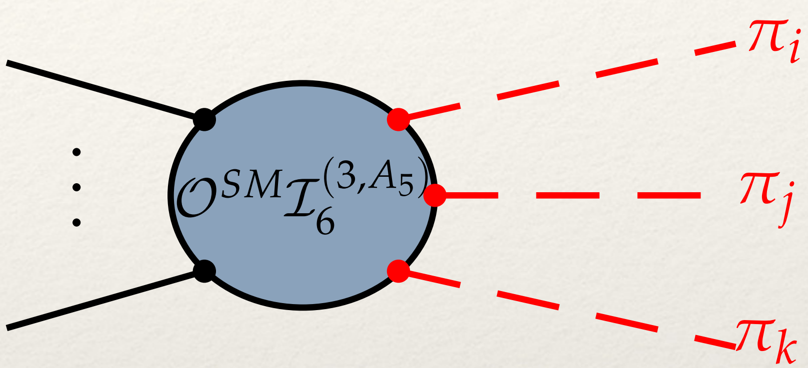
- Dominant invariant after SSB:

$$\mathcal{I}_6^{(\mathbf{3},A_5)} = \frac{32}{9}\, f^4 \left[\frac{31}{96} f^2 - \left(\pi_1^2 + \pi_2^2\right) + \frac{10\sqrt{2}}{24 f}\left(\pi_1^3 - 3\pi_1\pi_2^2\right) + \frac{\sqrt{30}}{4 f}\left(\pi_2^3 - 3\pi_1^2\pi_2\right) + \frac{31}{12 f^2}\left(\pi_1^2 + \pi_2^2\right)^2\right]$$

The Natural Minima for a $\mathbf{3}$ of A_5

- If the SM is a singlet of A_5:
  - ➡ Degeneracy
  - ➡ Simultaneous production
  - ➡ Specific production rates

- At low energies, around B, Z_3**-symmetric structure:**



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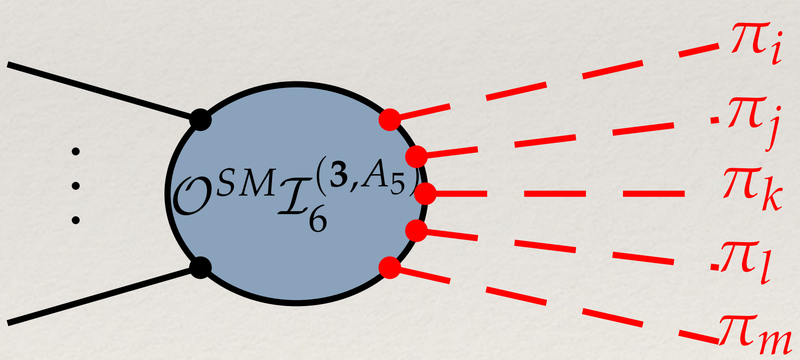
- At low energies, around A, Z_5 **-symmetric structure:**



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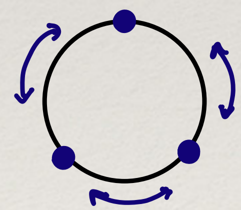
Conclusion

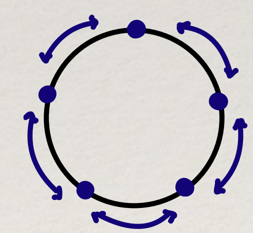
- **discrete Goldstone bosons** have enhanced protection from quadratic divergences → no Λ^2_{NP}

- The natural minima retain explicit symmetry → distinctive phenomenology
  - Degeneracy
  - Simultaneous production
  - Specific production rates

$A_5 \to Z_3$



$A_5 \to Z_5$



- Many possibilities, e.g., $\mathbf{3}$ of A_5, $\mathbf{3}$ or $\mathbf{3'}$ of S_4, $\mathbf{4}$ of A_5 explored in our paper

Thank you

Specific UV completion

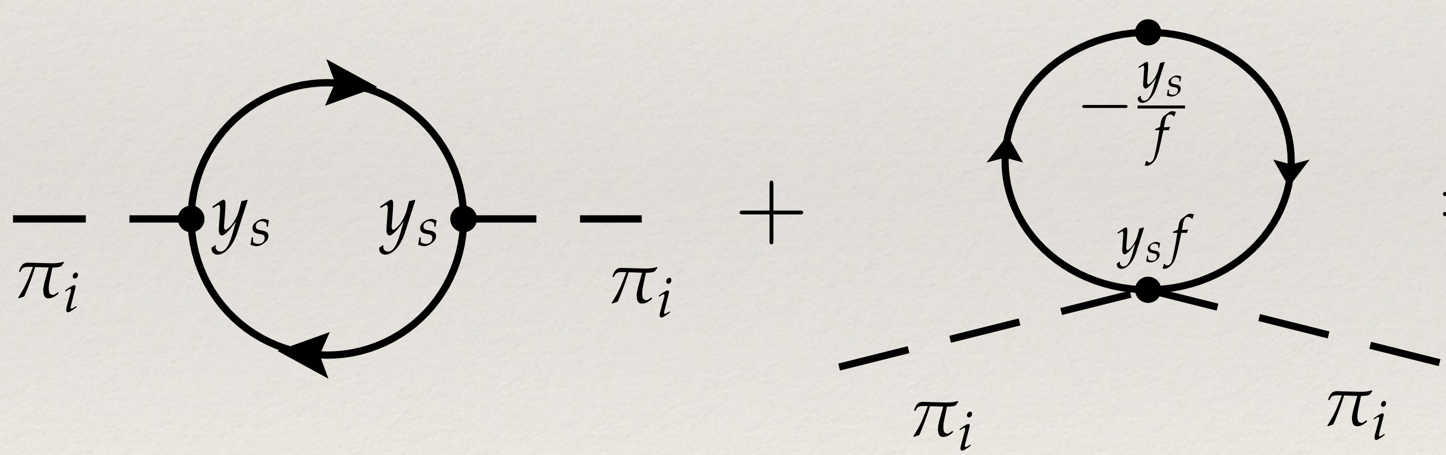
[Das, Hook, 2006.10767]

Upon SSB, $\left[SO(3) \rightarrow SO(2)\right]$: $\Phi(\pi_1, \pi_2) = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \exp\left[\frac{1}{f}\begin{pmatrix} 0 & 0 & \pi_1 \\ 0 & 0 & \pi_2 \\ -\pi_1 & -\pi_2 & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$

The interaction Lagrangian,

$$\mathcal{L}_{\rm int} = y_s\pi_1\left(\bar{\Psi}_2\Psi_3 + \bar{\Psi}_3\Psi_2\right) + y_s\pi_2\left(\bar{\Psi}_3\Psi_1 + \bar{\Psi}_1\Psi_3\right) + y_s f\left(1 - \frac{1}{2}\frac{\pi_1^2 + \pi_2^2}{f^2}\right)\left(\bar{\Psi}_1\Psi_2 + \bar{\Psi}_2\Psi_1\right)$$

$\delta m_h^2 \ni$



$= 0$

A cancellation similar to that in the Little Higgs model ($\ni Z_2$)

Arkani-Hamed, Cohen, Gregoire, Wacker (2002)
Arkani-Hamed, Cohen, Katz, Nelson (2002)

Specific UV completion

[Das, Hook, 2006.10767]

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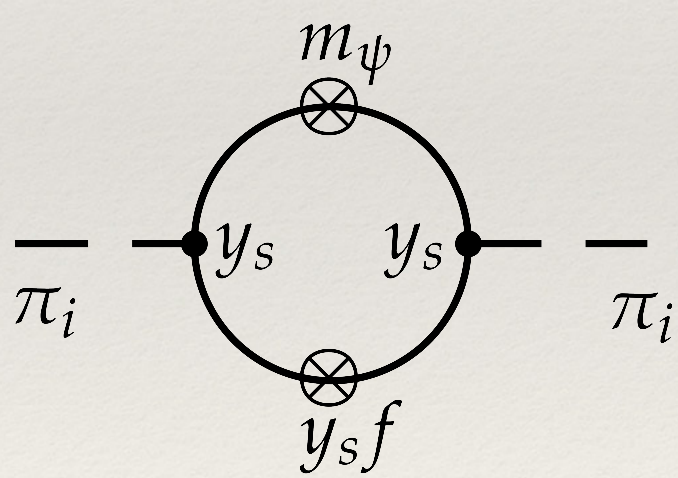
The interaction Lagrangian,

$$\mathcal{L}_{\rm int} = y_s \pi_1 \left(\bar{\Psi}_2 \Psi_3 + \bar{\Psi}_3 \Psi_2\right) + y_s \pi_2 \left(\bar{\Psi}_3 \Psi_1 + \bar{\Psi}_1 \Psi_3\right) + y_s f \left(1 - \frac{1}{2}\frac{\pi_1^2 + \pi_2^2}{f^2}\right)\left(\bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1\right)$$

$\delta m_h^2 \ni$



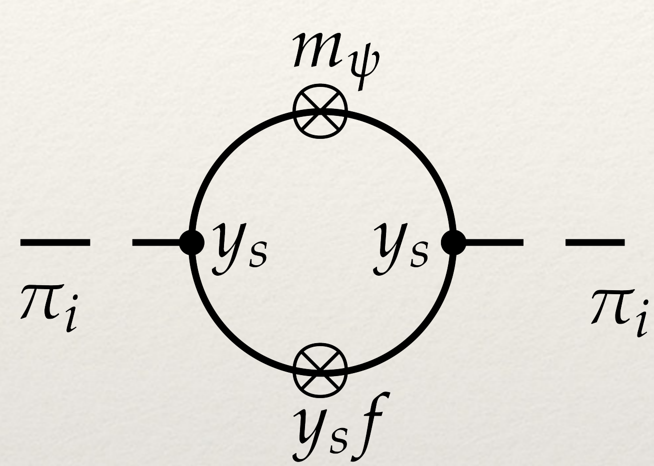
$\sim y^3 f\, m_\psi \log\left(\frac{m_\psi}{\mu}\right)$

Specific UV completions

- ϕ, ψ, triplets of A_4, $\mathcal{L}_{\rm int} = y_a \epsilon^{ijk} \bar{\psi}_i \psi_j \phi_k + y_s \tilde{\epsilon}^{ijk} \bar{\psi}_i \psi_j \phi_k \quad \tilde{\epsilon}_{ijk} = |\epsilon_{ijk}|$

[Das, Hook, 2006.10767]

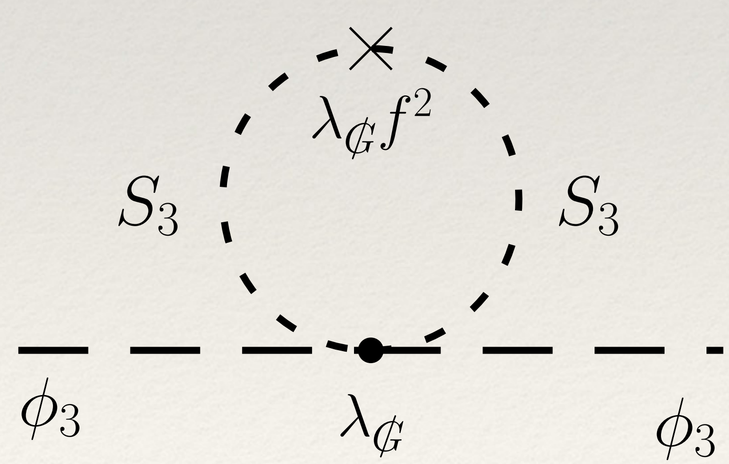
$\delta m_h^2 \ni$



$\sim y^3 f\, m_\psi \log\left(\frac{m_\psi}{\mu}\right)$

- ϕ, S , triplets of A_4, $\mathcal{L}_{\rm int} = \frac{\lambda}{4}\left(\phi_1^2 S_1^2 + \phi_2^2 S_2^2 + \phi_3^2 S_3^2\right)$

$\delta m_h^2 \ni$



$\sim \lambda^2 f^2 \log\left(\frac{m_S}{f}\right)$

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- The *Molien formalism*, $\mathcal{F}_{A_4}(\mathbf{1},\mathbf{3};\lambda) = \dfrac{1+\lambda^6}{(1-\lambda^2)(1-\lambda^3)(1-\lambda^4)}$

$\Longrightarrow$ three ***primary*** invariants:

$$\mathcal{I}_2 = \phi_i\phi_i = \phi_1^2 + \phi_2^2 + \phi_3^2$$ ← SO(3) invariant

$$\mathcal{I}_3 = \prod_{i<j<k} \phi_i\phi_j\phi_k = \phi_1\phi_2\phi_3$$

$$\mathcal{I}_4 = \sum_i \phi_i^4 = \phi_1^4 + \phi_2^4 + \phi_3^4$$

($\mathcal{I}_3$, $\mathcal{I}_4$): SO(3) breaking, A_4 preserving

$\Longrightarrow$ one ***secondary*** invariant: a non-polynomial combination of the primaries. The *syzygy* is:

$$4\mathcal{I}_6^2 = 2\mathcal{I}_4^3 - 5\mathcal{I}_4^2\mathcal{I}_2^2 + 4\mathcal{I}_4\mathcal{I}_2^4 - 36\mathcal{I}_4\mathcal{I}_3^2\mathcal{I}_2 - \mathcal{I}_2^6 + 20\mathcal{I}_3^2\mathcal{I}_2^3 - 108\mathcal{I}_3^4$$