Discrete Goldstone Bosons

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In Collaboration with Victor Enguita-Vileta and Belen Gavela (IFT),
Pablo Quilez (UCSD), arXiv:2205.09131

Collaborators on this Work

Giving a talk on Wednesday

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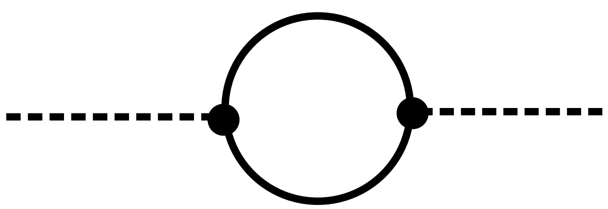
Let There Be Light

Higgs Naturalness

- The leading quantum correction to the Higgs mass:



$$\delta m_h^2 \sim y_t^2 \Lambda^2$$

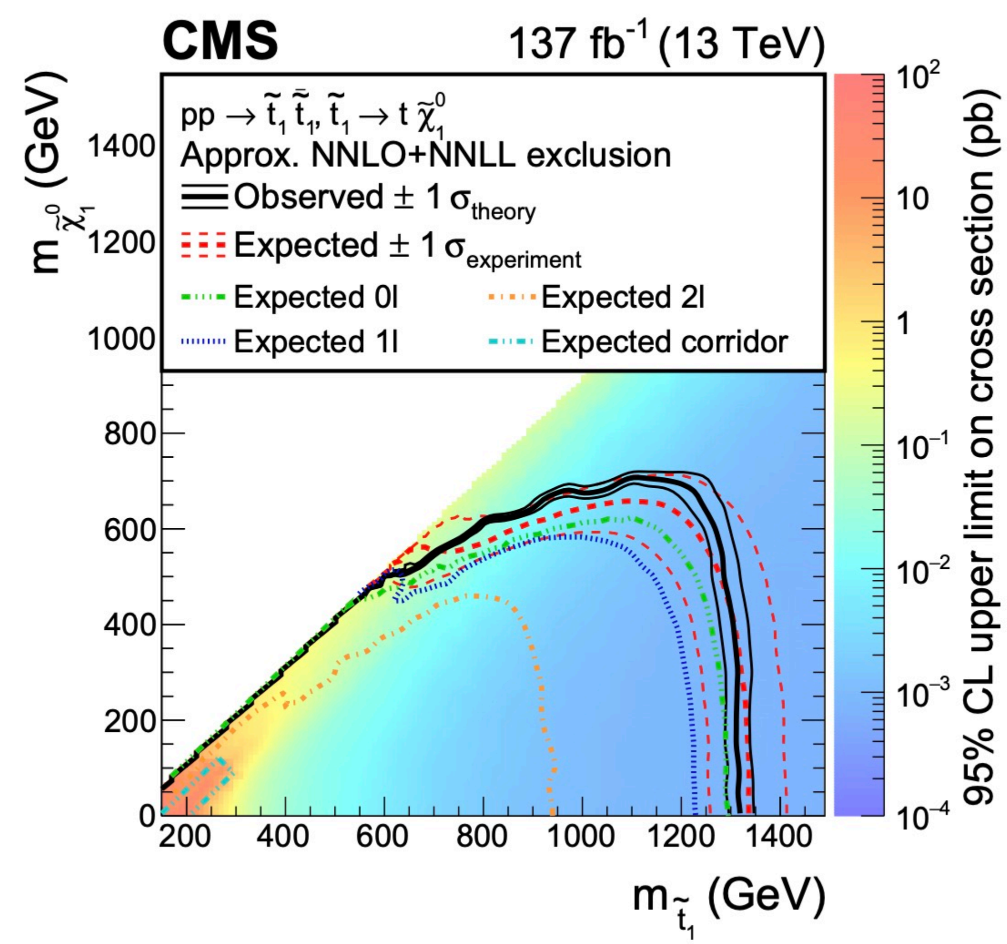
- Expect new physics at the TeV scale

Papucci, Ruderman, Weiler, arXiv:1110.6926

- Why is $m_h^2 \ll \Lambda_{\mathrm{NP}}^2$?

- More generally, classify ways to separate:

$$m_{\mathrm{scalar}}^2 \longleftrightarrow \Lambda_{\mathrm{NP}}^2$$

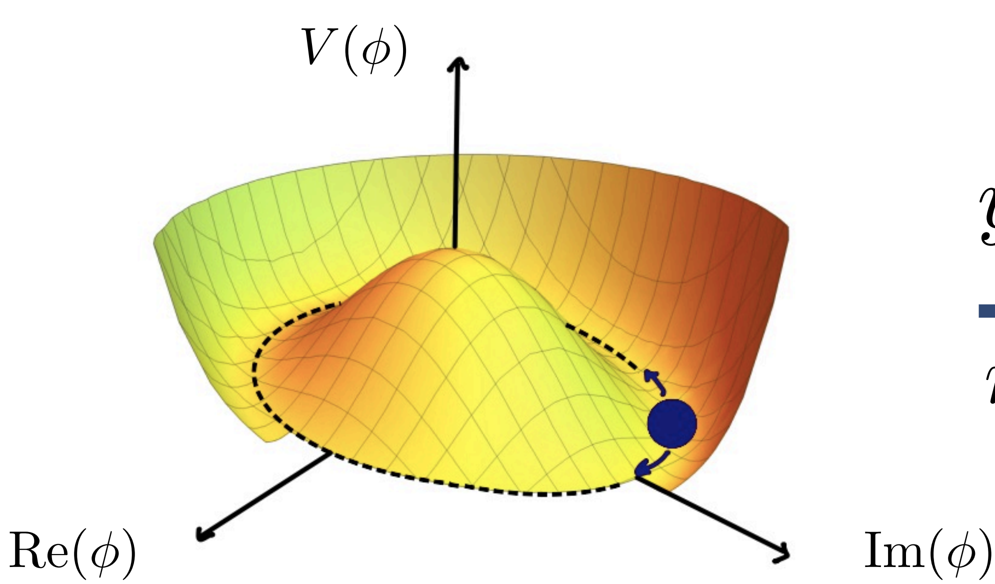


CMS, arXiv:2107.10892

Pseudo-Goldstone Bosons

- Many examples of pGBs:
    - SM pions $m_\pi \ll \Lambda_{\rm QCD}$
    - The axion $m_a f_a \approx m_\pi f_\pi$
- Many models identifying the Higgs as a pGB
  Kaplan, Georgi (1984)
  Dugan, Georgi, Kaplan (1985)
- Technically natural

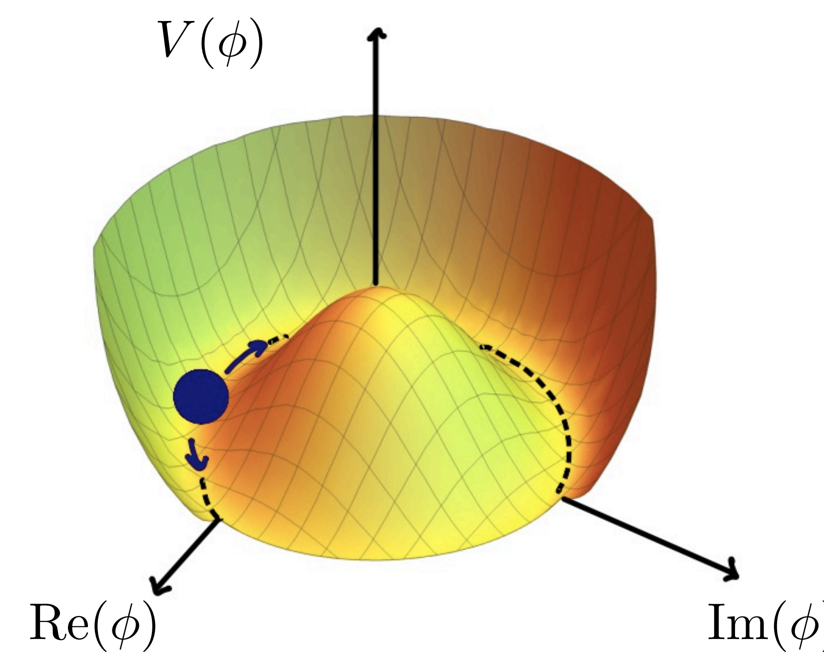
<u>Symmetry explicitly broken</u>



$y_{\rm break} \to 0$

$m_{\rm pGB} \to 0$

<u>Exact Symmetry restored</u>



pGB's and Discrete Symmetries

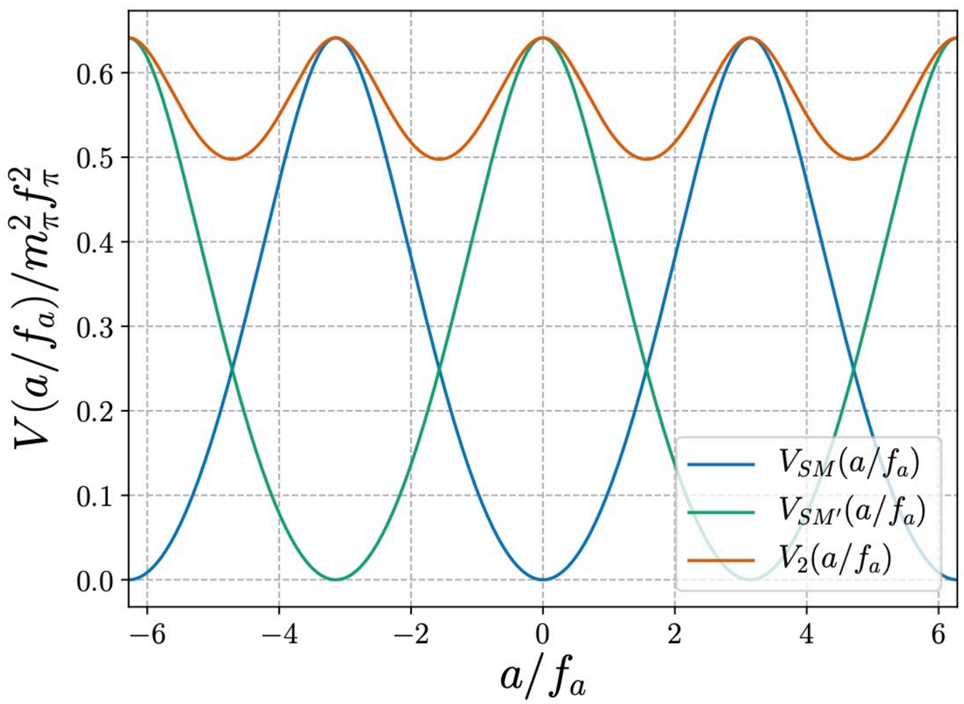
- Additional Z_N symmetries can enhance the mass protection of axionlike pGB fields
- Higher N leads to greater suppression:

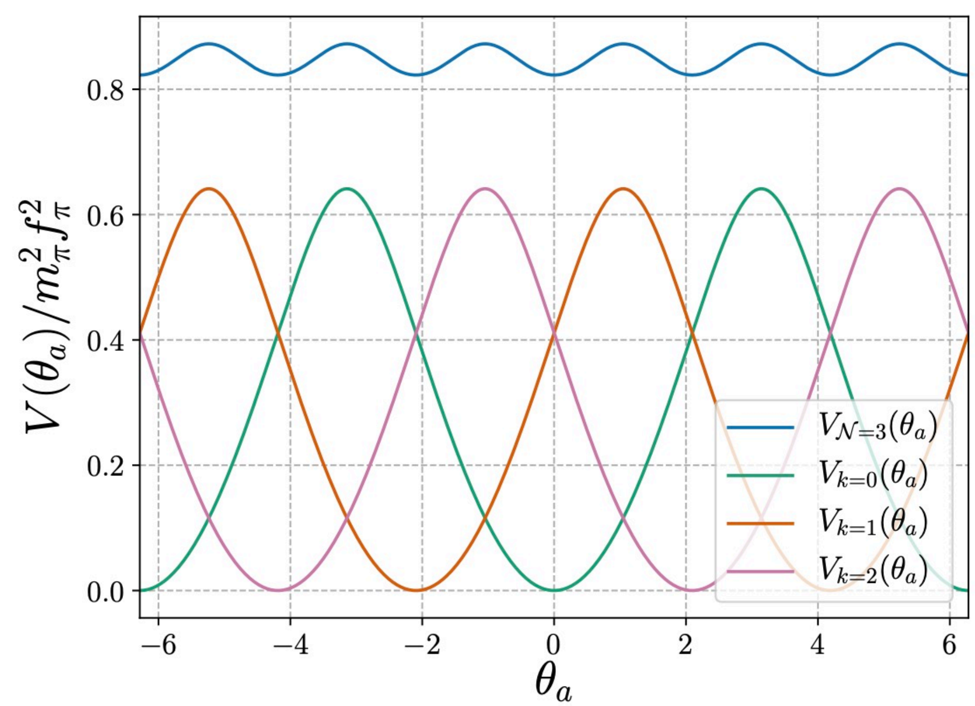
Hook, arXiv:1802.10093

Di Luzio, Gavela, Quilez, Ringwald, arXiv:2102.00012

Das, Hook, arXiv:2006.10767

$$Z_N \xrightarrow{N\to\infty} U(1) \qquad m_a \to 0$$





Plots lifted from Di Luzio, Gavela, Quilez, Ringwald, arXiv:2102.00012

Non-Abelian Discrete A_4

- A_4 is the simplest non-Abelian discrete group with a three-dimensional irreducible representation

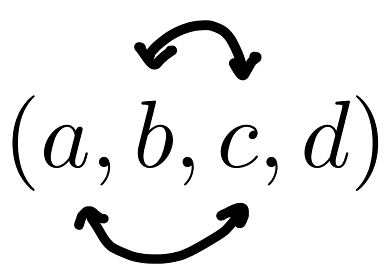
  ➡ useful in flavor model building, can contain three generations

Ma, Rajasekaran (2001)

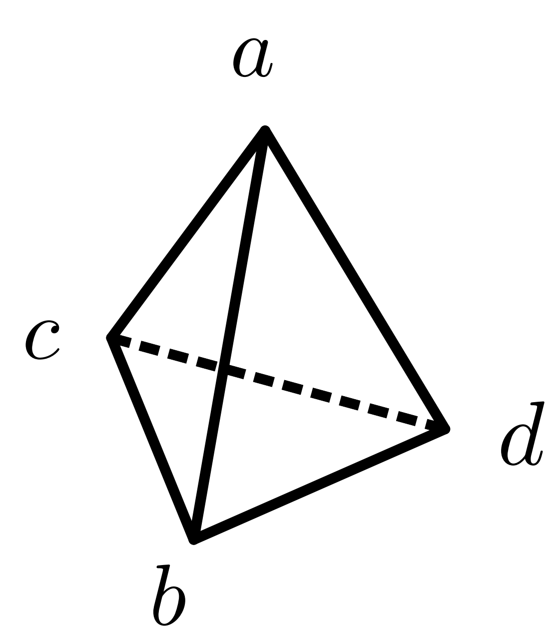
Babu, Ma, Valle (2002)

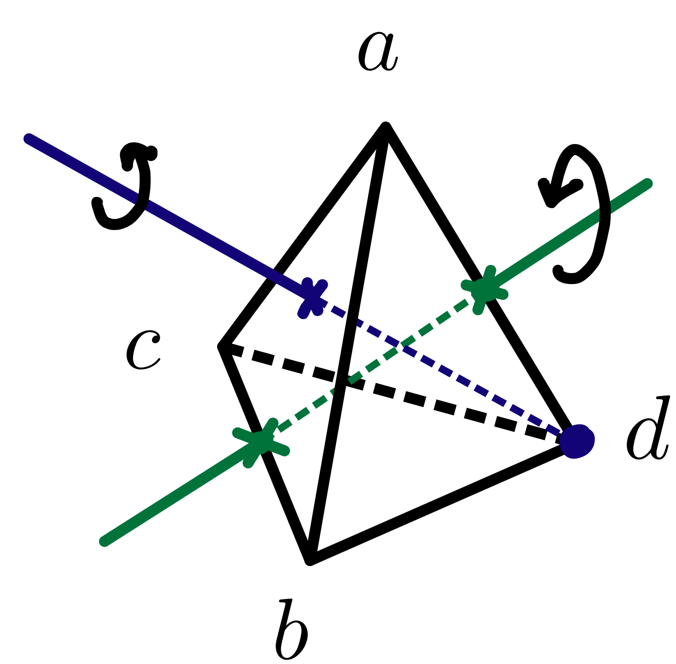
Altarelli, Feruglio (2005)

- A_4 group elements: all even permutations of four objects



Even # of swaps





Ishimori, *et al*, arXiv:1003.3552

Properties of the A_4 Group

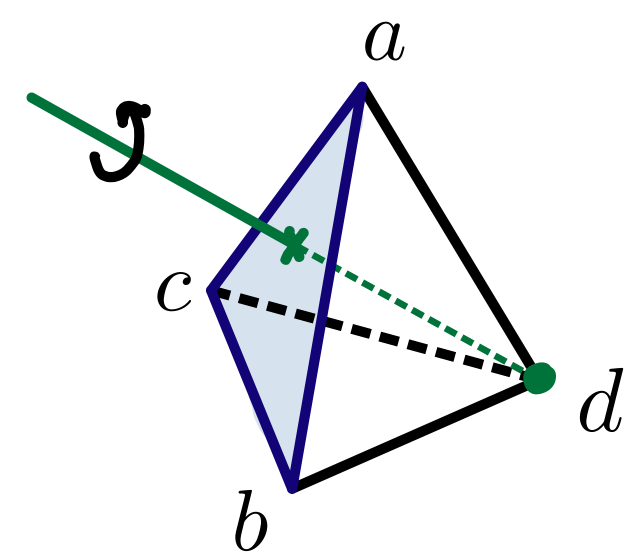
<u>Even permutations of four elements</u>

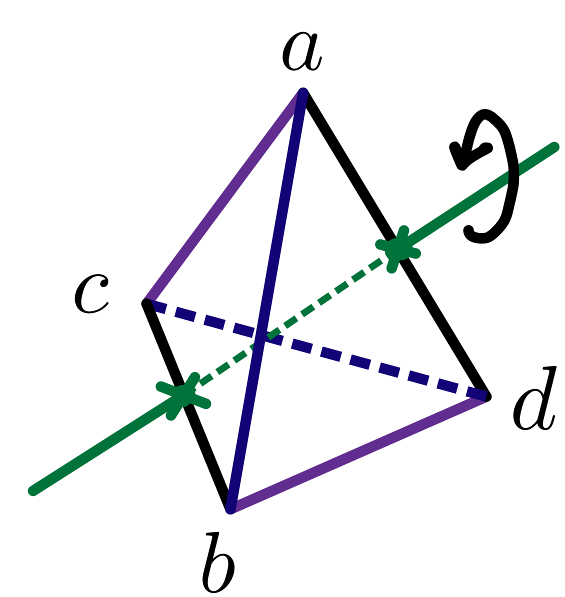
- t generator: $(a, b, c, d) \to (b, c, a, d)$
  - Eight "face rotations": (t, sts, st, ts)

    $(t^2, tst, st^2, t^2 s)$
- s generator: $(a, b, c, d) \to (b, a, d, c)$
  - Three "double flips": $(s, t^2 s, tst^2)$

➡ $4!/2 = 12$ total group elements

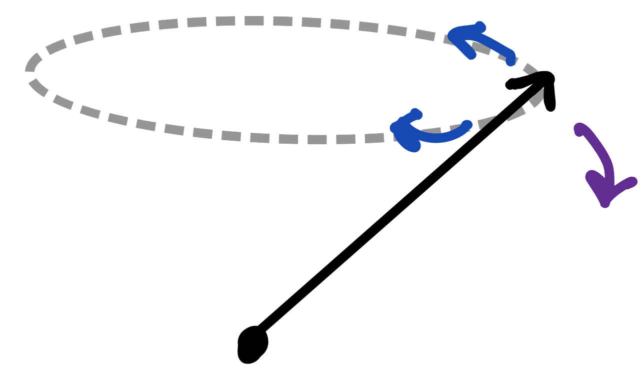
<u>Symmetries of the tetrahedron</u>





Properties of the A_4 Group

- Irreducible representations of A_4:
    - Trivial singlet (invariant)
    - Nontrivial singlets (pick up phases)
    - Triplets



- Of interest to discrete symmetry model builders:

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3'$$

$$A_3 \times B_3 \ni \begin{pmatrix} \{A_y, B_z\} \\ \{A_z, B_x\} \\ \{A_x, B_y\} \end{pmatrix}_3 + \begin{pmatrix} [A_y, B_z] \\ [A_z, B_x] \\ [A_x, B_y] \end{pmatrix}_3$$

$$\tilde{\epsilon}_{ijk} = |\epsilon_{ijk}|$$

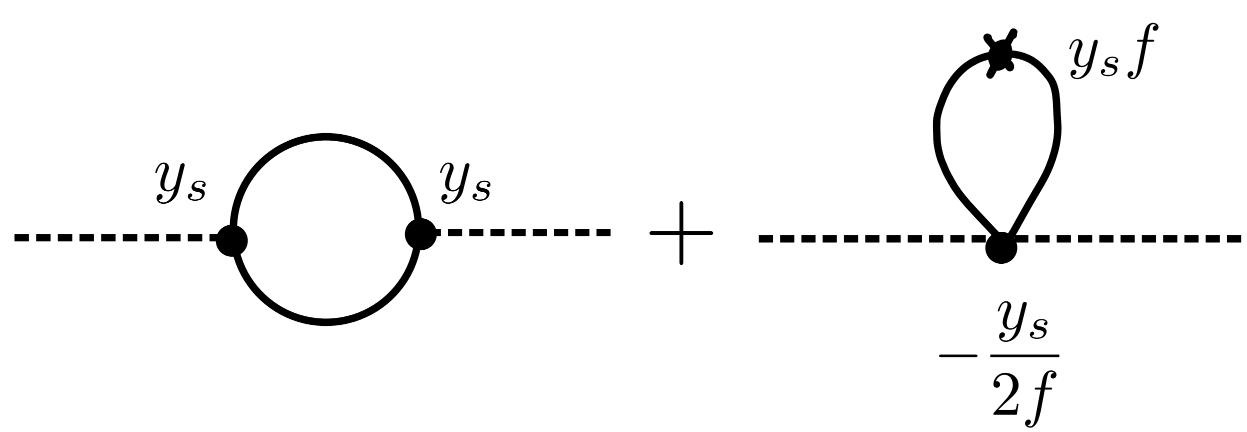
$$\tilde{\epsilon}_{ijk} A_i B_j \qquad \epsilon_{ijk} A_i B_j$$

Nonlinearly Realized Discrete A_4

Das, Hook, arXiv:2006.10767

- The quadratically divergent mass contributions stemming from $SO(3)$-breaking terms cancel:

$$\mathcal{L}_{\rm int} = y_s \pi_1 \left(\bar{\Psi}_2 \Psi_3 + \bar{\Psi}_3 \Psi_2\right) + y_s \pi_2 \left(\bar{\Psi}_3 \Psi_1 + \bar{\Psi}_1 \Psi_3\right) + y_s f \left(1 - \frac{1}{2}\frac{\pi_1^2 + \pi_2^2}{f^2}\right)\left(\bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1\right)$$

$\delta m_h^2 \ni$ 

- Similar to Little Higgs models

Arkani-Hamed, Cohen, Gregoire, Wacker (2002)

Arkani-Hamed, Cohen, Katz, Nelson (2002)

General Nonlinear Discrete Symmetries

- Consider a general scalar field Φ in an irreducible m-dimensional real representation of some discrete group D :

$$\Phi \equiv (\phi_1, \phi_2, ..., \phi_m)$$

- Nonlinearity constraint: $\Phi^T \Phi = \phi_1^2 + \phi_2^2 + ... + \phi_m^2 = f^2$

- Reduces the number of dof's by 1, resulting in a set of $m-1$ light spin-0 particles

    - These are the **discrete Goldstone Bosons (dGB)**

    - If D is embedded in a continuous group G, these are a class of pseudo-Goldstone bosons

The EFT for A_4 dGBs

- Consider a scalar field in the triplet of A_4: $\Phi \equiv (\phi_1, \phi_2, \phi_3)$
- The full A_4 invariant potential is a function of the primary invariants:

$$\mathcal{I}_2 = \phi_i \phi_i = \phi_1^2 + \phi_2^2 + \phi_3^2$$

← SO(3) invariant

$$\mathcal{I}_3 = \prod_{i<j<k} \phi_i \phi_j \phi_k = \phi_1 \phi_2 \phi_3$$

$$\mathcal{I}_4 = \sum_i \phi_i^4 = \phi_1^4 + \phi_2^4 + \phi_3^4$$

} SO(3) breaking

- We can expand this out in terms of invariants as: $(\Lambda \leq 4\pi f)$

$$V_{\mathrm{dGB}} = f(\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4) = f^2 \Lambda^2 \left[\hat{c}_3 \frac{\mathcal{I}_3}{f^3} + \hat{c}_4 \frac{\mathcal{I}_4}{f^4} + \hat{c}_6 \frac{\mathcal{I}_6}{f^6} + \hat{c}_7 \frac{\mathcal{I}_7}{f^7} + \ldots \right],$$

- Below Λ, the nonlinearity constraint holds: $\mathcal{I}_2 = f^2$

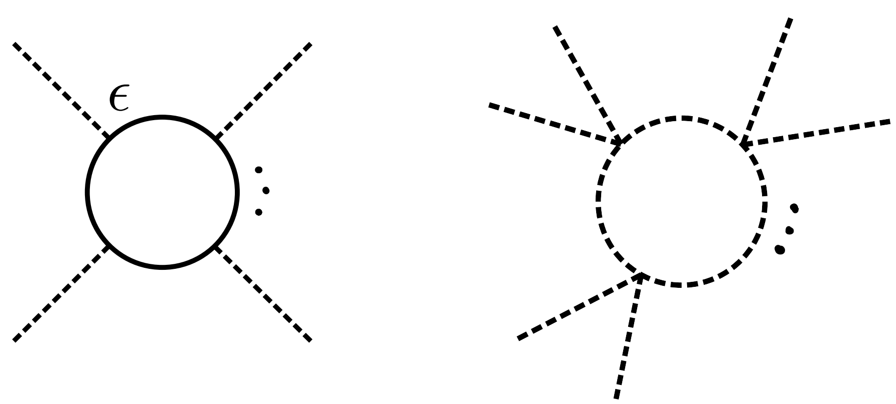
The dGB Potential

- The most general potential:

$$V_{\text{dGB}} = f^2\Lambda^2\left[\hat{c}_3\frac{\mathcal{I}_3}{f^3} + \hat{c}_4\frac{\mathcal{I}_4}{f^4} + \hat{c}_6\frac{\mathcal{I}_6}{f^6} + \hat{c}_7\frac{\mathcal{I}_7}{f^7} + \ldots\right]$$

- If the $\hat{c}_n$ are all $\mathcal{O}(1)$, then all terms will contribute equally
- It is easy, however, to arrange for lower order terms to dominate
- If the invariant operators are generated by renormalizable interactions:

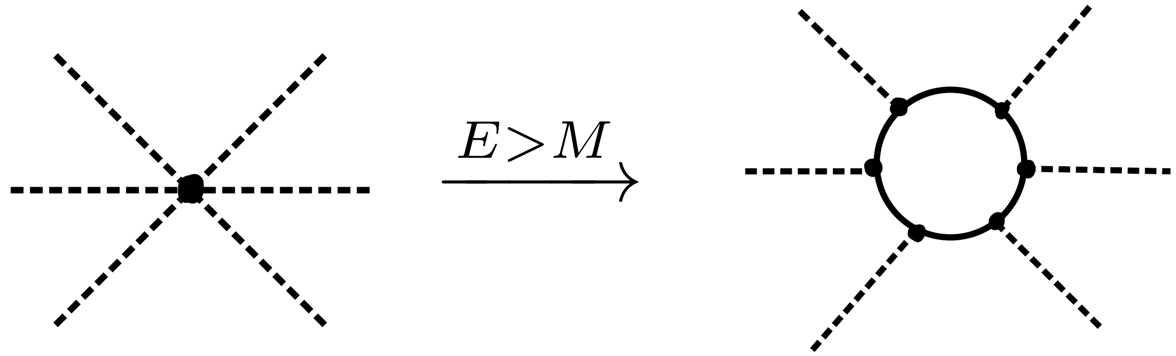
$$\hat{c}_n \sim \epsilon^n$$



- Being a *little* less agnostic about the UV theory can allow us to talk about scales that are not f

Additional $\hat{c}_n$ Suppression

- Consider that higher dimensional operator interactions are mediated by a fermion with mass M



$$\mathcal{L} = \frac{M^4}{16\pi^2}\sum_n \left(y\frac{\Phi}{M}\right)^n = \Lambda^2 f^2 \sum_n y^n \left(\frac{\Lambda}{M}\right)^{n-4}\left(\frac{\Phi}{\Lambda}\right)^n$$

- Allows us to estimate the sizes of $\hat{c}_n$: $\hat{c}_n \sim y^n \left(\frac{\Lambda}{M}\right)^{n-4}$

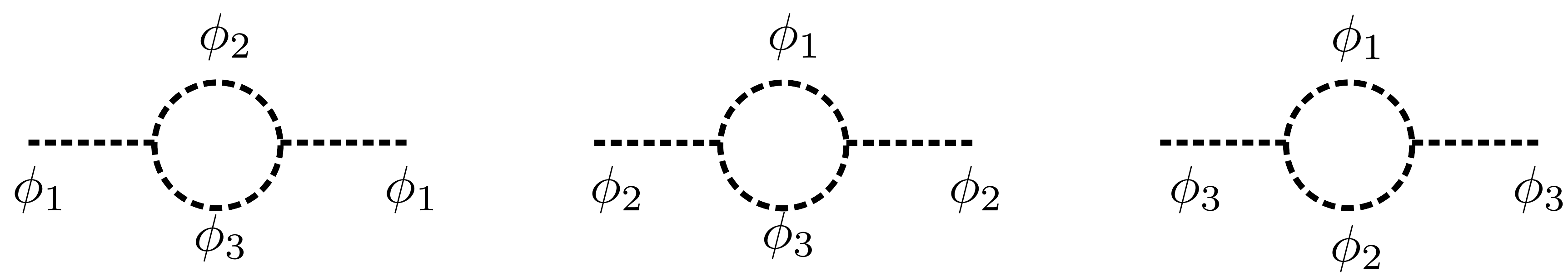
- We can say that the $n > 4$ terms will be subdominant so long as either:
  - $y < 1$
  - $M > \Lambda$

(for $\mathcal{I}_3 > \mathcal{I}_4$, we must assume $y < 1$)

The dGB Potential

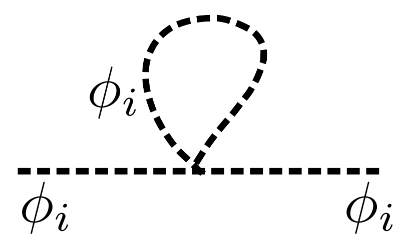
- Invariant interactions generate seemingly destabilizing mass contributions:

$\mathcal{I}_3$:



$\propto \mathcal{I}_2$

$\mathcal{I}_4$:



$\propto \mathcal{I}_2$

- These contributions are rendered harmless by the nonlinearity constraint

$$\Phi^T \Phi = \phi_1^2 + \phi_2^2 + \phi_3^2 = f^2$$

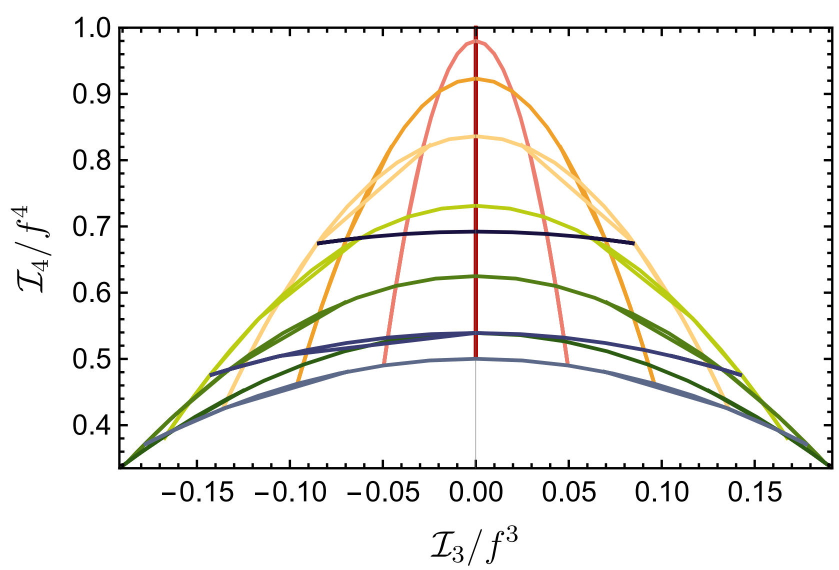
$$\Rightarrow \mathcal{I}_2 = f^2$$ ➡ Non-dynamical

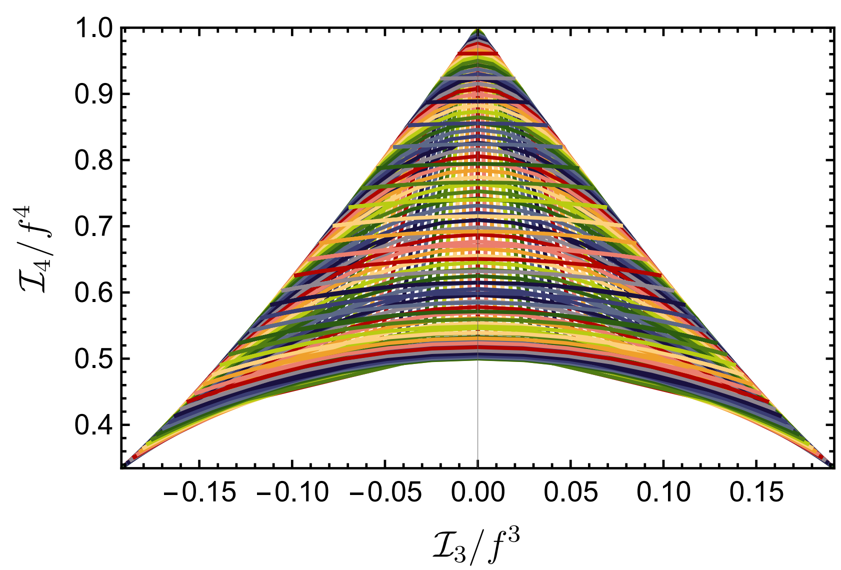
- Nothing proportional to $\mathcal{I}_2$ will generate mass terms for the pions (but other invariants will)

The Natural Minima

$$V_{\rm dGB} = f^2\Lambda^2\left[\hat{c}_3\frac{\mathcal{I}_3}{f^3} + \hat{c}_4\frac{\mathcal{I}_4}{f^4} + \hat{c}_6\frac{\mathcal{I}_6}{f^6} + \hat{c}_7\frac{\mathcal{I}_7}{f^7} + \ldots\right]$$ ➡ Complicated

- The space spanned by $\mathcal{I}_3$ and $\mathcal{I}_4$ given the nonlinearity constraint is bounded
- At the boundaries, the trajectories must reverse, $\partial\mathcal{I}_i/\partial\phi_j$ must change signs





- The natural extrema live on the boundaries. Where the edges meet at points give **maximally natural extrema**

The dGB Fields

- Once a minimum is identified, one can write down the invariants in terms of the dGB fields expanded about that minimum

$$I_3 = \frac{f}{\sqrt{3}}\left[-\frac{f^2}{3} + \pi_1^2 + \pi_2^2 - \frac{1}{3\sqrt{2}f}\left(\pi_1^3 - 3\pi_1\pi_2^2\right) - \frac{17}{24f^2}\left(\pi_1^2+\pi_2^2\right)^2\right] + \ldots$$

$$I_4 = \frac{4f^2}{3}\left[\frac{f^2}{4} + \pi_1^2 + \pi_2^2 + \frac{1}{\sqrt{2}f}\left(\pi_1^3 - 3\pi_1\pi_2^2\right) - \frac{29}{24f^2}\left(\pi_1^2+\pi_2^2\right)^2\right] + \ldots$$

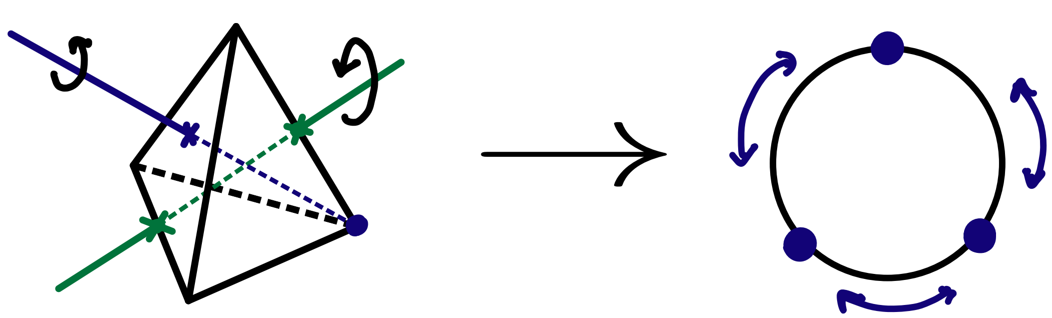
- The Z_3 symmetry is manifest, which can be seen by looking at the invariants of Z_3:

$$\mathcal{I}_2^{(2,Z_3)} = \pi_1^2 + \pi_2^2$$

$$\mathcal{I}_3^{(2,Z_3)} = \pi_1^3 - 3\pi_1\pi_2^2$$

➡ degenerate dGBs

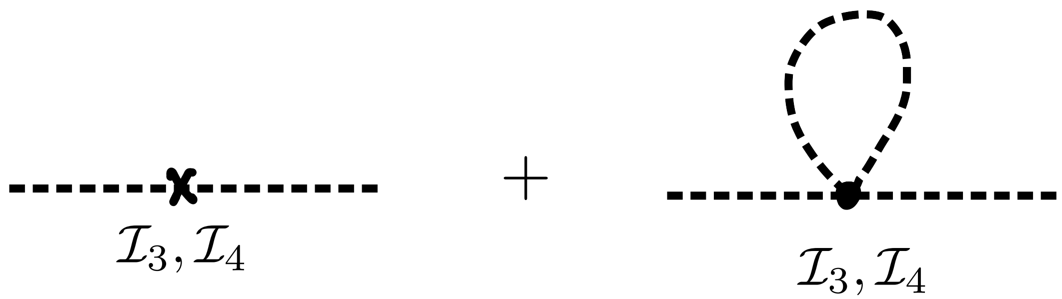
$A_4 \to Z_3$



The dGB Masses

- The $\mathcal{I}_3$ and $\mathcal{I}_4$ will source mass terms for the dGBs, and these will be quadratically sensitive to f

$$m_\pi^2 \ni$$



$$m_\pi^2 = \left(\frac{2}{\sqrt{3}}\hat{c}_3 + \frac{8}{3}\hat{c}_4\right) f^2 + \left(-\frac{17}{12\sqrt{3}\pi^2}\hat{c}_3 - \frac{29}{9\pi^2}\hat{c}_4\right) f^2$$

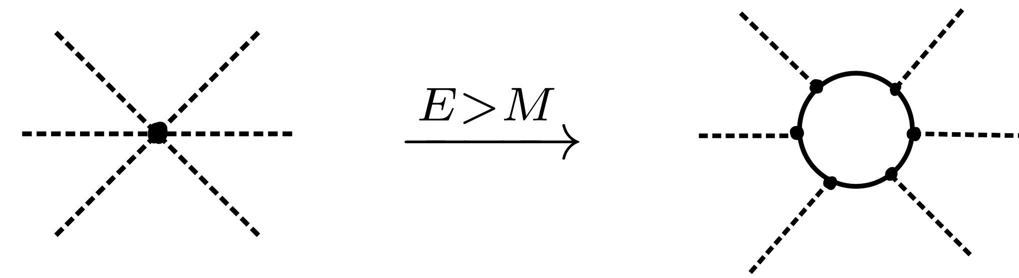
- What does it mean for the dGB to be light when f is the only scale present?

$$m_\pi^2 \sim \hat{c}_{3,4} f^2 \ , \qquad m_\pi^2\Big|_{\text{loop}} \sim 10\% \ m_\pi^2\Big|_{\text{tree}}$$

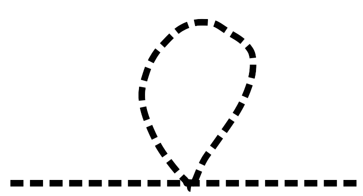
➡ Decently stable

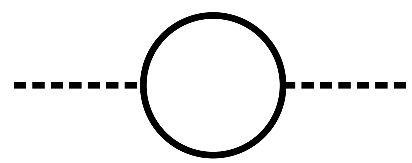
The dGB Mass Protection

- Consider again the theory above f with a single fermion with mass M:



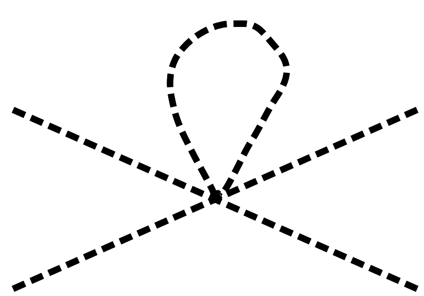
- Above M, the $> \log$ divergent diagrams we can make (at one loop) are:





$\propto \mathcal{I}_2$ ➡ Can't contribute to pion mass

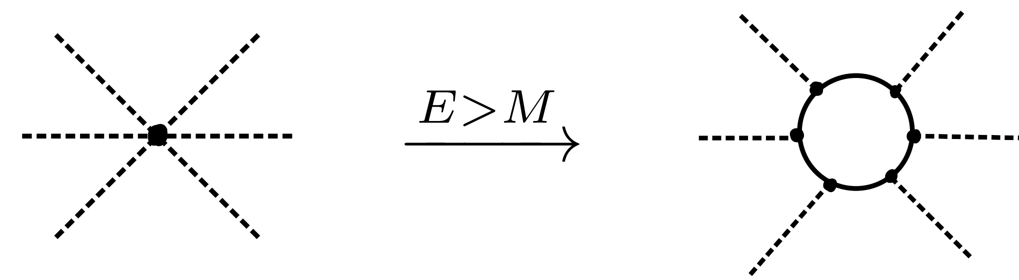
- Below M, we have more operators, and could be worried about diagrams like:



$\sim \hat{c}_6 \, \mathcal{I}_6 \frac{M^2}{f^2}$ ➡ Sourcing an $\mathcal{I}_4$, which we know contributes to the pion mass

The dGB Mass Protection

- Consider again the theory above f with a single fermion with mass M:



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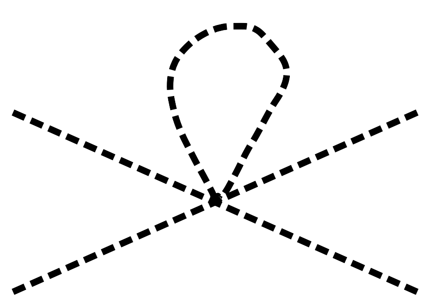
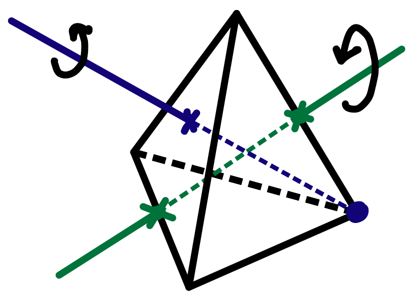


$$\sim \hat{c}_6 \, \mathcal{I}_6 \frac{M^2}{f^2} \sim y^6 \left(\frac{f}{M}\right)^2 \frac{M^2}{f^2}$$

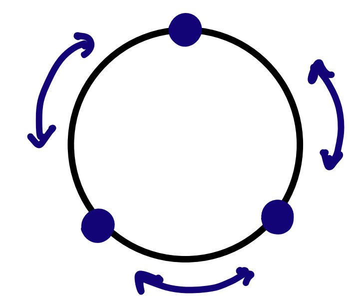
where $\hat{c}_n \sim y^n \left(\frac{f}{M}\right)^{n-4}$

➡ M sensitivity cancels out

Phenomenology of A_4 dGBs



$$A_4 \to Z_3$$



- Two degenerate dGB, guaranteed by the preserved Z_3 symmetry: $$m^2_{\pi_1} = m^2_{\pi_2}$$
- Assume the $\mathcal{I}_3$ operator is the leading term: $$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m}\mathcal{O}^{\text{SM}}\mathcal{I}_3$$
- If the SM production process is uncharged under the discrete group:

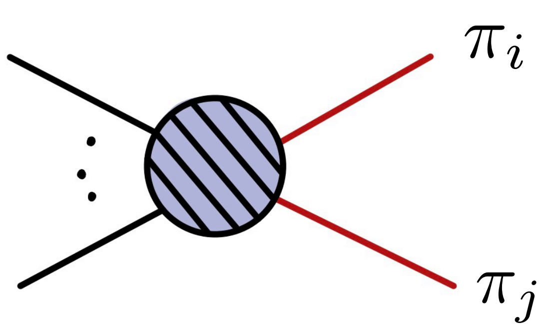
$$\mathcal{I}_3 \ni \frac{f}{\sqrt{3}}\left[\underbrace{\pi_1^2 + \pi_2^2}_{\text{2-pion production}} + \frac{1}{3\sqrt{2}f}\underbrace{\left(\pi_1^3 - 3\pi_1\pi_2^2\right)}_{\text{3-pion production}} - \frac{17}{24f^2}\underbrace{\left(\pi_1^2+\pi_2^2\right)^2}_{\text{4-pion production}}\right]$$

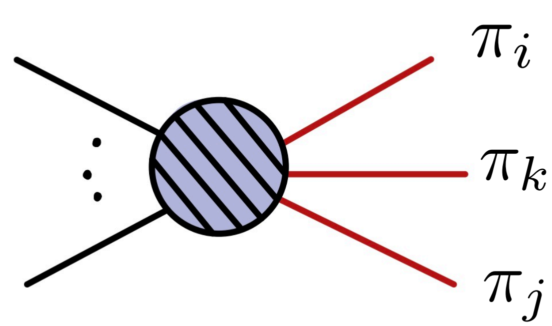
2-pion production | 3-pion production | 4-pion production

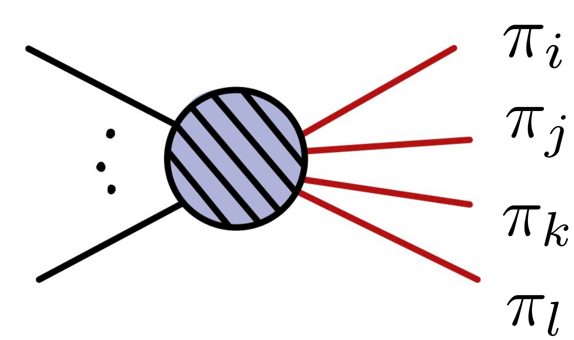
Phenomenology of A_4 dGBs

- Because these interactions all stem from the same operator, there is a consistent pattern of production cross sections

$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m} \mathcal{O}^{\text{SM}} \mathcal{I}_3$$





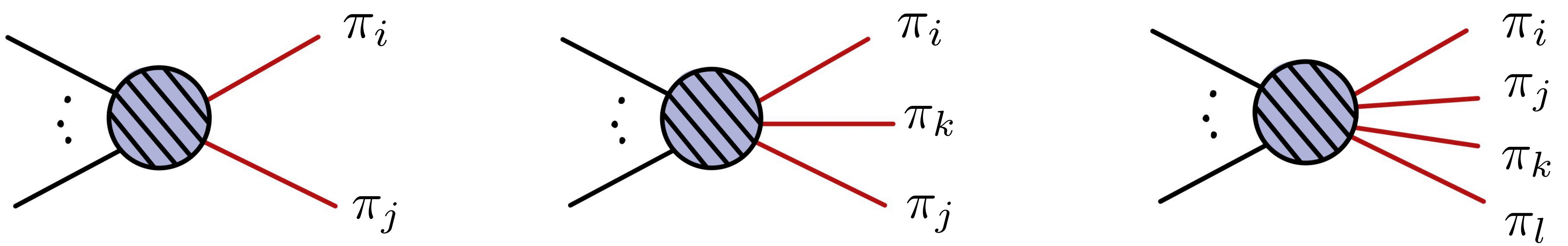


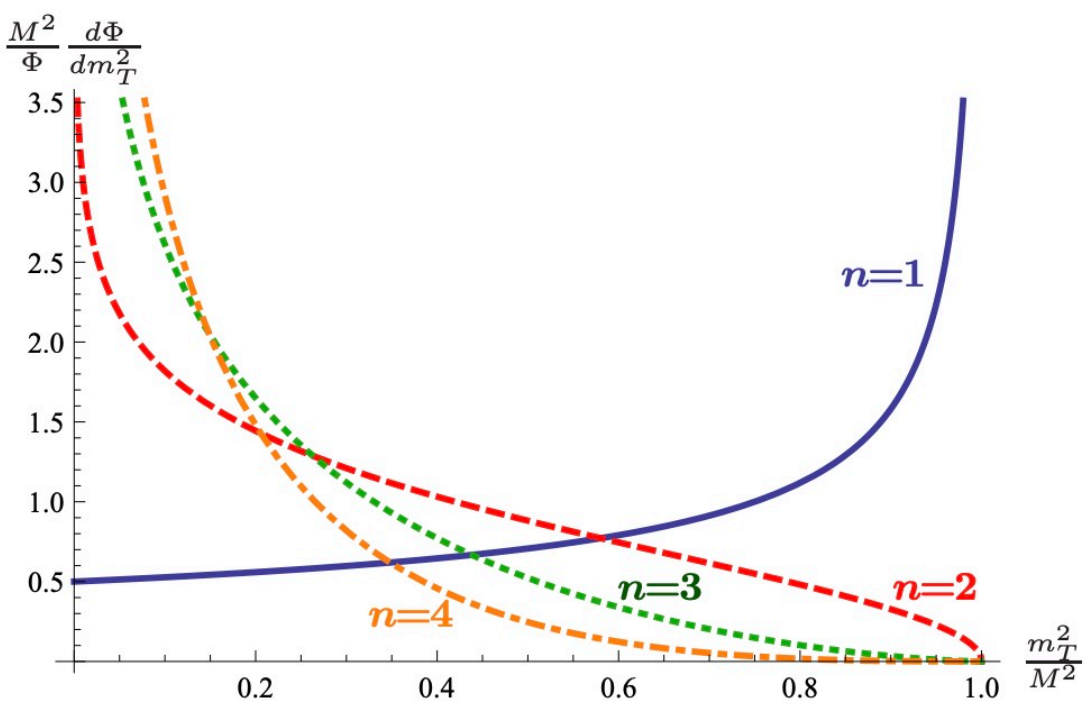
$$\frac{\sigma\left(\text{SM} \to 2\pi\right)}{\sigma\left(\text{SM} \to 3\pi\right)} = 2f^2 \frac{\Pi_2}{\Pi_3}$$

$$\frac{\sigma\left(\text{SM} \to 3\pi\right)}{\sigma\left(\text{SM} \to 4\pi\right)} = \frac{36 f^2}{19(17)^2} \frac{\Pi_3}{\Pi_4}$$

- This cross section information tells us about the A_4 symmetry, not just the preserved Z_3

Phenomenology of A_4 dGBs





Plot lifted from Giudice, Gripaios, Mahbubani, arXiv:1108.1800

- In a collider, we can disentangle multi-particle production via tails of kinematic variables like $\not{E}_T$
- Disentangling multicomponent DM in direct detection experiments is possible, but not promising for the degenerate case considered here

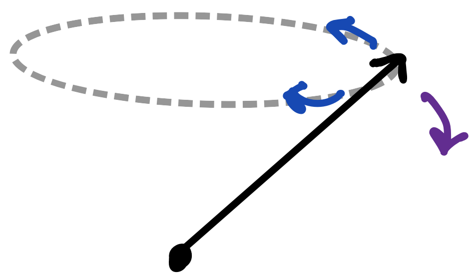
Herrero-Garcia, Scaffidi, White, Williams, arXiv:1709.01945

Higher Non-Abelian Discrete Symmetry Example

The Triplet of A_5

Even permutations of:
$(x_1, x_2, x_3, x_4, x_5)$

- Irreducible representations of A_5 : $\quad 1 \quad \textcircled{3} \quad 3' \quad 4 \quad 5$
- A_4 is a subgroup of A_5
- Consider the dGB's stemming from a scalar field in the **triplet** representation of A_5

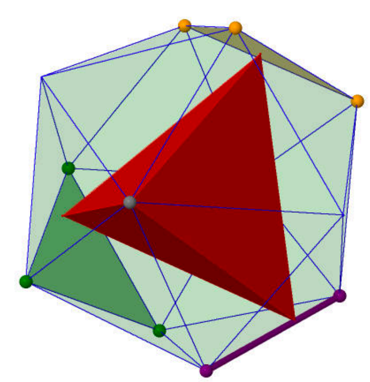


- The primary invariants of the triplet of A_5 :

$$\mathcal{I}_2^{(\mathbf{3},A_5)} = \mathcal{I}_2$$

$$\mathcal{I}_6^{(\mathbf{3},A_5)} = 22\,\mathcal{I}_3^2 + \mathcal{I}_2\mathcal{I}_4 - 2\sqrt{5}\mathcal{I}_6$$

$$\mathcal{I}_{10}^{(\mathbf{3},A_5)} = \mathcal{I}_2\mathcal{I}_4^2 + 38\,\mathcal{I}_3^2\mathcal{I}_4 - \frac{7}{11}\mathcal{I}_2^3\mathcal{I}_4 - \frac{128}{11\sqrt{5}}\mathcal{I}_2^2\mathcal{I}_6 + \frac{6}{\sqrt{5}}\mathcal{I}_4\mathcal{I}_6$$



where $(\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4, \mathcal{I}_6)$ are the invariants of A_4

The Lighter A_5 dGB

- The most general potential of the dGB's:

$$V_{\mathrm{dGB}} = f^2\Lambda^2\left[\hat{c}_6\frac{\mathcal{I}_6}{f^6} + \hat{c}_{10}\frac{\mathcal{I}_{10}}{f^{10}} + \hat{c}_{12}\frac{\mathcal{I}_6^2}{f^{12}} + \hat{c}_{15}\frac{\mathcal{I}_{15}}{f^{15}} + \ldots\right]$$

- Note that the A_5 symmetry forbids $\hat{c}_n$ for $n < 6$

➡ All terms in the potential come from higher dimensional operators

➡ This leads to an enhanced suppression for m_π :

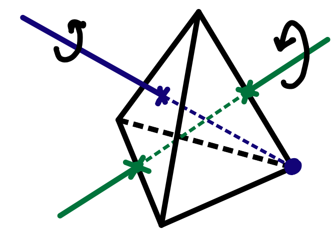
$$m_\pi^2 \sim \hat{c}_6 f^2 \qquad\qquad \hat{c}_n \sim y^n\left(\frac{f}{M}\right)^{n-4}$$

More suppressed ($\hat{c}_6$) — Still tied to f (f^2)

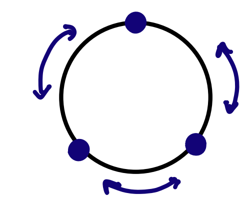
- Going to larger symmetry groups gives even stronger suppression

Conclusion

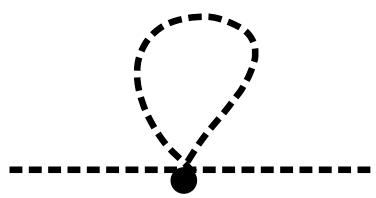
- Nonlinearly realized discrete symmetries produce **discrete Goldstone bosons**



$A_4 \to Z_3$

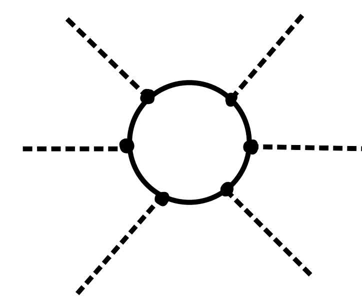


- dGBs have enhanced protection from quadratic divergences



$\mathcal{I}_4$

$\Rightarrow \mathcal{I}_2 = f^2$



$$\hat{c}_n \sim y^n \left(\frac{f}{M}\right)^{n-4}$$

- Wrote down an EFT of dGBs and classified its general features
  - Degenerate pions reflecting preserved low energy symmetries
  - Multi-particle invisible production
  - Cross section ratios give access to nonlinearly realized symmetry
- For more details, check out Victor's talk in the parallel session

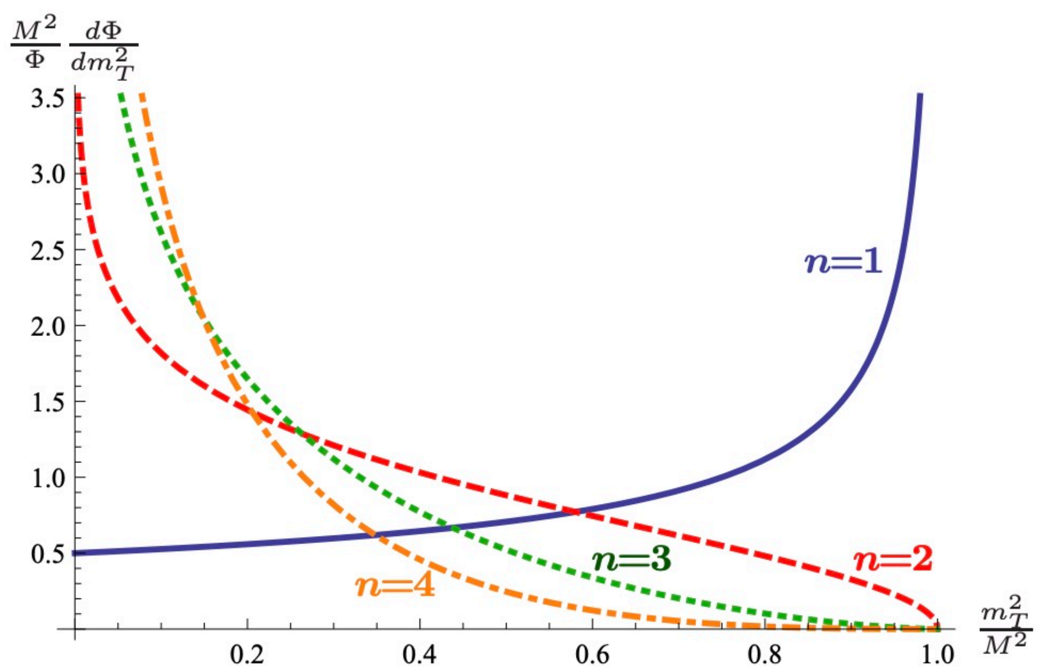
Thank you!

Back-up Slides

Phenomenology of A_4 dGBs

- In a collider, we can disentangle multi-particle production via tails of kinematic variables like $\not{E}_T$

$$m_T = m_V + \cancelto{0}{m_I^2} + 2\left(\sqrt{\not{p}_T^2\,(p_T^2 + m_V^2)} - \not{p}_T \cdot p_T\right)$$



Plot lifted from Giudice, Gripaios, Mahbubani, arXiv:1108.1800

- Works for heavy invisible as well:

Production	Observable	Invisibles	Exponent $\mu = 0$	Exponent $\mu \neq 0$
Single	m_T	n	$n - \frac{3}{2}$	$\frac{3n}{2} - 2$
Symmetric pair	m_{T2}	$n = k + l$	$k + l - \frac{3}{2}$	$\frac{3(k+l)}{2} - \frac{5}{2}$
Asymmetric pair	m_{T2}	$n = k + l, k < l$	$2k - 1$	-
-	m_V	n	$2n - 1$	$\frac{3n}{2} - 1$

- Disentangle scenarios by power law scaling, but there are some degeneracies that would need to be teased apart

Nonlinearly Realized Discrete A_4

Das, Hook, arXiv:2006.10767

- The field ϕ has an $SO(3)$ invariant potential in the UV: $$V(\phi) = -\frac{m^2}{2}\phi^T\phi + \frac{\lambda}{4}\left(\phi^T\phi\right)^2$$

- $SO(3)$ is broken by A_4 invariant Yukawa interactions:

$$\mathcal{L}_{\text{int}} = \underbrace{y_a \epsilon^{ijk}\bar{\psi}_i\psi_j\phi_k}_{\text{SO(3) invariant}} + \underbrace{y_s \tilde{\epsilon}^{ijk}\bar{\psi}_i\psi_j\phi_k}_{\text{SO(3) breaking}} \qquad \tilde{\epsilon}_{ijk} = |\epsilon_{ijk}|$$

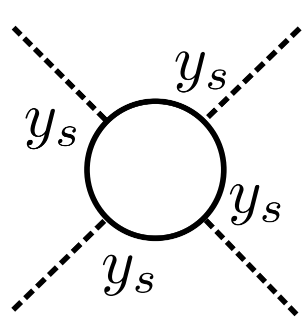
- Upon SSB: $SO(3) \to SO(2)$

$$\phi(\pi_1, \pi_2) = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \exp\left[\frac{1}{f}\begin{pmatrix} 0 & 0 & \pi_1 \\ 0 & 0 & \pi_2 \\ -\pi_1 & -\pi_2 & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

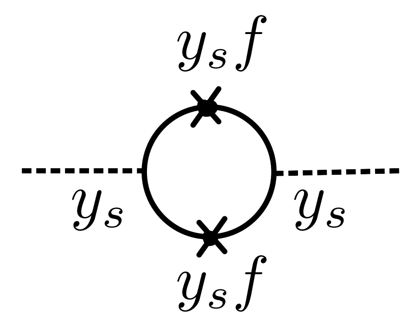
Nonlinearly Realized Discrete A_4

Das, Hook, arXiv:2006.10767

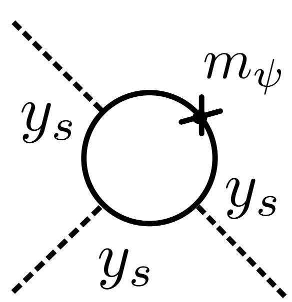
- The pions still have a nonzero mass from the $SO(3)$-breaking terms



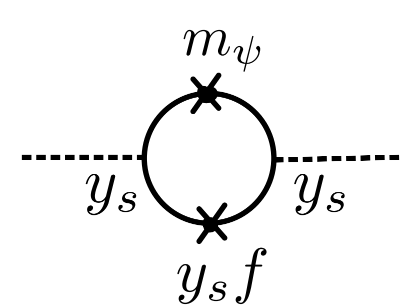
$\xrightarrow{SO(3)\to SO(2)}$ $\delta m_h^2 \ni$



$\sim y^4 f^2 \log\left(\frac{m_\psi}{\mu}\right)$



$\xrightarrow{SO(3)\to SO(2)}$ $\delta m_h^2 \ni$



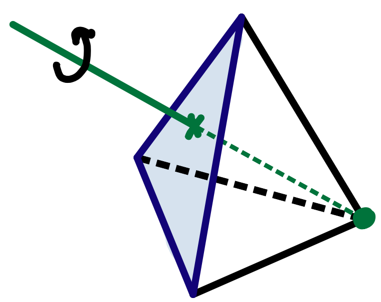
$\sim y^3 f m_\psi \log\left(\frac{m_\psi}{\mu}\right)$

- Nonlinearly realized A_4 offers protection and guarantees nonzero mass for associated light pGB fields

➡ How general is this?

More Discrete Groups: A_N , S_N

A_4 Tetrahedron



Even permutations of:

(x_1, x_2, x_3, x_4)

A_5 Icosahedron

Even permutations of:

$(x_1, x_2, x_3, x_4, x_5)$

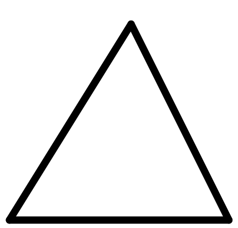
…

Higher A_N

Even permutations of N objects:

$(x_1, x_2, ..., x_N)$

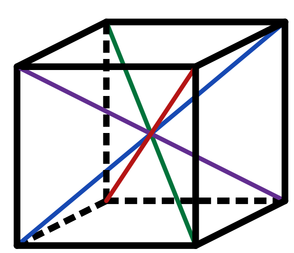
S_3 Triangle



All permutations of:

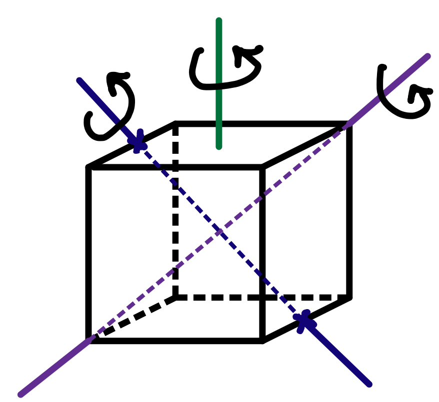
(x_1, x_2, x_3)

S_4 Cube



All permutations of:

(x_1, x_2, x_3, x_4)



…

Higher S_N

All permutations of N objects:

$(x_1, x_2, ..., x_N)$

The Minima of V_{dGB}

- Next, we want to parameterize the low energy theory after V_{dGB} takes a vev
- $V(\Phi)$ is a function of the primary invariants:

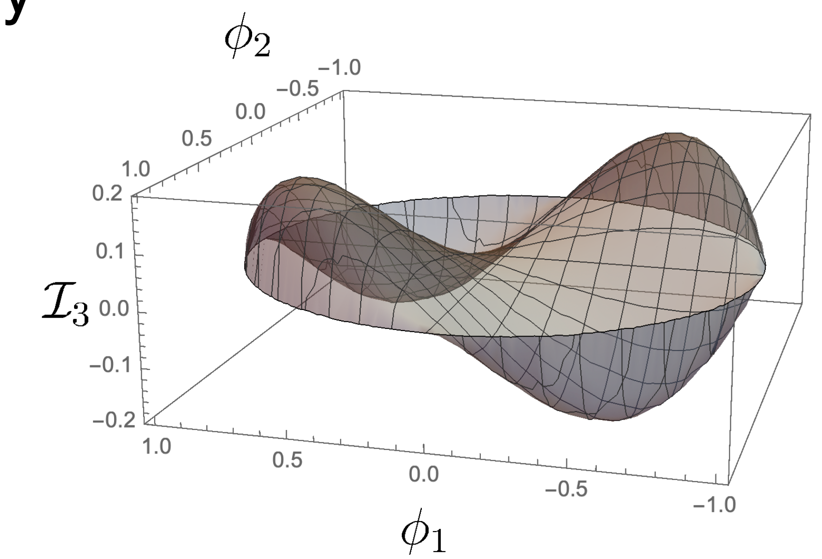
$$V(\Phi) = f(I_3, I_4)$$

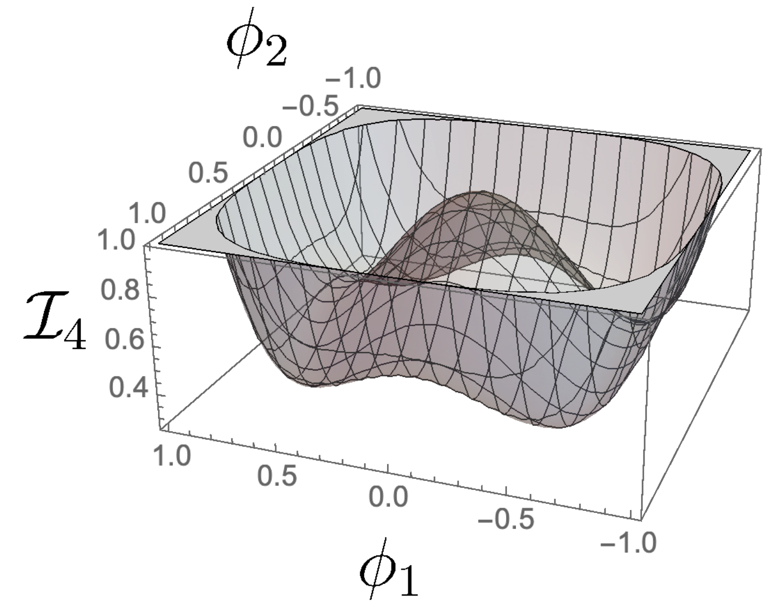
- The critical points of V occur when:

$$\frac{\partial V}{\partial \phi_j} = \sum_i \frac{\partial V}{\partial I_i} \frac{\partial I_i}{\partial \phi_j} = 0$$

Depends on the particular form of the potential

Depends on the structure of the invariants



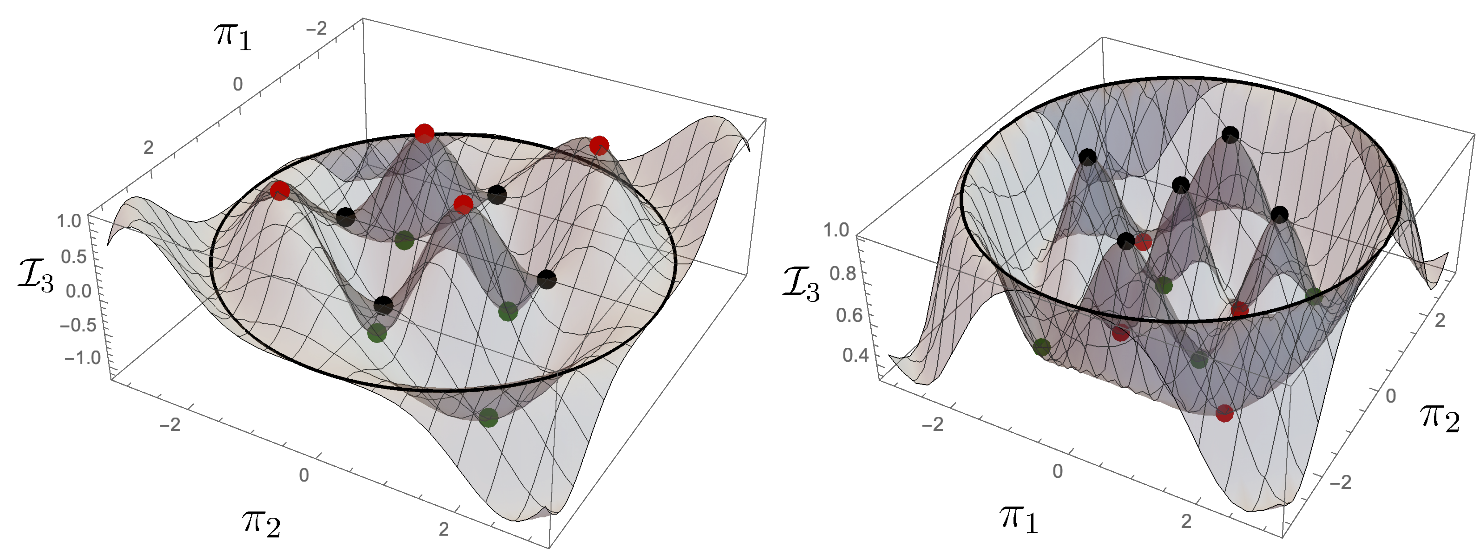


The dGB Fields

- Expanding ϕ around its minimum gives the dGB fields:

$$\phi(\pi_1, \pi_2) = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \exp\left[\frac{1}{f}\begin{pmatrix} 0 & 0 & \pi_1 \\ 0 & 0 & \pi_2 \\ -\pi_1 & -\pi_2 & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

Point	ϕ_1	ϕ_2	ϕ_3	Little group	Nature
A	0 0 ± 1	0 ± 1 0	± 1 0 0	Z_2	Saddles
B	$\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\pm\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\mp\frac{1}{\sqrt{3}}$	Z_3	Minima
C	$-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\pm\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\mp\frac{1}{\sqrt{3}}$	Z_3	Minima

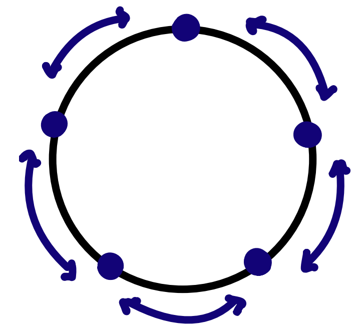


- Need to ensure the z-direction lines up with the unbroken generator:

$$\mathcal{I}'_i(\Phi) = \mathcal{I}_i(R^{-1}\Phi)\,, \qquad R \in SO(3) \qquad \text{such that} \qquad \Phi'_s = R\Phi_s = (0, 0, f)$$

Phenomenology of $A_5 \to Z_5$

$$A_5 \to Z_5$$



- Two degenerate dGB, guaranteed by the preserved Z_5 symmetry: $$m^2_{\pi_1} = m^2_{\pi_2}$$
- Assume the $\mathcal{I}_6$ operator is the leading term: $$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m} \mathcal{O}^{\text{SM}} \mathcal{I}_6^{(\mathbf{3}, A_5)} + \ldots$$
- If the SM production process is uncharged under the discrete group:

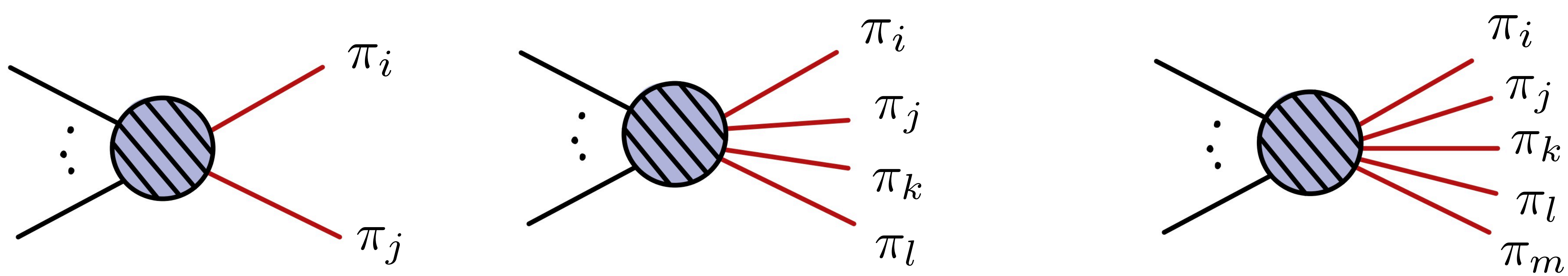
$$\mathcal{I}_6^{(\mathbf{3}, A_5)} = \frac{32}{5} f^4 \left[\frac{f^2}{32} + \pi_1^2 + \pi_2^2 - \frac{31}{12 f^2} \left(\pi_1^2 + \pi_2^2\right)^2 - \frac{1}{4 f^3} \left(\pi_1^5 - 10 \pi_1^3 \pi_2^2 + 5 \pi_1 \pi_2^4\right) \right]$$

2-pion production — 4-pion production — 5-pion production

Phenomenology of $A_5 \to Z_5$

- Because these interactions all stem from the same operator, there is a consistent pattern of production cross sections

$$\mathcal{L}_{\rm int} \propto \frac{1}{M^m} \mathcal{O}^{\rm SM} \mathcal{I}_6^{(\mathbf{3}, A_5)}$$



$$\frac{\sigma(\text{SM} \to 4\pi)}{\sigma(\text{SM} \to 5\pi)} = \frac{19(31)^2 f^2}{3(45)^2} \frac{\Pi_4}{\Pi_5} = \frac{19(31)^2 (8\pi)^2}{45^2} \frac{f^2}{E_{\rm CM}^2}$$

$$\frac{\sigma(\text{SM} \to 2\pi)}{\sigma(\text{SM} \to 4\pi)} = \frac{18 f^4}{19(31)^2} \frac{\Pi_2}{\Pi_4} = \frac{216(4\pi)^4}{19(31)^2} \frac{f^4}{E_{\rm CM}^4}$$

- This cross section information tells us about the A_5 symmetry, not just the preserved Z_5

The Quadruplet of A_5

Even permutations of:
$(x_1, x_2, x_3, x_4, x_5)$

- Irreducible representations of A_5: $\mathbf{1} \quad \mathbf{3} \quad \mathbf{3}' \quad \mathbf{4} \quad \mathbf{5}$

$$\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)$$

- This case will have a non-Abelian preserved symmetry

- The primary invariants of the quadruplet of A_5:

$$\mathcal{I}_2^{(\mathbf{4},A_5)} = \mathcal{I}_2 + \phi_4^2$$

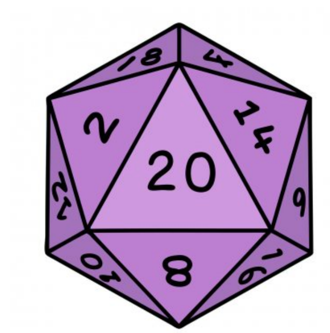
$$\mathcal{I}_3^{(\mathbf{4},A_5)} = \mathcal{I}_3 - \frac{\phi_4}{2\sqrt{5}}\mathcal{I}_2 + \frac{\phi_4^3}{2\sqrt{5}}$$

$$\mathcal{I}_4^{(\mathbf{4},A_5)} = \mathcal{I}_4 + \frac{12}{\sqrt{5}}\mathcal{I}_3\phi_4 + \frac{12}{5}\mathcal{I}_2\phi_4^2 + \frac{\phi_4^4}{5}$$

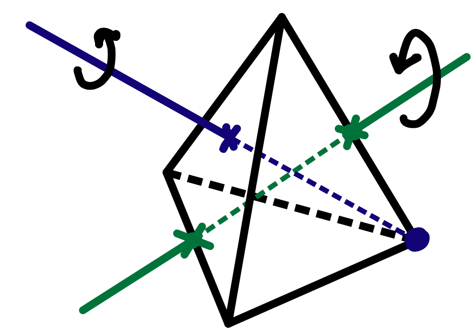
$$\mathcal{I}_5^{(\mathbf{4},A_5)} = \mathcal{I}_4\phi_4 - \frac{1}{2}\mathcal{I}_2\phi_4 - \frac{4}{\sqrt{5}}\mathcal{I}_3\phi_4^2 - \frac{\phi_4^3}{5}\mathcal{I}_2 + \frac{\mathcal{I}_4^5}{50}$$

where $(\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4)$ are the primary invariants of A_4

Phenomenology of $A_5 \to A_4$



$$A_5 \to A_4$$



- Three degenerate dGBs, guaranteed by the preserved A_4 symmetry:

$$m_{\pi_1} = m_{\pi_2} = m_{\pi_3}$$

- Assume the $\mathcal{I}_3$ operator is the leading term:

$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m} \mathcal{O}^{\text{SM}} \mathcal{I}_3$$

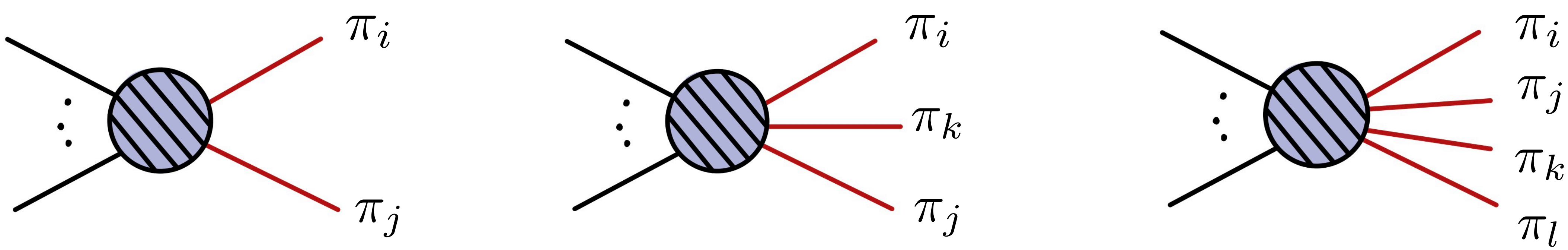
- If the SM production process is uncharged under the discrete group:

$$\mathcal{I}_3^{(\mathbf{4}, A_5)} = \frac{\sqrt{5} f}{4} \left[-\frac{2 f^2}{5} + \underbrace{\left(\pi_1^2 + \pi_2^2 + \pi_3^2\right)}_{\text{2-pion production}} - \underbrace{\frac{4}{\sqrt{5} f} \pi_1 \pi_2 \pi_3}_{\text{3-pion production}} - \underbrace{\frac{41}{60 f^2} \left(\pi_1^2 + \pi_2^2 + \pi_3^2\right)^2}_{\text{4-pion production}} \right]$$

Phenomenology of $A_5 \to A_4$

- Because these interactions all stem from the same operator, there is a consistent pattern of production cross sections

$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m}\mathcal{O}^{\text{SM}}\mathcal{I}_3^{(\mathbf{4},A_5)}$$



$$\frac{\sigma(\text{SM}\to 3\pi)}{\sigma(\text{SM}\to 4\pi)} = \frac{6f^2}{(41)^2}\frac{\Pi_3}{\Pi_4} = \left(\frac{24\pi}{41}\right)^2 \frac{f^2}{E_{\text{CM}}^2}$$

$$\frac{\sigma(\text{SM}\to 2\pi)}{\sigma(\text{SM}\to 3\pi)} = \frac{15f^2}{4}\frac{\Pi_2}{\Pi_3} = 120\pi^2\frac{f^2}{E_{\text{CM}}^2}$$

- This cross section information tells us about the A_5 symmetry, not just the preserved A_4