CPT and unitarity constraints for higher-order CP asymmetries at finite temperature

Peter Maták

In collaboration with Tomáš Blažek and Viktor Zaujec [JCAP 10 (2022) 042]

COMENIUS UNIVERSITY BRATISLAVA

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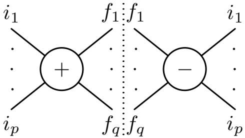
Outline of the talk

- Classical kinetic theory with **complete** perturbative description of particle interactions
- Holomorphic cutting rules and **unitarity constraints** at higher perturbative orders [Phys. Rev. D 103 (2021) L091302]
- **Quantum statistics from cylindrical diagrams** and CP asymmetries in quantum kinetic theory [J. Cosmol. Astropart. Phys. 10 (2022) 042]

Classical kinetic theory

change in # of particles $\quad\leftrightarrow\quad$ average # of interactions the particles participate in

$$\dot{n}_{i_1} + 3Hn_{i_1} = -\mathring{\gamma}_{fi} + \mathring{\gamma}_{if} + \dots$$



$$\boxed{\mathring{\gamma}_{fi} = -\frac{1}{\mathcal{V}_4}\int\left(\prod_{\forall i}[d\mathbf{p}_i]\mathring{f}_i(p_i)\right)\int\left(\prod_{\forall f}[d\mathbf{p}_f]\right)\mathrm{i}\,T^{\dagger}_{if}\,\mathrm{i}\,T_{fi}} \tag{1}$$

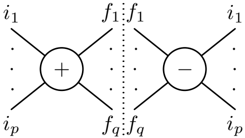
[Bernstein '88; Kolb, Turner '90]

$$[d\mathbf{p}] = \frac{d^3\mathbf{p}}{(2\pi)^3 2E_{\mathbf{p}}} \qquad |T_{fi}|^2 = \mathcal{V}_4(2\pi)^4\delta^{(4)}(p_f - p_i)|\mathring{M}_{fi}|^2 \qquad \mathcal{V}_4 = \mathcal{V}_3 \times \mathcal{T} \tag{2}$$

Classical kinetic theory

change in # of particles $\leftrightarrow$ average # of interactions the particles participate in

$$\dot{n}_{i_1} + 3Hn_{i_1} = -\mathring{\gamma}_{fi} + \mathring{\gamma}_{if} + \dots$$

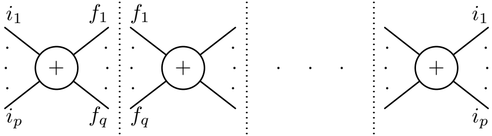


$$\boxed{\mathring{\gamma}_{fi} = -\frac{1}{\mathcal{V}_4} \int \left(\prod_{\forall i} [d\mathbf{p}_i] \mathring{f}_i(p_i) \right) \int \left(\prod_{\forall f} [d\mathbf{p}_f] \right) \mathrm{i}\, T^{\dagger}_{if} \mathrm{i}\, T_{fi}} \qquad (1)$$

[Bernstein ’88; Kolb, Turner ’90]

$\rightarrow$ typically leads to $\langle \sigma v \rangle \big(n^2 - n_{\mathrm{eq}}^2 \big)$ or $\langle \Gamma \rangle \big(n - n_{\mathrm{eq}} \big)$ (3)

Holomorphic cuts and higher orders



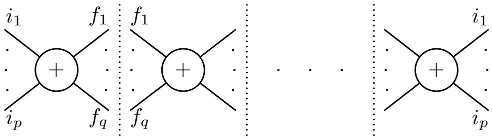
$$S^\dagger S = 1 \quad \rightarrow \quad \mathrm{i}\,T^\dagger = \mathrm{i}\,T - \mathrm{i}\,T\mathrm{i}\,T^\dagger \quad \text{for} \quad \mathrm{i}\,T = S - 1 \tag{4}$$

$$\left|T_{fi}\right|^2 = -\mathrm{i}\,T^\dagger_{if}\mathrm{i}\,T_{fi} = \boxed{-\mathrm{i}\,T_{if}\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \sum_{mn} \mathrm{i}\,T_{im}\mathrm{i}\,T_{mn}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} + \dots} \tag{5}$$

[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21; Blažek, Maták '21; Hannesdottir, Mizera '22]

$$1 - \mathrm{i}\,T^\dagger = (1 + \mathrm{i}\,T)^{-1} \quad \rightarrow \quad \mathrm{i}\,T^\dagger = \mathrm{i}\,T - (\mathrm{i}\,T)^2 + (\mathrm{i}\,T)^3 - \dots \tag{6}$$

Holomorphic cuts and higher orders



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[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21;
Blažek, Maták '21; Hannesdottir, Mizera '22]

$$\mathring{\gamma}_{fi} = \frac{1}{V_4}\int \prod_{\forall i} [d\mathbf{p}_i] \mathring{f}_i(p_i) \int \prod_{\forall f} [d\mathbf{p}_f] \Big(-\mathrm{i}\,T_{if}\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \ldots \Big) \tag{7}$$

CP violation and unitarity constraints

$$T_{fi} = C_{fi}^{\rm tree} K_{fi}^{\rm tree} + C_{fi}^{\rm loop} K_{fi}^{\rm loop} \quad \rightarrow \quad \Delta |T_{fi}|^2 \propto {\rm Im}\left[C_{fi}^{\rm tree} C_{fi}^{\rm loop*}\right] {\rm Im}\left[K_{fi}^{\rm tree} K_{fi}^{\rm loop*}\right] \tag{8}$$

$$\begin{aligned} \Delta |T_{fi}|^2 &= |T_{fi}|^2 - |T_{if}|^2 = -{\rm i}\, T^\dagger_{if}\, {\rm i}\, T_{fi} + {\rm i}\, T_{if}\, {\rm i}\, T^\dagger_{fi} \\ &= \boxed{\sum_n \left({\rm i}T_{in}{\rm i}T_{nf}{\rm i}T_{fi} - {\rm i}T_{if}{\rm i}T_{fn}{\rm i}T_{ni}\right)} \\ &\quad - \sum_{mn} \left({\rm i}T_{im}{\rm i}T_{mn}{\rm i}T_{nf}{\rm i}T_{fi} - {\rm i}T_{if}{\rm i}T_{fn}{\rm i}T_{nm}{\rm i}T_{mi}\right) \\ &\quad + \ldots \quad \rightarrow \quad \boxed{\sum_f \Delta |T_{fi}|^2 = 0} \end{aligned} \tag{9}$$

[Covi, Roulet, Vissani '98]

[Dolgov '79; Kolb, Wolfram '80]

CP violation and unitarity constraints

Lowest-order asymmetries

$$\Delta\mathring{\gamma}_{fi} = \frac{1}{V_4}\int\prod_{\forall i}[d\mathbf{p}_i]\mathring{f}_i(p_i)\int\prod_{\forall f}[d\mathbf{p}_f]\sum_n\Big(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{ni}\Big) \tag{10}$$

For $\Delta\mathring{\gamma}_{fi} \to \Delta\gamma_{fi}$ add statistical factors $1 \pm f$ for particles in $|n\rangle$, $|f\rangle$ states.

[Nanopoulos, Weinberg '79; Hook '11]

Higher-order asymmetries

$$\Delta|\,T_{fi}|^2 = \ldots - \boxed{\sum_{mn}\Big(\mathrm{i}\,T_{im}\mathrm{i}\,T_{mn}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{nm}\mathrm{i}\,T_{mi}\Big) + \ldots}$$

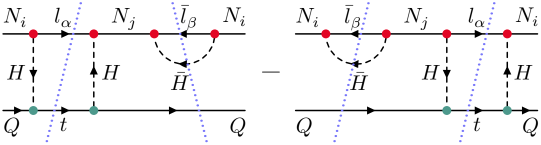
[J. Cosmol. Astropart. Phys. 10 (2022) 042]

Top-Yukawa corrections in leptogenesis

$$\mathcal{L} \supset -\frac{1}{2} M_i \bar{N}_i N_i - (\,Y_{\alpha i} \bar{N}_i P_L l_\alpha H + Y_t \bar{t} P_L Q H + \text{H.c.}) \tag{11}$$

[Pilaftsis, Underwood '04; '05; Abada, *et al.* '06; Nardi, Racker, Roulet '07; Racker '19; Giudice, *et al.* '04; Salvio, Lodone, Strumia '11]

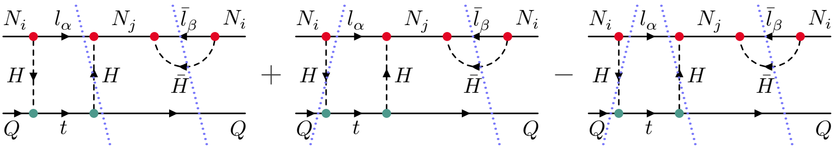
$$\Delta\mathring{\gamma}_{NQ\to lt} \quad \leftarrow \tag{12}$$



Higgs thermal mass from anomalous thresholds

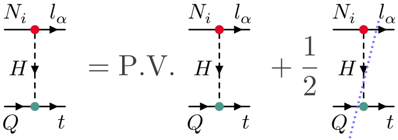
$$\mathcal{L} \supset -\frac{1}{2} M_i \bar{N}_i N_i - \left(Y_{\alpha i} \bar{N}_i P_L l_\alpha H + Y_t \bar{t} P_L Q H + \text{H.c.} \right) \tag{13}$$

[Pilaftsis, Underwood '04; '05; Abada, *et al.* '06; Nardi, Racker, Roulet '07; Racker '19; Giudice, *et al.* '04; Salvio, Lodone, Strumia '11]



(14)

$$\frac{1}{k^2 + \mathrm{i}\epsilon} = \text{P.V.} \frac{1}{k^2} - \mathrm{i}\pi\delta(k^2) \quad \rightarrow$$

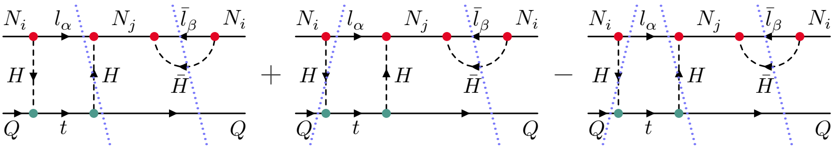


(15)

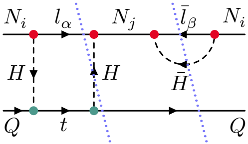
Higgs thermal mass from anomalous thresholds

$$\mathcal{L} \supset -\frac{1}{2} M_i \bar{N}_i N_i - \left(Y_{\alpha i} \bar{N}_i P_L l_\alpha H + Y_t \bar{t} P_L Q H + \text{H.c.} \right) \tag{13}$$

[Pilaftsis, Underwood ’04; ’05; Abada, *et al.* ’06; Nardi, Racker, Roulet ’07; Racker ’19; Giudice, *et al.* ’04; Salvio, Lodone, Strumia ’11]



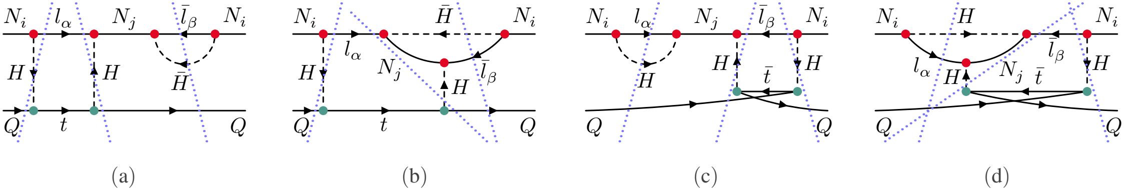
(14)

2P.V. 

$$\leftarrow \quad 2\delta_+(k^2)\,\text{P.V.}\frac{1}{k^2} = -\frac{1}{(k^0+|\mathbf{k}|)^2}\frac{\partial \delta(k^0-|\mathbf{k}|)}{\partial k^0} \tag{16}$$

[Frye, Hannesdottir, Paul, Schwartz, Yan ’19; Racker ’19]

Higgs thermal mass from anomalous thresholds



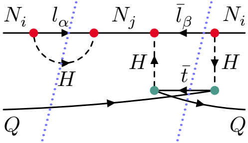
$$\Delta\mathring{\gamma}^{(a)}_{N_iQ\to lHQ} + \ldots = \frac{1}{4}\mathring{m}^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0}\Delta\mathring{\gamma}_{N_i\to lH} \tag{17}$$

[J. Cosmol. Astropart. Phys. 10 (2022) 042]

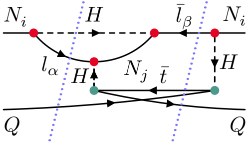
$$\mathring{m}^2_{H,Y_t}(T) = 12\,Y_t^2\int[d\mathbf{p}_Q]\exp\{-E_Q/T\} = \frac{3}{\pi^2}\,Y_t^2\,T^2 \tag{18}$$

Quantum statistics in classical kinetic theory

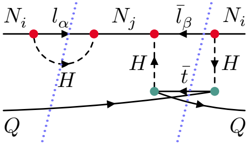
$$\Delta\mathring{\gamma}_{N_i(Q)\to lH(Q)} =$$



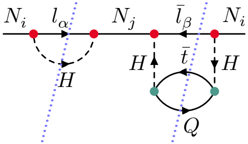
$+$



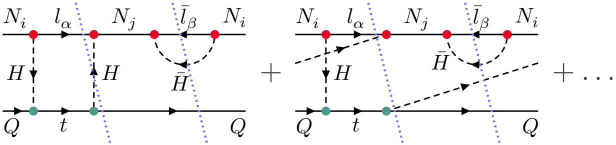
$$- \ \text{m.t.} \tag{19}$$



$$= -\mathring{f}_Q \times \tag{20}$$

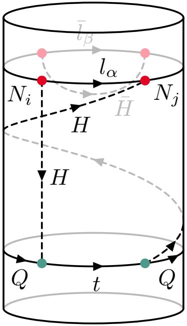


Quantum statistics in classical kinetic theory



(21)

$$\leftarrow \quad \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon} + 2\pi \sum_{w=1}^{\infty} \mathring{f}_H^w \theta(k^0)\delta(k^2) \tag{22}$$



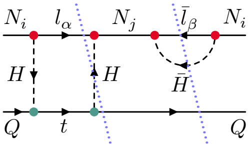
In thermal equilibrium

$$\sum_{w=1}^{\infty} \mathring{f}_H^w \quad \rightarrow \quad f_H = \frac{1}{\exp\{E_{\mathbf{k}}/T\} - 1} \tag{23}$$

[Eur. Phys. J. C 81 (2021) 1050]

Uncircled rate asymmetries

$$\Delta\gamma^{(a)}_{N_i Q\to lHQ} = 2\,\mathrm{P.V.}$$



$$+ \text{ all windings} \qquad (24)$$

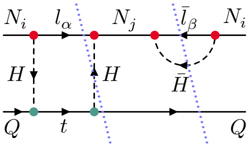
$$\mathring{f}_{N_i}\mathring{f}_Q \quad \to \quad f_{N_i} f_Q (1+f_H)(1-f_l)(1+f_{\bar{H}})(1-f_{\bar{l}}) \qquad (25)$$

$$\boxed{\frac{\partial}{\partial k^0}\bigg|_{k^0=|\mathbf{k}|} \frac{\mathcal{F}(k^0,\mathbf{k})}{(k^0+|\mathbf{k}|)^2} = \frac{\partial}{\partial m_H^2}\bigg|_{m_H=0} \frac{\mathcal{F}(E_\mathbf{k},\mathbf{k})}{2E_\mathbf{k}}} \quad \text{for} \quad E_\mathbf{k} = \sqrt{m_H^2+\mathbf{k}^2} \qquad (26)$$

[Eur. Phys. J. C 82 (2022) 214]

Uncircled rate asymmetries

$$\Delta\gamma^{(a)}_{N_i Q\to lHQ} = 2\text{P.V.}$$



$$+\ \text{all windings} \tag{24}$$

$$\mathring{f}_{N_i}\mathring{f}_Q \quad \to \quad f_{N_i} f_Q (1+f_H)(1-f_l)(1+f_{\bar{H}})(1-f_{\bar{l}}) \tag{25}$$

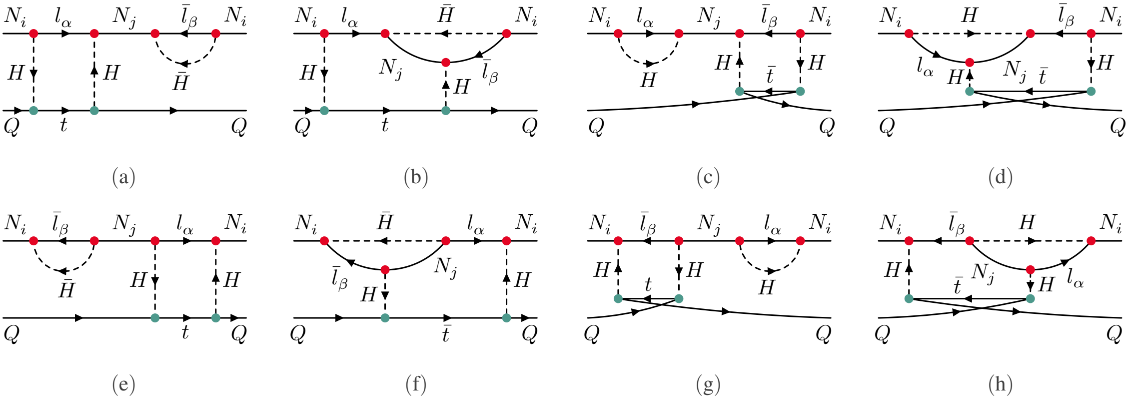
$$\boxed{\Delta\gamma_{N_i Q\to lHQ} = \Delta\gamma_{N_i(Q)\to lH(Q)} + \frac{1}{4} m^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0} \Delta\gamma_{N_i\to lH}} \tag{27}$$

[J. Cosmol. Astropart. Phys. 10 (2022) 042]

$$m^2_{H,Y_t}(T) = 12\,Y_t^2 \int [d\mathbf{p}_Q] f_Q = \frac{1}{4}\,Y_t^2\,T^2 \tag{28}$$

[Comelli, Espinosa '97; Giudice, *et al.* '04]

Unitarity constraints for NLO asymmetries

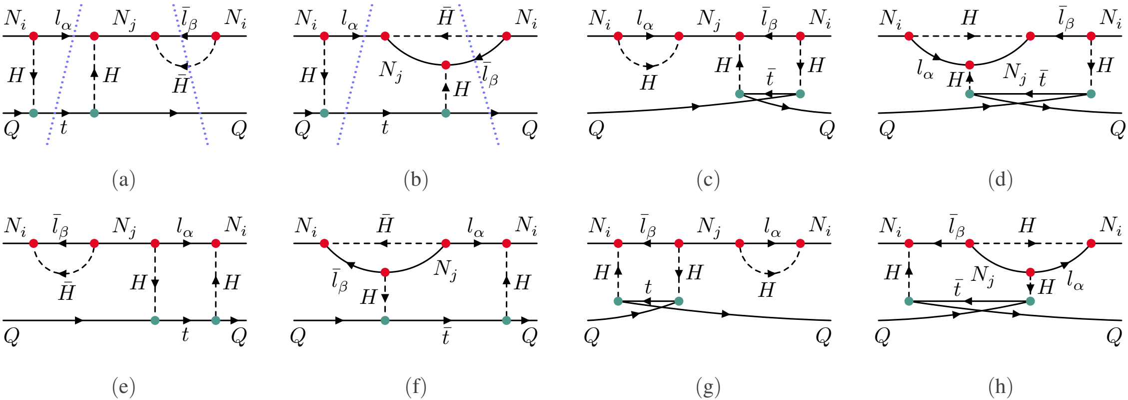


$$\boxed{\Delta\mathring{\gamma}_{N_iQ\to lt} + \Delta\mathring{\gamma}_{N_iQ\to lHQ} + \Delta\mathring{\gamma}_{N_iQ\to \bar l\bar H Q}} + \boxed{\Delta\mathring{\gamma}_{N_iQ\to \bar l Q Q\bar t}} = 0 \qquad (29)$$

[Pilaftsis, Underwood '04; '05; Abada, *et al.* '06; Nardi, Racker, Roulet '07; Racker '19]

[Blažek, Maták '21]

Unitarity constraints for NLO asymmetries

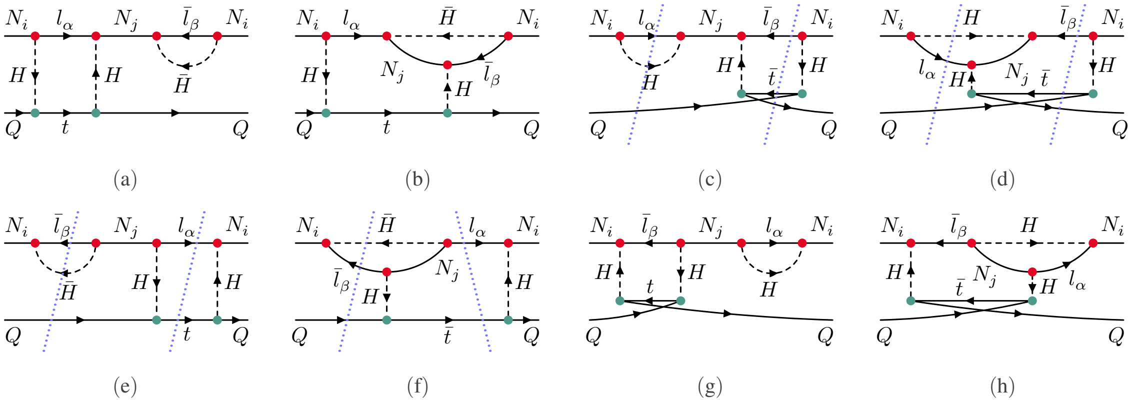


$$\boxed{\Delta\gamma_{N_iQ\to lt}} + \Delta\gamma_{N_i(Q)\to lH(Q)} + \Delta\gamma_{N_i(Q)\to \bar{l}\bar{H}(Q)} + \Delta\gamma_{N_iQ\to \bar{l}Q\bar{t}} = 0 \tag{30}$$

$$\frac{1}{4} m^2_{H,Y_t}(T) \frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0} \left(\Delta\gamma_{N_i\to lH} + \Delta\gamma_{N_i\to \bar{l}\bar{H}}\right) = 0 \tag{31}$$

[J. Cosmol. Astropart. Phys. 10 (2022) 042]

Unitarity constraints for NLO asymmetries



$$\Delta\gamma_{N_iQ\to lt} + \boxed{\Delta\gamma_{N_i(Q)\to lH(Q)} + \Delta\gamma_{N_i(Q)\to \bar{l}\bar{H}(Q)}} + \Delta\gamma_{N_iQ\to \bar{l}Q Q\bar{t}} = 0 \tag{30}$$

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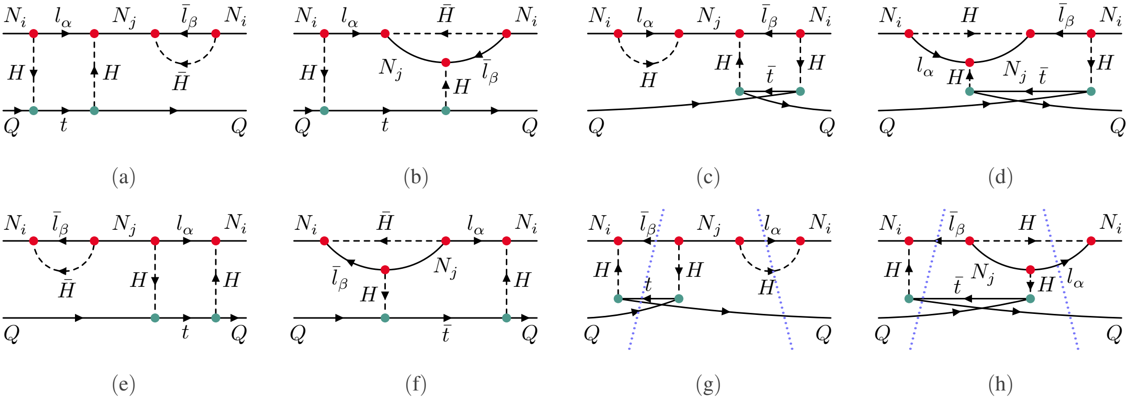
[J. Cosmol. Astropart. Phys. 10 (2022) 042]

Unitarity constraints for NLO asymmetries



$$\Delta\gamma_{N_i Q\to lt} + \Delta\gamma_{N_i(Q)\to lH(Q)} + \Delta\gamma_{N_i(Q)\to \bar{l}\bar{H}(Q)} + \boxed{\Delta\gamma_{N_i Q\to \bar{l}Q Q\bar{t}}} = 0 \qquad (30)$$

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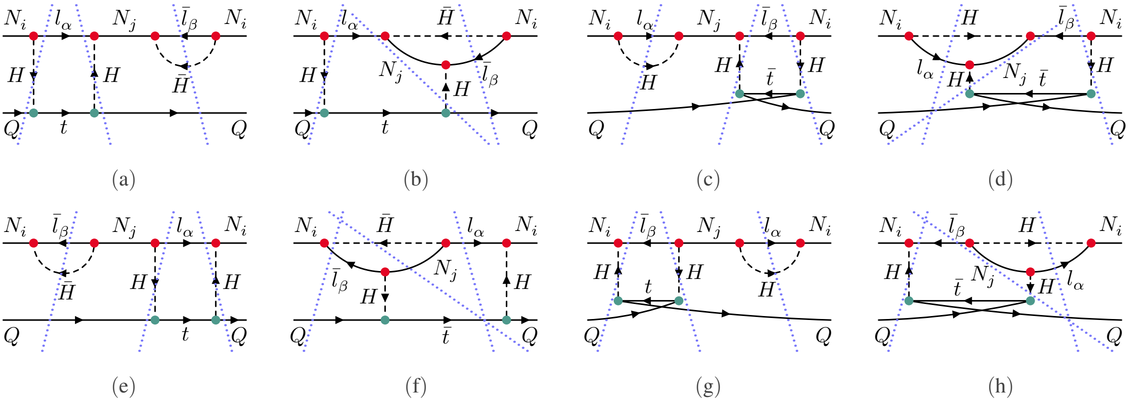
[J. Cosmol. Astropart. Phys. 10 (2022) 042]

Unitarity constraints for NLO asymmetries



$$\Delta\gamma_{N_i Q\to lt} + \Delta\gamma_{N_i(Q)\to lH(Q)} + \Delta\gamma_{N_i(Q)\to \bar{l}\bar{H}(Q)} + \Delta\gamma_{N_i Q\to \bar{l}QQ\bar{t}} = 0 \tag{30}$$

$$\boxed{\frac{1}{4} m^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0}\left(\Delta\gamma_{N_i\to lH} + \Delta\gamma_{N_i\to \bar{l}\bar{H}}\right)} = 0 \tag{31}$$

[J. Cosmol. Astropart. Phys. 10 (2022) 042]

Summary

- One cannot consistently include higher-order perturbative corrections to the interactions with no inclusion of quantum statistics.

- Winding of propagators represents higher occupation numbers in the Fock space.

- Cutting the diagrams with all possible windings of internal lines allows to formulate unitarity constraints for equlibrium rate asymmetries including thermal corrections.

Thank you!

Backup slides

General one-particle densities

The hermiticity and positive definiteness of $\hat{\rho}$ allows us to write

$$\hat{\rho} = \frac{1}{\mathcal{Z}} \exp\left\{-\hat{\mathcal{F}}\right\}, \mathcal{Z} = \operatorname{Tr}\exp\left\{-\hat{\mathcal{F}}\right\}, \tag{32}$$

assuming

$$\hat{\mathcal{F}} = \sum_p \mathcal{F}_p a_p^\dagger a_p. \tag{33}$$

$$\begin{aligned} \mathcal{Z} &= \sum_{\{i\}} \exp\left\{-\mathcal{F}_1 i_1 - \mathcal{F}_2 i_2 - \ldots\right\} = \prod_p \mathcal{Z}_p \quad \text{where} \quad \mathcal{Z}_p = \frac{\exp\{\mathcal{F}_p\}}{\exp\{\mathcal{F}_p\} - 1} \\ f_p &= \operatorname{Tr}\left[\hat{\rho} a_p^\dagger a_p\right] = \frac{1}{\exp\{\mathcal{F}_p\} - 1} \quad \rightarrow \quad \mathring{f}_p \overset{\text{def.}}{=} \exp\left\{-\mathcal{F}_p\right\} = \frac{f_p}{1 + f_p} \end{aligned} \tag{34}$$

General one-particle densities

$$\hat{\rho}' = S\hat{\rho}S^\dagger \quad \Rightarrow \quad \boxed{\hat{\rho}' - \hat{\rho} = T\hat{\rho}T^\dagger - \frac{1}{2}TT^\dagger\hat{\rho} - \frac{1}{2}\hat{\rho}TT^\dagger + \ldots} \tag{35}$$

[McKellar, Thomson '94]

Tracing with $a_p^\dagger a_p$ over $|i_1, i_2, \ldots\rangle$ we get

$$f_p' - f_p = \mathrm{Tr}\Big[a_p^\dagger a_p \Big(T\hat{\rho}T^\dagger - \hat{\rho}TT^\dagger\Big)\Big] = \ldots = \tag{36}$$

$$= \boxed{\frac{1}{\mathcal{Z}} \sum_{k=1}^{\infty} (-1)^k \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{i_1} \mathring{f}_2^{i_2} \ldots (\mathrm{i}\,T)^k_{in} \mathrm{i}\,T_{ni}}$$

leading to statistical factors as in equilibrium case. [Eur. Phys. J. C 81 (2021) 1050]