The Weinberg 3HDM potential

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Introduction

Weinberg 3HDM potential (1976) in notation of Ivanov and Nishi:

$$V = V_2 + V_4, \quad \text{with} \quad V_4 = V_0 + V_{\rm ph},$$

Insensitive to relative phases of fields ϕ_1, ϕ_2, ϕ_3:

$$V_2 = -[m_{11}(\phi_1^\dagger\phi_1) + m_{22}(\phi_2^\dagger\phi_2) + m_{33}(\phi_3^\dagger\phi_3)],$$

$$\begin{aligned} V_0 = &\lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{22}(\phi_2^\dagger\phi_2)^2 \\ &+ \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\ &+ \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2), \end{aligned}$$

Sensitive to phases:

$$V_{\rm ph} = \lambda_1(\phi_2^\dagger\phi_3)^2 + \lambda_2(\phi_3^\dagger\phi_1)^2 + \lambda_3(\phi_1^\dagger\phi_2)^2 + \text{ h.c.}$$

Introduction

Weinberg:

Natural flavour conservation and CPV can be arranged by complex potential. Branco (1980) showed that this could also be achieved with a real potential (and complex vevs). Case studied

$$V_2 = -[m_{11}(\phi_1^\dagger\phi_1) + m_{22}(\phi_2^\dagger\phi_2) + m_{33}(\phi_3^\dagger\phi_3)],$$

$$\begin{aligned} V_0 = &\lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{22}(\phi_2^\dagger\phi_2)^2 \\ &+ \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\ &+ \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2), \end{aligned}$$

$$V_{\text{ph}} = \lambda_1(\phi_2^\dagger\phi_3)^2 + \lambda_2(\phi_3^\dagger\phi_1)^2 + \lambda_3(\phi_1^\dagger\phi_2)^2 + \text{ h.c.}$$

Important element: $\mathbb{Z}_2 \times \mathbb{Z}_2$-symmetry (only even powers of fields)

$\phi_i \to -\phi_i$ for all three ϕ_i.

Introduction

Observation 1:

CP violation (good for baryogenesis) generated by

$$V_{\rm ph} = \lambda_1(\phi_2^\dagger\phi_3)^2 + \lambda_2(\phi_3^\dagger\phi_1)^2 + \lambda_3(\phi_1^\dagger\phi_2)^2 + \text{ h.c.}$$

Want CPV to be small, in view of SM-like Higgs boson at 125 GeV

Study limit:

$$V_{\rm ph} \to 0, \quad \{\lambda_1, \lambda_2, \lambda_3\} \to 0$$

Observation 2: $\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{U}(1)$ symmetry

$$V_2 = -[m_{11}(\phi_1^\dagger\phi_1) + m_{22}(\phi_2^\dagger\phi_2) + m_{33}(\phi_3^\dagger\phi_3)],$$

$$\begin{aligned} V_0 = {} & \lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{22}(\phi_2^\dagger\phi_2)^2 \\ & + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\ & + \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2), \end{aligned}$$

Observation 2 cont:

$V_2 + V_0$ is invariant under

$$\phi_1 \to e^{i\alpha_1}\phi_1, \quad \phi_2 \to e^{i\alpha_2}\phi_2, \quad \phi_3 \to e^{i\alpha_3}\phi_3$$

Observation 3:

Actually, one U(1) factor is combined with the hypercharge, when the U(1) $\times$ U(1) symmetry is broken by the vacuum, we are left with two Goldstone bosons

Conjecture:

With $V_{\rm ph} \neq 0$, or $\{\lambda_1, \lambda_2, \lambda_3\} \neq 0$,
two light states with a significant CP-odd content?

Minimization

Notation:

$$\phi_i = e^{i\theta_i} \begin{pmatrix} \phi_i^+ \\ (v_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, 3$$

$$\{w_1, w_2, w_3\} = \{v_1, v_2\, e^{i\theta_2}, v_3\, e^{i\theta_3}\}$$

Three minimization conditions with respect to moduli

$$\begin{aligned} m_{11} &= \lambda_{11} v_1^2 + \tfrac{1}{2}\bar{\lambda}_{12} v_2^2 + \tfrac{1}{2}\bar{\lambda}_{13} v_3^2 + \lambda_2 c_{2\theta_3} v_3^2 + \lambda_3 c_{2\theta_2} v_2^2, \\ m_{22} &= \lambda_{22} v_2^2 + \tfrac{1}{2}\bar{\lambda}_{12} v_1^2 + \tfrac{1}{2}\bar{\lambda}_{23} v_3^2 + \lambda_1 c_{(2\theta_3 - 2\theta_2)} v_3^2 + \lambda_3 c_{2\theta_2} v_1^2, \\ m_{33} &= \lambda_{33} v_3^2 + \tfrac{1}{2}\bar{\lambda}_{13} v_1^2 + \tfrac{1}{2}\bar{\lambda}_{23} v_2^2 + \lambda_1 c_{(2\theta_3 - 2\theta_2)} v_2^2 + \lambda_2 c_{2\theta_3} v_1^2, \end{aligned}$$

$c_x \equiv \cos x$

$$\bar{\lambda}_{12} \equiv \lambda_{12} + \lambda'_{12}, \quad \bar{\lambda}_{13} \equiv \lambda_{13} + \lambda'_{13}, \quad \bar{\lambda}_{23} \equiv \lambda_{23} + \lambda'_{23}$$

Minimization

Minimize with respect to phases:

$$\lambda_1 v_3^2 \sin(2\theta_2 - 2\theta_3) + \lambda_3 v_1^2 \sin 2\theta_2 = 0,$$
$$\lambda_1 v_2^2 \sin(2\theta_3 - 2\theta_2) + \lambda_2 v_1^2 \sin 2\theta_3 = 0.$$

Phases are related:

$$\lambda_3 v_2^2 \sin 2\theta_2 + \lambda_2 v_3^2 \sin 2\theta_3 = 0.$$

Relative sign of $\sin 2\theta_2$ and $\sin 2\theta_3$ is opposite of that of λ_2/λ_3.

Branco (1980)

$$\cos 2\theta_2 = \frac{1}{2}\left[\frac{D_{23}D_{31}}{D_{12}^2} - \frac{D_{31}}{D_{23}} - \frac{D_{23}}{D_{31}}\right]$$
$$\cos 2\theta_3 = \frac{1}{2}\left[\frac{D_{23}D_{12}}{D_{31}^2} - \frac{D_{12}}{D_{23}} - \frac{D_{23}}{D_{12}}\right]$$

$$D_{12} = \lambda_3 (v_1 v_2)^2, \quad D_{23} = \lambda_1 (v_2 v_3)^2, \quad D_{31} = \lambda_2 (v_3 v_1)^2$$

Minimization

Other (our) approach (retains sign information)

Free input: $v_1,\ v_2,\ v_3,\ \theta_2,\ \theta_3$

$$\lambda_2 = \frac{\lambda_1 v_2^2 \sin(2\theta_2 - 2\theta_3)}{v_1^2 \sin 2\theta_3},$$

$$\lambda_3 = -\frac{\lambda_1 v_3^2 \sin(2\theta_2 - 2\theta_3)}{v_1^2 \sin 2\theta_2}.$$

Next: masses

Masses

Neutral sector (5×5):

It is instructive to study the determinant:

$$D_{5\times 5} = \frac{\lambda_1^2 \sin^2(2\theta_2 - 2\theta_3)}{v^2 v_1^4 (v_2^2 + v_3^2)^5 \sin^5 2\theta_2 \sin^5 2\theta_3} F(\theta_2, \theta_3, \ldots),$$

$$\begin{aligned} F(\theta_2, \theta_3, \ldots) = {} & 64 \lambda_1^3 v_2^6 v_3^{10} w^2 \sin^2 2\theta_2 \sin^8 2\theta_3 \tilde{F}_{2,8} \\ & + \lambda_1^2 v_2^4 v_3^8 \sin^3 2\theta_2 \sin^7 2\theta_3 \tilde{F}_{3,7} \\ & + \lambda_1 v_2^2 v_3^6 \sin^4 2\theta_2 \sin^6 2\theta_3 \tilde{F}_{4,6} \\ & + v_2^4 v_3^4 \sin^5 2\theta_2 \sin^5 2\theta_3 \tilde{F}_{5,5} \\ & + \{(\theta_2, v_2, \lambda_{22}, \bar{\lambda}_{12}) \leftrightarrow (\theta_3, v_3, \lambda_{33}, \bar{\lambda}_{13}))\} \end{aligned}$$

Two powers of λ_1 for two light masses

Masses

Neutral sector (5×5):

general case

$$\mathcal{M}^2_{\rm neut} = \begin{pmatrix} X & X & X & 0 & 0 \\ X & X & X & 0 & x \\ X & X & X & x & 0 \\ 0 & 0 & x & x & x \\ 0 & x & 0 & x & x \end{pmatrix} \quad \begin{vmatrix} \eta_1^{\rm HB} \\ \eta_2^{\rm HB} \\ \eta_3^{\rm HB} \\ \chi_2^{\rm HB} \\ \chi_3^{\rm HB} \end{vmatrix}$$

$\theta_2 = \theta_3$

$$\begin{pmatrix} X & X & X & 0 & 0 \\ X & X & X & 0 & 0 \\ X & X & X & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x \end{pmatrix}$$

where x denotes a term vanishing with λ_1

$$\boxed{h_i = O_{ij}\varphi_j^{\rm HB}}$$

O only known numerically

$$m_1 < m_2 < m_3 < m_4 < m_5$$

Which one is 125 GeV?

Masses

Neutral sector (5×5):

general case

$$\mathcal{M}^2_{\rm neut} = \begin{pmatrix} X & X & X & 0 & 0 \\ X & X & X & 0 & x \\ X & X & X & x & 0 \\ 0 & 0 & x & x & x \\ 0 & x & 0 & x & x \end{pmatrix} \begin{vmatrix} \eta_1^{\rm HB} \\ \eta_2^{\rm HB} \\ \eta_3^{\rm HB} \\ \chi_2^{\rm HB} \\ \chi_3^{\rm HB} \end{vmatrix}$$

$\theta_2 = \theta_3$

$$\begin{pmatrix} X & X & X & 0 & 0 \\ X & X & X & 0 & 0 \\ X & X & X & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x \end{pmatrix}$$

simple case

$$\begin{pmatrix} X & X & 0 & 0 & 0 \\ X & X & X & 0 & 0 \\ 0 & X & x & 0 & 0 \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & X \end{pmatrix} \begin{vmatrix} \eta_1^{\rm HB} \\ \eta_2^{\rm HB} \\ \chi_3^{\rm HB} \\ \chi_2^{\rm HB} \\ \eta_3^{\rm HB} \end{vmatrix} \begin{matrix} \\ \\ \longleftarrow \\ \\ \longleftarrow \end{matrix}$$

where x denotes a term vanishing with λ_1

simple case: $\lambda_2 = \pm\lambda_3$, $\theta_2 = \mp\theta_3$, $v_2 = v_3$, $\lambda_{ii} = \lambda_{jj}$, $\bar{\lambda}_{ij} = \bar{\lambda}_{12}$

$$\boxed{h_i = O_{ij}\varphi_j^{\rm HB}}$$

O only known numerically

$$m_1 < m_2 < m_3 < m_4 < m_5$$

Which one is 125 GeV?

Gauge couplings

Cubic gauge-gauge-scalar part:

$$\mathcal{L}_{VVh} = \left(g m_W W^+_\mu W^{\mu -} + \frac{g m_Z}{2\cos\theta_W} Z_\mu Z^\mu \right) \sum_{i=1}^{5} O_{i1} h_i$$

Cubic gauge-scalar-scalar terms:

$$\mathcal{L}_{Vhh} = -\frac{g}{2\cos\theta_W} \sum_{i=1}^{5}\sum_{j=1}^{5} (O_{i2}O_{j4} + O_{i3}O_{j5})(h_i \overleftrightarrow{\partial}_\mu h_j) Z^\mu$$

$$+ \frac{g}{2} \sum_{i=1}^{5}\sum_{j=1}^{2} [(iO_{i\,j+1} + O_{i\,j+3}) \sum_{k=1}^{2} U_{jk} (h_k^+ \overleftrightarrow{\partial}_\mu h_i) W^{\mu -} + \text{h.c.}]$$

$$+ \left(ieA^\mu + \frac{ig\cos 2\theta_W}{2\cos\theta_W} Z^\mu \right) \sum_{j=1}^{2} (h_j^+ \overleftrightarrow{\partial}_\mu h_j^-),$$

Gauge couplings

The $h_j WW$ (and $h_j ZZ$) coupling is given by O_{j1}

How to measure different CP content?

Z is odd under CP, study the trilinear coupling $h_i h_j Z$

$$P_{ij} = (O_{i2}O_{j4} + O_{i3}O_{j5}) - (i \leftrightarrow j)$$

In the 2HDM, allowing for CP violation, the $h_i h_j Z$ couplings are essentially the same as the $h_k ZZ$ couplings, with i, j, k all different

Not the case in a 3HDM.

Scan

Scan over model parameters:

Want the Higgs-gauge coupling $h_{\rm SM}WW$ to be close to unity

$$|O_{j1}| \simeq 1, \quad \text{for some } j.$$

$$\begin{aligned} v_i &\in [0, v], & i &= 1,2,3, \quad \text{with } v_1^2 + v_2^2 + v_3^2 = v^2, \\ \theta_i &\in [-\pi, \pi], & i &= 2,3, \\ \lambda_{ii}, \lambda_{ij}, \lambda'_{ij}, \lambda_1 &\in [-4\pi, 4\pi], & i,j &= 1,2,3. \end{aligned}$$

Scan

Scan over model parameters:

For each $j = 1$ to 5:

1. check if the coupling O_{j1} to WW (or ZZ) is compatible with LHC measurements, 3σ ($\sigma = 0.12$) tolerance,
2. rescale all λs such that $m_j = m_{\text{SM}} = 125.25$ GeV [footnote]
3. check if all rescaled λs (including λ_2 and λ_3) are within the perturbative range,
4. check if the lightest charged scalar is above 80 GeV.

[footnote] masses squared are linear in λs

Distribution [in %] of SM-like h_j

h_1	h_2	h_3	h_4	h_5
0.32	38.05	28.22	22.83	10.58

Z affinity

The quantity

$$P_{ij} = (O_{i2}O_{j4} + O_{i3}O_{j5}) - (i \leftrightarrow j)$$

measures how “different” two states h_i and h_j are in terms of CP.

Recall the CP-conserving 2HDM: full-strength HAZ coupling, no hHZ coupling

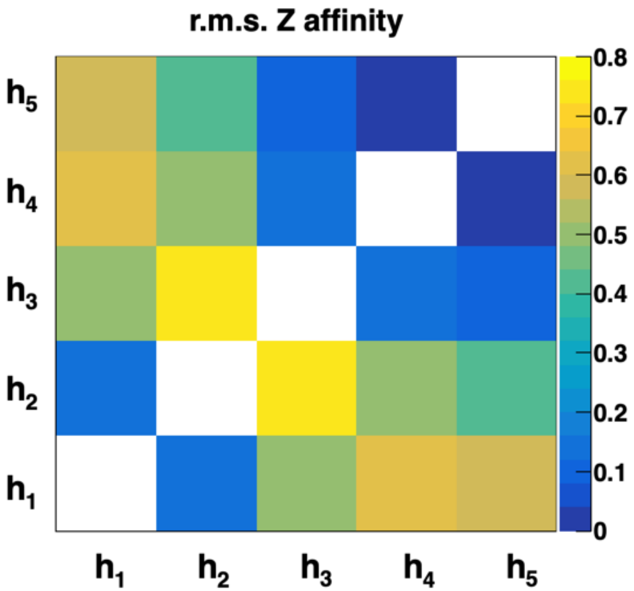
Because of alignment, no hAZ coupling either

As a reference, we analysed parameter points that were not subject to the experimental SM-like Higgs constraints

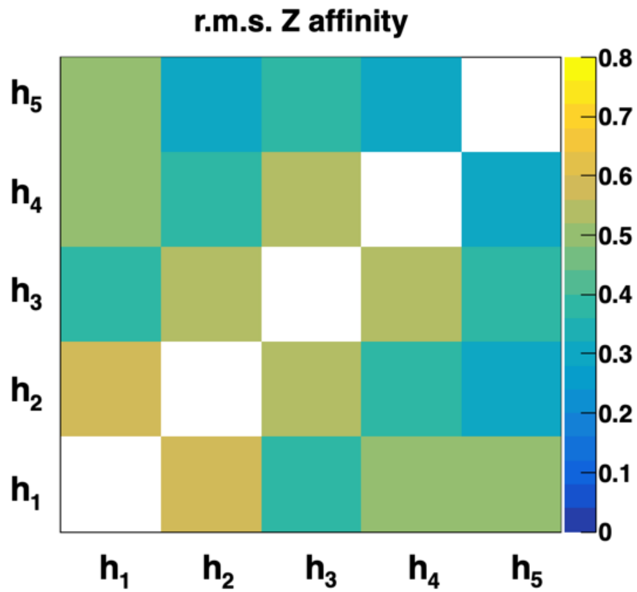
For this study, we define a “near $U(1) \times U(1)$ symmetry” condition

$$\max(|\lambda_1|, |\lambda_2|, |\lambda_3|) = 0.01$$

Z affinity



"near $U(1) \times U(1)$ symmetry":
h_1 and h_2 have low Z-affinity: similar CP. Likewise,
h_3, h_4 and h_5 have similar CP



No particular constraints on λ_1, λ_2, λ_3

Near the $U(1) \times U(1)$ limit we have two neutral states that are approximately odd under CP, and three that are approximately even.

Z affinity

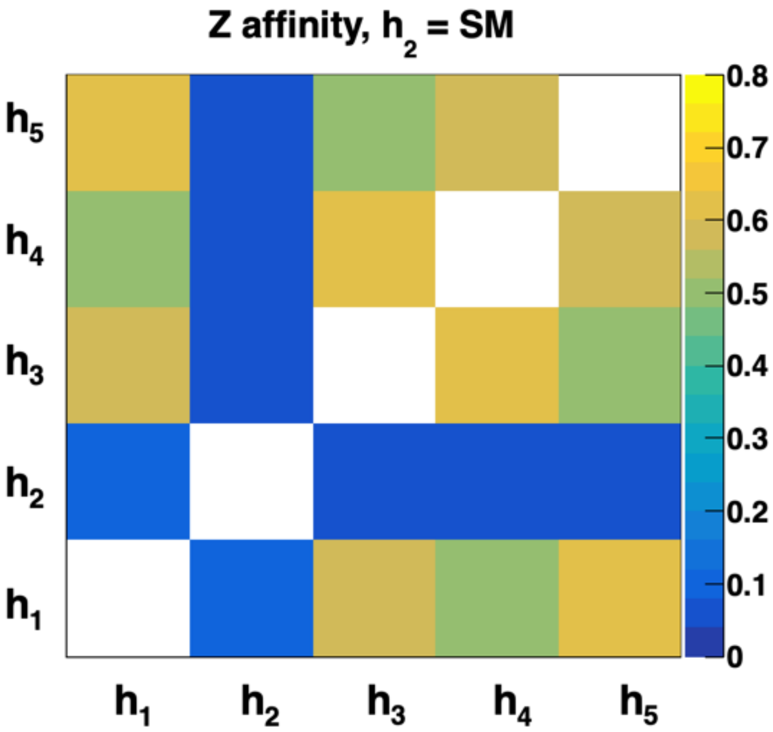
Consider the CP-conserving 2HDM: H and A have opposite CP (Z affinity $= 1$), as do h and A (but Z affinity $= 0$). Alignment!

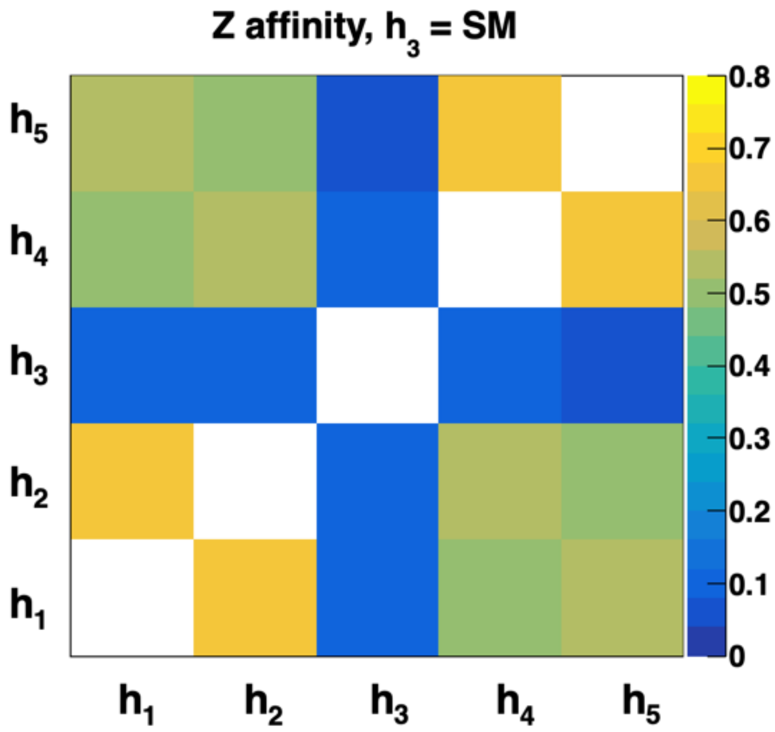
It is instructive to consider how the Z affinity is affected by alignment.
Let h_j be "aligned", meaning its coupling to WW is maximal, $O_{j1} = 1$.
By orthogonality, it follows that $O_{k1} = 0$ for $k \neq j$ and $O_{jk} = 0$, for $k \neq 1$. Then

$$P_{ij} = P_{ji} = 0 \quad \text{for all } i$$

Z affinity

Alignment examples





Scan points where h_2 (left) and h_3 (right) satisfy LHC SM constraint

Z affinity

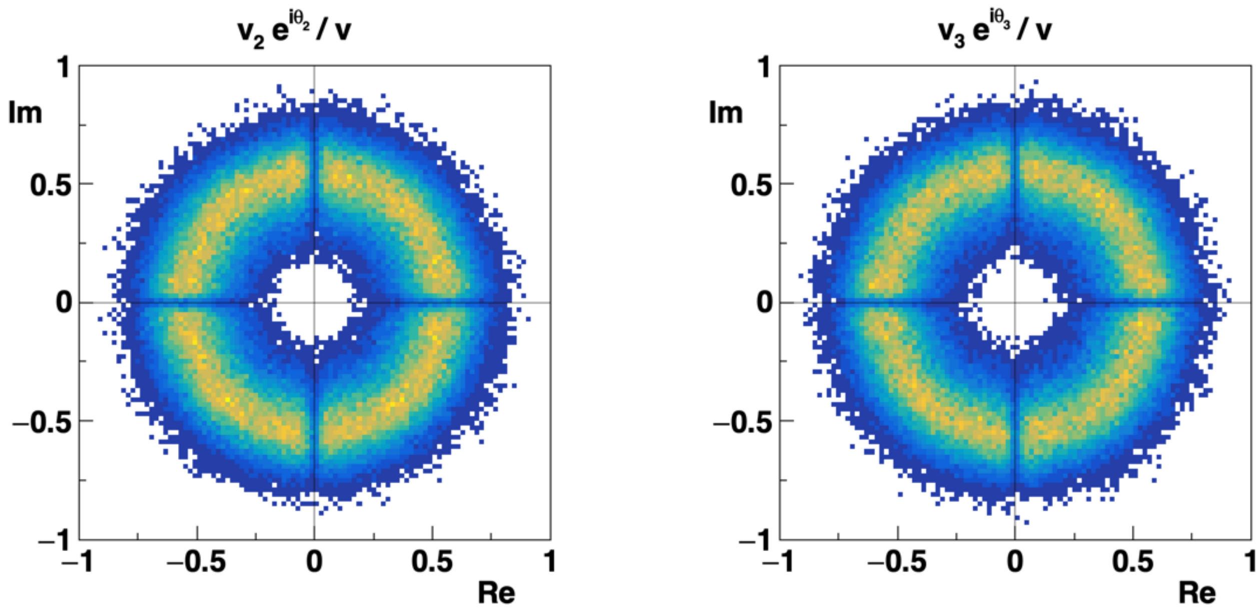
Attempt to circumvent the “alignment problem”

Normalized to the squared sum of even and odd couplings

$$\hat{P}_{ij} = \frac{P_{ij}}{\sqrt{\min(O_{i1}^2, O_{j1}^2) + P_{ij}^2}}$$

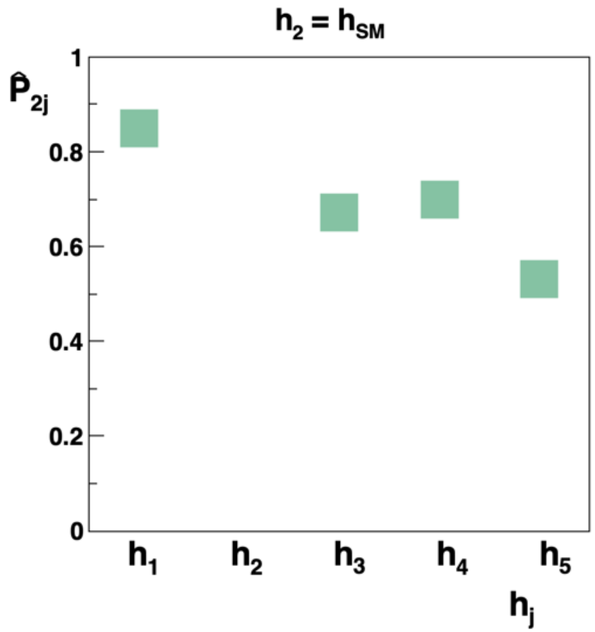
with O_{i1} representing the CP-even ZZh_i coupling

h_2 as h_{SM}



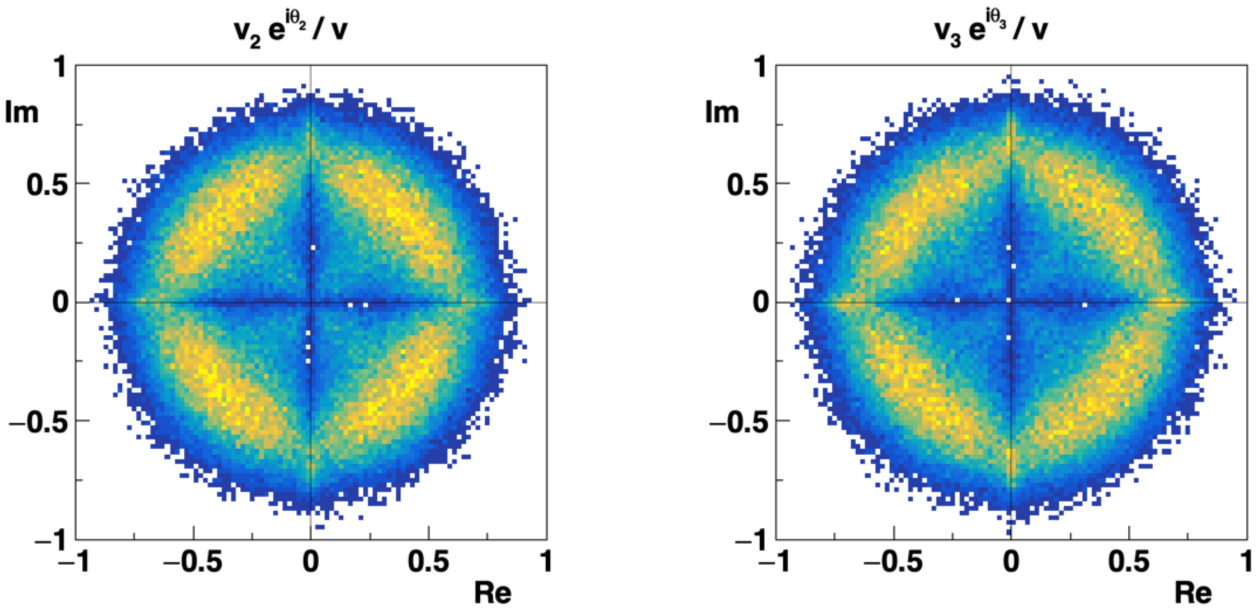
Complex vevs $v_2 e^{i\theta_2}/v$ and $v_3 e^{i\theta_3}/v$, for $h_2 = h_{\rm SM}$.
Yellow is high, dark blue is low. Arbitrary normalization.

h_2 as h_{SM}



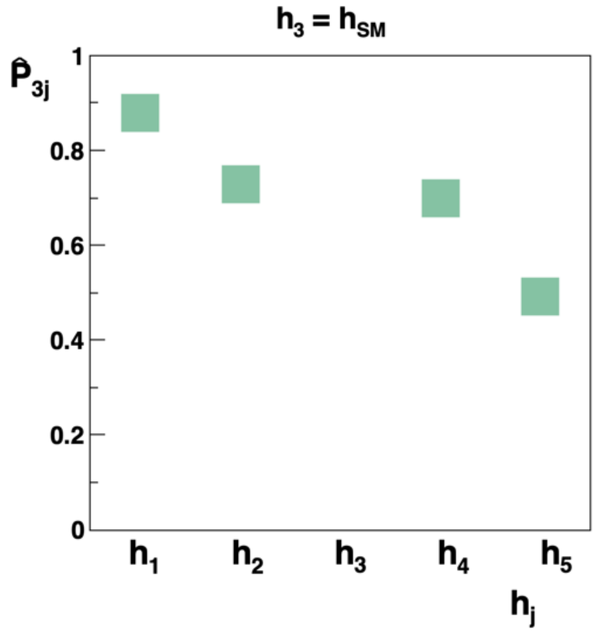
Relative strength of the $h_2 h_j Z$ couplings, in units of $g/(2\cos\theta_W)$ (root-mean-square, averaged over the scan).

h_3 as h_{SM}



Complex vevs $v_2 e^{i\theta_2}/v$ and $v_3 e^{i\theta_3}/v$, for $h_3 = h_{\rm SM}$.
Yellow is high, dark blue is low. Arbitrary normalization.

h_3 as h_{SM}



Relative strength of the $h_3 h_j Z$ couplings, in units of $g/(2\cos\theta_W)$.

Yukawa couplings

Example $\mathbb{Z}_2 \times \mathbb{Z}_2$ charges:

$$\phi_1 : (+1,+1) \qquad \phi_2 : (-1,+1) \qquad \phi_3 : (+1,-1)$$
$$u_R : (+1,+1) \qquad d_R : (-1,+1) \qquad e_R : (+1,-1)$$

Yukawa Lagrangian

$$\mathcal{L}_Y = Y^u \bar{Q}_L \tilde{\phi}_1 u_R + Y^d \bar{Q}_L \phi_2 d_R + Y^e \bar{E}_L \phi_3 e_R + \text{h.c.}$$

neutral interactions

$$\mathcal{L}_Y^{\text{neutral}} = \frac{1}{v_1} \bar{u} M^u (\eta_1 + i\chi_1\gamma_5) u + \frac{1}{v_2} \bar{d} M^d (\eta_2 + i\chi_2\gamma_5) d + \frac{1}{v_3} \bar{e} M^e (\eta_3 + i\chi_3\gamma_5) e.$$

The η_i and χ_i fields will mix.
Neutral physical scalar h_i and a fermion f:

$$\mathcal{L}_{h_i f f} = \frac{m_f}{v} h_i (\kappa^{h_i f f} \bar{f} f + i \tilde{\kappa}^{h_i f f} \bar{f} \gamma_5 f)$$

Yukawa couplings

$$\mathcal{L}_{h_i ff} = \frac{m_f}{v} h_i (\kappa^{h_i ff} \bar{f} f + i\tilde{\kappa}^{h_i ff} \bar{f} \gamma_5 f)$$

For $\tau\bar{\tau}$ final states, CMS has constrained mixing

$$\tan \alpha^{h_{\rm SM}\tau\tau} = \frac{\tilde{\kappa}^{h_{\rm SM}\tau\tau}}{\kappa^{h_{\rm SM}\tau\tau}}$$

Rotate from η_k and χ_k, via Higgs basis fields to physical fields:

$$Z_i^{(k)} = \tilde{\mathcal{R}}_{1k} O_{i1} + \tilde{\mathcal{R}}_{2k}(O_{i2} + iO_{i4}) + \tilde{\mathcal{R}}_{3k}(O_{i3} + iO_{i5})$$

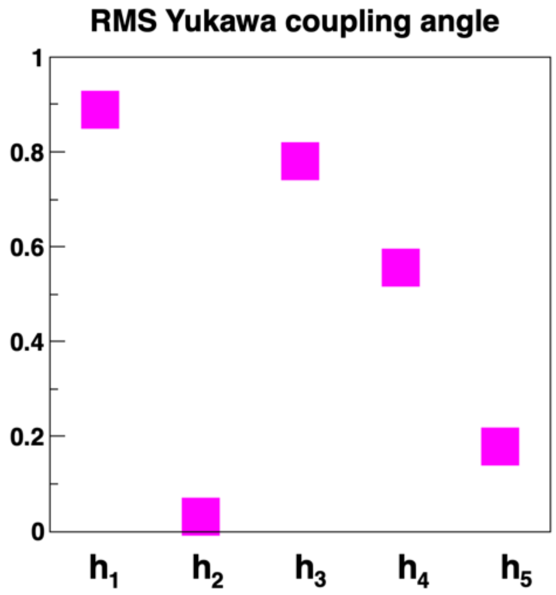
$$\alpha^{h_i\tau\tau} = \arg(Z_i^{(3)})$$

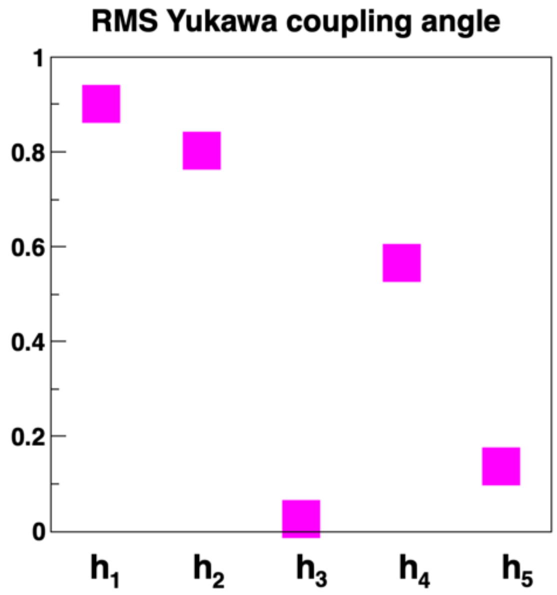
Yukawa couplings

h_2 as h_{SM}

h_3 as h_{SM}

having imposed cut on α for h_{SM}





EXPERIMENTAL ISSUE

If h_2 or h_3 plays the role of h_{SM} at 125 GeV

Why have not h_1 or h_2 been observed?

1. Reduced coupling for Bjorken process (LEP)
2. Reduced gamma-gamma BR

However, note suggestions by Heinemeyer et al, 96 GeV

2105.11189, 2203.13180, 2204.05975

CONCLUSIONS

We have reviewed the Weinberg 3HDM potential

- accommodates CP violation and NFC
- consequence: light states with a significant CP-odd content (below 125 GeV)
- Plea for LHC: keep searching!

BACKUP

Appendix: Limits of CPC

In special cases, no CP violation. Study CP-odd invariants

At the lowest non-trivial order, the invariants can be expanded in terms of

$S = \sin(2\theta_2 - 2\theta_3)$

and

$$
\begin{aligned}
X_a &= \lambda_{11}(\lambda_{12} - \lambda_{13}) + \lambda_{22}(\lambda_{23} - \lambda_{12}) + \lambda_{33}(\lambda_{13} - \lambda_{23}) \\
X_b &= \lambda_{11}(\lambda'_{12} - \lambda'_{13}) + \lambda_{22}(\lambda'_{23} - \lambda'_{12}) + \lambda_{33}(\lambda'_{13} - \lambda'_{23}) \\
X_c &= \lambda_{12}(\lambda'_{13} - \lambda'_{23}) + \lambda_{13}(\lambda'_{23} - \lambda'_{12}) + \lambda_{23}(\lambda'_{12} - \lambda'_{13})
\end{aligned}
$$

$$
\begin{aligned}
W_a &= (\lambda_{23} - \lambda_{13})v_1^4 v_2^4 \sin^2 2\theta_2 + (\lambda_{13} - \lambda_{12})v_2^4 v_3^4 \sin^2(2\theta_2 - 2\theta_3) \\
&\quad + (\lambda_{12} - \lambda_{23})v_1^4 v_3^4 \sin^2 2\theta_3, \\
W_b &= (\lambda'_{23} - \lambda'_{13})v_1^4 v_2^4 \sin^2 2\theta_2 + (\lambda'_{13} - \lambda'_{12})v_2^4 v_3^4 \sin^2(2\theta_2 - 2\theta_3) \\
&\quad + (\lambda'_{12} - \lambda'_{23})v_1^4 v_3^4 \sin^2 2\theta_3, \\
W_c &= (\lambda_{11} - \lambda_{22})v_1^4 v_2^4 \sin^2 2\theta_2 + (\lambda_{22} - \lambda_{33})v_2^4 v_3^4 \sin^2(2\theta_2 - 2\theta_3) \\
&\quad + (\lambda_{33} - \lambda_{11})v_1^4 v_3^4 \sin^2 3\theta_3).
\end{aligned}
$$

must all vanish…

Higgs basis

$$\mathcal{R}_2\mathcal{R}_1 \begin{pmatrix} v_1 \\ e^{i\theta_2} v_2 \\ e^{i\theta_3} v_3 \end{pmatrix} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathcal{R}_1 = \begin{pmatrix} 1 & 0 \\ 0 & R_1 \end{pmatrix}, \quad R_1 = \frac{1}{w} \begin{pmatrix} v_2 e^{-i\theta_2} & v_3 e^{-i\theta_3} \\ -v_3 e^{-i\theta_2} & v_2 e^{-i\theta_3} \end{pmatrix}, \quad w = \sqrt{v_2^2 + v_3^2}$$

Higgs basis

$$\mathcal{R}_2\mathcal{R}_1 \begin{pmatrix} v_1 \\ e^{i\theta_2} v_2 \\ e^{i\theta_3} v_3 \end{pmatrix} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathcal{R}_1 = \begin{pmatrix} 1 & 0 \\ 0 & R_1 \end{pmatrix}, \quad R_1 = \frac{1}{w} \begin{pmatrix} v_2 e^{-i\theta_2} & v_3 e^{-i\theta_3} \\ -v_3 e^{-i\theta_2} & v_2 e^{-i\theta_3} \end{pmatrix}, \quad w = \sqrt{v_2^2 + v_3^2}$$

$$\mathcal{R}_2 = \frac{1}{v} \begin{pmatrix} v_1 & w & 0 \\ -w & v_1 & 0 \\ 0 & 0 & v \end{pmatrix} \qquad \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \mathcal{R} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \tilde{\mathcal{R}} \begin{pmatrix} \phi_1 \\ e^{-i\theta_2}\phi_2 \\ e^{-i\theta_3}\phi_3 \end{pmatrix}$$

$$\tilde{\mathcal{R}} = \mathcal{R}_2 \frac{1}{w} \begin{pmatrix} w & 0 & 0 \\ 0 & v_2 & v_3 \\ 0 & -v_3 & v_2 \end{pmatrix} \qquad \text{in fact real.}$$

Higgs basis

$$\mathcal{R}_2\mathcal{R}_1 \begin{pmatrix} v_1 \\ e^{i\theta_2} v_2 \\ e^{i\theta_3} v_3 \end{pmatrix} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathcal{R}_1 = \begin{pmatrix} 1 & 0 \\ 0 & R_1 \end{pmatrix}, \quad R_1 = \frac{1}{w} \begin{pmatrix} v_2 e^{-i\theta_2} & v_3 e^{-i\theta_3} \\ -v_3 e^{-i\theta_2} & v_2 e^{-i\theta_3} \end{pmatrix}, \quad w = \sqrt{v_2^2 + v_3^2}$$

$$\mathcal{R}_2 = \frac{1}{v} \begin{pmatrix} v_1 & w & 0 \\ -w & v_1 & 0 \\ 0 & 0 & v \end{pmatrix} \qquad \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \mathcal{R} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \tilde{\mathcal{R}} \begin{pmatrix} \phi_1 \\ e^{-i\theta_2}\phi_2 \\ e^{-i\theta_3}\phi_3 \end{pmatrix}$$

$$\tilde{\mathcal{R}} = \mathcal{R}_2 \frac{1}{w} \begin{pmatrix} w & 0 & 0 \\ 0 & v_2 & v_3 \\ 0 & -v_3 & v_2 \end{pmatrix}$$

in fact real.

$$H_1 = \begin{pmatrix} G^+ \\ (v + \eta_1^{\mathrm{HB}} + iG_0)/\sqrt{2} \end{pmatrix}, \quad H_i = \begin{pmatrix} \varphi_i^{\mathrm{HB}\,+} \\ (\eta_i^{\mathrm{HB}} + i\chi_i^{\mathrm{HB}})/\sqrt{2} \end{pmatrix}, \quad i = 2, 3$$

$$\varphi_i^{\mathrm{HB}} = \{\eta_1^{\mathrm{HB}}, \quad \eta_2^{\mathrm{HB}}, \quad \eta_3^{\mathrm{HB}}, \quad \chi_2^{\mathrm{HB}}, \quad \chi_3^{\mathrm{HB}}\}, \quad i = 1, \dots 5$$

Masses

Charged sector:

$$(\mathcal{M}^2_{\rm ch})_{11} = -\frac{\lambda_1 v^2 \sin^2(2\theta_2 - 2\theta_3) v_2^2 v_3^2}{\sin 2\theta_2 \sin 2\theta_3 v_1^2 w^2} - (\lambda'_{12} v_2^2 + \lambda'_{13} v_3^2)\frac{v^2}{2w^2},$$

$$(\mathcal{M}^2_{\rm ch})_{12} = -\frac{\lambda_1 v v_1 v_2 v_3 \sin(2\theta_2 - 2\theta_3)}{\sin 2\theta_2 \sin 2\theta_3 v_1^2 w^2}(v_2^2 \sin 2\theta_2 e^{2i\theta_3} + v_3^2 \sin 2\theta_3 e^{2i\theta_2}) + \frac{v v_1 v_2 v_3}{2w^2}(\lambda'_{12} - \lambda'_{13}),$$

$$(\mathcal{M}^2_{\rm ch})_{21} = (\mathcal{M}^2_{\rm ch})^*_{12},$$

$$(\mathcal{M}^2_{\rm ch})_{22} = -\frac{\lambda_1}{\sin 2\theta_2 \sin 2\theta_3 w^2}(2\sin 2\theta_2 \sin 2\theta_3 \cos(2\theta_2 - 2\theta_3) v_2^2 v_3^2 + \sin^2 2\theta_2 v_2^4 + \sin^2 2\theta_3 v_3^4)$$
$$- \frac{1}{2w^2}\left[(\lambda'_{12} v_3^2 + \lambda'_{13} v_2^2) v_1^2 + \lambda'_{23} w^4\right].$$

terms proportional to λ_1 and to λ'_{ij}

$$h_i^+ = U_{ij} \varphi^{\rm HB\,+}_{j+1} \qquad U = \begin{pmatrix} \cos\gamma & \sin\gamma\, e^{i\phi} \\ -\sin\gamma\, e^{-i\phi} & \cos\gamma \end{pmatrix}$$

Masses

Neutral sector (5×5):

$$
\begin{aligned}
(\mathcal{M}^2_{\rm neut})_{11} &= \frac{4\lambda_1 v_2^2 v_3^2}{v^2 s_{2\theta_2} s_{2\theta_3}}[1 - c_{2\theta_2 - 2\theta_2} c_{2\theta_2} c_{2\theta_3}] \\
&+ \frac{2}{v^2}[\lambda_{11} v_1^4 + \lambda_{22} v_2^4 + \lambda_{33} v_3^4 + \bar{\lambda}_{12} v_1^2 v_2^2 + \bar{\lambda}_{13} v_1^2 v_3^2 + \bar{\lambda}_{23} v_2^2 v_3^2], \\
(\mathcal{M}^2_{\rm neut})_{12} &= \frac{-2\lambda_1 v_2^2 v_3^2}{v^2 w v_1 s_{2\theta_2} s_{2\theta_3}}[s^2_{2\theta_2 - 2\theta_3}(2w^2 - v^2) - 2c_{2\theta_2 - 2\theta_3} s_{2\theta_2} s_{2\theta_3} v_1^2] \\
&- \frac{v_1}{v^2 w}[2\lambda_{11} v_1^2 w^2 - 2\lambda_{22} v_2^4 - 2\lambda_{33} v_3^4 - (\bar{\lambda}_{12} v_2^2 + \bar{\lambda}_{13} v_3^2)(v^2 - 2w^2) - 2\bar{\lambda}_{23} v_2^2 v_3^2] \\
(\mathcal{M}^2_{\rm neut})_{13} &= \frac{2\lambda_1 v_2 v_3}{v w s_{2\theta_2} s_{2\theta_3}}[v_2^2 s^2_{2\theta_2} - v_3^2 s^2_{2\theta_3}] \\
&+ \frac{v_2 v_3 w}{v w^2}[-2\lambda_{22} v_2^2 + 2\lambda_{33} v_3^2 - \bar{\lambda}_{12} v_1^2 + \bar{\lambda}_{13} v_1^2 + \bar{\lambda}_{23}(v_2^2 - v_3^2)], \\
(\mathcal{M}^2_{\rm neut})_{22} &= \frac{4\lambda_1 v_2^2 v_3^2}{v^2 w^2 s_{2\theta_2} s_{2\theta_3}}[v_1^2 c_{2\theta_2 - 2\theta_3} s_{2\theta_2} s_{2\theta_3} - w^2 s^2_{2\theta_2 - 2\theta_3}] \\
&+ \frac{2v_1^2}{v^2 w^2}[\lambda_{11} w^4 + \lambda_{22} v_2^4 + \lambda_{33} v_3^4 - \bar{\lambda}_{12} v_2^2 w^2 - \bar{\lambda}_{13} v_3^2 w^2 + \bar{\lambda}_{23} v_2^2 v_3^2],
\end{aligned}
$$

Masses

Neutral sector (5×5):

$$(\mathcal{M}^2_{\text{neut}})_{23} = \frac{2\lambda_1 v_2 v_3}{v v_1 w^2 s_{2\theta_2} s_{2\theta_3}}[-w^2 s_{2\theta_2-2\theta_3}(v_2^2 s_{2\theta_2} c_{2\theta_3} + v_3^2 s_{2\theta_3} c_{2\theta_2}) + v_1^2(v_2^2 - v_3^2) c_{2\theta_2-2\theta_3} s_{2\theta_2} s_{2\theta_3}]$$
$$+ \frac{v_1 v_2 v_3}{v w^2}[-2\lambda_{22} v_2^2 + 2\lambda_{33} v_3^2 + (\bar{\lambda}_{12} - \bar{\lambda}_{13}) w^2 + \bar{\lambda}_{23}(v_2^2 - v_3^2)],$$
$$(\mathcal{M}^2_{\text{neut}})_{25} = \frac{2\lambda_1 v v_2 v_3}{v_1} s_{2\theta_2-2\theta_3},$$
$$(\mathcal{M}^2_{\text{neut}})_{33} = \frac{-4\lambda_1 v_2^2 v_3^2}{w^2} c_{2\theta_2-2\theta_3} + \frac{2 v_2^2 v_3^2}{w^2}[\lambda_{22} + \lambda_{33} - \bar{\lambda}_{23}],$$
$$(\mathcal{M}^2_{\text{neut}})_{34} = \frac{-2\lambda_1 v v_2 v_3}{v_1} s_{2\theta_2-2\theta_3},$$
$$(\mathcal{M}^2_{\text{neut}})_{44} = \frac{-2\lambda_1 v^2 v_2^2 v_3^2}{v_1^2 w^2 s_{2\theta_2} s_{2\theta_3}} s^2_{2\theta_2-2\theta_3},$$
$$(\mathcal{M}^2_{\text{neut}})_{45} = \frac{-2\lambda_1 v v_2 v_3}{v_1 w^2 s_{2\theta_2} s_{2\theta_3}} s_{2\theta_2-2\theta_3}[v_2^2 s_{2\theta_2} c_{2\theta_3} + v_3^2 s_{2\theta_3} c_{2\theta_2}],$$
$$(\mathcal{M}^2_{\text{neut}})_{55} = \frac{-2\lambda_1}{w^2 s_{2\theta_2} s_{2\theta_3}}[2 v_2^2 v_3^2 c_{2\theta_2-2\theta_3} s_{2\theta_2} s_{2\theta_3} + v_2^4 s^2_{2\theta_2} + v_3^4 s^2_{2\theta_3}],$$
$$(\mathcal{M}^2_{\text{neut}})_{14} = (\mathcal{M}^2_{\text{neut}})_{15} = (\mathcal{M}^2_{\text{neut}})_{24} = (\mathcal{M}^2_{\text{neut}})_{35} = 0.$$