## The Weinberg 3HDM potential

Per Osland

University of Bergen

DISCRETE 2022

Baden-Baden, Nov 2022

Robin Plantey, Marius Solberg (both NTNU, Trondheim) Odd Magne Ogreid, Gui Rebelo, P.O.: 2208.13594, 2209.06499

## **1 Introduction** where *V*<sup>2</sup> and *V*<sup>0</sup> are insensitive to independent rephasing of the Higgs doublets,

<sup>2</sup>2) + 13(*†*

11)(*†*

<sup>3</sup>3) + 22(*†*

<sup>2</sup>2)

11)(*†*

11)<br>11) - Johann Barnes<br>11) - Johann Barnes

Weinberg 3HDM potential (1976) in notation of Ivanov and Nishi: *V* = *V*<sup>2</sup> + *V*4*,* with *V*<sup>4</sup> = *V*<sup>0</sup> + *V*ph*,* (0.1) *V*ph = 1(*†* <sup>2</sup>3) <sup>2</sup> + 2(*†* <sup>2</sup> + 3(*†* <sup>2</sup> + h.c. (0.3) *V*<sup>0</sup> = 11(*†* <sup>1</sup>1) <sup>2</sup> + 12(*†* 11)(*†* <sup>2</sup>2) + 13(*†* 11)(*†* <sup>3</sup>3) + 22(*†*

 $v = v_2 + v_4$ , with  $v_4 = v_0 + v_{ph}$ ,  $V = V_2 + V_4$ , with  $V_4 = V_0 + V_{\text{ph}}$ ,  $v = v_2 + v_4$ , with  $v_4 = v_0 + v_0$ ,  $v_0$ + 23(*†* 22)(*†* <sup>3</sup>3) + 33(*†* <sup>3</sup>3)

 $V_2 = -[m_{11}(\phi_1^{\dagger}\phi_1) + m_{22}(\phi_2^{\dagger}\phi_2) + m_{33}(\phi_3^{\dagger}\phi_3)],$  $V_0 = \lambda_{11}(\phi_1^{\dagger}\phi_1)^2 + \lambda_{12}(\phi_1^{\dagger}\phi_1)(\phi_2^{\dagger}\phi_2) + \lambda_{13}(\phi_1^{\dagger}\phi_1)(\phi_3^{\dagger}\phi_3) + \lambda_{22}(\phi_2^{\dagger}\phi_2)^2$  $+ \lambda_{23}(\phi_2^{\dagger}\phi_2)(\phi_3^{\dagger}\phi_3) + \lambda_{33}(\phi_3^{\dagger}\phi_3)^2$  $+ \lambda'_{12}(\phi_1^{\intercal}\phi_2)(\phi_2^{\intercal}\phi_1) + \lambda'_{13}(\phi_1^{\intercal}\phi_3)(\phi_3^{\intercal}\phi_1) + \lambda'_{23}(\phi_2^{\intercal}\phi_3)(\phi_3^{\intercal}\phi_2),$  $V = \int_{\infty}^{\infty} (\phi^{\dagger} \phi) + \infty (\phi^{\dagger} \phi) + \infty (\phi^{\dagger} \phi)$  $\lambda_{23}(\phi_{2}^{\dagger}\phi_{2})(\phi_{3}^{\dagger}\phi_{3}) + \lambda_{33}(\phi_{3}^{\dagger}\phi_{3})$  $(93)^{2}$ <sup>3</sup>3) + 22(*†*  $+ \lambda'_{12}(\phi_1^{\prime}\phi_2)(\phi_2^{\prime}\phi_1) + \lambda'_{13}(\phi_1^{\prime}\phi_3)(\phi_3^{\prime}\phi_1) + \lambda'_{23}(\phi_2^{\prime}\phi_3)(\phi_3^{\prime}\phi_2),$  $V_2 = -[m_{11}(\phi_1^{\dagger} \phi_1) + m_{22}(\phi_2^{\dagger} \phi_2) + m_{33}(\phi_3^{\dagger} \phi_3)],$ Insensitive to relative phases of fields  $\phi_1, \phi_2, \phi_3$ :  $\frac{1}{2}$   $\frac{1}{4}$   $\frac{1}{2}$   $\mathbf{p}_{\mathbf{H}}$  $\frac{v}{\sqrt{2}}$  $\lambda_{12}(\phi_{1}^{'}\phi_{1})(\phi_{2}^{'}\phi_{2}) + \lambda_{13}(\phi_{1}^{'}\phi_{1})(\phi_{3}^{'}\phi_{3}) + \lambda_{22}(\phi_{2}^{'}\phi_{2})$  $\int$  - h.c. (1)

> + <sup>0</sup> 12(*†* 12)(*†* <sup>2</sup>1) + <sup>0</sup> Sensitive to phases:

 $= \lambda_1(\phi_2^{\dagger} \phi_3)^2 + \lambda_2(\phi_3^{\dagger} \phi_1)^2 + \lambda_3(\phi_1^{\dagger} \phi_2)^2 + \text{ h.c.}$ 13(*†* 13)(*†* <sup>3</sup>1) + <sup>0</sup> 23(*†* 23)(*†*  $V_{\rm ph} = \lambda_1(\phi_2^{\dagger}\phi_3)^2 + \lambda_2(\phi_3^{\dagger}\phi_1)^2 + \lambda_3(\phi_1^{\dagger}\phi_2)^2 + \text{ h.c.}$ 

#### Introduction  $Introduction$ <sup>2</sup>2) + 13(*†* 22)(*†* <sup>3</sup>3) + 33(*†* <sup>3</sup>3) <sup>3</sup>1) + <sup>0</sup> 23(*†* 23)(*†* <sup>3</sup>2)*,* (0.2b) Insensitive to relative phases of fields 1, 2, 3:

2

11)(*†*

<sup>3</sup>3) + 22(*†*

<sup>2</sup>2)

### 12)(*†* 21) + 01<br>21) + 02<br>21) + 02 Weinberg:

<sup>3</sup>3) + 33(*†*

22)(*†*

+ 23(*†*

12(*†*

*V*<sup>0</sup> = 11(*†*

13)(*†*

2

<sup>3</sup>3)

13(*†*

+ 23(*†*

<sup>1</sup>1)

**Weinberg:**<br>Natural flavour conservation and CPV can be arranged by complex potential. Branco (1980) showed that this could also be achieved with a real potential ford complex represented and the doublets of the Higgs doublets and  $\alpha$ *V*<sub>2</sub> = *M*<sup>2</sup>  $\frac{1500}{2000}$  showed that this codid also <sup>3</sup>3)]*,* (0.2a) editally Branco (1980) showed that this could also be achieved with a real potential Natural flavour conservation and CPV can be arranged by complex potential. (and complex vevs). Case studied

(and complex vevs). Case studied  
\n
$$
V_2 = -[m_{11}(\phi_1^{\dagger}\phi_1) + m_{22}(\phi_2^{\dagger}\phi_2) + m_{33}(\phi_3^{\dagger}\phi_3)],
$$
\n
$$
V_0 = \lambda_{11}(\phi_1^{\dagger}\phi_1)^2 + \lambda_{12}(\phi_1^{\dagger}\phi_1)(\phi_2^{\dagger}\phi_2) + \lambda_{13}(\phi_1^{\dagger}\phi_1)(\phi_3^{\dagger}\phi_3) + \lambda_{22}(\phi_2^{\dagger}\phi_2)^2
$$
\n
$$
+ \lambda_{23}(\phi_2^{\dagger}\phi_2)(\phi_3^{\dagger}\phi_3) + \lambda_{33}(\phi_3^{\dagger}\phi_3)^2
$$
\n
$$
+ \lambda'_{12}(\phi_1^{\dagger}\phi_2)(\phi_2^{\dagger}\phi_1) + \lambda'_{13}(\phi_1^{\dagger}\phi_3)(\phi_3^{\dagger}\phi_1) + \lambda'_{23}(\phi_2^{\dagger}\phi_3)(\phi_3^{\dagger}\phi_2),
$$

 $V_{\rm ph} = \lambda_1(\phi_2^{\dagger} \phi_3)^2 + \lambda_2(\phi_3^{\dagger} \phi_1)^2 + \lambda_3(\phi_1^{\dagger} \phi_2)^2 + \text{ h.c.}$  $\begin{bmatrix} \mathbf{E} & \mathbf{$  $\gamma$  ph  $\gamma$   $\gamma$  ( $\gamma$ <sub>2</sub> $\gamma$ <sub>3</sub>)  $\sqrt{2}$  version of  $\sqrt{2}$  $V_{\rm ph}=\lambda_1$ *v*<sub>1</sub>(*†* 273) + *'* '2(*†* 371) + ' ''3(*†* 1*†* 2)

ant element:  $\mathbb{Z}_2 \times \mathbb{Z}_2$ -symmetry (only even powers of fields) Important element:  $\mathbb{Z}_2 \times \mathbb{Z}_2$ -symmetry (only even powers of fields)  $\frac{2}{7}$   $\frac{11.0}{7}$  $\phi_i \rightarrow -\phi_i$  for all three  $\phi_i$ . <sup>2</sup>3) <sup>3</sup>1) <sup>1</sup>2)  $\mathcal{L} = \mathcal{L} \mathcal$ 

#### Introduction Important element: Z2-symmetry and Z2-symmetry and Z2-symmetry and Z2-symmetry and Z2-symmetry and Z2-symmetry CP violation (good for baryogenesis) generated by <u>Important elements and the symmetry of the sy</u> Important element: Z2-symmetry and Z2-symmetry and Z2-symmetry and Z2-symmetry and Z2-symmetry and Z2-symmetry<br>The contract of the contract o CP violation (good for baryogenesis) generated by CP violation (good for baryogenesis) generated by <sup>2</sup> + 2(*†* <sup>3</sup>1) <sup>2</sup> + 3(*†*

<sup>1</sup>2)

 $2 \times 10^{-10}$  ,  $2 \times 10^{-10}$ 

### Observation 1:  $O$ boorvotion 1. *ve*rvation<br>*v*

(and complex vevs)

CP violation (good for baryogenesis) generated by  $\mathbf v$ CP violation (good for baryogenesis) generated by od for baryogenesis) generated by  $\left($   $\begin{array}{c} 2 \ 1 \end{array} \right)$ CP violation (good for baryogenesis) generated by

<sup>2</sup>3)

<u>Important elements elements and a symmetry of the symmetry of</u>

*V*ph = 1(*†*

 $V_{\rm ph} = \lambda_1 (\phi_2^{\dagger} \phi_3)^2 + \lambda_2 (\phi_3^{\dagger} \phi_1)^2 + \lambda_3 (\phi_1^{\dagger} \phi_2)^2 + \text{ h.c.}$  $v_{\rm ph} = \alpha_1(\varphi_2\varphi_3) + \alpha_2(\varphi_3\varphi_1) + \alpha_3(\varphi_1\varphi_2) + \cdots$ 

*Vant CPV to be small, in view of SM-like Higgs boson at 125 GeV.*  $eV$ Want CPV to be small, in view of SM-like Higgs boson at 125 GeV. *V*ph ! 0*, {*1*,* 2*,* 3*}* ! 0 (0.5)

Study limit:  $\mathcal{L}$ 

(and complex vevs)

(only even powers of fields)

 $\{ \lambda_1, \lambda_2, \lambda_3 \} \rightarrow 0$  $V_{\rm ph} \rightarrow 0, \quad {\lambda_1, \lambda_2, \lambda_3} \rightarrow 0$ 

Observation 2:  $\Omega$ beemstiep  $\Omega$ <sup>1  $V(1)$ </sup>  $\cdot$   $V(1)$   $\cdot$   $V(1)$  $V_2 = -[m_{11}(\phi_1^{\dagger}\phi_1) + m_{22}(\phi_2^{\dagger}\phi_2) + m_{33}(\phi_3^{\dagger}\phi_3)],$  $V_2 = -[m_{11}(\varphi_1^{\dagger}\varphi_1) + m_{22}(\varphi_2\varphi_2) + m_{33}(\varphi_3\varphi_3)],$ <br>  $V_0 = \lambda_{11}(\varphi_1^{\dagger}\varphi_1)^2 + \lambda_{12}(\varphi_1^{\dagger}\varphi_1)(\varphi_2^{\dagger}\varphi_2) + \lambda_{13}(\varphi_1^{\dagger}\varphi_1)(\varphi_3^{\dagger}\varphi_3) + \lambda_{22}(\varphi_2^{\dagger}\varphi_2)^2$  $+ \lambda_{23}(\phi_2^{\dagger}\phi_2)(\phi_3^{\dagger}\phi_3) + \lambda_{33}(\phi_3^{\dagger}\phi_3)^2$  $+\lambda'_{12}(\phi_1^{\dagger}\phi_2)(\phi_2^{\dagger}\phi_1)+\lambda'_{13}(\phi_1^{\dagger}\phi_3)(\phi_3^{\dagger}\phi_1)+\lambda'_{23}(\phi_2^{\dagger}\phi_3)(\phi_3^{\dagger}\phi_2),$  $V(1) \times U(1) \times U(1)$  symmetry  $U = \frac{\lambda_{12}(\varphi_1\varphi_1)(\varphi_2\varphi_2) + \lambda_{13}(\varphi_1\varphi_1)(\varphi_3\varphi_3) + \lambda_{22}(\varphi_2\varphi_2)}{\mu_1\mu_2\mu_3}$  $U_{1}^{1}(\varphi_{1}\varphi_{1}) + m_{22}(\varphi_{2}\varphi_{2}) + m_{33}(\varphi_{3}\varphi_{3})$ ,  $U(1) \times U(1) \times U(1)$  symmetry

## Introduction *V*ph ! 0*, {*1*,* 2*,* 3*}* ! 0 (0.5) *V*ph ! 0*, {*1*,* 2*,* 3*}* ! 0 (0.5)

## Observation 2 cont:  $\mathcal{V}$  contracting (1)  $\mathcal{V}$

 $V_2 + V_0$  is invariant under  $V + V$  is inv  $+$   $V_0$  is invariant under

<sup>1</sup> ! *<sup>e</sup><sup>i</sup>*↵<sup>1</sup> 1*,* <sup>2</sup> ! *<sup>e</sup><sup>i</sup>*↵<sup>2</sup> 2*,* <sup>3</sup> ! *<sup>e</sup><sup>i</sup>*↵<sup>3</sup> 3*,* (0.7)  $\phi_1 \rightarrow e^{i\alpha_1}\phi_1, \quad \phi_2 \rightarrow e^{i\alpha_2}\phi_2, \quad \phi_3 \rightarrow e^{i\alpha_3}\phi_3$  $\frac{i\alpha_2}{\alpha_3}$   $\frac{i\alpha_3}{\alpha_4}$  $\varphi_1 \to e^{i\alpha_1} \varphi_1, \quad \varphi_2 \to e^{i\alpha_2} \varphi_2, \quad \varphi_3 \to e^{i\alpha_3} \varphi_3$  $U_{1}$   $\vdots$   $U_{n}$   $\vdots$ 

## Observation 3:

Actually, one  $U(1)$  factor is combined with the hypercharge, we are left with two Goldstone bosons when the  $U(1) \times U(1)$  symmetry is broken by the vacuum,

#### Conjecture:  $A(\mathbf{r})$  factor is combined with the hypercharge,  $\mathbf{r}$ we are left with two Goldstone bosons with two Goldstone bosons with the contract of the contr  $A_n = \frac{1}{n}$  for  $n = 1$ ye are left with two Goldstone bosons with two Goldstone bosons with two Goldstone bosons with two Goldstone bo *<u>1*  $\frac{1}{2}$  *= 0 6*  $\frac{1}{2}$  *= 0*  $\frac{1}{2}$  *= 0 \frac{1}{2* $$

With  $V_{\text{ph}} \neq 0$ , or  $\{\lambda_1, \lambda_2, \lambda_3\} \neq 0$ , two light states with a significant CP-odd content?

two light states with a significant CP-odd content?

## Minimization *{w*1*, w*2*, w*3*}* <sup>=</sup> *{v*1*, v*<sup>2</sup> *<sup>e</sup><sup>i</sup>*✓<sup>2</sup> *, v*<sup>3</sup> *<sup>e</sup><sup>i</sup>*✓<sup>3</sup> *}.* (0.8) *{w*1*, w*2*, w*3*}* <sup>=</sup> *{v*1*, v*<sup>2</sup> *<sup>e</sup><sup>i</sup>*✓<sup>2</sup> *, v*<sup>3</sup> *<sup>e</sup><sup>i</sup>*✓<sup>3</sup> *}.* (0.8) *{w*1*, w*2*, w*3*}* <sup>=</sup> *{v*1*, v*<sup>2</sup> *<sup>e</sup><sup>i</sup>*✓<sup>2</sup> *, v*<sup>3</sup> *<sup>e</sup><sup>i</sup>*✓<sup>3</sup> *}.* (0.8)

 $N_{\alpha}$ tat Notation:

 $c_x =$ 

$$
\phi_i = e^{i\theta_i} \left( \frac{\phi_i^+}{(v_i + \eta_i + i\chi_i)/\sqrt{2}} \right), \quad i = 1, 2, 3
$$

$$
\{w_1, w_2, w_3\} = \{v_1, v_2 e^{i\theta_2}, v_3 e^{i\theta_3}\}
$$

 $T$  , and the minimization conditions with respect to  $\mathcal{L}$  and  $\mathcal{L}$  and  $\mathcal{L}$ Three minimization conditions with respect to moduli

Three minimization conditions with respect to moduli  
\n
$$
m_{11} = \lambda_{11}v_1^2 + \frac{1}{2}\overline{\lambda}_{12}v_2^2 + \frac{1}{2}\overline{\lambda}_{13}v_3^2 + \lambda_2c_{2\theta_3}v_3^2 + \lambda_3c_{2\theta_2}v_2^2,
$$
\n
$$
m_{22} = \lambda_{22}v_2^2 + \frac{1}{2}\overline{\lambda}_{12}v_1^2 + \frac{1}{2}\overline{\lambda}_{23}v_3^2 + \lambda_1c_{(2\theta_3 - 2\theta_2)}v_3^2 + \lambda_3c_{2\theta_2}v_1^2,
$$
\n
$$
m_{33} = \lambda_{33}v_3^2 + \frac{1}{2}\overline{\lambda}_{13}v_1^2 + \frac{1}{2}\overline{\lambda}_{23}v_2^2 + \lambda_1c_{(2\theta_3 - 2\theta_2)}v_2^2 + \lambda_2c_{2\theta_3}v_1^2,
$$
\n
$$
c_x \equiv \cos x
$$

$$
\bar{\lambda}_{12}\equiv\lambda_{12}+\lambda'_{12},\quad \bar{\lambda}_{13}\equiv\lambda_{13}+\lambda'_{13},\quad \bar{\lambda}_{23}\equiv\lambda_{23}+\lambda'_{23}
$$

### Minimization <sup>12</sup>*,* ¯<sup>13</sup> ⌘ <sup>13</sup> <sup>+</sup> <sup>0</sup> Minimize with respect to phases: ¯<sup>12</sup> ⌘ <sup>12</sup> <sup>+</sup> <sup>0</sup> <sup>12</sup>*,* ¯<sup>13</sup> ⌘ <sup>13</sup> <sup>+</sup> <sup>0</sup> <sup>13</sup>*,* ¯<sup>23</sup> ⌘ <sup>23</sup> <sup>+</sup> <sup>0</sup> <sup>12</sup>*,* ¯<sup>13</sup> ⌘ <sup>13</sup> <sup>+</sup> <sup>0</sup> Minimize with respect to phases: <u>120, Minimization</u>

<sup>13</sup>*,* ¯<sup>23</sup> ⌘ <sup>23</sup> <sup>+</sup> <sup>0</sup>

<sup>13</sup>*,* ¯<sup>23</sup> ⌘ <sup>23</sup> <sup>+</sup> <sup>0</sup>

<sup>13</sup>*,* ¯<sup>23</sup> ⌘ <sup>23</sup> <sup>+</sup> <sup>0</sup>

<sup>23</sup>*.* (0.13)

<sup>23</sup>*.* (0.13)

Minimize with respect to phases:  $M_{\rm eff}$  and  $M_{\rm eff}$  with respect to phases:  $\mu_{\rm eff}$  $\frac{1}{2}$  Minimize with respect to phases: Minimize with respect to phases: Minimize with respect to phases:

¯<sup>12</sup> ⌘ <sup>12</sup> <sup>+</sup> <sup>0</sup>

¯<sup>12</sup> ⌘ <sup>12</sup> <sup>+</sup> <sup>0</sup>

$$
\lambda_1 v_3^2 \sin(2\theta_2 - 2\theta_3) + \lambda_3 v_1^2 \sin 2\theta_2 = 0,
$$
  
\n
$$
\lambda_1 v_2^2 \sin(2\theta_3 - 2\theta_2) + \lambda_2 v_1^2 \sin 2\theta_3 = 0.
$$

Phases are related: Phases are related:

¯<sup>12</sup> ⌘ <sup>12</sup> <sup>+</sup> <sup>0</sup>

 $M_{\rm eff}$  minimize  $\sim$  10  $\mu$  minimizes:  $\mu$  minimizes:  $\mu$ 

$$
\lambda_3 v_2^2 \sin 2\theta_2 + \lambda_2 v_3^2 \sin 2\theta_3 = 0.
$$

of  $\sin 2\theta_2$  and  $\sin 2\theta_3$  is opposite of that of  $\lambda_2/\lambda_3$ . Branco (1980)<br> $\frac{1980}{2000}$ Branco (1980)<br><sup>1⊥1 and a</sup><sup>1⊥</sup>2 and <sup>2</sup><sup>1</sup> Relative sign of sin 2✓<sup>2</sup> and sin 2✓<sup>3</sup> is opposite of that of 2*/*3. Relative sign of sin 2✓<sup>2</sup> and sin 2✓<sup>3</sup> is opposite of that of 2*/*3. Relative sign of  $\sin 2\theta_2$  and  $\sin 2\theta_3$  is opposite of that of  $\lambda_2/\lambda_3$ . Branco (1980) Branco (1980)

$$
\cos 2\theta_2 = \frac{1}{2} \left[ \frac{D_{23}D_{31}}{D_{12}^2} - \frac{D_{31}}{D_{23}} - \frac{D_{23}}{D_{31}} \right]
$$
  
\n
$$
\cos 2\theta_3 = \frac{1}{2} \left[ \frac{D_{23}D_{12}}{D_{31}^2} - \frac{D_{12}}{D_{23}} - \frac{D_{23}}{D_{12}} \right]
$$
  
\n
$$
D_{12} = \lambda_3 (v_1 v_2)^2, \quad D_{23} = \lambda_1 (v_2 v_3)^2, \quad D_{31} = \lambda_2 (v_3 v_1)^2
$$

### Minimization *D*<sup>2</sup> 31 *D*<sup>12</sup> = 3(*v*1*v*2) *, D*<sup>23</sup> = 1(*v*2*v*3) *D*<sup>23</sup> 2

*D*12

*.* (0.20)

*, D*<sup>31</sup> = 2(*v*3*v*1)

2

 $\frac{1}{2}$  (*p*<sup>2</sup>) approach (*vetains sign information*) 2 Free input:  $v_1$ ,  $v_2$ ,  $v_3$ ,  $\theta_2$ ,  $\theta_3$ Other (our) approach (retains sign information)

2

$$
\lambda_2 = \frac{\lambda_1 v_2^2 \sin(2\theta_2 - 2\theta_3)}{v_1^2 \sin 2\theta_3},
$$
  

$$
\lambda_3 = -\frac{\lambda_1 v_3^2 \sin(2\theta_2 - 2\theta_3)}{v_1^2 \sin 2\theta_2}.
$$

Next: masses

## 1.2 Neutral sector Masses

<sup>1</sup> *v*<sup>6</sup>

2*v*8

1*v*4

2*v*<sup>10</sup>

<sup>3</sup> *w*<sup>2</sup> sin<sup>2</sup> 2✓<sup>2</sup> sin<sup>8</sup> 2✓3*F*˜2*,*<sup>8</sup>

<sup>3</sup> sin<sup>3</sup> 2✓<sup>2</sup> sin<sup>7</sup> 2✓<sup>3</sup> *F*˜3*,*<sup>7</sup>

Neutral sector  $(5 \times 5)$ :

 $\frac{D}{2}$  and  $\frac{D}{2}$  and  $\frac{D}{2}$ It is also instructive to study the determinant: It is instructive to study the determinant:

$$
D_{5\times 5} = \frac{\lambda_1^2 \sin^2(2\theta_2 - 2\theta_3)}{v^2 v_1^4 (v_2^2 + v_3^2)^5 \sin^5 2\theta_2 \sin^5 2\theta_3} F(\theta_2, \theta_3, \ldots),
$$
  
\n
$$
F(\theta_2, \theta_3, \ldots) = 64 \lambda_1^3 v_2^6 v_3^{10} w^2 \sin^2 2\theta_2 \sin^8 2\theta_3 \tilde{F}_{2,8}
$$
  
\n
$$
+ \lambda_1^2 v_2^4 v_3^8 \sin^3 2\theta_2 \sin^7 2\theta_3 \tilde{F}_{3,7}
$$
  
\n
$$
+ \lambda_1 v_2^2 v_3^6 \sin^4 2\theta_2 \sin^6 2\theta_3 \tilde{F}_{4,6}
$$
  
\n
$$
+ v_2^4 v_3^4 \sin^5 2\theta_2 \sin^5 2\theta_3 \tilde{F}_{5,5}
$$
  
\n
$$
+ \{(\theta_2, \nu_2, \lambda_{22}, \bar{\lambda}_{12}) \leftrightarrow (\theta_3, \nu_3, \lambda_{33}, \bar{\lambda}_{13})\}
$$

Two t *vws*2✓<sup>2</sup> *s*2✓<sup>3</sup>  $\lambda_1$  for two lig with *F*˜*mn* regular, homogeneous expansions in the 's and powers of the vevs, as well as Two powers of  $\lambda_1$  for two light masses 2<sup>*x*</sup> two light masses <sup>3</sup> sin<sup>5</sup> 2✓<sup>2</sup> sin<sup>5</sup> 2✓<sup>3</sup> *F*˜5*,*<sup>5</sup> (2✓<sup>2</sup> 2✓3). Two powers of  $\lambda_1$  for two light masses

### Masses 1.2 Neutral sector BBBB@ *XXX* 0 *x x* 123557  $\mathsf{F}^{\text{max}}$  and  $\mathsf{M}$  and  $\mathsf{M}$  and  $\mathsf{M}$  we have  $\mathsf{M}$

*<sup>M</sup>*<sup>2</sup>

⌘HB

HB

i<br>L ī

 

CCCCA

neut =

l<br>L ī

  BBBB@

*XXX x* 0

C

l  ⌘HB

HB

0 *x* 0 *x x*

#### Neutral sector  $(5 \times 5)$ :  $\begin{pmatrix} X & X & X \\ X & X & X \end{pmatrix}$  $\begin{bmatrix} 0 \\ x \end{bmatrix}$   $\begin{bmatrix} \eta_1^{\text{m}} \\ \eta_2^{\text{h}} \end{bmatrix}$ *v*<sub>2</sub> 0  $\eta_3^{\text{HB}}$ <br> $\eta_3^{\text{HB}}$  $\begin{bmatrix} X & X & X & 0 & 0 \\ X & X & X & 0 & 0 \end{bmatrix}$  $\mathbf{J}$  $\left| \chi_3^{\text{HB}} \right|$  (0 0 0 *v*) general case  $\theta_2 = \theta_3$  $\mathcal{M}^2_{\rm neut} =$  $\sqrt{ }$  $\overline{\phantom{a}}$ *XXX* 0 0 *XXX* 0 *x XXX x* 0 0 0 *x x x* 0 *x* 0 *x x*  $\setminus$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \hline \end{array}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\eta_1^{\rm HB}$  $\eta_{2}^{\rm HB}$  $\eta_{3}^{\rm HB}$  $\chi^{\rm HB}_{2}$  $\chi_3^\mathrm{HB}$  $\begin{array}{c} \hline \end{array}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\begin{bmatrix} 1 & X & X & X & 0 \ X & X & X & 0 \ X & X & X & 0 \end{bmatrix}$  $N$ eutral sector  $(5 \times 5)$  $\begin{pmatrix} X & X & X & 0 & 0 \ X & X & X & 0 & 0 \ X & X & X & 0 & 0 \ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$  $\theta_2=\theta_3$ *X X X X* 0 0  $0 \t 0 \t 0 \t 0 \t 0$  $\begin{pmatrix} 0 & 0 & 0 & 0 & x \\ 0 & 0 & 0 & 0 & x \end{pmatrix}$  $\theta_2 = \theta_3$  $X$  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *XXX* 0 0  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ *x* 1  $\frac{1}{2}$  *x x*  $(5 \times 5)$ : eneut)<br>45  $\frac{1}{2}$ *x*<sub>2</sub>*x*<sub>3</sub>*x*<sub>3</sub> *s*<sub>2</sub>  $\theta_2 = \theta_3$ <br>  $\theta_3 = \theta_3$ <br>  $\theta_1 = \theta_3$ <br>  $\theta_2 = \theta_3$  $\mathcal{M}^2_{\text{neut}} = \begin{bmatrix} X & 1 \\ X & 1 \end{bmatrix}$ *s*<sup>1</sup>**b**<sup>1</sup>*s***<sub>2</sub> <b>s**<sup>2</sup> $\left($  **b**<sup>2</sup> $\left($  **b**<sup>2</sup> $\left($  **b**<sup>2</sup> $\left($  **b**<sup>2</sup> $\left($  **b**<sup>2</sup> $\left($  **b**<sup>2</sup> $\left($  **c**<sup>2</sup> $\left($  $\begin{bmatrix} X & x & 0 \\ x & x & x \\ x^{\text{HB}} & x^{\text{HB}} \end{bmatrix}$   $\begin{bmatrix} \tilde{m} \\ \tilde{m} \\ \tilde{m} \end{bmatrix}$   $\begin{bmatrix} X & X & X & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ sector  $(5 \times 5)$ :  $\sqrt{ }$  $\overline{\phantom{a}}$ *XXX* 0 0 *XXX* 0 0 *XXX* 0 0  $0 \t 0 \t 0 \t 0$ 0 0 00 *x*  $\setminus$  $\begin{array}{c} \hline \end{array}$  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$  $\begin{bmatrix} X & X & 0 & 0 \end{bmatrix}$   $\begin{bmatrix} \eta_1^{\text{HB}} \\ \eta_2^{\text{HB}} \end{bmatrix}$  $\left[\begin{array}{ccc}X & X & 0 & x \X & X & x & 0\end{array}\right] = \left[\begin{array}{c}\eta_2 \\ \eta_2^{\text{HB}}\end{array}\right].$ ⌘HB  $\begin{pmatrix} X \\ Y \end{pmatrix}$  $\begin{bmatrix} X & A \\ X & A \end{bmatrix}$  $\frac{1}{2}$  $\overset{\sim}{X}$ X<br>V  $\frac{X}{X}$  $\chi$  $\left( \begin{matrix} 0 & x & 0 & x & x \end{matrix} \right)$   $\left| \chi_3^{\text{m}} \right|$  and  $\left| \begin{matrix} 0 & 0 & 0 & x \end{matrix} \right|$ which is also block diagonal, having interchanged rows (and columns) 3 and 5, i.e.,  $\sum_{i=1}^{n}$  $\mathbb{R}^{\text{HB}}_3$  and  $\left(0 \quad 0 \quad 0 \quad 0 \quad x\right)$

*<sup>M</sup>*<sup>2</sup>

1<br>1<br>1

 $\chi$  denotes a term vanishing  $\chi$  $V$ <sup>2</sup> where  $\mathbf x$  denotes a term vanishing with  $\mathbf x$ where  $x$  denotes a term vanishing with  $\lambda_1$ swapped ⌘HB  $3 \cdot 1$ sning with  $\lambda$ 

*s*2✓22✓<sup>3</sup> *,*

BBBB@

neut)<sup>34</sup> <sup>=</sup> 21*vv*2*v*<sup>3</sup>

 $\frac{1}{2}$ 

Finally, for the "simple model" we have

$$
m_1 < m_2 < m_3 < m_4 < m_5
$$
  $h_i = O_{ij} \varphi^{\rm HB}_j$   $O$  only known numerically  
Which one is 125 GeV?

Finally, for the "simple model" we have

*<sup>M</sup>*<sup>2</sup>  $rac{1}{\sqrt{2}}$ Which one is 125 GeV? *XXX* 0 0

#### Masses 1.2 Neutral sector 1.2 Ne 0 **x** Masses *x* 123557  $\mathsf{F}^{\text{max}}$  and  $\mathsf{M}$  and  $\mathsf{M}$  and  $\mathsf{M}$  we have  $\mathsf{M}$

### Neutral sector  $(5 \times 5)$ :  $N$ eutral sector  $(5 \times 5)$  $\frac{1}{2}$  *x x*  $(5 \times 5)$ :  $\text{sector } (5 \times 5)$ :

*s*2✓22✓<sup>3</sup> *,*

neut)<sup>34</sup> <sup>=</sup> 21*vv*2*v*<sup>3</sup>

 $\frac{1}{2}$ 

Finally, for the "simple model" we have

 $m_1 < m_2 <$ 

BBBB@



*x*<br>De car x denotes a term vanis<sup>7</sup><br>ase:  $\lambda_2 = +\lambda_2$ ,  $\theta_2 = \pm \theta_2$ .  $v_3, \theta_2 = \mp \theta_3, v_2$  $\lambda_2$  and  $\lambda_3$  (*s*<sub>2</sub> =  $\pm \lambda_3$ , *b*<sub>2</sub> =  $\mp \theta_3$ , *v*<sub>2</sub> = *v*<sub>3</sub>,  $\lambda_{ii} = \lambda_{jj}$ ,  $\bar{\lambda}_{ij} = \bar{\lambda}_{12}$ where  $\mathbf x$  denotes a term vanishing with  $\mathbf x$ where *x* denotes a term vanishing with  $\lambda_1$  $= +c$ ble case:  $\lambda_2 = \pm \lambda_3$ ,  $\theta_2 = \mp \theta_3$ ,  $v_2 = v_3$ ,  $\lambda_{ii} = \lambda_{jj}$ ,  $\bar{\lambda}_{ij} = \bar{\lambda}_{12}$ Finally,  $\sigma_2 = \pm \lambda_3$ ,  $\sigma_2 = \pm \sigma_3$ ,  $\sigma_2 = \sigma_3$ ,  $\lambda_{ii} = \lambda_{jj}$ ,  $\lambda_{ij} = \lambda_{12}$ simple case:  $\lambda_2 = \pm \lambda_3$ ,  $\theta_2 = \mp \theta_3$ ,  $v_2 = v_3$ ,  $\lambda_{ii} = \lambda_{jj}$ ,  $\bar{\lambda}_{ij} = \bar{\lambda}_{12}$ swapped ⌘HB <sup>3</sup> and HB <sup>3</sup> . where  $x$  d motos a torm vanishing with ).  $\overline{X}$   $\overline{X}$  $\mathbf{x}, \lambda_{ii} - \lambda_{jj}, \lambda_{ij}$  $\overline{\cdot}$  $\bar{\lambda}$  $\bigg)$  $\frac{1}{10}$  $\frac{12}{2}$  $\frac{115}{8}$  with  $\lambda$  $\frac{3}{1}$  .  $v_3,\,\lambda_{ii}=$  $\frac{1}{2}$  =  $\frac{1}{3}$ ,  $\frac{1}{2}$  =  $\frac{1}{3}$ ,  $\frac{1}{2}$  =  $\frac{1}{3}$ ,  $\frac{1}{2}$  =  $\frac{1}{2}$ swapped ⌘HB  $3 \cdot 1$ sning with  $\lambda$  $\lambda_{i}, v_{2} = v_{3}, \lambda_{ii} = \lambda_{jj}, \bar{\lambda}_{ij} = \bar{\lambda}_{12}$ 

*<sup>M</sup>*<sup>2</sup>

1<br>1<br>1

⌘HB

$$
m_1 < m_2 < m_3 < m_4 < m_5
$$
\n
$$
m_1 < m_2 < m_3 < m_4 < m_5
$$
\nWhich one is 125 GeV?

*<sup>M</sup>*<sup>2</sup>

i<br>L ī

 

*<sup>M</sup>*<sup>2</sup>

⌘HB

HB

neut = 1<br>neutralis

CCCCA

HB

neut =

0

BBBB@

i<br>L ī

  BBBB@

*XXX* 0 0

*XXX* 0 0

*XXX x* 0

1

 $\overline{a}$ 

⌘HB 1

 $\overline{a}$ 

 

⌘HB 3

  C

l  ⌘HB

HB

0 *x* 0 *x x*

CCCCA

 $\frac{1}{2}$ Which one is 125 GeV? *XXX* 0 0

#### **Gauge couplings** *i*=1*,*2*,*3 (*Dµi*) *†* (*D<sup>µ</sup>i*)*.* (2.1) *<u>Rauge coupling</u>* **L**<br>**L**<br>**G**aud *i*=1*,*2*,*3 (*Dµi*) *†* (*D<sup>µ</sup>i*)*.* (2.1) *gm<sup>W</sup> W*<sup>+</sup> *<u>µ <i>w w w w w w y v w w y a*</u> *ZµZ<sup>µ</sup>*

*ZµZ<sup>µ</sup>*

→<br>XX<br>XX

*Oi*1*hi,* (2.2)

*Oi*1*hi,* (2.2)

Cubic gauge-gauge-scalar part: Cubic gauge-gauge-scalar part: *LVVh* =

Cubic gauge-gauge-scalar part:

$$
\mathcal{L}_{VVh} = \left(g m_W W^+_\mu W^{\mu -}_\mu + \frac{g m_Z}{2 \cos \theta_W} Z_\mu Z^\mu\right) \sum_{i=1}^5 O_{i1} h_i
$$

white gauge-statal-statal strills. Cubic gauge-scalar-scalar terms: cubic gauge-scalar-scalar term

*LVVh* =

✓

$$
\mathcal{L}_{Vhh} = -\frac{g}{2\cos\theta_{W}} \sum_{i=1}^{5} \sum_{j=1}^{5} (O_{i2}O_{j4} + O_{i3}O_{j5})(h_{i}\overleftrightarrow{\partial_{\mu}}h_{j})Z^{\mu} \n+ \frac{g}{2} \sum_{i=1}^{5} \sum_{j=1}^{2} [(iO_{i j+1} + O_{i j+3}) \sum_{k=1}^{2} U_{jk}(h_{k}^{+} \overleftrightarrow{\partial_{\mu}}h_{i})W^{\mu-} + \text{h.c.}] \n+ \left( ieA^{\mu} + \frac{ig\cos 2\theta_{W}}{2\cos\theta_{W}} Z^{\mu} \right) \sum_{j=1}^{2} (h_{j}^{+} \overleftrightarrow{\partial_{\mu}}h_{j}^{-}),
$$

#### Gauge couplings *Gauc* ◆X X *Ujkhih H Gauge* <u>a cos </u> *W*<sup>+</sup> *<sup>µ</sup> Z<sup>µ</sup>*  $\overline{\text{base}}$ *i*=1 *<sup>A</sup>µA<sup>µ</sup>* <sup>+</sup> *<sup>g</sup>*<sup>2</sup> cos<sup>2</sup> <sup>2</sup>✓*<sup>W</sup> <sup>Z</sup>µZ<sup>µ</sup>* <sup>+</sup> *eg* cos 2✓*<sup>W</sup>*

2 cos ✓*<sup>W</sup>*

5

XII<br>XIII<br>XIII

2

*i*=1

*j,k*=1

*AµZ<sup>µ</sup>*

*<sup>k</sup>* (*Oij*+1 <sup>+</sup> *iOij*+3) + h.c.

→<br>XX<br>XX

*h*+

*<sup>k</sup>* (*Oij*+1 <sup>+</sup> *iOij*+3) + h.c.

#### The *hjWW* (and *hjZZ*) coupling is given by *Oj*<sup>1</sup> How to measure different CP content? How to measure different  $\cup$ P content?  $\overline{U}$   $\overline{V}$   $\overline{V$  $\mathsf{r}$ ✓*eg*  $\ddot{\phantom{0}}$ ر<br>aure different C <sub>2</sub> i U i L *<i>P*  $co$ ntant? 5 2 *<sup>k</sup>* (*Oij*+1 <sup>+</sup> *iOij*+3) + h.c.

 $\boldsymbol{Z}$  is odd under CP, study the trilinear coupling  $h_ih_j\boldsymbol{Z}$ 

2

8 cos<sup>2</sup> ✓*<sup>W</sup>*

2 cos ✓*<sup>W</sup>*

*<sup>µ</sup> W<sup>µ</sup>* + *e*<sup>2</sup>

*<sup>µ</sup> <sup>A</sup><sup>µ</sup> <sup>g</sup>*<sup>2</sup> sin<sup>2</sup> ✓*<sup>W</sup>*

*W*<sup>+</sup>

<sup>4</sup> *<sup>W</sup>*<sup>+</sup>

2

✓*eg*

✓*g*<sup>2</sup>

✓*eg*

2

*W*<sup>+</sup>

$$
P_{ij} = (O_{i2}O_{j4} + O_{i3}O_{j5}) - (i \leftrightarrow j)
$$

In the 2HDM, allowing for CP violation, the  $h_i h_j Z$  couplings are essentially  $\alpha$ In the zindivity, and will generally violation, the  $n_i n_j z$  couplings are essentially the same as the  $h_k ZZ$  couplings, with i, i, k all different Not the case in a 3HDM. In the 2HDM, allowing for CP violation, the *hihjZ* couplings are essentially the same as the  $h_kZZ$  couplings, with  $i, j, k$  all different

## **Scan**

### Scan over model parameters: 5 Parameter scans 5 Parameter scans

Want the Higgs-gauge coupling *h*SM*WW* to be close to unity

 $\overline{\phantom{a}}$ 

Want the Higgs-gauge coupling  $h_{\text{SM}}WW$  to be close to unity Want the Higgs-gauge coupling  $h_{SM}WW$  to be close to unity

> *|Oj*1*|* ' 1*,* for some *j.* (5.1)  $|O_{j1}| \simeq 1$ , for some *j.*

$$
v_i \in [0, v], \t i = 1, 2, 3, \t with v_1^2 + v_2^2 + v_3^2 = v^2,
$$
  

$$
\theta_i \in [-\pi, \pi], \t i = 2, 3,
$$
  

$$
\lambda_{ii}, \lambda_{ij}, \lambda'_{ij}, \lambda_1 \in [-4\pi, 4\pi], \t i, j = 1, 2, 3.
$$

#### Scan them. The neutral mass eigenvalues are ordered as  $\mathcal{C}_{\mathsf{CQCD}}$ From these parameters one can reconstruct the mass-squared matrices and diagonalize the neutral mass eigenvalues are ordered as  $\sim$ From these parameters one can reconstruct the mass-squared matrices and diagonalize them. The neutral mass eigenvalues are ordered as  $\mathsf{S}\mathsf{c}$ 1. check if the coupling *Oj*<sup>1</sup> to *WW* (or *ZZ*) is compatible with LHC measurements, 1. check if the coupling *Oj*<sup>1</sup> to *WW* (or *ZZ*) is compatible with LHC measurements,

### Scan over model parameters: *m*<sup>1</sup> *< m*<sup>2</sup> *< m*<sup>3</sup> *< m*<sup>4</sup> *< m*5*.* (5.5) *m*<sup>1</sup> *< m*<sup>2</sup> *< m*<sup>3</sup> *< m*<sup>4</sup> *< m*5*.* (5.5)  $\overline{c}$

For each  $j = 1$  to 5: For each  $i = 1$  to For each  $j = 1$  to 5:

For each *j* = 1 to 5:

- 1. check if the coupling  $O_j$ <sup>1</sup> to *WW* (or *ZZ*) is compatible with LHC measurements,<br> $3\sigma$  ( $\sigma = 0.12$ ) tolerance. 1. check if the coupling  $O_{j1}$  to  $WW$  (or  $ZZ$ ) is compatible with LHC measurements,  $3\sigma$  ( $\sigma = 0.12$ ) tolerance, 1. check if the coupling  $O_{11}$  to  $WW$  (or  $ZZ$ ) is compatible with LHC measurements  $3\sigma (\sigma = 0.12)$  tolerance,
- 3 ( = 0*.*12) tolerance, 2. rescale all  $\lambda$ s such that  $m_j = m_{\text{SM}} = 125.25 \text{ GeV}$  [footnote] 2. rescale all  $\lambda$ s such that  $m_j = m_{\text{SM}} = 125.25 \text{ GeV}$  [for
	- 3. check if all rescaled  $\lambda$ s (including  $\lambda_2$  and  $\lambda_3$ ) are within the perturbative range,  $\sigma$ . Check it all research  $\lambda$ s (meruding  $\lambda_2$
- 4. check if the lightest charged scalar is above 80 GeV.

[footnote] masses squared are linear in  $\lambda$ s<br>  $\sum_{i=1}^{n}$  of  $\sum_{i=1}^{n}$  of  $\sum_{i=1}^{n}$  of  $\sum_{i=1}^{n}$  of  $\lambda$ 

 $\sum_{i=1}^{\infty}$  check if the lightest charged scalar is above  $\sum_{i=1}^{\infty}$  is above  $\sum_{i=1}^{\infty}$ Distribution [in  $\%$ ] of SM-like  $h_j$ 



*h*<sup>1</sup> *h*<sup>2</sup> *h*<sup>3</sup> *h*<sup>4</sup> *h*<sup>5</sup>

## Z affinity The *hjWW* (and *hjZZ*) coupling is given by *Oj*<sup>1</sup>

 $\sum_{i=1}^{n}$ The quantity

$$
P_{ij} = (O_{i2}O_{j4} + O_{i3}O_{j5}) - (i \leftrightarrow j)
$$

measures how "different" two states  $h_i$  and  $h_j$  are in terms of CP. Recall the CP-conserving 2HDM: full-strength  $HAZ$  coupling, no  $hHZ$  coupling<br>Because of alignment, no  $hAZ$  coupling either Because of alignment, no *hAZ* coupling either

Because of anginiem, no *1812* coupling entier SM-like Higgs constraints As a reference, we analysed parameter points that were not subject to the experimental<br>CM ii. II: light states, as determined from the gauge couplings, and the gauge couplings, and then subsequently study then subsequently s SM-like Higgs constraints As a reference we analysed parameter points

 $\ddot{\text{line}}$ or this study we defin For this study, we define a "hear  $U(1) \times U(1)$  symmetry" condition For this study, we define a "near  $U(1) \times U(1)$  symmetry" condition For this study, we define a "near *U*(1) ⇥ *U*(1) symmetry" condition For this study, we define a "near  $U(1) \times U(1)$  symmetry" condition

 $\max(|\lambda_1|, |\lambda_2|, |\lambda_3|) = 0.01$ max(*|*1*|, |*2*|, |*3*|*)=0*.*01 (4.6)

## Z affinity measures how "di↵erent" two states *h<sup>i</sup>* and *h<sup>j</sup>* are in terms of CP. measures how "di↵erent" two states *h<sup>i</sup>* and *h<sup>j</sup>* are in terms of CP. The quantity



The quantity

 $\frac{u_{\text{noor}}}{\sqrt{1}} I(1) \times I(1)$  symmetry".  $h$  and  $h$  have similar CP. Likewise,  $h_3$ ,  $h_4$  and  $h_5$  have similar CP  $\frac{1}{2}$ ,  $\frac{1}{2}$ "near  $U(1) \times U(1)$  symmetry":  $h_1$  and  $h_2$  have low *Z*-affinity:



No particular constraints on  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ 

*Near the*  $U(1) \times U(1)$  *limit we have two neutral states that are approximately odd under*  $CP$ , and three that are approximately even.  $\mathcal{L} = \mathcal{L} \times \mathcal{N}$  ,  $\mathcal{L} = \mathcal{L} \times \mathcal{N}$  ,  $\mathcal{L} = \mathcal{L} \times \mathcal{N}$  ,  $\mathcal{L} = \mathcal{L} \times \mathcal{N}$ *CP, and three that are approximately even.*  $neutral\ states\ that\ are\ approximately\ odd\ under$  $\overline{n}$ .

### Z affinity *Near the U*(1) ⇥ *U*(1) *limit we have two neutral states that are approximately odd under CP, and three that are approximately even.* No particular constraints on 1, 2, <sup>3</sup> *Near the U*(1) ⇥ *U*(1) *limit we have two neutral states that are approximately odd under*

Consider the CP-conserving 2HDM: *H* and *A* have opposite CP (*Z* affinity = 1), as do *h* and *A* (but *Z* affinity = 0). Alignment! *CP, and three that are approximately even.*

It is instructive to consider how the  $Z$  affinity is affected by alignment. Let  $h_j$  be "aligned", meaning its coupling to *WW* is maximal,  $O_{j1} = 1$ . By orthogonality, it follows that  $O_{k1} = 0$  for  $k \neq j$  and  $O_{jk} = 0$ , for  $k \neq 1$ . Then

$$
P_{ij} = P_{ji} = 0 \quad \text{for all } i
$$

## Z affinity Not "near *U*(1) ⇥ *U*(1) symme-

have similar CP

## Alignment examples



Scan points where  $h_2$  (left) and  $h_3$  (right) satisfy LHC SM constraint

### Z affinity *Pij* = (*Oi*2*Oj*<sup>4</sup> + *Oi*3*Oj*5) (*i* \$ *j*)*.* (2.5) *Z* is odd under CP, study the trilinear coupling *hihjZ Pij* = (*Oi*2*Oj*<sup>4</sup> + *Oi*3*Oj*5) (*i* \$ *j*)*.* (2.5) *Z* is odd under CP, study the trilinear coupling *hihjZ*

### Attempt to circumvent the "alignment prol Not the case in a 3HDM. the same as the *hkZZ* couplings, with *i, j, k* all di↵erent Attempt to circumvent the "alignment problem"

Normalized to the squared sum of even and odd couplings.  $N_{\text{orm}}$  alized to the squared sum of even and odd malized to the squared sum of even and odd coupling

$$
\hat{P}_{ij} = \frac{P_{ij}}{\sqrt{\min(O_{i1}^2, O_{j1}^2) + P_{ij}^2}}
$$

with  $O_i$ <sup>1</sup> representing the CP-even  $ZZn_i$  coupling. with  $O_{i1}$  representing the CP-even  $ZZh_i$  coupling.

## $h_2$  as hsm



Complex vevs  $v_2 e^{i\theta_2}/v$  and  $v_3 e^{i\theta_3}/v$ , for  $h_2 = h_{\text{SM}}$ . Yellow is high, dark blue is low. Arbitrary normalization.

## $h_2$  as hsm



Relative strength of the  $h_2h_jZ$  couplings, in units of  $g/(2\cos\theta_W)$ (root-mean-square, averaged over the scan).

### $h_3$  as hsm 6.1 *h*<sup>2</sup> as *h*SM



Complex vevs  $v_2e^{i\theta_2}/v$  and  $v_3e^{i\theta_3}/v$ , for  $h_3 = h_{\text{SM}}$ . Yellow is high, dark blue is low. Arbitrary normalization.

## $\mathsf{S}_{\mathsf{S}}$  as hSM as hSM as hSM as hSM as how in figure  $\mathsf{S}_{\mathsf{S}}$  as how in figure  $\mathsf{S}_{\mathsf{S}}$  as hSM as  $\mathsf{S}_{\mathsf{S}}$  as hSM as  $\mathsf{S}_{\mathsf{S}}$  and  $\mathsf{S}_{\mathsf{S}}$  and  $\mathsf{S}_{\mathsf{S}}$  are distributed by  $\mathsf{S}_{$ butions of the complex vevs *v*2*e<sup>i</sup>*✓<sup>2</sup> and *v*3*e<sup>i</sup>*✓<sup>3</sup> . Superimposed on circular structures with

(root-mean-square, averaged over the scan).



Relative strength of the  $h_3h_jZ$  couplings, in units of  $g/(2\cos\theta_W)$ .

### **THE SERVICE SE**  $\mathsf{Y}$  yukawa couplings of  $\mathsf{Y}$ 3 Yukawa couplings <sup>1</sup> : (+1*,* +1) <sup>2</sup> : (1*,* +1) <sup>3</sup> : (+1*,* 1) (3.1) *u<sup>R</sup>* : (+1*,* +1) *d<sup>R</sup>* : (1*,* +1) *e<sup>R</sup>* : (+1*,* 1) (3.2)

Example  $\mathbb{Z}_2 \times \mathbb{Z}_2$  charges: Example  $\mathbb{Z}$ Example  $\mathbb{Z}_2 \times \mathbb{Z}$  $\text{Liam}_{\mathcal{F}} \times \mathbb{Z}_2 \times \mathbb{Z}_2$  charges.  $\Omega$  :  $\mathbb{Z}_2 \times \mathbb{Z}_2$  charges:  $1e \mathbb{Z}_{2} \times \mathbb{Z}_{2}$  charges *Example*  $\mathbb{Z}_2 \times \mathbb{Z}_2$  charges:

$$
\begin{array}{cccc}\n\phi_1: (+1, +1) & \phi_2: (-1, +1) & \phi_3: (+1, -1) \\
u_R: (+1, +1) & d_R: (-1, +1) & e_R: (+1, -1)\n\end{array}
$$
\nValues for a graph of the following equations:

*u<sup>R</sup>* : (+1*,* +1) *d<sup>R</sup>* : (1*,* +1) *e<sup>R</sup>* : (+1*,* 1) (3.2) Yukawa Lagrangian *Q*¯*L*2*d<sup>R</sup>* + *Y <sup>e</sup> <sup>L</sup><sup>Y</sup>* <sup>=</sup> *<sup>Y</sup> <sup>u</sup>Q*¯*L*˜1*u<sup>R</sup>* <sup>+</sup> *<sup>Y</sup> <sup>d</sup>* physical fermion fields, we obtain

3 Yukawa couplings

 $\mathbb{E}\left[\mathbb$ 

 $\mathbb{E}\left[\mathbb$ 

Example Z<sup>2</sup> ⇥ Z<sup>2</sup> charges:

 $\mathcal{L}_Y = Y^u \bar{Q}_L \tilde{\phi}_1 u_R + Y^d \bar{Q}_L \phi_2 d_R + Y^e \bar{E}_L \phi_3 e_R + \text{h.c.}$  $\frac{1}{2}$  $\mathcal{L} = \nabla \phi^2 \cos \phi^2$ neutral interactions  $\overline{v}$  $\mathcal{L}_Y = Y^u \bar{Q}_L \tilde{\phi}_1 u_R + Y^d \bar{Q}_L \phi_2 d_R + Y^e \bar{E}_L \phi_3 e_R + \text{h.c.}$  $E = \frac{1}{\sqrt{2}}$  $\mathcal L_Y=Y$  " $Q_L \rho_1 u_R$  $r$ al interactions  $\frac{1}{\sqrt{2}}$  $\frac{1}{2}$ 

neutral interactions  
\n
$$
\mathcal{L}_Y^{\text{neutral}} = \frac{1}{v_1} \bar{u} M^u (\eta_1 + i \chi_1 \gamma_5) u + \frac{1}{v_2} \bar{d} M^d (\eta_2 + i \chi_2 \gamma_5) d + \frac{1}{v_3} \bar{e} M^e (\eta_3 + i \chi_3 \gamma_5) e.
$$

*<sup>Y</sup>* = The  $\eta_i$  and  $\chi_i$  fields will mix. *v*1  $\frac{u}{u}$  and  $\lambda_i$  notes with interests  $h_i$  $\alpha$  *d*<sub>*d*</sub>  $\alpha$  is the multimum.<br> *d*  $\alpha$  fermion *f*: The  $\eta_i$  and  $\chi_i$  fields will mix. The  $\eta_i$  and  $\chi_i$  fields will mix. Neutral physical scalar  $h_i$  and a fermion f: The  $\eta_i$  and  $\chi_i$  fields will mix.<br>Neutral physical scalar *h* and a fermion

$$
\mathcal{L}_{hiff} = \frac{m_f}{v} h_i(\kappa^{h_i ff} \bar{f} f + i \tilde{\kappa}^{h_i ff} \bar{f} \gamma_5 f)
$$

#### Yukawa couplings Neutral physical scalar *h<sup>i</sup>* and a fermion *f*: *L<sup>h</sup>if f* = *m<sup>f</sup> hi*(*<sup>h</sup>if f* ¯*ff* + *i*˜*<sup>h</sup>if f* ¯*f*5*f*)*.* (3.5) *hi*(*<sup>h</sup>if f* ¯*ff* + *i*˜*<sup>h</sup>if f* ¯*f*5*f*)*.* (3.5) by "undoing" the transformation to the Higgs basis, Eq. (1.26), writing the inverse, for with  $R$ <sup>2</sup> given by Eq. (1.27). Next, the  $R$ <sup>2</sup> given by Eq. (1.27). Next, the  $R$ according to Eq. (1.29), can be expressed in terms of the physical states  $\mathbf{r}$  via Eq. (1.32). The physical states  $\mathbf{r}$

*<sup>i</sup>* and HB

*<sup>i</sup>* , collectively referred to as 'HB

$$
\mathcal{L}_{hiff} = \frac{m_f}{v} h_i (\kappa^{h_i ff} \bar{f} f + i \tilde{\kappa}^{h_i ff} \bar{f} \gamma_5 f)
$$

by "undoing" the transformation to the Higgs basis, Eq. (1.26), writing the inverse, for

TOF 77 IIIIal states, CIVIS has constrained inixing  $\tan \alpha^{h_{\text{SMTT}}} = \frac{K^{h_{\text{SMTT}}}}{h_{\text{SMTT}}}$ For  $\tau\bar{\tau}$  final states, CMS has constrained mixing  $t_{\text{SMT}}$ <sup>7</sup> For  $\tau\bar{\tau}$  final states, CMS has constrained m  $\tilde{\kappa}^{h_{\text{SM}}\tau\tau}$  $\frac{h}{\kappa h_{\text{SMTT}}}$  $\tan \alpha^{h_{\rm SM} \tau \tau} = \frac{\tilde{\kappa}^{h_{\rm SM} \tau \tau}}{h_{\rm max}}$  $\kappa^{h_{\rm SM}\tau\tau}$  $\tan \alpha^{h_{\text{SMTT}}} = \frac{\ddot{K}^{h_{\text{SMTT}}}}{h_{\text{SMTT}}}$  $\kappa^{n_{\rm SM}\tau\tau}$  $v \sim \frac{1}{2}$  ,  $v \sim \frac{1}{2}$  $\tan \alpha^{h_{\text{SM}}\tau\tau} = \frac{\kappa^{h_{\text{SM}}}}{h}$ 

The ⌘*<sup>i</sup>* and *<sup>i</sup>* fields will mix.

the neutral fields, in the form  $\mathcal{O}(n)$  is the form of  $\mathcal{O}(n)$ 

*L<sup>h</sup>if f* =

ia Higgs basis *h*  $\frac{1}{2}$ *k h* and *m sical neids:*<br> $\approx$  $\frac{6}{1}$ In order the in  $\eta_k$  and  $\chi_k$ , was neglected the fields to physical fields. Rotate from  $\eta_k$  and  $\chi_k$ , via Higgs basis fields to physical fields:  $\chi$ <sub>*l*</sub> , via Higgs basis fields to *v* rsical field<br> *i*  $\alpha$ *, (3.9) ,*  $\alpha$ 

$$
Z_i^{(k)} = \tilde{\mathcal{R}}_{1k}O_{i1} + \tilde{\mathcal{R}}_{2k}(O_{i2} + iO_{i4}) + \tilde{\mathcal{R}}_{3k}(O_{i3} + iO_{i5})
$$

$$
\alpha^{h_i \tau \tau} = \arg(Z_i^{(3)})
$$

## Yukawa couplings





## having imposed cut on α for hs<sub>M</sub>





## **EXPERIMENTAL ISSUE**

If  $h_2$  or  $h_3$  plays the role of  $h_{SM}$  at 125 GeV

Why have not  $h_1$  or  $h_2$  been observed?

1. Reduced coupling for Bjorken process (LEP) 2. Reduced gamma-gamma BR

However, note suggestions by Heinemeyer et al, 96 GeV 2105.11189, 2203.13180, 2204.05975

## **CONCLUSIONS**

We have reviewed the Weinberg 3HDM potential

- accommodates CP violation and NFC
- consequence: light states with a significant CP-<br>odd content (below 125 GeV)
- Plea for LHC: keep searching!

## BACKUP

# Appendix: Limits of CPC conservation of CP conserva

In special cases, no CP violation. Study CP-odd invariants At the lowest non-trivial order, the invariants can be expanded in terms of At the lowest non-trivial order in terms of the invariants can be expanded in terms of the international can be expanded in terms of the international can be expanded in terms of the international can be expanded in terms *', the invariants can be expanded in terms* Acted and a CD wiclotion Ctudy CD add *s* the invariants can be expanded in the In special cases, no  $CP$  violation. Study  $CP$ -odd invariants  $\blacksquare$ 

$$
S = \sin(2\theta_2 - 2\theta_3)
$$

and

and 
$$
X_a = \lambda_{11}(\lambda_{12} - \lambda_{13}) + \lambda_{22}(\lambda_{23} - \lambda_{12}) + \lambda_{33}(\lambda_{13} - \lambda_{23})
$$

$$
X_b = \lambda_{11}(\lambda'_{12} - \lambda'_{13}) + \lambda_{22}(\lambda'_{23} - \lambda'_{12}) + \lambda_{33}(\lambda'_{13} - \lambda'_{23})
$$

$$
X_c = \lambda_{12}(\lambda'_{13} - \lambda'_{23}) + \lambda_{13}(\lambda'_{23} - \lambda'_{12}) + \lambda_{23}(\lambda'_{12} - \lambda'_{13})
$$

$$
W_{a} = (\lambda_{23} - \lambda_{13})v_{1}^{4}v_{2}^{4}\sin^{2}2\theta_{2} + (\lambda_{13} - \lambda_{12})v_{2}^{4}v_{3}^{4}\sin^{2}(2\theta_{2} - 2\theta_{3})
$$
  
+  $(\lambda_{12} - \lambda_{23})v_{1}^{4}v_{3}^{4}\sin^{2}2\theta_{3},$   

$$
W_{b} = (\lambda'_{23} - \lambda'_{13})v_{1}^{4}v_{2}^{4}\sin^{2}2\theta_{2} + (\lambda'_{13} - \lambda'_{12})v_{2}^{4}v_{3}^{4}\sin^{2}(2\theta_{2} - 2\theta_{3})
$$
  
+  $(\lambda'_{12} - \lambda'_{23})v_{1}^{4}v_{3}^{4}\sin^{2}2\theta_{3},$   

$$
W_{c} = (\lambda_{11} - \lambda_{22})v_{1}^{4}v_{2}^{4}\sin^{2}2\theta_{2} + (\lambda_{22} - \lambda_{33})v_{2}^{4}v_{3}^{4}\sin^{2}(2\theta_{2} - 2\theta_{3})
$$
  
+  $(\lambda_{33} - \lambda_{11})v_{1}^{4}v_{3}^{4}\sin^{2}3\theta_{3}.$  **must all vanish...**

#### **Higgs basis** *e<sup>i</sup>*✓<sup>2</sup> *v*<sup>2</sup>  $\overline{P}$  $\overline{\phantom{a}}$  $\overline{\mathbf{s}}$  $\overline{\phantom{0}}$

A *.* (0.23)

$$
\mathcal{R}_2 \mathcal{R}_1 \begin{pmatrix} v_1 \\ e^{i\theta_2} v_2 \\ e^{i\theta_3} v_3 \end{pmatrix} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}
$$
  

$$
\mathcal{R}_1 = \begin{pmatrix} 1 & 0 \\ 0 & R_1 \end{pmatrix}, \quad R_1 = \frac{1}{w} \begin{pmatrix} v_2 e^{-i\theta_2} & v_3 e^{-i\theta_3} \\ -v_3 e^{-i\theta_2} & v_2 e^{-i\theta_3} \end{pmatrix}, \quad w = \sqrt{v_2^2 + v_3^2}
$$

0.1 Rotating to a Higgs basis

 $A \rightarrow \infty$  suitable Higgs basis is reached by the transformation

#### **Higgs basis** *e<sup>i</sup>*✓<sup>2</sup> *v*<sup>2</sup>  $\overline{P}$  $\overline{\phantom{a}}$  $\overline{\mathbf{s}}$  $\overline{\phantom{0}}$ A *.* (0.23) *, R*<sup>1</sup> = *v*3 *basis* Thus, the Higgs basis (with SU(2) doublets *H*1, *H*<sup>2</sup> and *H*3) is reached by *R* ⌘ *R*2*R*1,

*, w* =

*v*2

<sup>2</sup> + *v*<sup>2</sup>

*H*<sup>1</sup>

1<br>1<br>1

0<br>1900<br>1900

<sup>3</sup>*,* (0.24)

$$
\mathcal{R}_{2}\mathcal{R}_{1}\begin{pmatrix}v_{1} \\ e^{i\theta_{2}}v_{2} \\ e^{i\theta_{3}}v_{3}\end{pmatrix} = \begin{pmatrix}v_{1} \\ 0 \\ 0\end{pmatrix} \n\mathcal{R}_{1} = \begin{pmatrix}1 & 0 \\ 0 & R_{1}\end{pmatrix}, \quad R_{1} = \frac{1}{w}\begin{pmatrix}v_{2}e^{-i\theta_{2}} & v_{3}e^{-i\theta_{3}} \\ -v_{3}e^{-i\theta_{2}} & v_{2}e^{-i\theta_{3}}\end{pmatrix}, \quad w = \sqrt{v_{2}^{2} + v_{3}^{2}} \n\mathcal{R}_{2} = \frac{1}{v}\begin{pmatrix}v_{1} & w & 0 \\ -w & v_{1} & 0 \\ 0 & 0 & v\end{pmatrix} \qquad \begin{pmatrix}H_{1} \\ H_{2} \\ H_{3}\end{pmatrix} = \mathcal{R}\begin{pmatrix}\phi_{1} \\ \phi_{2} \\ \phi_{3}\end{pmatrix} = \tilde{\mathcal{R}}\begin{pmatrix}\phi_{1} \\ e^{-i\theta_{2}}\phi_{2} \\ e^{-i\theta_{3}}\phi_{3}\end{pmatrix} \n\tilde{\mathcal{R}} = \mathcal{R}_{2}\frac{1}{w}\begin{pmatrix}w & 0 & 0 \\ 0 & v_{2} & v_{3} \\ 0 & -v_{3} & v_{2}\end{pmatrix} \qquad \text{in fact real}.
$$

0.1 Rotating to a Higgs basis

 $A \rightarrow \infty$  suitable Higgs basis is reached by the transformation

*R*2*R*<sup>1</sup>

0

*v*1

*R*<sup>1</sup> =

1

A =

*v*<sup>1</sup> *w* 0

0

*v*

1

1<br>1<br>1

0

@

*e<sup>i</sup>*✓<sup>2</sup> *v*<sup>2</sup>

@

0<br>1900 - 1910<br>1910 - 1910 - 1910

#### **Higgs basis** *e<sup>i</sup>*✓<sup>2</sup> *v*<sup>2</sup>  $\overline{P}$  $\overline{\phantom{a}}$  $\overline{\mathbf{s}}$  $\overline{\phantom{0}}$ A *.* (0.23) *, R*<sup>1</sup> = *<i><u>i*</del>*basis*</u> Thus, the Higgs basis (with SU(2) doublets *H*1, *H*<sup>2</sup> and *H*3) is reached by *R* ⌘ *R*2*R*1, **b** Dasis A *.* (0.25) Thus, the Higgs basis (with SU(2) doublets *H*1, *H*<sup>2</sup> and *H*3) is reached by *R* ⌘ *R*2*R*1,

*, w* =

*v*2

<sup>2</sup> + *v*<sup>2</sup>

*H*<sup>1</sup>

1<br>1<br>1

0<br>1900<br>1900

<sup>3</sup>*,* (0.24)

0.1 Rotating to a Higgs basis

 $A \rightarrow \infty$  suitable Higgs basis is reached by the transformation

*R*2*R*<sup>1</sup>

0

*v*1

*R*<sup>1</sup> =

1

A =

*v*<sup>1</sup> *w* 0

0

*v*

1

1<br>1<br>1

0

@

*e<sup>i</sup>*✓<sup>2</sup> *v*<sup>2</sup>

@

0<br>1900 - 1910<br>1910 - 1910 - 1910

$$
\mathcal{R}_{2}\mathcal{R}_{1}\begin{pmatrix} v_{1} \\ e^{i\theta_{2}}v_{2} \\ e^{i\theta_{3}}v_{3} \end{pmatrix} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \n\mathcal{R}_{1} = \begin{pmatrix} 1 & 0 \\ 0 & R_{1} \end{pmatrix}, \quad R_{1} = \frac{1}{w} \begin{pmatrix} v_{2}e^{-i\theta_{2}} & v_{3}e^{-i\theta_{3}} \\ -v_{3}e^{-i\theta_{2}} & v_{2}e^{-i\theta_{3}} \end{pmatrix}, \quad w = \sqrt{v_{2}^{2} + v_{3}^{2}}.
$$
\n
$$
\mathcal{R}_{2} = \frac{1}{v} \begin{pmatrix} v_{1} & w & 0 \\ -w & v_{1} & 0 \\ 0 & 0 & v \end{pmatrix} \qquad \begin{pmatrix} H_{1} \\ H_{2} \\ H_{3} \end{pmatrix} = \mathcal{R} \begin{pmatrix} \phi_{1} \\ \phi_{2} \\ \phi_{3} \end{pmatrix} = \tilde{\mathcal{R}} \begin{pmatrix} \phi_{1} \\ e^{-i\theta_{2}}\phi_{2} \\ e^{-i\theta_{3}}\phi_{3} \end{pmatrix}
$$
\n
$$
\tilde{\mathcal{R}} = \mathcal{R}_{2} \frac{1}{w} \begin{pmatrix} w & 0 & 0 \\ 0 & v_{2} & v_{3} \\ 0 & -v_{3} & v_{2} \end{pmatrix} \qquad \text{in fact real.}
$$
\n
$$
H_{1} = \begin{pmatrix} G^{+} \\ (v + \eta_{1}^{HB} + iG_{0})/\sqrt{2} \end{pmatrix}, \quad H_{i} = \begin{pmatrix} \varphi_{i}^{HB} + \\ (\eta_{i}^{HB} + i\chi_{i}^{HB})/\sqrt{2} \end{pmatrix}, \quad i = 2, 3
$$
\n
$$
\varphi_{i}^{HB} = \{\eta_{1}^{HB}, \quad \eta_{2}^{HB}, \quad \eta_{3}^{HB}, \quad \chi_{2}^{HB}, \quad \chi_{3}^{HB}, \quad i = 1, \dots 5
$$

### Masses and enumerate the neutral fields *{*1,2,3,4,5*}* according to the following sequence: <sup>2</sup> *,* ⌘HB <sup>3</sup> *,* HB <sup>2</sup> *,* HB <sup>2</sup> *,* ⌘HB <sup>3</sup> *,* HB <sup>2</sup> *,* HB  $\overline{\phantom{a}}$  Masses ch)11 = 1*v*2 sin2(2)<br>*v*2 sin2(2)*v* 2*v*<sup>2</sup>

and enumerate the neutral fields *{*1,2,3,4,5*}* according to the following sequence:

<sup>3</sup> *}, i* = 1*,...* 5*.* (1.29)

<sup>3</sup>) *<sup>v</sup>*<sup>2</sup>

<sup>3</sup> *}, i* = 1*,...* 5*.* (1.29)

Charged sector: Charged sector: Charged sectors in the second sector in the second sector in the second sector in the second sector. The second sector in the second second sector in the second sector in the second sector in the second sector in the secon  $\mathbf{r}$ <sup>+</sup>

'<br>'HBC<br>'HBC

'HB

*<sup>i</sup>* <sup>=</sup> *{*⌘HB

*<sup>i</sup>* <sup>=</sup> *{*⌘HB

Charged sector:

<sup>1</sup> *,* ⌘HB

<sup>1</sup> *,* ⌘HB

$$
\begin{split}\n(\mathcal{M}_{ch}^2)_{11} &= -\frac{\lambda_1 v^2 \sin^2(2\theta_2 - 2\theta_3) v_2^2 v_3^2}{\sin 2\theta_2 \sin 2\theta_3 v_1^2 w^2} - (\lambda'_{12} v_2^2 + \lambda'_{13} v_3^2) \frac{v^2}{2w^2}, \\
(\mathcal{M}_{ch}^2)_{12} &= -\frac{\lambda_1 v v_1 v_2 v_3 \sin(2\theta_2 - 2\theta_3)}{\sin 2\theta_2 \sin 2\theta_3 v_1^2 w^2} (v_2^2 \sin 2\theta_2 e^{2i\theta_3} + v_3^2 \sin 2\theta_3 e^{2i\theta_2}) + \frac{v v_1 v_2 v_3}{2w^2} (\lambda'_{12} - \lambda'_{13}), \\
(\mathcal{M}_{ch}^2)_{21} &= (\mathcal{M}_{ch}^2)_{12}^*, \\
(\mathcal{M}_{ch}^2)_{22} &= -\frac{\lambda_1}{\sin 2\theta_2 \sin 2\theta_3 w^2} (2 \sin 2\theta_2 \sin 2\theta_3 \cos(2\theta_2 - 2\theta_3) v_2^2 v_3^2 + \sin^2 2\theta_2 v_2^4 + \sin^2 2\theta_3 v_3^4) \\
&- \frac{1}{2w^2} [(\lambda'_{12} v_3^2 + \lambda'_{13} v_2^2) v_1^2 + \lambda'_{23} w^4].\n\end{split}
$$

<sub>ገ</sub><br>ገ terms proportional to  $\lambda_1$  and to  $\lambda'_{ij}$ 

$$
h_i^+ = U_{ij} \varphi_{j+1}^{\text{HB}+} \qquad U = \begin{pmatrix} \cos \gamma & \sin \gamma e^{i\phi} \\ -\sin \gamma e^{-i\phi} & \cos \gamma \end{pmatrix}
$$

## Masses

## Neutral sector: Neutral sector  $(5 \times 5)$ :

1.2 Neutral sector

$$
(\mathcal{M}_{\text{neut}}^{2})_{11} = \frac{4\lambda_{1}v_{2}^{2}v_{3}^{2}}{v^{2} s_{2\theta_{2}} s_{2\theta_{3}}}[1 - c_{2\theta_{2}-2\theta_{2}} c_{2\theta_{2}} c_{2\theta_{3}}] + \frac{2}{v^{2}}[\lambda_{11}v_{1}^{4} + \lambda_{22}v_{2}^{4} + \lambda_{33}v_{3}^{4} + \bar{\lambda}_{12}v_{1}^{2}v_{2}^{2} + \bar{\lambda}_{13}v_{1}^{2}v_{3}^{2} + \bar{\lambda}_{23}v_{2}^{2}v_{3}^{2}], (\mathcal{M}_{\text{neut}}^{2})_{12} = \frac{-2\lambda_{1}v_{2}^{2}v_{3}^{2}}{v^{2} w v_{1} s_{2\theta_{2}} s_{2\theta_{3}}}[s_{2\theta_{2}-2\theta_{3}}^{2}(2w^{2} - v^{2}) - 2c_{2\theta_{2}-2\theta_{3}} s_{2\theta_{2}} s_{2\theta_{3}} v_{1}^{2}] - \frac{v_{1}}{v^{2} w}[2\lambda_{11}v_{1}^{2}w^{2} - 2\lambda_{22}v_{2}^{4} - 2\lambda_{33}v_{3}^{4} - (\bar{\lambda}_{12}v_{2}^{2} + \bar{\lambda}_{13}v_{3}^{2})(v^{2} - 2w^{2}) - 2\bar{\lambda}_{23}v_{2}^{2}v_{3}^{2}] (\mathcal{M}_{\text{neut}}^{2})_{13} = \frac{2\lambda_{1}v_{2}v_{3}}{v w s_{2\theta_{2}} s_{2\theta_{3}}}[v_{2}^{2} s_{2\theta_{2}}^{2} - v_{3}^{2} s_{2\theta_{3}}^{2}] + \frac{v_{2}v_{3}w}{vw^{2}}[-2\lambda_{22}v_{2}^{2} + 2\lambda_{33}v_{3}^{2} - \bar{\lambda}_{12}v_{1}^{2} + \bar{\lambda}_{13}v_{1}^{2} + \bar{\lambda}_{23}(v_{2}^{2} - v_{3}^{2})], (\mathcal{M}_{\text{neut}}^{2})_{22} = \frac{4\lambda_{1}v_{2}^{2}v_{3}^{2}}{v^{2} w^{2} s_{2\
$$

### Masses 1.2 Neutral sector 1.2 Ne <sup>1</sup>*c*2✓22✓<sup>3</sup> *<sup>s</sup>*2✓<sup>2</sup> *<sup>s</sup>*2✓<sup>3</sup> *<sup>w</sup>*2*s*<sup>2</sup> <sup>2</sup>✓22✓<sup>3</sup> ] <sup>3</sup> ¯12*v*<sup>2</sup> <sup>2</sup>*w*<sup>2</sup> ¯13*v*<sup>2</sup> <sup>3</sup>*w*<sup>2</sup> <sup>+</sup> ¯23*v*<sup>2</sup> *<sup>v</sup>*2*w*<sup>2</sup> [11*w*<sup>4</sup> <sup>+</sup> 22*v*<sup>4</sup> <sup>2</sup> + 33*v*<sup>4</sup>

2*v*2 3]*,*

## Neutral sector  $(5 \times 5)$ :

 $\frac{1}{2}$ 

neut)<sup>22</sup> <sup>=</sup> <sup>4</sup>1*v*<sup>2</sup>

2*v*<sup>2</sup> 1

[*v*2

<sup>2</sup> + 33*v*<sup>4</sup>

*v*2*w*2*s*2✓<sup>2</sup> *s*2✓<sup>3</sup>

*<sup>v</sup>*2*w*<sup>2</sup> [11*w*<sup>4</sup> <sup>+</sup> 22*v*<sup>4</sup>

(*M*<sup>2</sup>

WELLIAI SECHOI (J X J).  
\n
$$
(\mathcal{M}_{\text{neut}}^2)_{23} = \frac{2\lambda_1 v_2 v_3}{v v_1 w^2 s_{2\theta_2 s_{2\theta_3}} [ -w^2 s_{2\theta_2 - 2\theta_3} (v_2^2 s_{2\theta_2} c_{2\theta_3} + v_3^2 s_{2\theta_3} c_{2\theta_2}) + v_1^2 (v_2^2 - v_3^2) c_{2\theta_2 - 2\theta_3} s_{2\theta_2} s_{2\theta_3}] + \frac{v_1 v_2 v_3}{w v^2} [-2\lambda_{22} v_2^2 + 2\lambda_{33} v_3^2 + (\bar{\lambda}_{12} - \bar{\lambda}_{13}) w^2 + \bar{\lambda}_{23} (v_2^2 - v_3^2)],
$$
\n
$$
(\mathcal{M}_{\text{neut}}^2)_{25} = \frac{2\lambda_1 v v_2 v_3}{v_1} s_{2\theta_2 - 2\theta_3},
$$
\n
$$
(\mathcal{M}_{\text{neut}}^2)_{34} = \frac{-4\lambda_1 v_2^2 v_3^2}{v_1} c_{2\theta_2 - 2\theta_3} + \frac{2v_2^2 v_3^2}{w^2} [\lambda_{22} + \lambda_{33} - \bar{\lambda}_{23}],
$$
\n
$$
(\mathcal{M}_{\text{neut}}^2)_{44} = \frac{-2\lambda_1 v v_2 v_3}{v_1} s_{2\theta_2 - 2\theta_3},
$$
\n
$$
(\mathcal{M}_{\text{neut}}^2)_{45} = \frac{-2\lambda_1 v^2 v_2^2 v_3^2}{v_1 w^2 s_{2\theta_2} s_{2\theta_3}} s_{2\theta_2 - 2\theta_3} [v_2^2 s_{2\theta_2} c_{2\theta_3} + v_3^2 s_{2\theta_3} c_{2\theta_2}],
$$
\n
$$
(\mathcal{M}_{\text{neut}}^2)_{45} = \frac{-2\lambda_1 v_2 v_3}{v_1 w^2 s_{2\theta_2} s_{2\theta_3}} [2v_2^2 v_3^2 c_{2\theta_2 - 2\theta_3} s_{2\theta_2 - 2\theta_3}
$$