Theory of CP Violation in Charm Physics

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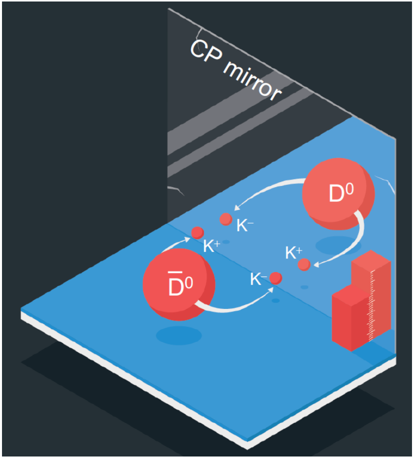
Charm CP Violation:
New unique gate to flavor structure of up-type quarks.

[LHCb 1903.08726, HFLAV 2021]

$$a_{CP}^{\text{dir}}(D^0 \to K^+K^-) - a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = (-0.161 \pm 0.028)\% \,.$$

The problem: Is it SM?



[CERN]

Please note:

This is my personal list, so the overview is biased towards my own work.

Direct CP Violation is an Interference Effect

$$a_{CP}^{\mathrm{dir}}(f) \equiv \frac{|\mathcal{A}(D^0 \to f)|^2 - |\mathcal{A}(\overline{D}^0 \to f)|^2}{|\mathcal{A}(D^0 \to f)|^2 + |\mathcal{A}(\overline{D}^0 \to f)|^2} \approx 2(r_{\mathrm{CKM}} \sin \varphi_{\mathrm{CKM}})\,(r_{\mathrm{QCD}} \sin \delta_{\mathrm{QCD}})\,.$$

f = CP-eigenstate.

The decay amplitude:

$$\mathcal{A} = 1 + r_{\mathrm{CKM}}\, r_{\mathrm{QCD}}\, e^{i(\varphi_{\mathrm{CKM}}+\delta_{\mathrm{QCD}})}$$

- r_{CKM} : real ratio of CKM matrix elements.
- φ_{CKM} : weak phase.
- r_{QCD} : real ratio of hadronic matrix elements.
- δ_{QCD} : strong phase.

Where does the interference come from?

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^- \xrightarrow{\text{QCD}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^- \xrightarrow{\text{QCD}} K^+K^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^-$$

$KK \leftrightarrow \pi\pi$ rescattering into same final state.

Weak and strong factors

$$\frac{\mathcal{A}(D \to \pi\pi \to KK)}{\mathcal{A}(D \to KK)} = \left(r_{\text{CKM}} e^{i\varphi_{\text{CKM}}}\right)\left(r_{\text{QCD}} e^{i\delta_{\text{QCD}}}\right)$$

- r_{QCD}: ratio of rescattering amplitudes.
- $\delta_{\text{QCD}} = \mathcal{O}(1)$: strong phase.
- $r_{\text{CKM}} = 1$: ratio of CKM factors, $\left|V_{cd}^* V_{ud}/(V_{cs}^* V_{us})\right|$
- $\varphi_{\text{CKM}} \approx 6 \cdot 10^{-4}$: deviation from 2×2 unitarity.

Prediction

$$\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{\text{QCD}}$$

- U-spin decomposition: $r_{\text{QCD}} = \mathcal{A}^{\Delta U=0}/\mathcal{A}^{\Delta U=1}$.

Can we overcome soft QCD in Charm?

Expansion parameters

- In kaon decays we have m/Λ.
- In B decays we have Λ/m.
- In charm. . . ?

We have to find new ways to do predictions.

The three $\Delta I = 1/2$ rules for $P \to \pi\pi$

- Relevant ratio of strong isospin matrix elements:

$r_{QCD}^{\Delta I=1/2} \equiv A^{\Delta I=1/2}/A^{\Delta I=3/2}$	Kaon	Charm	Beauty
Data	22	2.5	1.5
"No QCD" limit	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
Enhancement	$\mathcal{O}(10)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$

[D: Franco Mishima Silvestrini 2012, B: Grinstein Pirtskhalava Stone Uttayarat 2014]

- Rescattering most important in K decays, less important but still significant in D decays, and small in B decays.

Back to U-spin

$$\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{\text{QCD}}\,, \quad r_{\text{QCD}} = \mathcal{A}^{\Delta U=0}/\mathcal{A}^{\Delta U=1}$$

Assuming the SM, the data implies $r_{\text{QCD}} \sim 1$.

What is r_{QCD}?

- Light Cone Sum Rules (LCSR)

  [Petrov Khodjamirian 1706.07780, Chala Lenz Rusov Scholtz 1903.10490]

$$r_{QCD} \sim \mathcal{O}\left(\frac{\alpha_s}{\pi}\right) \sim 0.1.$$

- Low energy QCD, rescattering is $\mathcal{O}(1)$

  [Grossman StS 1903.10952, Brod Kagan Zupan 1111.5000]

$$r_{QCD} \sim \mathcal{O}(1).$$

Same pattern as in charm $\Delta I = 1/2$ rule.

The jury is still out: Is it SM or not?

- No matter what it is, we learn sth new.
- We have a good argument why it is QCD.
- Assumption of large rescattering at low energy agrees with the data.

$$\text{Loop/Tree} = \mathcal{O}(1)$$

A_{CP} Sum Rules: Overconstrain the SM

Challenge for predicting CP asymmetries

- New hadronic quantities appear.
- These cannot be extracted from $\mathcal{B}$ measurements.

Solution

Make up $SU(3)_F$ sum rules in which these cancel.

$SU(3)_F$ limit sum rules

$$a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) + a_{CP}^{\text{dir}}(D^0 \to K^+K^-) = 0\,,$$
$$a_{CP}^{\text{dir}}(D_s^+ \to K_S\pi^+) + a_{CP}^{\text{dir}}(D^+ \to K_SK^+) = 0\,.$$

Key Measurements for $D \to PP'$.

A_{CP} sum rules including breaking effects [Müller Nierste StS 1506.04121]

- SM sum rule 1: $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, $D^0 \to \pi^0\pi^0$.
- SM sum rule 2: $D^+ \to K_SK^+$, $D_s^+ \to K_S\pi^+$, $D_s^+ \to K^+\pi^0$.

Isospin Analysis [Grossman Kagan Zupan 1204.3557]

- Extract $\Delta I = 1/2$ and $\Delta I = 3/2$ MEs from

$$D^0 \to \pi^+\pi^-, D^+ \to \pi^+\pi^0, D^0 \to \pi^0\pi^0.$$

- $a_{CP}^{\mathrm{dir}}(D^+ \to \pi^+\pi^0) = 0.$ Higher orders < sensitivity.

What next?

- Measurements of CP asymmetries in all SCS $D \to PP'$ decays.
- Need sum rules for multi-body decays at higher order in $SU(3)_F$.

SU(3)-flavor

- SU(3): Approximate symmetry for the light quarks u, d, s.
- Very useful, but $\mathcal{O}(30\%)$ breaking from corrections.
- Going to higher order: complicated.

$$(\mathbf{15}) \otimes (\mathbf{8}) = (\mathbf{42}) \oplus (\mathbf{24}) \oplus (\mathbf{15_1}) \oplus (\mathbf{15_2}) \oplus (\mathbf{15'}) \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3})$$
$$(\bar{\mathbf{6}}) \otimes (\mathbf{8}) = (\mathbf{24}) \oplus (\mathbf{15}) \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3})$$

Decay d	$B_1^{3_1}$	$B_1^{3_2}$	$B_8^{3_1}$	$B_8^{3_2}$	$B_8^{\bar{6}_1}$	$B_8^{\bar{6}_2}$	$B_8^{15_1}$	...
$D^0 \to K^+K^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$\frac{1}{10}$	$-\frac{1}{10\sqrt{2}}$	$-\frac{7}{10\sqrt{122}}$	...
$D^0 \to \pi^+\pi^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$-\frac{1}{10}$	$\frac{1}{10\sqrt{2}}$	$-\frac{11}{10\sqrt{122}}$	...
$D^0 \to \bar{K}^0K^0$	$-\frac{1}{4\sqrt{10}}$	$-\frac{1}{8}$	$\frac{1}{5\sqrt{2}}$	$\frac{1}{2\sqrt{5}}$	0	0	$-\frac{9}{5\sqrt{122}}$	...
$D^0 \to \pi^0\pi^0$	$-\frac{1}{8\sqrt{5}}$	$-\frac{1}{8\sqrt{2}}$	$-\frac{1}{20}$	$-\frac{1}{4\sqrt{10}}$	$\frac{1}{10\sqrt{2}}$	$-\frac{1}{20}$	$\frac{11}{20\sqrt{61}}$	...
...	...	...	...	...	...	...	...	...

Solving the Problem of Higher Order U-spin

[Gavrilova Grossman StS, 2205.12975]

We proved several theorems enabling calculations to arbitrary order.

- We are able to determine a priori up to which order sum rules exist.
- We do not need explicit Clebsches. Big complexity reduction.
- Hope: Opens the door for precision in hadronic decays.
- Close a gap between theory and experiment.

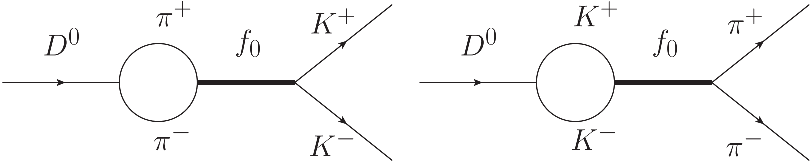
Take advantage of precision data on nonleptonic decays.

What next? Generalization to $\text{SU}(3)_F$, implications for observables.

What next? Check dynamical mechanism from data.

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^- \xrightarrow{\text{QCD}} \pi^+\pi^-$$



Assumptions

[StS and A. Soni, 2110.07619]

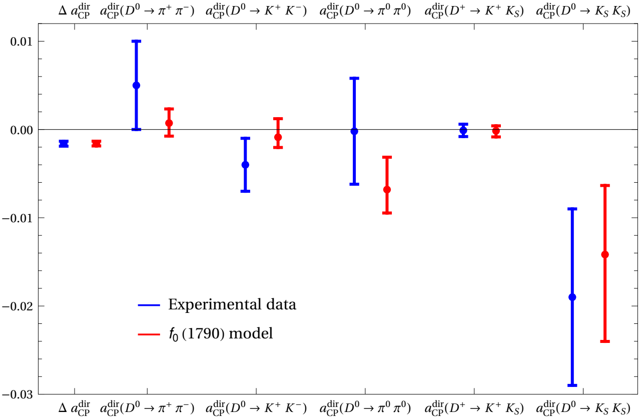
- Amplitudes to $I = 0$ states dominated by f_0 close to D^0 mass.
- Amplitudes into $I = 1$ states relatively suppressed.

Resonance structure can also be incorporated in future LCSR calculations.

[Khodjamirian Petrov 1706.07780]

Fit to Scalar Resonance Model

[StS and A. Soni, 2110.07619]



More on rescattering: [Franco Mishima Silvestrini 1203.3131]
[Bediaga Frederico Magalhaes 2203.04056]

What next? Study of $\Delta U = 0$ in three-body decays

[Dery Grossman StS Soffer 2101.02560]

$$\mathcal{A}(D^0 \to \pi^+\rho^-) = -\lambda\, T^{P_1V_2} - V_{cb}^* V_{ub}\, R^{P_1V_2}$$
$$\mathcal{A}(D^0 \to \pi^-\rho^+) = -\lambda\, T^{P_2V_1} - V_{cb}^* V_{ub}\, R^{P_2V_1}$$

- Time-integrated CP asym. of 2-body decays give only combinations

$$|\widetilde{R}^{P_1V_2}|\sin(\delta_{P_1V_2}) \qquad \text{and} \qquad |\widetilde{R}^{P_2V_1}|\sin(\delta_{P_2V_1})\,,$$

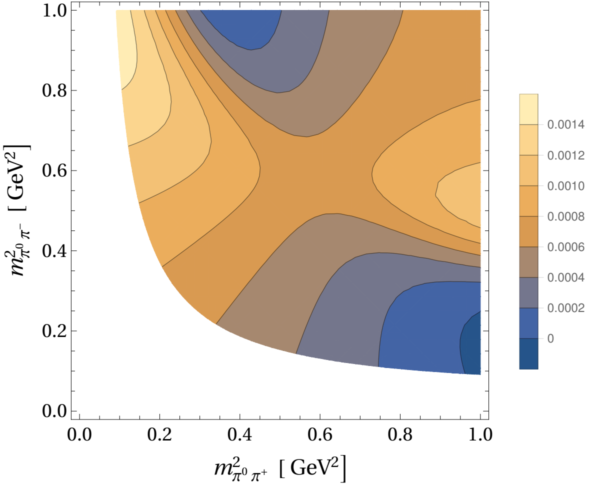
  but **not** magnitudes and phases separately.

- Three body decay changes 2 things:
  - We have additional kinematic dependences.
  - Only in a three-body decay we have interference between $D^0 \to \pi^+(\rho^- \to \pi^-\pi^0)$ and $D^0 \to \pi^-(\rho^+ \to \pi^+\pi^0)$.

➥ Extraction of all parameters from time-integrated CP meas.

Local $a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-\pi^0)$ in overlap region of $\rho^\pm$

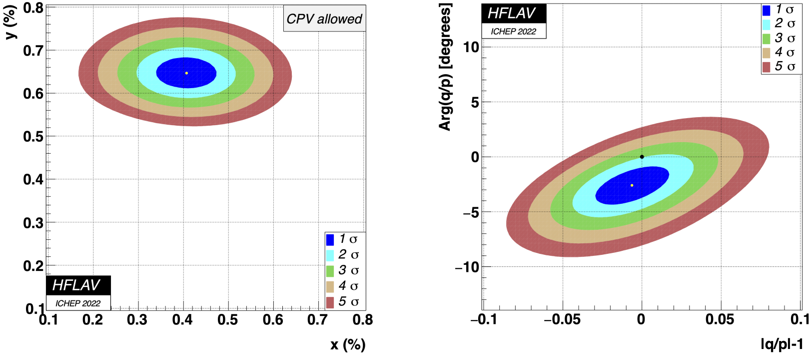
[Dery Grossman StS Soffer 2101.02560]



Numerical example: $\widetilde{R}^{P_1V_2} = \exp(i\pi/2), \quad \widetilde{R}^{P_2V_1} = \frac{1}{4}\exp(i\pi/3)$

Charm Mixing and Indirect CP Violation

Charm Mixing

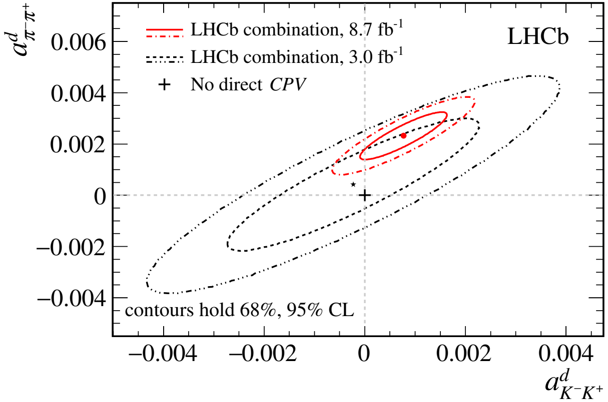


- Mixing parameters $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/(2\Gamma)$.
- $|q/p| \neq 1$ would indicate CPV in mixing.
- $\mathrm{Arg}(q/p) \neq 0$ would indicate CPV from interference mixing/decay.
- No Mixing $(x, y) = (0, 0)$ excluded at more than 11.5σ.
- No CP violation $(|q/p| - 1, \phi) = (0, 0)$ excluded at 2.1σ.
- SM: hard to calculate. Qualitative agreement with SM.

Beyond ΔA_{CP}

Going beyond Δa_{CP}^{dir}

[LHCb, 2209.03179]



- First evidence of direct CPV in single decay, $D^0 \to \pi^+\pi^-$: 3.8σ.
- $a_{CP}^{\text{dir}}(D^0 \to K^+K^-) = (7.7 \pm 5.7) \cdot 10^{-4}$,
  $a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = (23.2 \pm 6.1) \cdot 10^{-4}$.

Violation of a U-spin limit sum rule

- Separate measurement of both CP asymmetries allows test of U-spin expansion in subleading amplitude contributions which are relatively CKM-suppressed.

U-spin limit sum rule: Broken at 2.7σ [LHCb, 2209.03179]

$$\Sigma a_{CP}^{dir} \equiv a_{CP}^{dir}(D^0 \to K^+K^-) + a_{CP}^{dir}(D^0 \to \pi^+\pi^-) \overset{\text{U-spin}}{=} 0$$

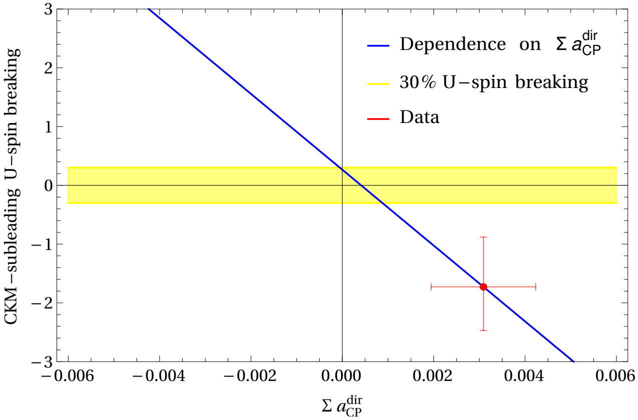
Improved U-spin limit sum rule: Broken at 2.1σ [StS, 2207.08539]

$$-\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} \frac{a_{CP}^{\text{dir}}(D^0 \to K^+K^-)}{a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-)} = -0.93^{+0.62}_{-0.41} \neq +1.$$

- U-spin breaking is expected: Only approximate symmetry.
- Amount goes beyond SM expectations of $m_s/\Lambda_{QCD} \sim 30\%$ at 1.9σ.

U-spin breaking in the CKM-subleading amplitude: $\left(173^{+85}_{-74}\right)\%$

[StS 2207.08539]



- $\Sigma a_{CP}^{\rm dir} \equiv a_{CP}^{\rm dir}(D^0 \to K^+K^-) + a_{CP}^{\rm dir}(D^0 \to \pi^+\pi^-)$.
- 1.95σ from SM: $\mathcal{O}(30\%)$.

Model-Independent Predictions

- Large U-spin breaking indicates large $\Delta U = 1$ operator(s).
- It follows $\mathcal{O}(1)$ breaking of U-spin limit sum rule:

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = -\frac{a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-)}{a_{CP}^{\text{dir}}(D^0 \to K^+K^-)} \quad \text{broken at } \mathcal{O}(1)\,,$$

- Connected to wider class of decays via $SU(3)$-flavor symmetry.

$$\text{Expect} \quad \frac{\Gamma(D^+ \to \overline{K}^0 K^+)}{\Gamma(D_s^+ \to K^0\pi^+)} = -\frac{a_{CP}^{\text{dir}}(D_s^+ \to K^0\pi^+)}{a_{CP}^{\text{dir}}(D^+ \to \overline{K}^0 K^+)} \quad \text{also broken at } \mathcal{O}(1).$$

- Improved versions of these sum rules: [Müller Nierste StS 1506.04121]

$$a_{CP}^{dir}(D^0 \to K^+K^-)\,, \quad a_{CP}^{dir}(D^0 \to \pi^+\pi^-)\,, \quad a_{CP}^{dir}(D^0 \to \pi^0\pi^0)\,, \quad \text{and}$$
$$a_{CP}^{dir}(D^+ \to K_S K^+)\,, \quad a_{CP}^{dir}(D_s^+ \to K_S\pi^+)\,, a_{CP}^{dir}(D_s^+ \to K^+\pi^0)\,.$$

These should also be broken at $\mathcal{O}(1)$.

Explanations beyond the SM: "$\Delta U = 1$ models"

- NP models with $\Delta U = 1$ operators can explain breaking of U-spin limit sum rules. [Hiller Jung StS 1211.3734]

- Additional operators with flavor content $\bar{s}c\bar{u}s$ and/or $\bar{d}c\bar{u}d$ with non-universal coefficients.

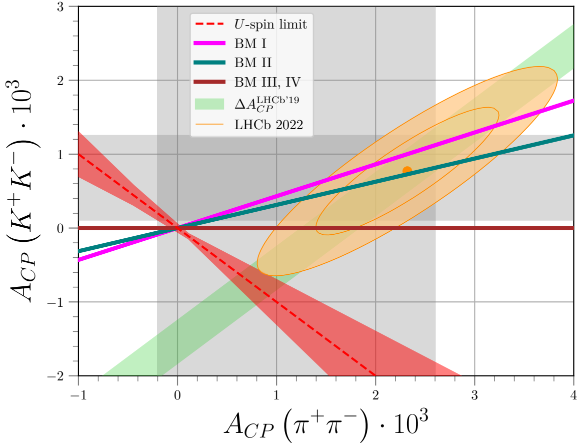
- Example: Z' models where fermion charges depend on generation.

[Bause Gisbert Golz Hiller 2004.01206, Bause Gisbert Hiller Höhne Litim Steudtner 2210.16330]

$$\begin{aligned}\mathcal{L}_{Z'} \supset \Big(& g_L^{uc}\bar{u}_L\gamma^\mu c_L Z'_\mu + g_R^{uc}\bar{u}_R\gamma^\mu c_R Z'_\mu + \text{h.c.}\Big) \\ & + g_L^{d}\bar{d}_L\gamma^\mu d_L Z'_\mu + g_R^{d}\bar{d}_R\gamma^\mu d_R Z'_\mu \\ & + g_L^{s}\bar{s}_L\gamma^\mu s_L Z'_\mu + g_R^{s}\bar{s}_R\gamma^\mu s_R Z'_\mu \\ & + g_L^{ll}\bar{l}_L\gamma^\mu l_L Z'_\mu + g_R^{ll}\bar{l}_R\gamma^\mu l_R Z'_\mu\end{aligned}$$

Z' model predictions

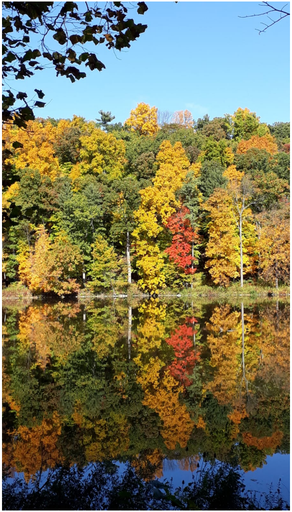
[Bause Gisbert Hiller Höhne Litim Steudtner 2210.16330]



- Viable models with leptophobic Z' below $\mathcal{O}(20\,\mathrm{GeV})$.
- Pattern of CP violation in $D \to \pi\pi$, including $a_{CP}^{dir}(D^+ \to \pi^+\pi^0) \neq 0$.

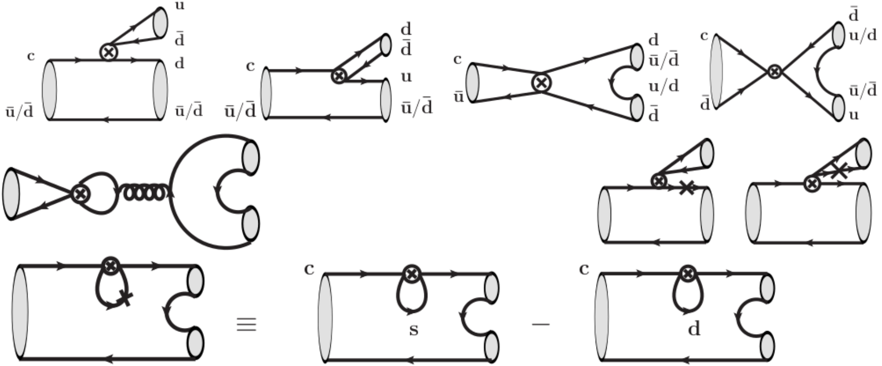
Conclusions

- This is just the beginning of the exploration of charm CPV.
- Charm is a unique gate to flavor structure of up-type quarks.
- Necessary to benefit from insights of flavor symmetry sum rules.
- No matter what, we will learn sth new: QCD or New Physics.



BACK-UP

Charm: Non-perturbative Diagrams



Systematics of U-spin breaking

- U-spin breaking from mass difference of strange and down quarks:

$$\varepsilon = \frac{m_s - m_d}{\Lambda_{\mathrm{QCD}}} \sim 0.3\,.$$

- Parametrized by triplet-operator H_ε:

$$\mathcal{H}_{\mathrm{eff}} = \sum_{m,b} f_{u,m} \left(H_m^u \otimes H_\varepsilon^{\otimes b} \right), \qquad H_\varepsilon^{\otimes b} \equiv \underbrace{H_\varepsilon \otimes \cdots \otimes H_\varepsilon}_{b}\,.$$

- Any system can be constructed from tensor products of doublets.
- Moving irreps ("crossing sym.") does not affect structure of sum rules.
- Without loss of generality, consider doublet-only system with

$$0 \to \left(\frac{1}{2}\right)^{\otimes n}$$ and singlet Hamiltonian.

Properties of U-spin pairs

[Gavrilova Grossman StS, 2205.12975]

- Amplitude:

$$A_j = \underbrace{(-,-,+,-,+,\dots,+)}_{n} = \sum_\alpha C_{j\alpha} X_\alpha \,.$$

- U-spin conjugated amplitude (complete interchange $s \leftrightarrow d$):

$$\overline{A}_j = \underbrace{(+,+,-,+,-,\dots,-)}_{n} = (-1)^p \sum_\alpha (-1)^b C_{j\alpha} X_\alpha \,.$$

- Notation: Abbreviate m-quantum number: $\pm 1/2 \mapsto \pm$.
  X_α: Reduced matrix element. $C_{j\alpha}$: Clebsches.
- Define (anti-)symmetric combinations of U-spin pairs:

$$a_j \equiv \underbrace{A_j - (-1)^p \overline{A}_j}_{\text{odd in } b}\,, \qquad s_j \equiv \underbrace{A_j + (-1)^p \overline{A}_j}_{\text{even in } b}\,.$$

Results: Sum Rules at any order of U-spin breaking

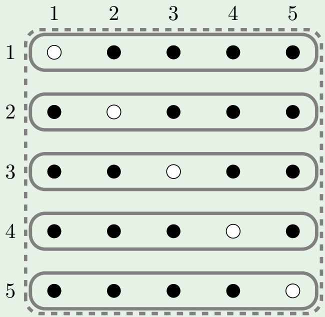
[Gavrilova Grossman StS, 2205.12975]

All sum rules at any order b can be written as:

$$\sum_j a_j = 0\,, \qquad \sum_j s_j = 0\,.$$

Example: $n = 6$ doublets. Dimension of lattice $d = n/2 - 1 = 2$.

- Each node ⇔ U-spin pair.
- Each node (points):
  a-type sum rule valid to $b = 0$.
- Sums of nodes in lines:
  s-type sum rules valid to $b = 1$.
- Sum of all nodes in plane:
  a-type sum rule valid up to $b = 2$.



Parametrization of D^0 Decays

[Brod Grossman Kagan Zupan 1203.6659]

$$\mathcal{A}(\overline{D}^0 \to K^+\pi^-) = V_{cs}V_{ud}^*\left(t_0 - \frac{1}{2}t_1\right), \qquad \text{(CF)}$$

$$\mathcal{A}(\overline{D}^0 \to \pi^+\pi^-) = -\Sigma^*\left(t_0 + s_1 + \frac{1}{2}t_2\right) - \lambda_b^*\left(p_0 - \frac{1}{2}p_1\right), \qquad \text{(SCS)}$$

$$\mathcal{A}(\overline{D}^0 \to K^+K^-) = \Sigma^*\left(t_0 - s_1 + \frac{1}{2}t_2\right) - \lambda_b^*\left(p_0 + \frac{1}{2}p_1\right), \qquad \text{(SCS)}$$

$$\mathcal{A}(\overline{D}^0 \to \pi^+K^-) = V_{cd}V_{us}^*\left(t_0 + \frac{1}{2}t_1\right), \qquad \text{(DCS)}$$

$$\Sigma \equiv \frac{V_{cs}^*V_{us} - V_{cd}^*V_{ud}}{2}, \qquad -\frac{\lambda_b}{2} \equiv -\frac{V_{cb}^*V_{ub}}{2} = \frac{V_{cs}^*V_{us} + V_{cd}^*V_{ud}}{2}.$$

Direct CP asymmetry:

$$a_{CP}^{\mathrm{dir}} = \mathrm{Im}\left(\frac{\lambda_b}{\Sigma}\right)\mathrm{Im}\left(\frac{A_b}{A_\Sigma}\right).$$

Solving for Parameters to $\mathcal{O}(\varepsilon^2)$

[Grossman StS 1903.10952, StS 2207.08539]

$$\mathcal{A}(\overline{D}^0 \to \pi^+\pi^-) = -\Sigma^* \left(t_0 + s_1 + \frac{1}{2}t_2\right) - \lambda_b^* \left(p_0 - \frac{1}{2}p_1\right) \qquad \text{(SCS)}$$

$$\mathcal{A}(\overline{D}^0 \to K^+K^-) = \Sigma^* \left(t_0 - s_1 + \frac{1}{2}t_2\right) - \lambda_b^* \left(p_0 + \frac{1}{2}p_1\right) \qquad \text{(SCS)}$$

Solution for parameters: $\tilde{s}_1 = s_1/t_0, \quad \tilde{p}_0 = p_0/t_0, \quad \tilde{p}_1 = p_1/t_0$.

$$\tilde{s}_1 = -\frac{1}{2} R_{KK,\pi\pi}$$

$$\mathrm{Im}(\tilde{p}_0) = \frac{1}{4\,\mathrm{Im}(\lambda_b/\Sigma)} \Delta a_{CP}^{\mathrm{dir}}$$

$$\mathrm{Im}(\tilde{p}_1) = \frac{1}{2\,\mathrm{Im}(\lambda_b/\Sigma)} \left(\Sigma a_{CP}^{\mathrm{dir}} + \frac{1}{2} R_{KK,\pi\pi} \Delta a_{CP}^{\mathrm{dir}}\right)$$

Branching ratio combination: $R_{KK,\pi\pi} \equiv \dfrac{|A(KK)|^2 - |A(\pi\pi)|^2}{|A(KK)|^2 + |A(\pi\pi)|^2}$.

U-spin breaking in the CKM-subleading amplitude

[StS 2207.08539]

- To order $\mathcal{O}(\varepsilon^2)$:

$$\frac{1/2\,\mathrm{Im}(\tilde{p}_1)}{\mathrm{Im}(\tilde{p}_0)} = \frac{\Sigma a_{CP}^{dir}}{\Delta a_{CP}^{dir}} + \frac{1}{2} R_{KK,\pi\pi}\,.$$

- We have no sensitivity yet to the corresponding real parts.
  ↳Need very precise measurements of time-dependent CP violation.

Assumption

Due to non-perturbative rescattering, the phases of $\tilde{p}_0$ and $\tilde{p}_1$ are $\mathcal{O}(1)$, resulting in

$$\left|\frac{\mathrm{Im}(\tilde{p}_1)}{\mathrm{Im}(\tilde{p}_0)}\right| \approx \frac{|\tilde{p}_1|}{|\tilde{p}_0|}\,.$$

Exclusive Approach: Hadron-Level

$$\Gamma_{12}^D = \sum_n \rho_n \left\langle \overline{D^0} \right| \mathcal{H}_{eff}^{\Delta C=1} |n\rangle \langle n| \mathcal{H}_{eff}^{\Delta C=1} \left| D^0 \right\rangle ,$$

$$M_{12}^D = \sum_n \left\langle \overline{D^0} \right| \mathcal{H}_{eff}^{\Delta C=2} \left| D^0 \right\rangle + \mathcal{P} \sum_n \frac{\left\langle \overline{D^0} \right| \mathcal{H}_{eff}^{\Delta C=1} |n\rangle \langle n| \mathcal{H}_{eff}^{\Delta C=1} \left| D^0 \right\rangle}{m_D^2 - E_n^2}$$

- n: all possible hadronic states. ρ_n: density of state. $\mathcal{P}$: principal value.
- Result: $y \sim 1\%$, agreeing with measurements.

What next?

- More experimental input needed (BRs and phases).
- Theory: Need to take into account more $\mathrm{SU}(3)_F$ breaking effects.
- Long-term: Lattice predictions?

Inclusive Approach: Quark-Level

- Heavy-Quark Expansion (HQE), motivated by $\tau(D^+)/\tau(D^0)$.
- Needed non-perturbative matrix elements from sum rules or Lattice
- Severe GIM-cancellations may take place.

Recent Developments

[Lenz Piscopo Vlahos 2007.03022]

- GIM depends on scales entering different box contributions.
  These contain different amounts of strangeness.
- No need that these scales are the same $\Rightarrow$ GIM cancellation broken.
- HQE uncertainty gets larger, including y^{exp}.

What next?

- Higher orders in HQE expansion.
- After Γ_{12} also M_{12}, e.g. with dispersion relations.