Spontaneous CP violation and the Strong CP problem

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Based on 2105.09122 in collaboration with Luca Vecchi

8th Symposium on Prospects in the Physics of Discrete Symmetries
Baden-Baden, 9 November 2022

Outline

The Strong CP problem

Colored sector of the SM:

$$\mathcal{L}_{\rm C}^{\rm SM} = \mathcal{L}_{\rm kin} + \frac{g_{\rm C}^2}{32\pi^2}\theta\, G_{\mu\nu}^a \widetilde{G}^{a\,\mu\nu} - Y_u\, qHu - Y_d\, q\tilde{H}d$$

$$\text{Irreducible } \not{C}\text{P} : \begin{cases} \delta_{\rm CKM} \subset V_{\rm CKM} \\ \bar{\theta} = \theta + \arg\det Y_u Y_d \end{cases}$$

$\boldsymbol{\delta_{\rm CKM} \approx 1.2}$

PDG (2022)

$\boldsymbol{\bar{\theta} \lesssim 10^{-10}}$

C. Abel et al. (2020)

The Strong CP problem:

Why is CP violation in flavor-conserving processes so suppressed?
A naturalness question

Outline

Spontaneous CP violation

Spontaneous $\not{CP}$: CP *exact* in the UV, then spontaneously broken

"good" UV symmetry: no quality problem!

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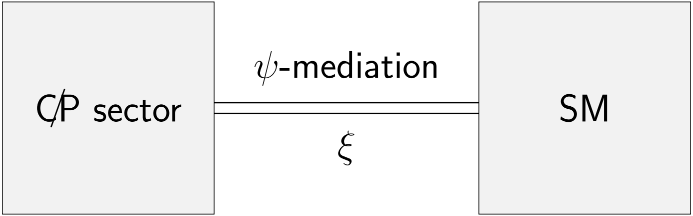
Nelson-Barr models

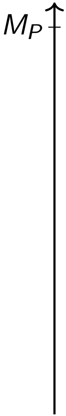
A. Nelson (1984), S. Barr (1984)



$\not{CP}$ entirely encoded in ξ

Vector-like mediator $\psi^c \sim u, d$ (q excluded L. Vecchi (2014))

$$-\mathcal{L}^{d-\text{med}} \supset y_u\, qHu + y_d\, q\widetilde{H}d + y^t\, \Sigma\psi d + m_\psi\, \psi\psi^c + \text{hc}$$



CP exact:

$y_u, y_d, y, m_\psi \in \mathbb{R}$ and $\theta = 0$

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M_P

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$y_u, y_d, y, m_\psi \in \mathbb{R}$ and $\theta = 0$

$\langle\Sigma\rangle$

Spontaneous C̸P: $\xi^\dagger = y^t\,\langle\Sigma\rangle \to \mathcal{M}_d = \begin{pmatrix} v\, y_d & 0 \\ \xi^\dagger & m_\psi \end{pmatrix}$

$$\bar{\theta}_{\text{tree}} = \theta_{\text{tree}} + \arg\det \mathcal{M}_{d,\text{tree}} + \arg\det y_{u,\text{tree}} = 0$$

Vector-like mediator $\psi^c \sim u, d$ (q excluded L. Vecchi (2014))

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Spontaneous $\not{C}$P: $\quad \xi^\dagger = y^t\,\langle\Sigma\rangle \rightarrow \mathcal{M}_d = \begin{pmatrix} v\, y_d & 0 \\ \xi^\dagger & m_\psi \end{pmatrix}$

$$\boldsymbol{\bar{\theta}_{\text{tree}} = \theta_{\text{tree}} + \arg\det\mathcal{M}_{d,\text{tree}} + \arg\det y_{u,\text{tree}} = 0}$$

m_ψ

SM + $\mathcal{O}(1/m_\psi)$ (SMEFT)

Key questions:

1. is ξ able to reproduce Standard Model $\not{CP}$?

   and ...

2. is it compatible with $\bar{\theta}_{\text{rad}} \lesssim 10^{-10}$?
3. is it compatible with experimental bounds on ψ?

If one point is not satisfied, **Strong CP problem is not solved!**

Outline

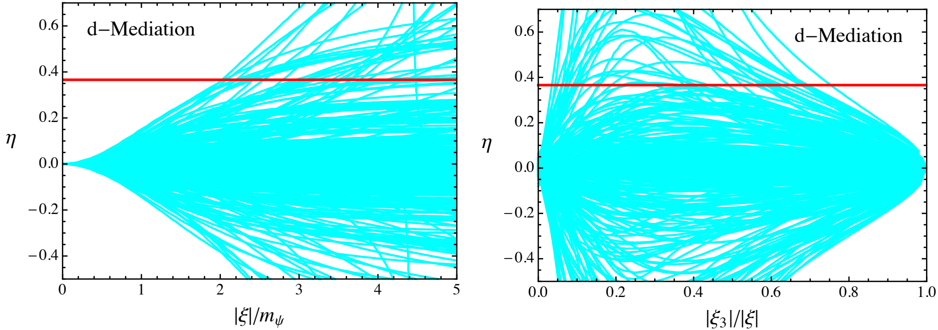
1. Standard Model $\not{C}\not{P}$ and quark masses

$\mathcal{M}_d = \begin{pmatrix} v\, y_d & 0 \\ \xi^\dagger & m_\psi \end{pmatrix}$: for $|\xi| \gg v$, leading order diagonalization:

$$\mathcal{L} \supset \mathcal{L}^{\text{SM}} - Y\, q\tilde{H}\psi^c - M\,\psi\psi^c$$

$$\begin{cases} Y_d = y_d \left[1 - \frac{\xi\xi^\dagger}{|\xi|^2}\left(1 - \frac{m_\psi}{M}\right)\right] \\ Y_u = y_u \end{cases} \qquad \begin{cases} M = \sqrt{|\xi|^2 + m_\psi^2} \\ Y = y_d \frac{\xi}{M} = Y_d \frac{\xi}{m_\psi} \end{cases}$$

$$Y_d Y_d^\dagger = y_d \left(1 - \frac{\xi\xi^\dagger}{M^2}\right) y_d^t \equiv V_{\text{CKM}}^* \, \widehat{Y}_d^2 \, V_{\text{CKM}}^t \longrightarrow \frac{|\xi|}{m_\psi} \sim \mathcal{O}(1)$$



$$\frac{|\xi|}{m_\psi} \gtrsim 2 \qquad (d\text{−med})$$

Perturbativity:

$$Y_i = \left(Y_d \frac{\xi}{m_\psi} \right)_i \ll 4\pi \longrightarrow \frac{|\xi_3|}{m_\psi} \ll \frac{4\pi}{\hat{y}_b}$$

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$$2 \lesssim \frac{|\xi|}{m_\psi} \ll 10^3 \qquad (d\text{−med})$$

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$$2 \lesssim \frac{|\xi|}{m_\psi} \ll 10^3 \qquad (d\text{−med})$$

$$20 \lesssim \frac{|\xi|}{m_\psi} \ll 300 \quad \& \quad |\xi_3| \sim \lambda_c^2 |\xi_2| \qquad (u\text{−med})$$

Fine-tuning that must be addressed by realistic UV completions!

Outline

2. Radiative corrections to $\bar{\theta}$

i) *Reducible*: no threat

ii) *Irreducible*: generated by ψ itself

$\bar{\theta}_{\text{rad,irr}} \lesssim 10^{-10}$ compatible with bounds on $|\xi|/m_\psi$?

2. Radiative corrections to $\bar{\theta}$

$\bar{\theta}$: CP-odd flavor invariant. Leading contribution:

$$\bar{\theta}_{\text{rad,irr}} \sim \left(\frac{1}{16\pi^2}\right)^3 \text{Im}\,\text{tr}\left(Y^\dagger \left[Y_d Y_d^\dagger, Y_u Y_u^\dagger\right] Y\right) \quad \left(Y = Y_{u,d}\frac{\xi}{m_\psi}\right)$$

$$\approx \begin{cases} 10^{-18}\,\text{Im}\left(\frac{\xi_i \xi_j^*}{m_\psi^2}\right) & (d\text{-med}) \\ 10^{-15}\,\text{Im}\left(\frac{\xi_i \xi_j^*}{m_\psi^2}\right) & (u\text{-med}) \end{cases}$$

$\rightarrow$ Compatible with $1 \lesssim \frac{|\xi|}{m_\psi} \ll 10^3$ (d-med) and $20 \lesssim \frac{|\xi|}{m_\psi} \ll 300$ (u-med) ✓

2. Radiative corrections to $\bar{\theta}$

- $N_{\text{med}} \geq 2$: AV, L. Vecchi (2021)

$$\xi_i \to \xi_{ij}, \quad m_\psi \to (m_\psi)_{ij}, \quad \frac{|\xi|}{m_\psi} \to |Y_{u,d}^{-1} Y|$$

Additional flavor violation = additional invariants

$$\bar{\theta}_{N_{\text{med}} \geq 2} \sim \left(\frac{1}{16\pi^2}\right)^3 \text{Im}\,\text{tr}\left(\left[Y^\dagger Y_u Y_u^\dagger Y, Y^\dagger Y\right] F(M^\dagger M)\right)$$

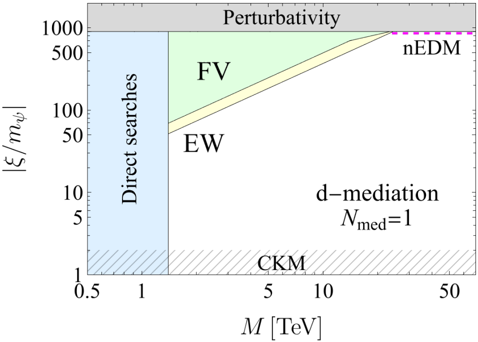
$$\approx \begin{cases} 10^{-18}\,|Y_d^{-1} Y|^4 & (d\text{-med}) \\ 10^{-12}\,|Y_u^{-1} Y|^4 & (u\text{-med}) \end{cases}$$

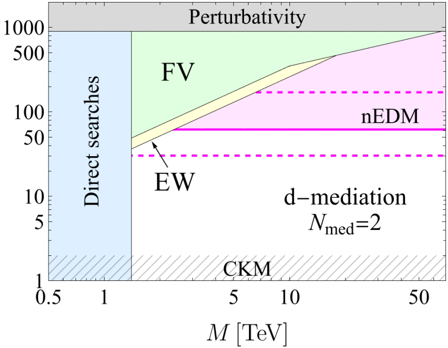
Strongly constraining, $u-$med severely disfavored

Outline

$d-$mediation

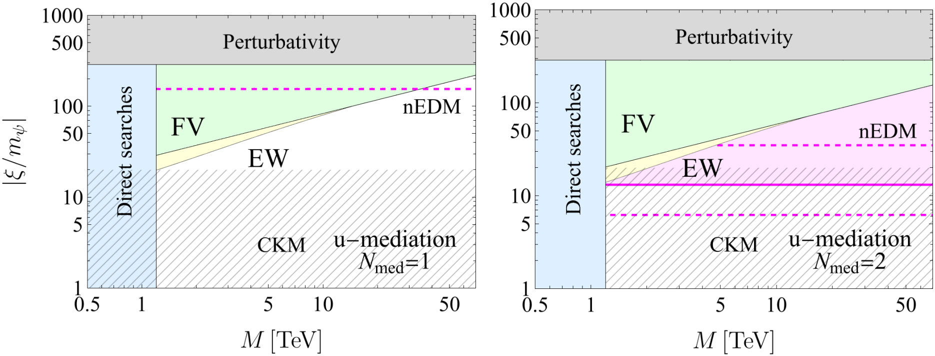
$$\mathcal{L}_\psi \supset i\psi^\dagger \not{D}\psi + i\psi^{c\,\dagger}\not{D}\psi^c - M\psi\psi^c - Y\, q\tilde{H}\psi + \text{hc}, \qquad Y = Y_d\frac{\xi}{m_\psi}$$





$u-$mediation

$$\mathcal{L}_\psi \supset i\psi^\dagger \not{D}\psi + i\psi^{c\,\dagger}\not{D}\psi^c - M\psi\psi^c - Y\,qH\psi + \text{hc}, \quad Y = Y_u\frac{\xi}{m_\psi}$$



Outline

Conclusion

- Spontaneous CP violation can solve the Strong CP problem **without any quality problem**
- Nelson-Barr models:
  - $q-$mediation: strongly disfavored   L. Vecchi (2014)
  - $u-$mediation: disfavored   this work
  - $d-$mediation: fine once $|\xi| \sim m_\psi$ is addressed   this work
- Addressing the coincidence in explicit UV models is possible and provides testable phenomenological signatures ($M \lesssim 10$'s TeV) AV, L. Vecchi (2021)

Thank you for your attention!

Backup slides

Standard Model $\not{C}\not{P}$: semi-analytical approach

$$J \simeq \eta A^2 \lambda_c^6 \propto \det\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right] = \det\left[y_u y_u^t, y_d\left(1 - \frac{\xi\xi^\dagger}{M^2}\right) y_d^t\right]$$

For $|\xi\xi^\dagger| \ll M^2$, $y_d \approx Y_d|_{\eta=0}$:

$$J = A(1-\rho)\frac{m_s}{m_b}\lambda_c^4 \, \mathrm{Im}\left(\frac{\xi_2\xi_3^\dagger}{M^2}\right)\left[1 + \mathcal{O}\left(\frac{|\xi|^2}{M^2}, \lambda_c\right)\right] \approx 10^{-5}\, \mathrm{Im}\left(\frac{\xi_2\xi_3^\dagger}{M^2}\right)$$

$\rightarrow$ need $|\xi|/m_\psi \sim O(1)$ with $\xi_i \sim \xi_j$. Expansion not trustable.

$u-$med, $m_s/m_b \rightarrow m_c/m_t \approx \lambda_c^4$: expansion even less trustable

Standard Model $\not{CP}$: semi-analytical approach

Way out:

$$J \simeq \eta A^2 \lambda_c^6 \propto \det\left[H_u, H_d\right] = \det\left[h_u, h_d - YY^\dagger\right]$$
$$= \dots$$
$$= I_{2,1} + Y^\dagger Y\, I_{1,2} + Y^\dagger H_u^2 Y\, I_{1,0} - Y^\dagger H_u Y\, I_{1,1}$$
$$= F\left(V_{CKM}, \hat{Y}_d, \hat{Y}_u, \frac{\xi}{m_\psi}\right)$$

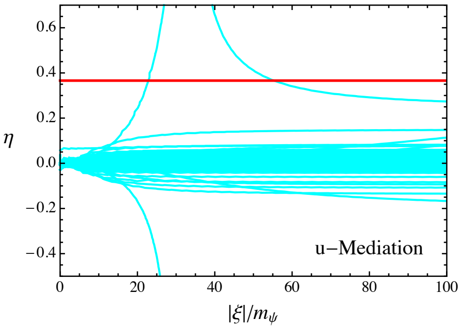
with

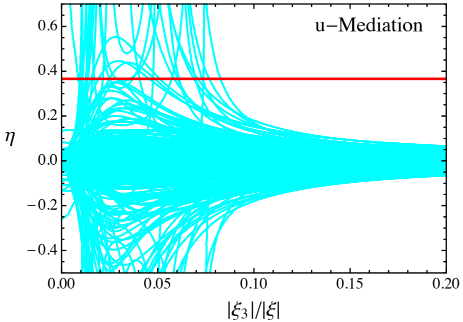
$$I_{2,1} = Y^\dagger[H_u, [H_u, H_d]^2]Y, \qquad I_{1,2} = Y^\dagger H_u[H_u, H_d]H_u Y$$
$$I_{1,0} = Y^\dagger[H_u, H_d]Y, \qquad I_{1,1} = Y^\dagger\{H_u, [H_u, H_d]\}Y$$

$Y = Y_d \frac{\xi}{m_\psi}$

Standard Model $\not{C}P$ for u-mediation

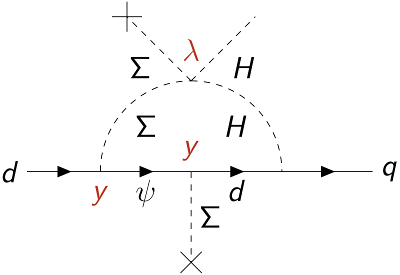
$$Y_u Y_u^\dagger = y_u \left(1 - \frac{\xi\xi^\dagger}{M^2}\right) y_u^t \equiv V_{\rm CKM}^* \, \widehat{Y}_u^2 \, V_{\rm CKM}^t$$





$$\frac{|\xi|}{m_\psi} \gtrsim 20 \quad \& \quad |\xi_3| \sim \lambda_c^2 |\xi_2| \qquad (u\text{−med})$$

Reducible corrections



$$\bar{\theta} \sim \frac{1}{16\pi^2} \text{Im tr} \left[\langle \Sigma \rangle^\dagger \lambda \frac{1}{m_\Sigma^2} y y^t \langle \Sigma \rangle \right] \lesssim 10^{-10} \text{ for } \lambda \ll 1 \text{ or } y \ll 1$$

Also *avoid* $q\tilde{H}\psi^c$ breaking $U(1)_\psi$: $(\psi, \psi^c, \Sigma) \sim (1, 1, -1)$