Universität Zürich$^{\text{UZH}}$

Implications of the $(g-2)_\mu$ anomaly on the flavor structure of New Physics

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Based on work with: Gino Isidori, Julie Pagès
[2111.13724]

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Flavor anomalies

- Muon magnetic moment $(g-2)_\mu$ anomaly $(4.2\,\sigma) \rightarrow$ assumption: due to NP

  Muon $(g-2)$ Collaboration [hep-ex/0602035, 2104.03281]; Aoyama et al. [2006.04822]

  See also lattice results: Borsanyi et al. [2002.12347]; Alexandrou et al. [2206.15084]; Cè et al. [2206.06582]

- B-meson anomalies (LFUV)
  - Neutral current: (3.9σ)

    $$b \rightarrow s\ell\ell, \left[\, R\left(K^{(*)}\right), \, B_s \rightarrow \mu\mu, \, \ldots \right]$$

    LHCb [2103.11769], CMS [1910.12127], CMS [PAS-BPH-21-006], Lancierini et al. [2104.05631], …
  - Charged current: $(\sim 3\sigma)$

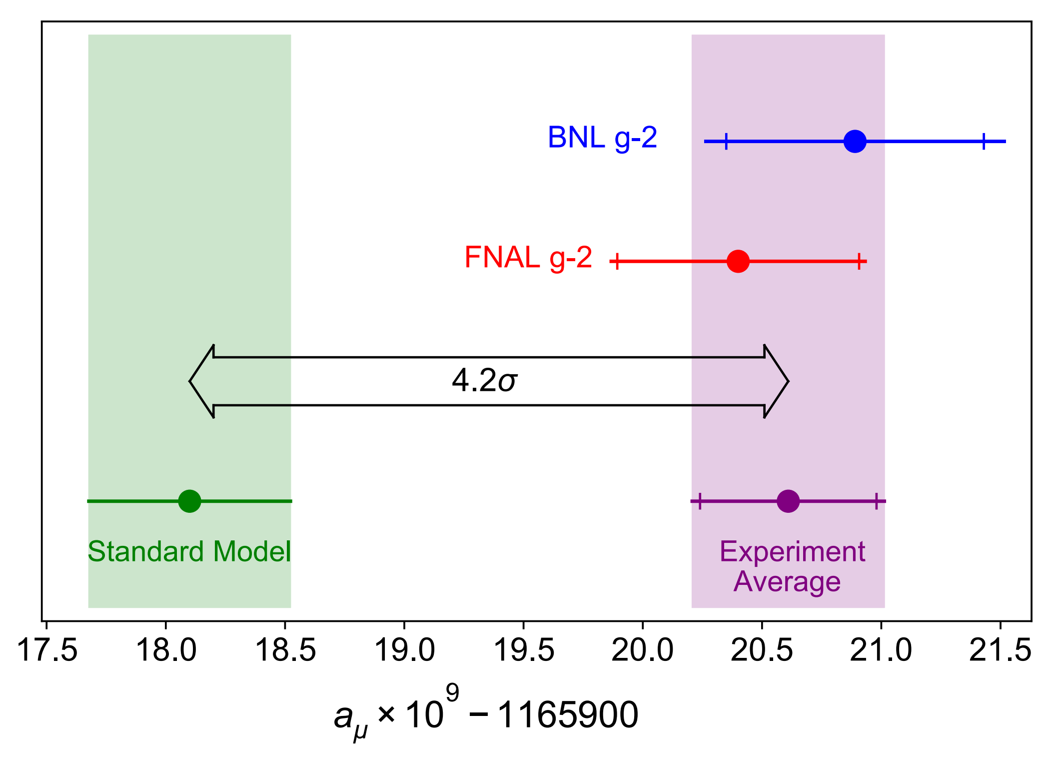
    $$b \rightarrow c\ell\nu, \left[R\left(D^{(*)}\right), \, \ldots\right]$$

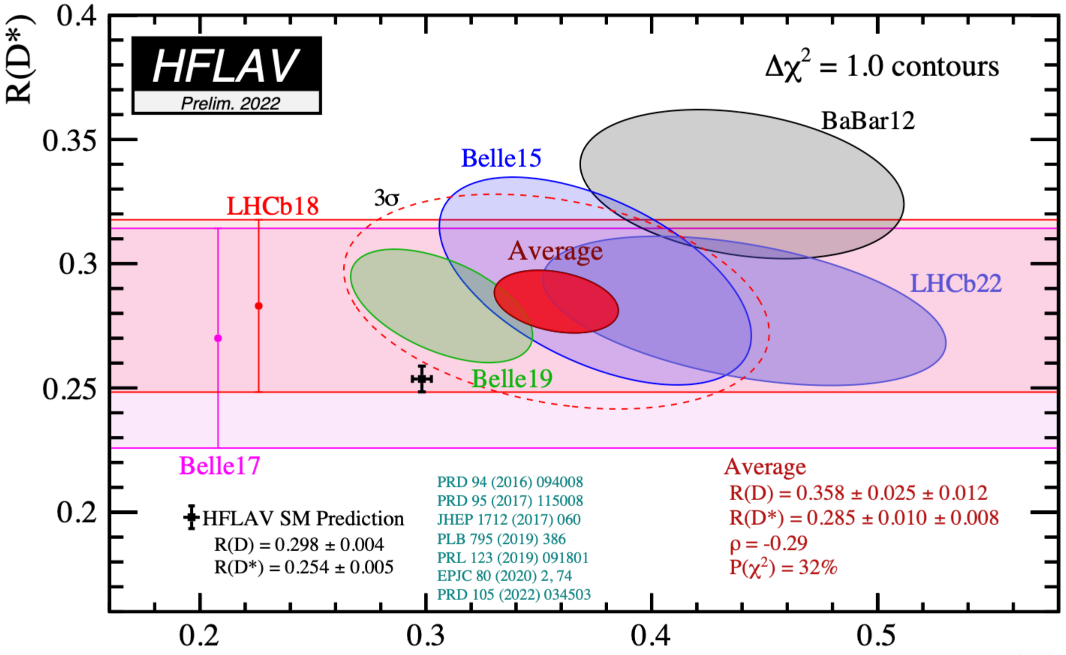
    Belle [1904.08794], BaBar [1303.0571], LHCb [https://indico.cern.ch/event/1187939/], …

- Combined explanation with 4-fermion operators?

  Marzocca, Trifinopoulos [2104.05730], Greljo et al. [2103.13991], Crivellin et al. [1912.04224], …

- How do both types of anomalies fit together?





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$(g-2)_\mu$ and $\mu \to e\gamma$

- We work in the SMEFT with the hypothesis of heavy NP: $\Lambda_{\mathrm{NP}} \gg v$
- Electromagnetic dipole operator: $[Q_{e\gamma}]_{\alpha\beta} = (v/\sqrt{2})\left(\bar{e}^L_\alpha \sigma^{\mu\nu} e^R_\beta\right) F_{\mu\nu}$

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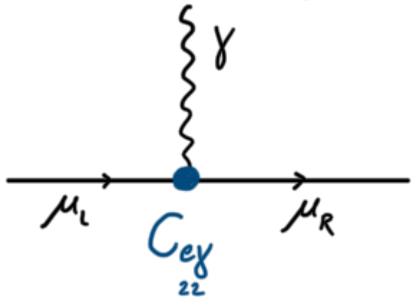

$$\Delta a_\mu = a_\mu^{\rm Exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$$

[hep-ex/0602035, 2104.03281, 2006.04822]

Hints at non-vanishing dipole operator

$[C_{e\gamma}]_{22} \neq 0$

$[C_{e\gamma}]_{22} = 1.0 \times 10^{-5}\,{\rm TeV}^{-2}$

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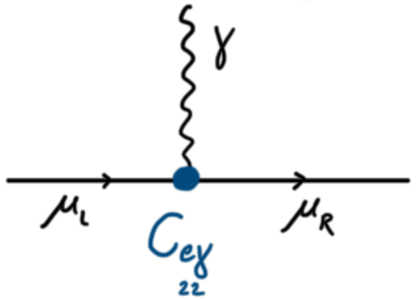

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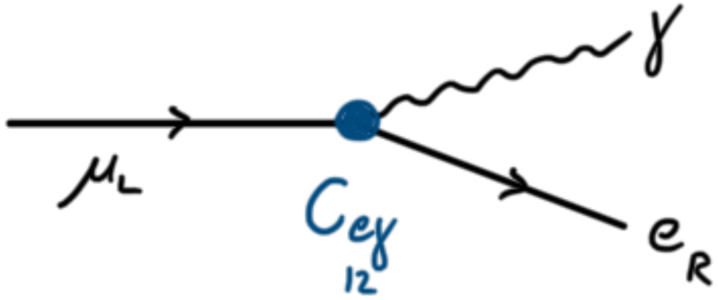

Non-observation of radiative LFV decays $\mathscr{B}(\mu^+ \to e^+\gamma) \leq 4.2 \times 10^{-13}$ (90% C.L.)

MEG [1605.05081]

Implies strongly suppressed off-diagonal couplings $[C_{e\gamma}]_{12(21)} \ll [C_{e\gamma}]_{22}$

$$|\,[C_{e\gamma}]_{12(21)}\,| \leq 2.1 \times 10^{-10}\,\mathrm{TeV}^{-2}$$

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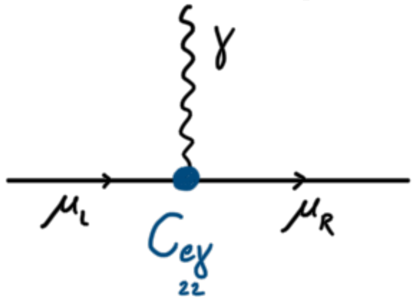

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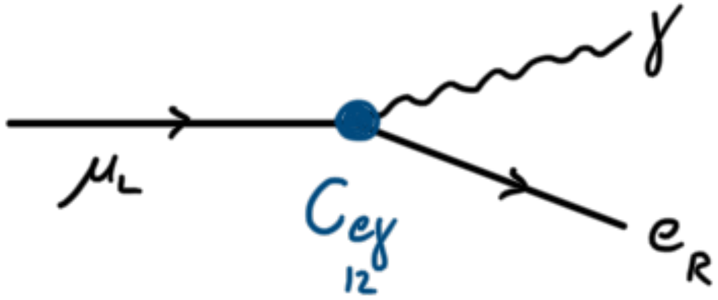

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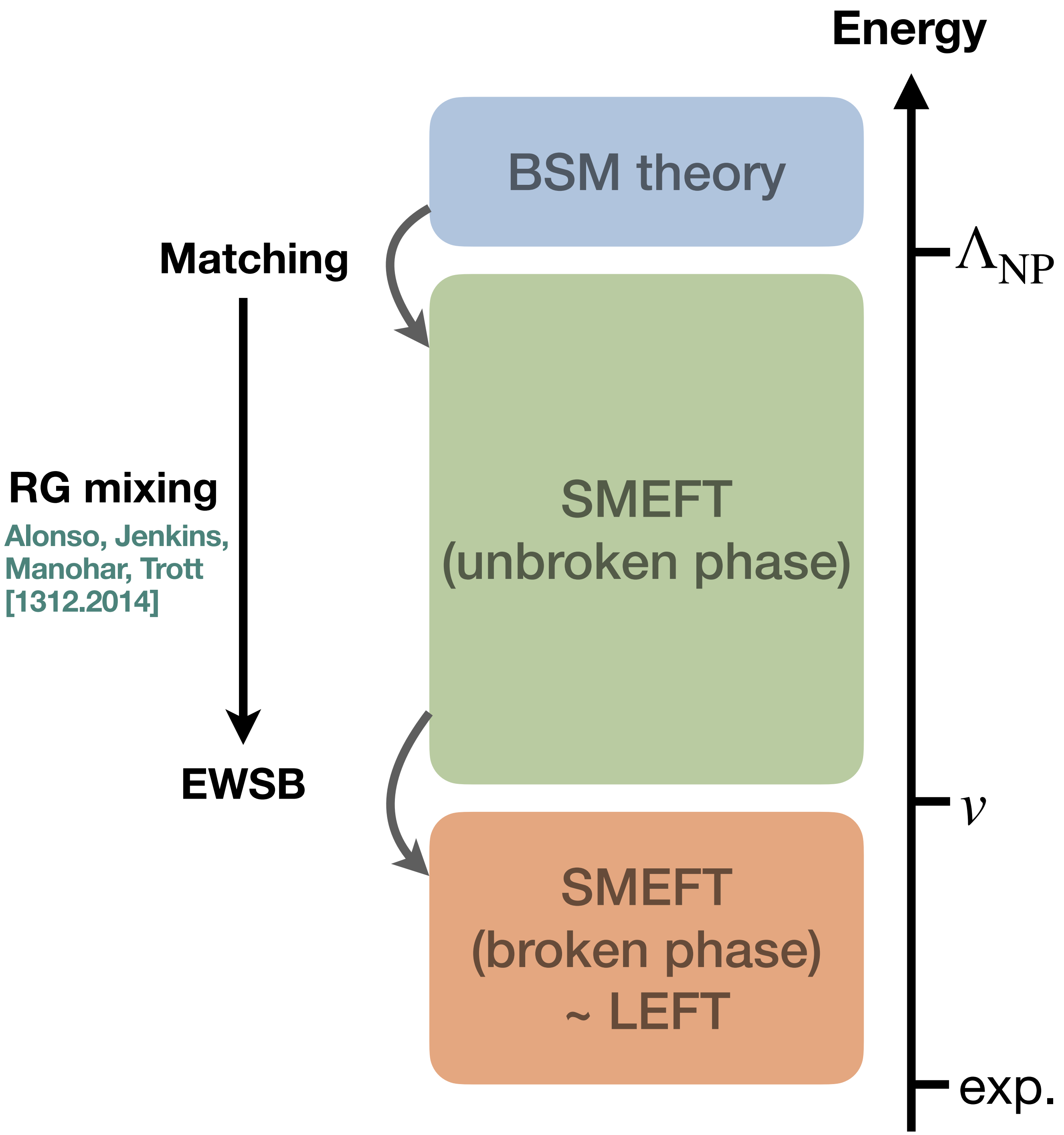
Misalignment: $\epsilon_{12}^{L(R)} \equiv \left| [C_{e\gamma}]_{12(21)} / [C_{e\gamma}]_{22} \right| \leq 2 \times 10^{-5}$

RG evolution



Energy

BSM theory

$\Lambda_{\rm NP}$

Matching

RG mixing

Alonso, Jenkins, Manohar, Trott [1312.2014]

SMEFT (unbroken phase)

EWSB

v

SMEFT (broken phase) ~ LEFT

exp.

RG evolution

SMEFT operators with $(\bar\ell_\alpha \Gamma e_\beta)$ structure:

$[Q^{(1)}_{lequ}]_{\alpha\beta ij} = (\bar\ell^n_\alpha e_\beta)\epsilon_{nm}(\bar q^m_i u_j)$

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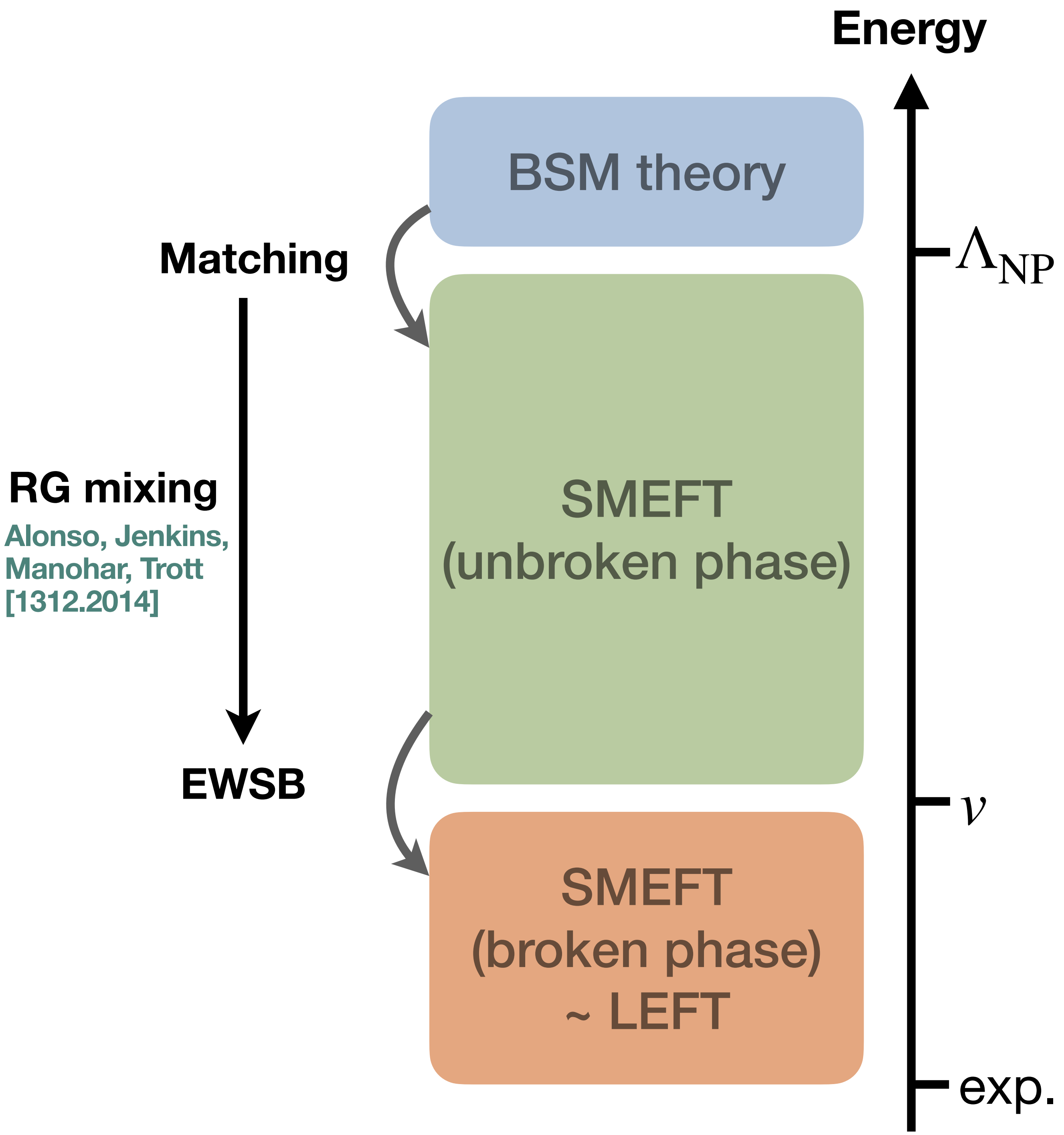
$[Q_{eW}]_{\alpha\beta} = (\bar\ell_\alpha \sigma^{\mu\nu} e_\beta)\tau^I HW^I_{\mu\nu}$

$[Q_{eH}]_{\alpha\beta} = (H^\dagger H)(\bar\ell_\alpha e_\beta H)$



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The broken phase

$$\begin{pmatrix} C_{\underset{rs}{e\gamma}} \\ C_{\underset{rs}{eZ}} \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ -s_\theta & -c_\theta \end{pmatrix} \begin{pmatrix} C_{\underset{rs}{eB}} \\ C_{\underset{rs}{eW}} \end{pmatrix}$$

$$\begin{pmatrix} [\mathcal{Y}_e]_{rs} \\ [\mathcal{Y}_{eh}]_{rs} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} \\ 1 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} [Y_e]_{rs} \\ v^2 C_{\underset{rs}{eH}} \end{pmatrix}$$

$$c_\theta = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{e}{g_1}$$

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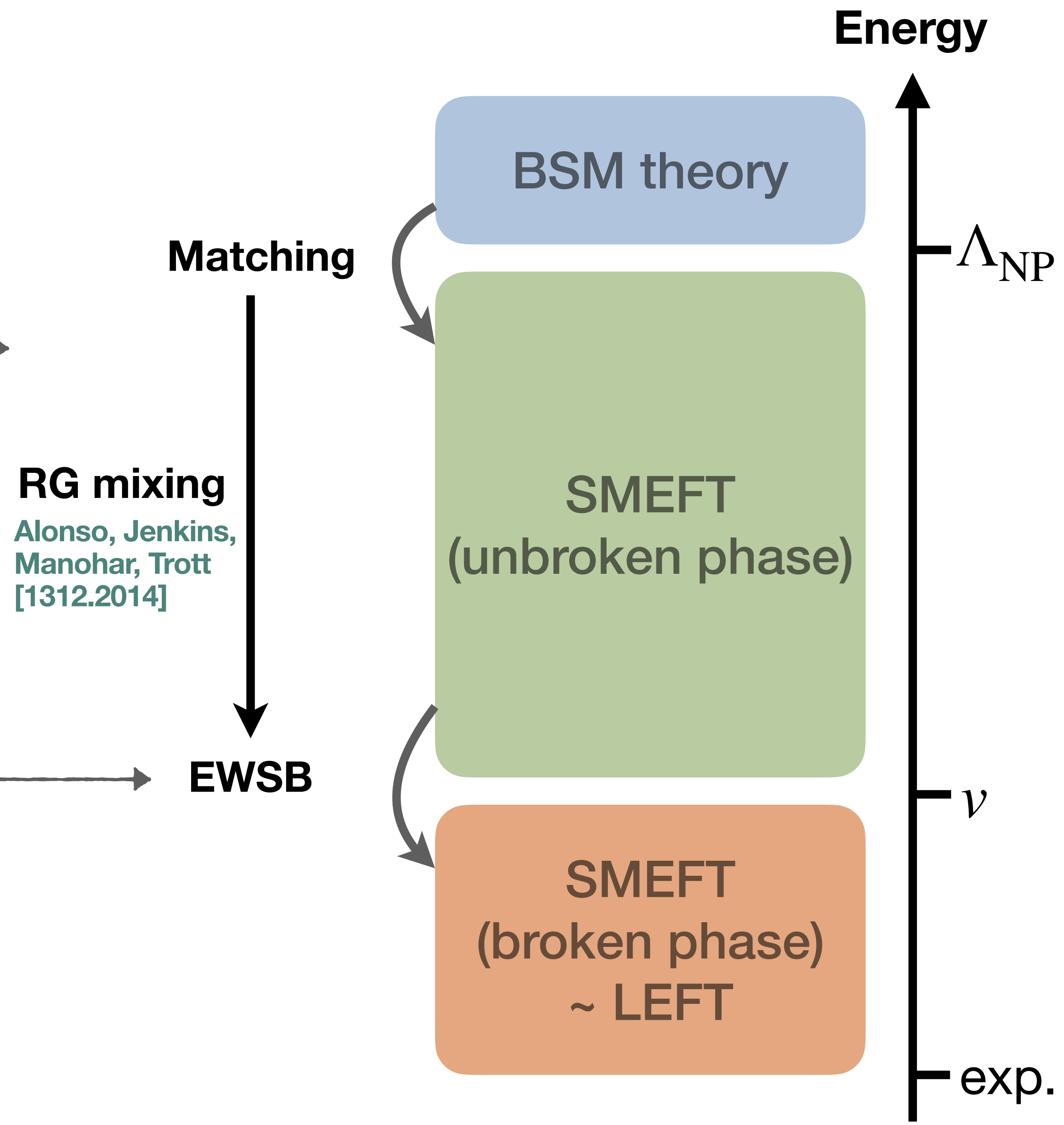

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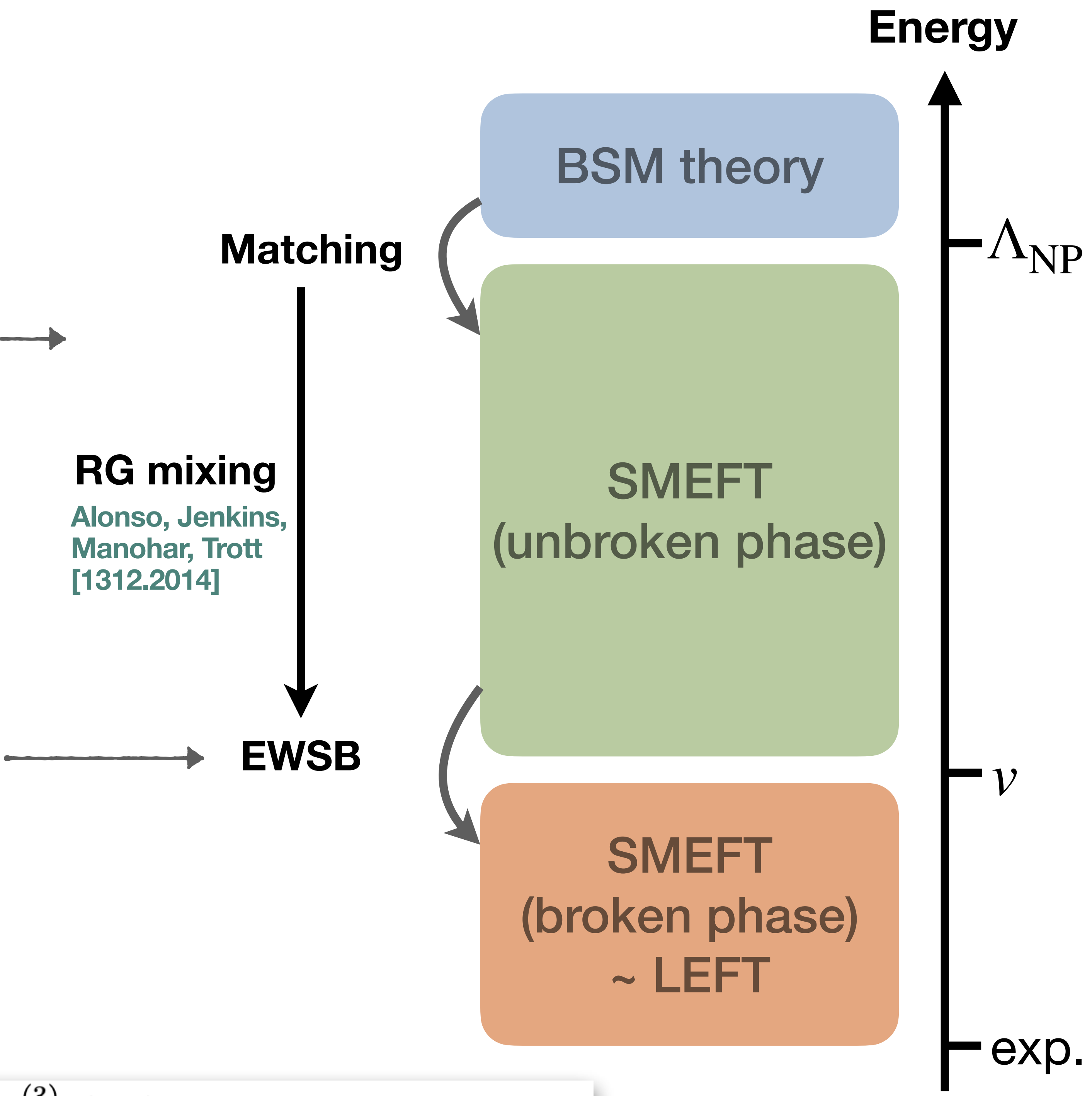

Alonso, Jenkins, Manohar, Trott [1312.2014]

y_t and y_b Yukawa enhanced running:

$$\underset{\alpha\beta}{C_{e\gamma}}(\mu_L) = \left[1 - 3L\left(y_t^2 + y_b^2\right)\right] \underset{\alpha\beta}{C_{e\gamma}}(\mu_H) - \left[16 L y_t e\right] \underset{\alpha\beta 33}{C^{(3)}_{lequ}}(\mu_H)$$

$$[\mathcal{Y}_e]_{\alpha\beta}(\mu_L) = [Y_e]_{\alpha\beta}(\mu_H) - \frac{v^2}{2}\underset{\alpha\beta}{C_{eH}}(\mu_H) + 6v^2 L \left[y_t^3 \underset{\alpha\beta 33}{C^{(1)}_{lequ}} - y_b^3 \underset{\alpha\beta 33}{C_{ledq}} + \frac{3}{4}(y_t^2 + y_b^2)\underset{\alpha\beta}{C_{eH}} \right]_{\mu_H}$$

Mass basis

- Diagonalize lepton Yukawa $[\mathscr{Y}_e]$ with rotation: $\theta^{\mathscr{Y}}_{L(R)} = -[\mathscr{Y}_e]_{12(21)}/[\mathscr{Y}_e]_{22}\Big|_{\mu_L}$
- Dipole in the mass basis:

$$C'_{\underset{12(21)}{e\gamma}}(\mu_L) = C_{\underset{12(21)}{e\gamma}}(\mu_L) + \theta^{\mathscr{Y}}_{L(R)} C_{\underset{22}{e\gamma}}(\mu_L), \qquad C'_{\underset{22}{e\gamma}}(\mu_L) = C_{\underset{22}{e\gamma}}(\mu_L)$$

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- LFV dipole in the mass basis:

$$\boxed{\begin{aligned} C'_{\underset{12}{e\gamma}}(\mu_L) &= (\theta_L^{e\gamma} - \theta_L^{Y}) C'_{\underset{22}{e\gamma}}(\mu_L) + (\theta_L^{e\gamma} - \theta_L^{u_3})(16Ley_t) C^{(3)}_{\underset{2233}{lequ}}(\mu_H) \\ &\quad + \left[(\theta_L^{Y} - \theta_L^{u_1}) C^{(1)}_{\underset{2233}{lequ}}(\mu_H) + (\theta_L^{d} - \theta_L^{Y}) \frac{y_b^3}{y_t^3} C_{\underset{2233}{ledq}}(\mu_H) \right] 6Lv^2 y_t^3 \frac{1}{y_\mu} C'_{\underset{22}{e\gamma}}(\mu_L) \\ &\quad + (\theta_L^{eH} - \theta_L^{Y}) \frac{1 - 9L(y_t^2 + y_b^2)}{2} C_{\underset{22}{eH}}(\mu_H) v^2 \frac{1}{y_\mu} C'_{\underset{22}{e\gamma}}(\mu_L)\ . \end{aligned}}$$

- Flavor phases / angles:

$$\theta_L^{Y} = \frac{[Y_e]_{12}}{[Y_e]_{22}}\Bigg|_{\mu_H}, \quad \theta_L^{e\gamma} = \frac{C_{\underset{12}{e\gamma}}}{C_{\underset{22}{e\gamma}}}\Bigg|_{\mu_H}, \quad \theta_L^{eH} = \frac{C_{\underset{12}{eH}}}{C_{\underset{22}{eH}}}\Bigg|_{\mu_H}, \quad \theta_L^{u_i} = \frac{C^{(i)}_{\underset{1233}{lequ}}}{C^{(i)}_{\underset{2233}{lequ}}}\Bigg|_{\mu_H}, \quad \theta_L^{d} = \frac{C_{\underset{1233}{ledq}}}{C_{\underset{2233}{ledq}}}\Bigg|_{\mu_H}$$

- $C'_{\underset{22}{e\gamma}} \neq 0$ due to Δa_μ anomaly
- C_{ledq} has negligible coefficient $(y_b/y_t)^3$
- LFV Higgs couplings $(\theta_L^{eH} - \theta_L^{Y})$ is tightly constrained

Blankenburg, Ellis, Isidori [1202.5704]
Harnik, Kopp, Zupan [1209.1397]

➡ **We have $C'_{\underset{12}{e\gamma}}$ sufficiently small only if all flavor phases θ_L^X are aligned!**

Alignment conditions & mechanism

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$$\boxed{\epsilon^L_{12} = (\theta_L^{e\gamma} - \theta_L^{Y}) + (\theta_L^{u_3} - \theta_L^{e\gamma})\Delta_3 + (\theta_L^{u_1} - \theta_L^{Y})\Delta_1}$$

- $C'_{\underset{12}{e\gamma}}(\mu_L) \neq 0$ is generated if $\theta_L^X \neq 0$ for any X even if $C_{\underset{12}{e\gamma}}(\mu_H) = 0$

- All phases θ_L^X have to be aligned to get $C'_{\underset{12}{e\gamma}}(\mu_L) \sim 0$ and thus not violating the $\mu \to e\gamma$ bound

$$\epsilon_{12}^{L(R)} \equiv \left| C'_{\underset{12}{e\gamma}} \Big/ C'_{\underset{22}{e\gamma}} \right|_{\mu_L} \leq 2 \times 10^{-5}$$

$$\theta_L^i = \left(C_{\underset{12}{i}} \Big/ C_{\underset{22}{i}} \right)_{\mu_H}$$

$$\Delta_3 \equiv \frac{-16 L e y_t C^{(3)}_{\underset{2233}{lequ}}(\mu_H)}{C'_{\underset{22}{e\gamma}}(\mu_L)} = \mathcal{O}(1)$$

$$\Delta_1 \simeq 0.4 \times 10^{-2} \left[\frac{C^{(1)}_{\underset{2233}{lequ}}}{4 C^{(3)}_{\underset{2233}{lequ}}} \right]_{\mu_H} \times \Delta_3$$

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Case	Dynamical hypothesis	Alignment condition
$\Delta_3 = \mathcal{O}(1)$	**I)** Dipole operator radiatively generated with $C^{(3)}_{lequ}$	$\theta_L^{e\gamma} = \theta_L^{u_3}$
	II) $C^{(1)}_{lequ}$ and $C^{(3)}_{lequ}$ from same UV dynamics	$\theta_L^{u_1} = \theta_L^{u_3}$
	III) y_μ radiatively generated with $C^{(1)}_{lequ}$	$\theta_L^{Y} = \theta_L^{u_1}$

Flavor symmetries

Can the alignment be achieved by flavor symmetries?

- Individual lepton number conservation $U(1)^3$ $\Rightarrow$ all flavor phases θ_L^X vanish

$$U(1)^3 = U(1)_{L_e} \otimes U(1)_{L_\mu} \otimes U(1)_{L_\tau} = U(1)_L \otimes U(1)_{L_e - L_\mu} \otimes U(1)_{L_\tau - L_\mu}$$

see e.g.: Greljo, Stangl, Thomsen [2103.13991]; Arcadi, Calibbi, Fedele, Mescia [2104.03228]; Cen, Cheng, He, Sun [2104.05006]; …

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- Minimally broken $U(2)$ symmetry acting on the 2 light generations

$$U(2)^2 = U(2)_L \otimes U(2)_E$$

Barbieri et al. [1105.2296, 1203.4218]
Blankenburg et al. [1204.0688]

light generations: doublets,
e.g. $L = \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix} \sim 2_L$

third generation: singlet,
e.g. $\ell_3 \sim 1$

  - Lepton Yukawa: $Y_e = y_\tau \begin{pmatrix} \Delta_e & x_\tau V_\ell \\ 0 & 1 \end{pmatrix}$ $U(2)_L$ $U(2)_E$
  - Spurions $V_\ell \sim (2,1)$, $\Delta_e \sim (2,\bar{2})$ parametrized by:

$$V_\ell = e^{i\bar{\phi}_\ell} \begin{pmatrix} 0 \\ \epsilon_\ell \end{pmatrix}, \quad \Delta_e = O_e^{\mathrm{T}} \begin{pmatrix} \delta_e' & 0 \\ 0 & \delta_e \end{pmatrix}, \quad O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}$$

Bordone, Isidori, Trifinopoulos [1702.07238]

  - Hierarchy from Yukawas: $1 \gg \epsilon_i \gg \delta_i \gg \delta_i' > 0$

Fuentes-Martín, Isidori, Pagès, Yamamoto [1909.02519]

$$\epsilon_i = \mathcal{O}\left(10^{-1}\right), \quad \delta_i = \mathcal{O}\left(10^{-2}\right), \quad \delta_i' = \mathcal{O}\left(10^{-3}\right), \quad s_e = \mathcal{O}\left(\sqrt{m_e/m_\mu}\right) \geq 10^{-2}$$

$e - \mu$ alignment in $U(2)^5$

- Relevant leptonic structure: $(\bar{\ell}_\alpha \, \Gamma \, e_\beta)$
  - Leading breaking $(\bar{L}\Delta_e E)$ is universal ⇒ no misalignment
  - Sub-leading breaking $(\bar{L}V_\ell V_\ell^\dagger \Delta_e E)$ can generate LFV
- Breaking term $X^n_{\alpha\beta} = \kappa_n(\Delta_e)_{\alpha\beta} + \eta_n(V_\ell)_\alpha(V_\ell^\dagger)_\gamma(\Delta_e)_{\gamma\beta}$

  diagonalized for each operator Q_n with rotation

$$\theta^n_L = \frac{s_e}{c_e}\left(1 - a_n|\epsilon_\ell|^2\right), \quad \theta^n_R = -\frac{s_e}{c_e}\frac{\delta'_e}{\delta_e}, \quad \text{where } a_n = \eta_n / \kappa_n = \mathcal{O}(1)$$

  - Automatic alignment in the $U(2)_E$ space
  - Alignment constraint in the $U(2)_L$ space

$$\theta^{e\gamma}_L - \theta^Y_L = \frac{s_e}{c_e}|\epsilon_\ell|^2(a_Y - a_{e\gamma}) \leq \epsilon^L_{12} \sim 10^{-5}$$

$\epsilon_\ell \sim \mathcal{O}(10^{-1})$

➡ Alignment constraint in $e - \mu$ sector $\left|s_e(a_Y - a_{e\gamma})\right| \lesssim 10^{-3}$

Natural expectation:
$s_e = \mathcal{O}\left(\sqrt{m_e/m_\mu}\right)$

An explicit NP model

- $S_1 \sim (\bar{3},1)_{1/3}$ scalar leptoquark + heavy Higgs $\Phi \sim (1,2)_{1/2}$ model:

$$\begin{aligned}\mathcal{L}_{\mathrm{S}_1} \;=\;& \mathcal{L}_{\mathrm{SM}} + (D_\mu S_1)^\dagger (D^\mu S_1) - M_{S_1}^2 S_1^\dagger S_1 - \left[\lambda^L_{i\alpha}(\bar{q}^c_i \epsilon \ell_\alpha) S_1 + \lambda^R_{i\alpha}(\bar{u}^c_i e_\alpha) S_1 + \mathrm{h.c.}\right] \\ & + (D_\mu \Phi)^\dagger (D^\mu \Phi) - M_\Phi^2 \Phi^\dagger \Phi - \left[\lambda^e_{\alpha\beta}(\bar{\ell}_\alpha e_\beta)\Phi + \lambda^u_{ij}(\bar{q}_i u_j)\tilde{\Phi} + \mathrm{h.c.}\right] \,,\end{aligned}$$

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Matching:

$$C^{(1)}_{\substack{lequ \\ \alpha\beta ij}} = \frac{\lambda^{L\,*}_{i\alpha}\lambda^R_{j\beta}}{2M^2_{S_1}} - \frac{\lambda^e_{\alpha\beta}\lambda^u_{ij}}{M^2_\Phi} , \qquad C^{(3)}_{\substack{lequ \\ \alpha\beta ij}} = -\frac{\lambda^{L\,*}_{i\alpha}\lambda^R_{j\beta}}{8M^2_{S_1}}$$

$$\begin{aligned}\mathcal{C}_{\substack{e\gamma \\ \alpha\beta}}(\mu_H) = \frac{e}{16\pi^2 M^2_{S_1}} \Bigg\{ &-\frac{1}{8}\left[(\lambda^L)^\dagger \lambda^L Y_e\right]_{\alpha\beta} - \frac{1}{8}\left[Y_e (\lambda^R)^\dagger \lambda^R\right]_{\alpha\beta} \\ &+ \left(\frac{7}{4} + \log\frac{\mu_H^2}{M^2_{S_1}}\right)\left[(\lambda^L)^\dagger Y_u^* \lambda^R\right]_{\alpha\beta} \Bigg\} ,\end{aligned}$$

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$$C_{\substack{lequ \\ \alpha\beta ij}}^{(1)} = \frac{\lambda_{i\alpha}^{L\,*} \lambda_{j\beta}^R}{2 M_{S_1}^2} - \frac{\lambda_{\alpha\beta}^e \lambda_{ij}^u}{M_\Phi^2} \,, \qquad C_{\substack{lequ \\ \alpha\beta ij}}^{(3)} = -\frac{\lambda_{i\alpha}^{L\,*} \lambda_{j\beta}^R}{8 M_{S_1}^2}$$

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Flavor phases:

$$\begin{aligned}\theta_L^{u_1} &= \frac{\lambda_{31}^{L\,*} \lambda_{32}^R + 2 \lambda_{12}^e \lambda_{33}^u M_{S_1}^2 / M_\Phi^2}{\lambda_{32}^{L\,*} \lambda_{32}^R + 2 \lambda_{22}^e \lambda_{33}^u M_{S_1}^2 / M_\Phi^2} \,, \\ \theta_L^{u_3} &= \frac{\lambda_{31}^{L\,*}}{\lambda_{32}^{L\,*}} \,, \\ \theta_L^{e\gamma} &= \frac{(Y_e)_{1\alpha} \lambda_{i\alpha}^{R\,*} \lambda_{i2}^R + \lambda_{i1}^{L\,*} \lambda_{i\alpha}^L (Y_e)_{\alpha 2} - 14\, y_t \lambda_{31}^{L\,*} \lambda_{32}^R}{(Y_e)_{2\alpha} \lambda_{i\alpha}^{R\,*} \lambda_{i2}^R + \lambda_{i2}^{L\,*} \lambda_{i\alpha}^L (Y_e)_{\alpha 2} - 14\, y_t \lambda_{32}^{L\,*} \lambda_{32}^R}\end{aligned}$$

θ_L^Y is a free parameter *(Yukawa is a marginal operator)*

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Flavor phases:

$$\begin{aligned}\theta_L^{u_1} &= \frac{\lambda^{L\,*}_{31}\lambda^R_{32} + 2\lambda^e_{12}\lambda^u_{33}M^2_{S_1}/M^2_\Phi}{\lambda^{L\,*}_{32}\lambda^R_{32} + 2\lambda^e_{22}\lambda^u_{33}M^2_{S_1}/M^2_\Phi} , \\ \theta_L^{u_3} &= \frac{\lambda^{L\,*}_{31}}{\lambda^{L\,*}_{32}} , \\ \theta_L^{e\gamma} &= \frac{(Y_e)_{1\alpha}\lambda^{R\,*}_{i\alpha}\lambda^R_{i2} + \lambda^{L\,*}_{i1}\lambda^L_{i\alpha}(Y_e)_{\alpha 2} - 14\,y_t\lambda^{L\,*}_{31}\lambda^R_{32}}{(Y_e)_{2\alpha}\lambda^{R\,*}_{i\alpha}\lambda^R_{i2} + \lambda^{L\,*}_{i2}\lambda^L_{i\alpha}(Y_e)_{\alpha 2} - 14\,y_t\lambda^{L\,*}_{32}\lambda^R_{32}}\end{aligned}$$

θ_L^Y is a free parameter *(Yukawa is a marginal operator)*

- $\theta_L^{e\gamma} \simeq \theta_L^{u_1} \simeq \theta_L^{u_3}$ obtained for:
  $M_\Phi^2 \gg M_{S_1}^2, \quad \lambda^L_{i\alpha} \ll \lambda^L_{3\alpha}, \quad (\lambda^R)^2 \sim 0$

- Hard to conceive dynamical alignment of θ_L^Y

➡ $U(2)^2$ hypothesis:

$$\theta_L^{e\gamma} = \frac{\lambda^{L\,*}_{31}}{\lambda^{L\,*}_{32}} = \frac{(V_\ell)_1}{(V_\ell)_2} \longrightarrow 0 , \qquad \theta_L^Y = \frac{(\Delta_e)_{12}}{(\Delta_e)_{22}} \longrightarrow s_e$$

➡ Unnatural alignment $|s_e| < \mathcal{O}(10^{-5})$

Conclusions

- In combination the $(g-2)_\mu$ anomaly and the non-observation of $\mu \to e\gamma$ imply a tight alignment on the lepton dipole operator $Q_{e\gamma}$ in flavor space
- By RG mixing several 4-fermion operators have to satisfy similar alignment constraints
- Alignment mechanisms:
  - Dynamical alignment $\rightarrow$ alignment of Yukawa θ_L^Y especially difficult
  - Flavor symmetries $\rightarrow$ $U(1)^3$ sets all flavor phases θ_L^X to zero

➡ If $(g-2)_\mu$ anomaly is a sign of NP the lepton sector must feature enhanced symmetries

➡ The quark and lepton sectors must behave quite differently beyond the SM

Thank you for your attention!