$B^0 - \overline{B}^0$ entanglement for an ideal experiment on the direct CP violation ϕ_3/γ phase

Francisco J. Botella (with J. Bernabeu, M. Nebot, and A. Segarra,
arXiv::2204.05567 [hep-ph], PRD 106, 054026 (2022))

IFIC-U.Valencia & CSIC

November 2022

Introduction on phases I

- The quark charged current couplings are parametrized by the Cabibbo-Kobayashi-Maskawa (CKM) matrix V

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix}_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^{\mu\dagger}$$

- We can eliminate 5 phases out of 9, $\left(q_L \to e^{i\phi_q} q_L\right)$:

$$V = \begin{pmatrix} |V_{ud}| & |V_{us}|\, e^{i\chi'} & |V_{ub}|\, e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|\, e^{-i\beta} & -|V_{ts}|\, e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

- γ **is the unique non small phase that can appear at tree level**

$$\gamma = \phi_3 = \arg\left(-V_{ud} V_{ub}^* V_{cd}^* V_{cb}\right) \sim \mathcal{O}(1)$$ (**tree level** $B^{\pm,0}$decays)

Introduction on phases II

- Measured for example in

$$A^- \equiv A\left(B^- \to \underset{\hookrightarrow f}{D} + K^-\right) =$$

$$= \quad A\left(D^0 \to f\right) A\left(B^- \to D^0 K^-\right) + A\left(\overline{D}^0 \to f\right) A\left(B^- \to \overline{D}^0 K^-\right)$$

that for $f = K^+ K^-$ gives

$$\left|A^-\right|^2 - \left|A^+\right|^2 \propto \mathrm{Im}\left(ab^*\right) \underbrace{\mathrm{Im}\left(V_{cb} V_{cs}^* V_{ub}^* V_{us}\right)}_{\propto \sin(\gamma+\chi') \sim \sin(\gamma)}$$

needed of strong phases in $\mathrm{Im}\left(ab^*\right)$. Gronau, London, Wyler (GLW), Atwood, Dunietz, Soni (ADS), Giri, Grossman, Soffer, Zupan (GGSZ) and many more.

Using entanglement for gamma I

- The use of the EPR correlation to study CP violation was proposed by Wolfenstein, Gavela et al, Falk and Petrov and Alvarez and Bernabeu among others for several decay channels in the B factories. The method **for γ consists in the observation of the coherent double decay of $\Upsilon(4s)(1^{--})$ to the CP eigenstates $(\mathbf{f};\mathbf{g})$**, with $f = J/\psi K_S(0^{-+}), J/\psi K_L(0^{--})$ (in short S or L) and $g = h^+h^-$, h^0h^0 $(0^{++}$, and $h = \pi, \rho_L)$ .In such a way that

$$\Upsilon(4s) \rightarrow (J/\psi K_S)_B (hh)_B \quad ; \quad \text{is CP allowed}$$
$$\Upsilon(4s) \rightarrow (J/\psi K_L)_B (hh)_B \quad ; \quad \text{is CP forbidden}$$

Using entanglement for gamma II

- The necessary interference between amplitudes is automatic from the two terms of the entangled $B^0 - \overline{B}^0$ system (from $\Upsilon(4s)$):

$$\left|\Psi_0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|B_d^0\right\rangle\left|\overline{B}_d^0\right\rangle - \left|\overline{B}_d^0\right\rangle\left|B_d^0\right\rangle\right)$$
$$= \frac{1}{2\sqrt{2}pq}\left(\left|B_L\right\rangle\left|B_H\right\rangle - \left|B_H\right\rangle\left|B_L\right\rangle\right)$$

with $B_H = pB^0 + q\overline{B}^0$ $B_L = pB^0 - q\overline{B}^0$ and $A^f_{H,L} = \langle f|\mathcal{T}\left|B_{H,L}\right\rangle$ where $A_f = \langle f|\mathcal{T}\left|B^0\right\rangle$; $\overline{A}_f = \langle f|\mathcal{T}\left|\overline{B}^0\right\rangle$, $\left(\lambda_f = q\overline{A}_f/pA_f\right)$ as can be seen in

$$\langle f, t_0; g, t_0 + t|\mathcal{T}\left|\Psi_0\right\rangle = \frac{e^{-i(\mu_L+\mu_H)t_0}}{2\sqrt{2}pq}\left(e^{-i\mu_H t}A^f_L A^g_H - e^{-i\mu_L t}A^f_H A^g_L\right)$$

Using entanglement for gamma III

- The double decay rate to the state f at t_0 and to the state g at $t_0 + t$ integrated for t_0 is given by
  $\Upsilon(4s) \to \left(B^0\overline{B}^0 - \overline{B}^0 B^0\right) \to (f, t_0; g, t_0 + t)$

$$I(f, g; t) = \frac{e^{-\Gamma|t|}}{16\Gamma\,|pq|^2} \left| \begin{array}{c} \cos\left(\frac{\Delta M t}{2}\right)\left(A_L^f A_H^g - A_H^f A_L^g\right) \\ -i\sin\left(\frac{\Delta M t}{2}\right)\left(A_L^f A_H^g + A_H^f A_L^g\right) \end{array} \right|^2$$

$$\widehat{I}(f, g; t) \equiv \frac{\Gamma}{\langle\Gamma_f\rangle\langle\Gamma_g\rangle} I(f, g; t) =$$

$$= \quad e^{-\Gamma|t|}\left[I_d^{fg}\cos^2\left(\frac{\Delta M t}{2}\right) + I_m^{fg}\sin^2\left(\frac{\Delta M t}{2}\right) + I_{od}^{fg}\sin(\Delta M t)\right]$$

Important consistency properties I

- The relation among reversed order in time (f,g) and (g,f) is:

$$I_d^{fg} = I_d^{gf}\,;\, I_m^{fg} = I_m^{gf}\,;\, I_{od}^{fg} = -I_{od}^{gf}$$

- The relation among the CP allowed (S,g) and forbidden (L,g)

$$\boxed{I_d^{Sg} + I_d^{Lg} = 1\ ;\, I_m^{Sg} + I_m^{Lg} = 1\ ;\, I_{od}^{Sg} + I_{od}^{Lg} = 0}$$

One can measure all three observables $\mathbf{I}_{d,m,od}^{\substack{L\\S}g}$ for all the (f,g) and (g,f) channels $f = J/\psi K_S, J/\psi K_L$ and $g = (\rho_L^+\rho_L^-)\,,(\rho_L^0\rho_L^0)\,,(\pi^+\pi^-)\,,(\pi^0\pi^0)$. **Just measuring ratios** $\mathbf{I}_{m,od}^{\substack{L\\S}g}/\mathbf{I}_d^{\substack{L\\S}g}$!!!!

- They are $\mathbf{I}^{L\,g}_{S\,d,m,od}$ for each g channel in both time ordering . Our parameters are

$$\boxed{\frac{\overline{A}_g}{A_g} = \boldsymbol{\rho}_g\ \mathbf{e}^{-2i\phi_g}}\ ;\ \left(\frac{\overline{A}_{(\pi\pi)_{I=2}}}{A_{(\pi\pi)_{I=2}}} = e^{-2i\gamma}\right)\ ;\ \left(\frac{q}{p}\right)_B = e^{-2i\phi_M}$$

$$\boxed{\frac{\overline{A}_S}{A_S} = -\ \frac{\overline{A}_L}{A_L} = \mathbf{-1}}\ ;\ \lambda_L = -\lambda_S = -e^{-2i\phi_M}\ ;\ \phi_M = \beta$$

- For the observable we get

$$I^{L\,g}_{S\,d} = \frac{\left|\left(\frac{\overline{A}_g}{A_{g-}} - \frac{\overline{A}_f}{A_{f-}}\right)\right|^2}{\left(1+|\lambda_f|^2\right)\left(1+|\lambda_g|^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g \cos\left(2\phi_g\right)}{\left(1+\rho_g^2\right)}\right]$$

The observables II

It is present at $t = 0$. It is sensible to the decay phases in g and f. If there were not penguin pollution $\left(\rho_g = 1\right)$ in the decays all ϕ_g would be $\phi_g = \gamma$ and

$$I_d^{Lg} = \sin^2\gamma \text{ and } I_d^{Sg} = \cos^2\gamma$$

For the other observables we get

$$I_m^{\substack{L\\S}g} = \frac{\left|(1-\lambda_g\lambda_f)\right|^2}{\left(1+|\lambda_f|^2\right)\left(1+|\lambda_g|^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g \cos\left(4\phi_M + 2\phi_g\right)}{\left(1+\rho_g^2\right)}\right]$$

$$I_{od}^{\substack{L\\S}g} = \frac{2\,\mathrm{Im}\left[\left(\lambda_g^* - \lambda_f^*\right)(1-\lambda_g\lambda_f)\right]}{\left(1+|\lambda_f|^2\right)\left(1+|\lambda_g|^2\right)} = (\mp)\,\frac{\left(1-\rho_g^2\right)}{\left(1+\rho_g^2\right)}\sin\left(2\phi_M\right)$$

The observables III

- The quantities to be extracted from $I^{L_S g}_{d,m,od}$, for each channel g, are ϕ_g, ρ_g and ϕ_M. The one **we are mainly interested** is $\boldsymbol{\phi}_g$. The way of getting γ from ϕ_g is by the Gronau and London isospin analysis".

Isospin analysis I

- In general we will have for each $g = h^+h^-$ or h^0h^0 channel a departure from the universal γ value

$$\epsilon_g = \gamma - \phi_g$$

- The charged decay amplitudes $A_{+0} = A\left(B^+ \to h^+h^0\right)$ and $\overline{A}_{+0} = A\left(B^- \to h^-h^0\right)$ have a final $\left(h^\pm h^0\right)$ isospin 2 state and, therefore, only the $\Delta I = 3/2$ tree-level amplitude contributes with the weak phase γ: $\overline{A}_{+0}/A_{+0} = e^{-2i\gamma}$. Then we have

$$a_g = \frac{A_g}{A_{+0}}\ ;\ \bar{a}_g = \frac{\overline{A}_g}{\overline{A}_{+0}} \ \Rightarrow \frac{\bar{a}_g}{a_g} = \rho_g e^{-2i\phi_g} e^{i2\gamma} = \rho_g e^{2i\epsilon_g}$$

Isospin analysis II

- The isospin triangular relations with these complex ratios are

$$\frac{1}{\sqrt{2}} a_{+-} = 1 - a_{00} \; ; \; \frac{1}{\sqrt{2}} \bar{a}_{+-} = 1 - \bar{a}_{00}$$

therefore we can get a_g and $\bar{a}_g$ from the branching ratios of the processes $B^{\pm} \to h^{\pm} h^0$;$B^0, \overline{B}^0 \to h^+ h^-, h^0 h^0$ fixing ϵ_g and ρ_g.

- The summary of our isospin analysis with the present PDG data is

g	ρ_g	ϵ_g
$\rho_L^+ \rho_L^-$	1.007 ± 0.076	$\mathbf{0.008 \pm 0.091}$
$\rho_L^0 \rho_L^0$	0.972 ± 0.241	0.007 ± 0.345
$\pi^+ \pi$	1.392 ± 0.062	$\pm (0.307 \pm 0.170)$
$\pi^0 \pi^0$	1.306 ± 0.206	$\pm (0.427 \pm 0.172)$

Because the $\rho_L^+ \rho_L^-$ is the one with largest branching ratio, $\delta\epsilon_{\rho_L^+ \rho_L^-} = 0.091 = 5.2^o$ gives us an estimate of the uncertainty, due to

Isospin analysis III

the present knowledge of the penguin pollution, in the determination of γ/ϕ_3. (Important improvements are expected from Belle II and LHCb).

Potential estimate of the method I

- The intrinsic accuracy of the proposed method is controlled by the ability to extract ϕ_g. Under the assumption that Belle II can collect 1000 $\rho_L^+\rho_L^-$ events in the categories $\left(L,\rho_L^+\rho_L^-\right)$, $\left(S,\rho_L^+\rho_L^-\right)$, $\left(\rho_L^+\rho_L^-,L\right)$, $\left(\rho_L^+\rho_L^-,S\right)$, 50 $\rho_L^0\rho_L^0$, 200 $\pi^+\pi^-$ and 50 $\pi^0\pi^0$ we generate simulated data.
- For each g, we generate values of t, the events, distributed according to the four double-decay intensities. To incorporate the experimental time resolution, each t is randomly displaced following a normal distribution with zero mean and $\sigma = 1ps$. Efficiencies are not included (but remember only ratio measurements). Generation proceeds until the chosen number of events. Events are binned. The procedure is repeated in order to obtain mean values and standard deviations in each bin: these constitute our simulated data. Results are shown for 20 bins, there are no significant differences if one considers, for example, 15 or 10 bins.

Potential estimate of the method II

- Our fits to the simulated data result in:

g	ϕ_g	ρ_g
$\rho_L^+\rho_L^-$	$\mathbf{1.222 \pm 0.020}$	1.00 ± 0.06
$\rho_L^0\rho_L^0$	1.22 ± 0.09	1.00 ± 0.24
$\pi^+\pi$	1.57 ± 0.12	1.35 ± 0.12
$\pi^0\pi^0$	1.57 ± 0.18	1.35 ± 0.24
	$\phi_M = 0.384 \pm 0.031$	

We conclude that, since $\gamma = \phi_g + \epsilon_g$, **the error $\delta\phi_{\rho_L^+\rho_L^-} = 0.020 = 1.1^o$ gives an idea of the intrinsic statistical limiting error we would expect in the determination of γ for the assumed number of events.**

Conclusions I

- $B^0 - \overline{B}^0$ entanglement from $\Upsilon(4s)$ gives two decay paths to measure interfering phases.
- With decays to CP eigenstates we can chose CP allowed and CP forbidden decays. The possibility of measuring γ.
- $f = J/\psi K_S$ or $J/\psi K_L$ and $g = (\pi\pi)$ or $(\rho_L \rho_L)$ have a tree level common γ phase. $\rho_L^+ \rho_L^-$ is the benchmark channel.
- General constraints allow the full measurement combining (S, g) and (L, g).
- To extract γ, the proposal has to be completed with an isospin analysis (IA) of $B \to \rho_L \rho_L$ or $\pi\pi$.
- The intrinsic accuracy we estimate is 1^o. The accuracy associated to IA is 5^o with actual data (to be improved)

Back material I