Theory of electric dipole moments

Yohei Ema

University of Minnesota

DISCRETE 2022 @ Baden-Baden 09.Nov.2022

Based on 2108.05398, 2202.10524, 2205.11532 and 2207.01679

with T. Gao and M. Pospelov

CKM and muon EDM contributions to paramagnetic EDMs

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Electric dipole moment

- Electric dipole moment of a particle is proportional to spin:

$$\mathscr{H} = -\vec{B}\cdot\vec{\mu} - \vec{E}\cdot\vec{d} = -2\vec{s}\cdot\left(\mu\vec{B} + d\vec{E}\right).$$

* μ : magnetic dipole moment, d :electric dipole moment.

EDM violates P and T (or CP).

	$\vec{B}$	$\vec{E}$	$\vec{s}$
P	$+$	$-$	$+$
T	$-$	$+$	$-$

- Flavor diagonal: standard model contribution extremely suppressed.

e.g. $d_e^{(\mathrm{equiv})}(\delta_{\mathrm{CKM}}) \simeq 10^{-35}\, e\mathrm{cm}$ (← will see it later)

c.f. the current best limit $|d_e(\mathrm{ThO})| < 1.1\times 10^{-29}\, e\mathrm{cm}$ [ACME 18]

Background free probe of CP-odd new physics.

$$|d_e| < 1.1\times 10^{-29}\, e\,\mathrm{cm} \quad \Rightarrow \quad \Lambda \gtrsim 50\,\mathrm{TeV} \ \text{ if } \ \frac{d_e}{e} \sim \frac{\alpha}{\pi}\frac{m_e}{\Lambda^2}.$$

- CP violation motivated by baryogenesis, BSM such as 2HDM, SUSY, ...

EDM experiments

Neutron

- Sensitive to hadronic CP violation: $\theta_{\rm QCD}, \quad d_q, \quad \tilde{d}_q, \quad w, \quad \cdots .$
- $|d_n| < 1.8 \times 10^{-26}\, e\,{\rm cm}\,.$ [PSI-nEDM 20]

Diamagnetic atom (all electrons paired)

- Sensitive to hadronic CP violation (not only to d_N but to CP-odd pion nucleon interactions).
- $^{199}{\rm Hg} : |d_{\rm Hg}| < 7.4 \times 10^{-30}\, e\,{\rm cm}\,.$ [Graner+ 16]
- $^{129}{\rm Xe} : |d_{\rm Xe}| < 1.5 \times 10^{-27}\, e\,{\rm cm}\,.$ [Allmendinger+ 19]

* Sensitivity of ^{199}Hg to d_n comparable to the direct one due to Schiff screening, with different systematics.

Paramagnetic atom/molecule (unpaired electron)

- Sensitive to leptonic CP violation (not only to d_e but to CP-odd electron nucleon interactions).
- ${\rm ThO} : |d_e^{\rm equiv}| < 1.1 \times 10^{-29}\, e\,{\rm cm}\,.$ [ACME 18]
- ${\rm HfF}^+ : |d_e^{\rm equiv}| < 1.3 \times 10^{-28}\, e\,{\rm cm}\,.$ [Cairncross+ 17]

Today: CKM and muon EDM contributions to paramagnetic atomic EDM.

EDM experiments

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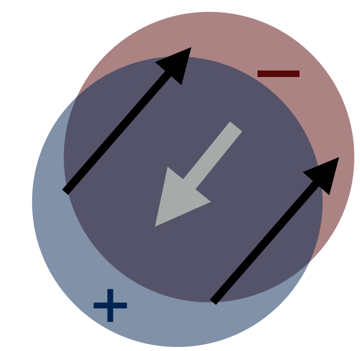
Paramagnetic atomic EDM

- Atom couples to $\vec{E}_{\rm ext}$ as

$$\mathscr{H}_A \sim -\sum_k \left(\vec{d}_k + e_k\vec{r}_k\right)\cdot\vec{E}_{\rm ext},$$

➡ Two contributions to atomic EDM: $\vec{d}_A \sim \langle\Psi|\sum_k\left(\vec{d}_k + e_k\vec{r}_k\right)|\Psi\rangle$

1. directly from constituent particle's EDM
2. wave function mixing in $|\Psi\rangle$ due to P, CP-odd interaction



* these two cancel for non-relativistic neutral point particle's EDM: shielding theorem.

- Paramagnetic atomic/molecular EDM sensitive only to a linear combination of

$$-\frac{id_e}{2}\bar{e}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu} e \quad \text{and} \quad C_S\frac{G_F}{\sqrt{2}}\bar{e}i\gamma_5 e\,\bar{N}N.$$

➡ $$d_e^{\rm equiv} = d_e + C_S \times 1.5\times 10^{-20}\, e{\rm cm}\ \ \text{for}\ \ {\rm ThO}.$$

- Experimental constraint: $|d_e^{\rm equiv}| < 1.1\times 10^{-29}\, e{\rm cm}\,.$ [ACME 18]

Outline

1. Introduction

2. CKM contribution to paramagnetic EDM

3. Indirect limits on muon EDM

4. Summary

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SM value of paramagnetic EDM

- SM breaks CP by CKM phase → generate non-zero EDM.

- CKM contribution to electron EDM:

  - Short distance contribution appears at four-loop, $d_e^{(\text{short})} \sim \mathcal{O}(10^{-44})\, e\,\text{cm}\,.$ [Pospelov, Ritz 13]

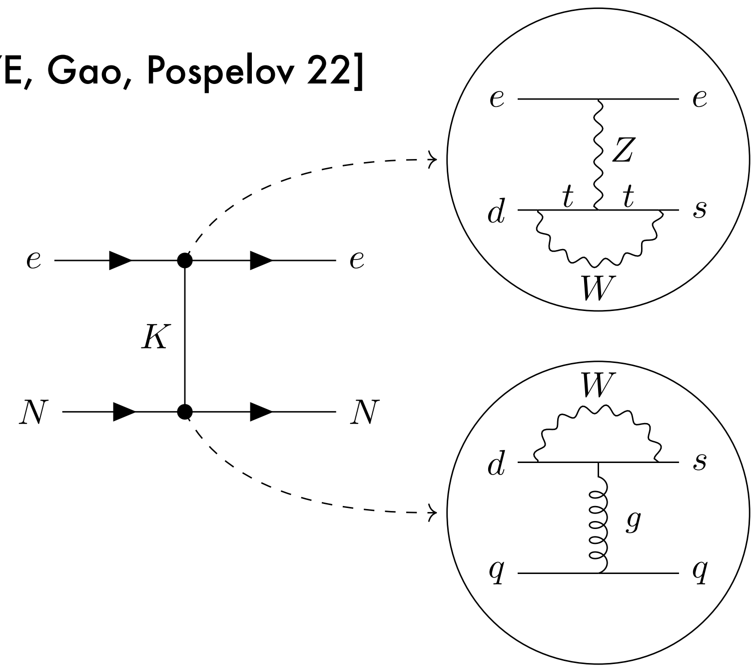
  - Long distance contribution may be lager $d_e^{(\text{long})} \sim 6 \times 10^{-40}\, e\,\text{cm}\,.$ [Yamaguchi, Yamanaka 20]

- CKM contribution to C_S:

  - Two photon exchange $d_e^{(\text{equiv})} \sim 10^{-38}\, e\,\text{cm}.$ [Pospelov, Ritz 13]

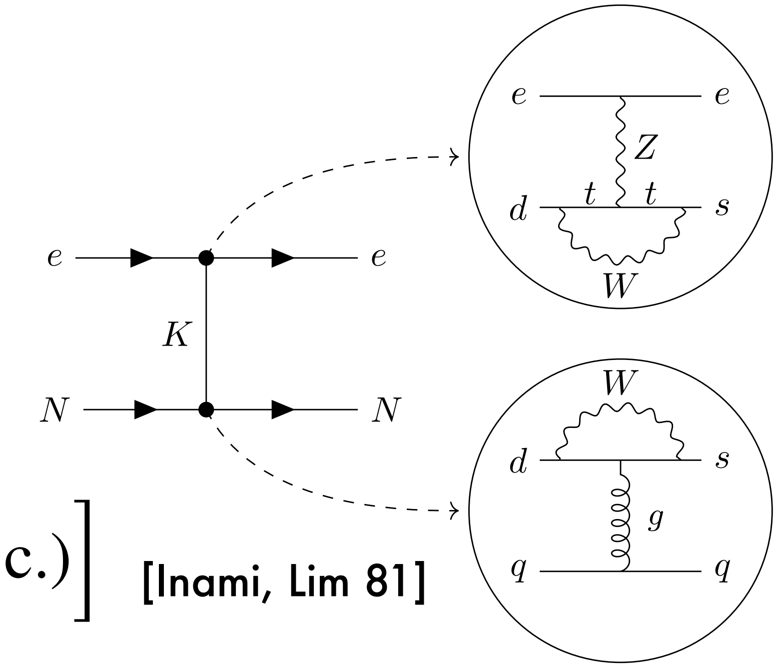
  - Kaon exchange $d_e^{(\text{equiv})} = 1.0 \times 10^{-35}\, e\,\text{cm}\,.$ [YE, Gao, Pospelov 22]

    * still well below experimental sensitivity $\sim 10^{-29}\, e\,\text{cm}\,.$



- Calculable within ChPT (NLO calculated as $30 \sim 40\,\%$).

Kaon exchange



- EW penguin + W box induce

$$\mathscr{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi s_W^2} \left[\bar{e}\gamma^\mu(1-\gamma_5)e\right] \sum_{i=c,t} I(x_i)\left[V_{is}^* V_{id} \bar{s}\gamma_\mu(1-\gamma_5)d + (\text{h.c.})\right]$$ [Inami, Lim 81]

$$\Rightarrow \mathscr{L}_{\text{eff}} = \frac{\alpha}{2\pi s_W^2} G_F f_K m_e I(x_t) \text{Im}[V_{ts}^* V_{td}] \times K_S \, \bar{e} i\gamma_5 e + \cdots,$$ below QCD scale.

- Weak hyperon non-leptonic decay with $\Delta I = 1/2$ rule:

$$\mathscr{L}_{\text{eff}} = -a\text{Tr}_F\left[\bar{B}\{\xi^\dagger h \xi, B\}\right] - b\text{Tr}_F\left[\bar{B}[\xi^\dagger h \xi, B]\right] = -\frac{\sqrt{2}K_S}{f_\pi}\left[(b-a)\bar{p}p + 2b\bar{n}n\right] + \cdots.$$

[Bijnens+ 85; Jenkins 91; ...]

- Combining them we obtain

$$C_S = J \times \frac{N + 0.7Z}{A} \times \frac{13[m_\pi^2] f_\pi m_e G_F}{m_K^2} \times \frac{\alpha I(x_t)}{\pi s_W^2}.$$

* Reduced Jarlskog invariant $J = \text{Im}[V_{ts}^* V_{td} V_{ud}^* V_{us}]$ after summing over K_S and K_L.

$$\Rightarrow d_e^{(\text{equiv})}(\text{ThO}) = 1.0 \times 10^{-35} \, e\,\text{cm}$$ (after including NLO corrections).

[YE, Gao, Pospelov 22]

- $1/m_K^2 \propto 1/m_s$ singularity in chiral limit → distinct from other contributions.

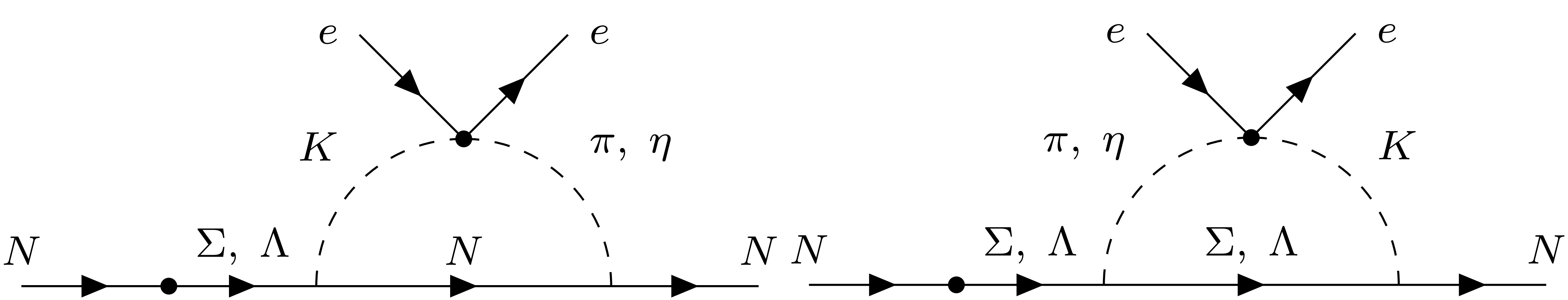
NLO correction

- NLO correction at $m_K/\Delta m_B \sim \sqrt{m_s/\Lambda_{\rm had}}$ from baryon pole diagrams.
- Calculable within heavy baryon ChPT.
- Numerically we found

$$\frac{C_S^{\rm NLO}}{C_S^{\rm LO}}(p) \simeq 30\,\%\,, \quad \frac{C_S^{\rm NLO}}{C_S^{\rm LO}}(n) \simeq 40\,\%\,.$$

[YE, Gao, Pospelov 22]

➡ corrections under control.



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Muon EDM

- Recently FNAL confirms BNL muon g-2 result:

$$a_\mu(\text{exp}) - a_\mu(\text{SM}) = \left(251 \pm 59\right) \times 10^{-11} \quad (4.2\sigma)$$

[FNAL muon g-2 21]

- Muon g-2 and EDM can be closely related:

$$\mathscr{L} = -\frac{c}{2}\bar{\psi}_R\, \sigma \cdot F\, \psi_L + \text{h.c.} \quad \Rightarrow \quad \text{Re}[c] = \frac{e\Delta a_\mu}{2m_\mu}, \;\; \text{Im}[c] = d_\mu\,.$$

$$d_\mu \simeq 2 \times 10^{-22}\, e\,\text{cm} \times \tan\phi \left(\frac{\Delta a_\mu}{2.5 \times 10^{-9}}\right) \text{ with } c = |c|e^{i\phi}\,.$$

[See e.g. Crivellin+ 18]

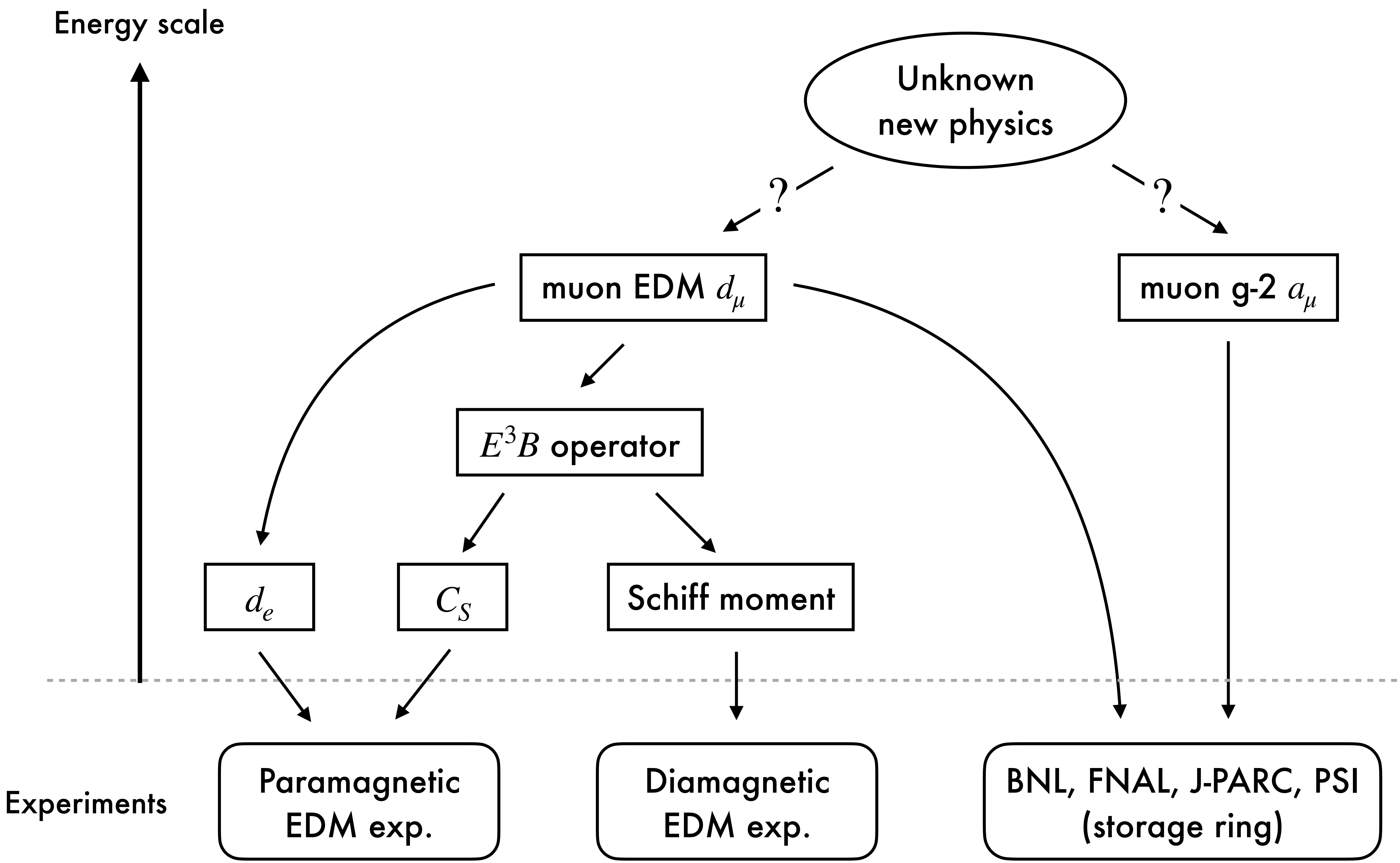
- Current/future direct measurements of muon EDM:

  - Brookhaven (existing limit): $|d_\mu| < 1.8 \times 10^{-19}\, e\,\text{cm}\,.$
  - Fermilab, J-PARC: $|d_\mu| \lesssim 10^{-21}\, e\,\text{cm}\,.$
  - PSI ("frozen spin"): $|d_\mu| < 6 \times 10^{-23}\, e\,\text{cm}\,.$

➡ understand indirect limits from atomic/molecular EDM experiments.

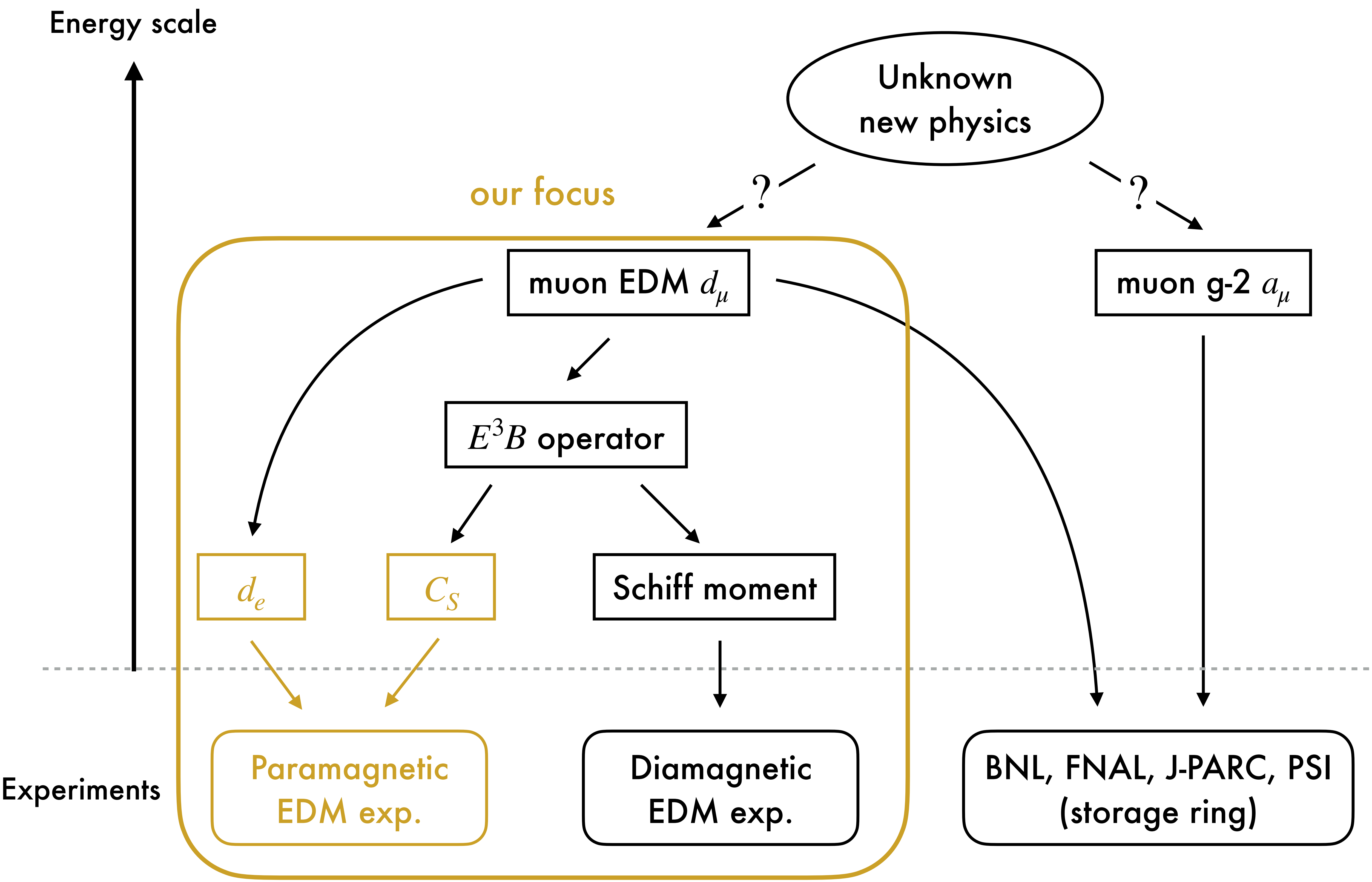
Toward observables

Many observables and many paths to get them



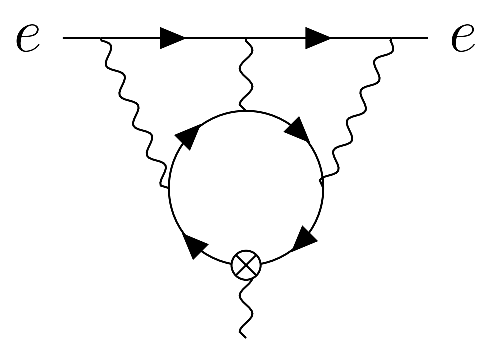
Toward observables

Many observables and many paths to get them



Electron EDM

- Muon EDM induces electron EDM at three-loop:



+ permutations * cross-dot: EDM operator insertion

- There are two types of contributions: * [Grozin, Khriplovich, Rudenko 08] computed only $S^{(1)}$.

$$i\mathcal{M} = i\tilde{F}^{\mu\nu}\,\bar{e}(p)\left[S^{(1)}m_e\sigma_{\mu\nu} + S^{(2)}\left\{\sigma_{\mu\nu}, \not{p}\right\}\right]e(p)\,.$$

- Combining two, the result is $\sim 40\,\%$ larger: [YE, Gao, Pospelov 22]

$$d_e = 2.75 \times d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \sim 2 \times 10^{-10} \times d_\mu \quad \text{(UV finite).}$$

- Paramagnetic atom sensitive only to linear combination of d_e and C_S:

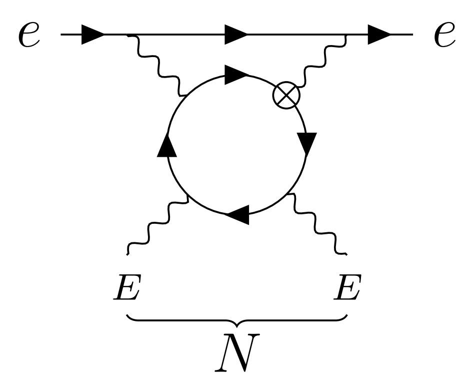
$$d_e^{\mathrm{equiv}} = d_e + C_S \times 1.5 \times 10^{-20}\, e\,\mathrm{cm} \ \text{ for } \ \mathrm{ThO}.$$

➡ Need to evaluate semi-leptonic CP-odd operator C_S .

Semi-leptonic CP-odd operator

- Paramagnetic atom EDM depends on C_S : $\mathscr{L} = C_S \frac{G_F}{\sqrt{2}} \bar{e} i \gamma_5 e \, \bar{N} N \,.$

- Muon EDM induces



$$\Rightarrow \quad d_\mu \times \bar{e} i \gamma_5 e \times E_N^2 \,.$$

* We evaluated this with leading-log accuracy.

- Nuclear electric field E_N^2 localized around nucleus.

$\bar{e} i \gamma_5 e \times E_N^2 \;\sim\; \bar{e} i \gamma_5 e \times \bar{N} N$: equivalent to C_S.

* Fudge factor included in our actual computation.

- ACME experiment: $|\, d_e^{\mathrm{equiv}} \,| < 1.1 \times 10^{-29} \, e \, \mathrm{cm}$ [ACME 18]

$|\, d_\mu(\mathrm{ThO}) \,| < 1.7 \times 10^{-20} \, e \, \mathrm{cm}$ [YE, Gao, Pospelov 21, 22]

* C_S dominates over d_e by a factor 4, better than direct bound by BNL $|\, d_\mu \,| < 1.8 \times 10^{-19} \, e \, \mathrm{cm}$.

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CKM contribution to paramagnetic EDM:

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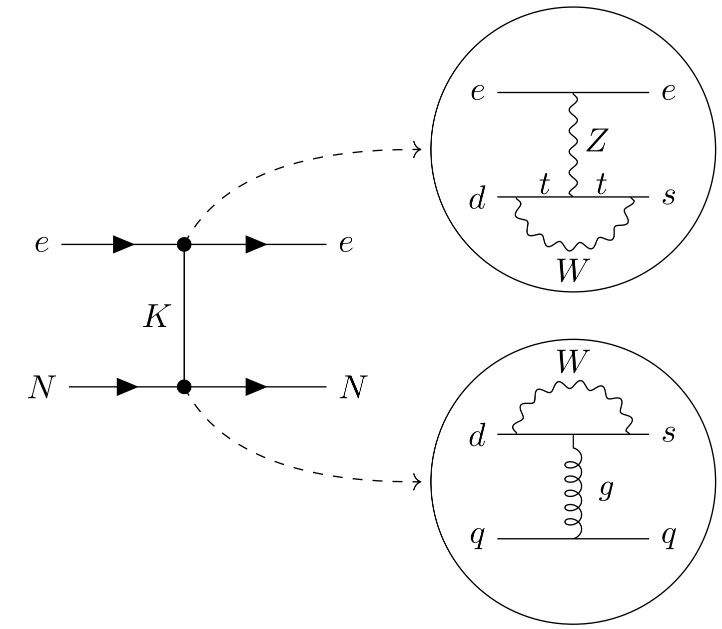
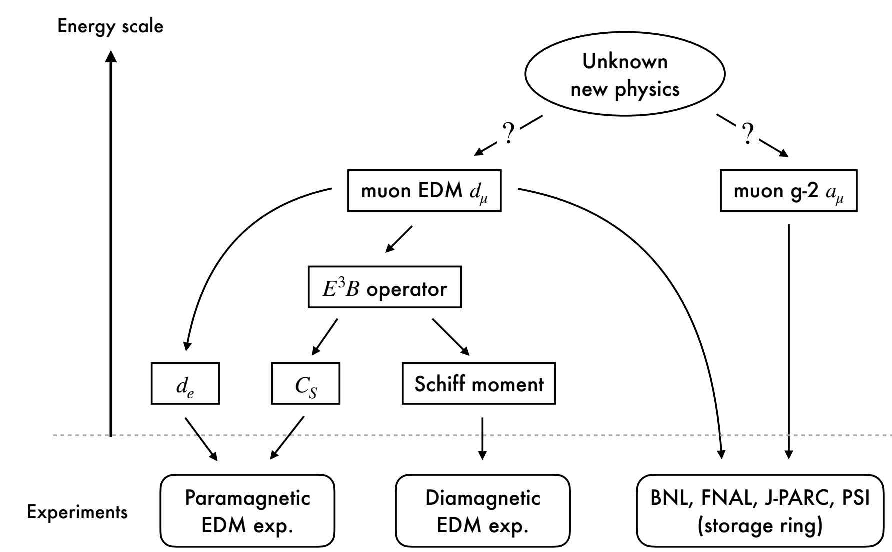
- paramagnetic EDM sensitive to both d_e and C_S.
- Kaon exchange induces $d_e^{(\text{equiv})} = 1.0 \times 10^{-35}\, e\,\text{cm}$.
- Distinct $1/m_s$ structure in the chiral limit.
- NLO correction calculable and under control.



Indirect constraints on muon EDM:

2108.05398, 2207.01679

- Muon EDM is interesting, given the muon $g-2$ anomaly.
- From ThO we get $|d_\mu(\text{ThO})| < 1.7 \times 10^{-20}\, e\text{cm}$.
- From ^{199}Hg we get $|d_\mu(\text{Hg})| < 6.4 \times 10^{-20}\, e\text{cm}$.
- Similar constraints on tau/charm/bottom EDMs.



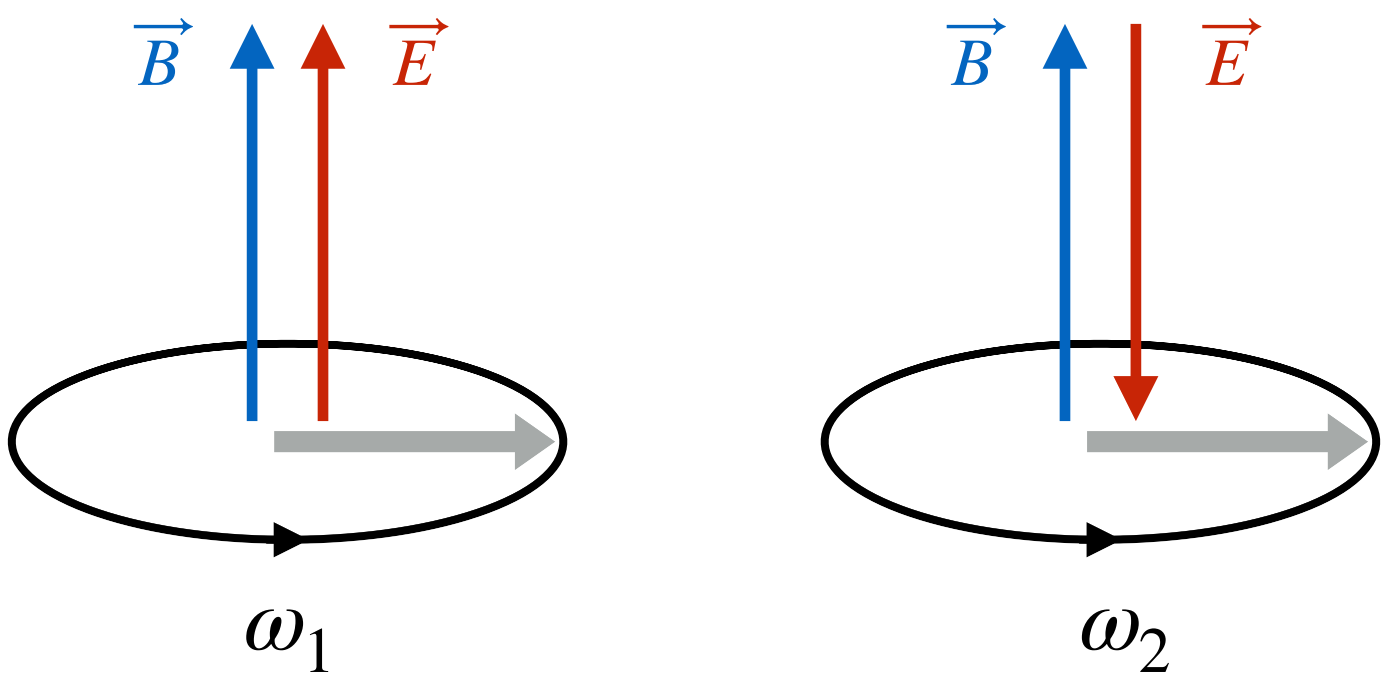
Back up

Spin precession

- EDM observable: spin precession

$$\frac{d\vec{s}}{dt} = \vec{\omega} \times \vec{s}, \quad \vec{\omega} = 2\mu \vec{B} + 2d\vec{E}\,.$$

- Extract EDM by flipping $\vec{E}$:

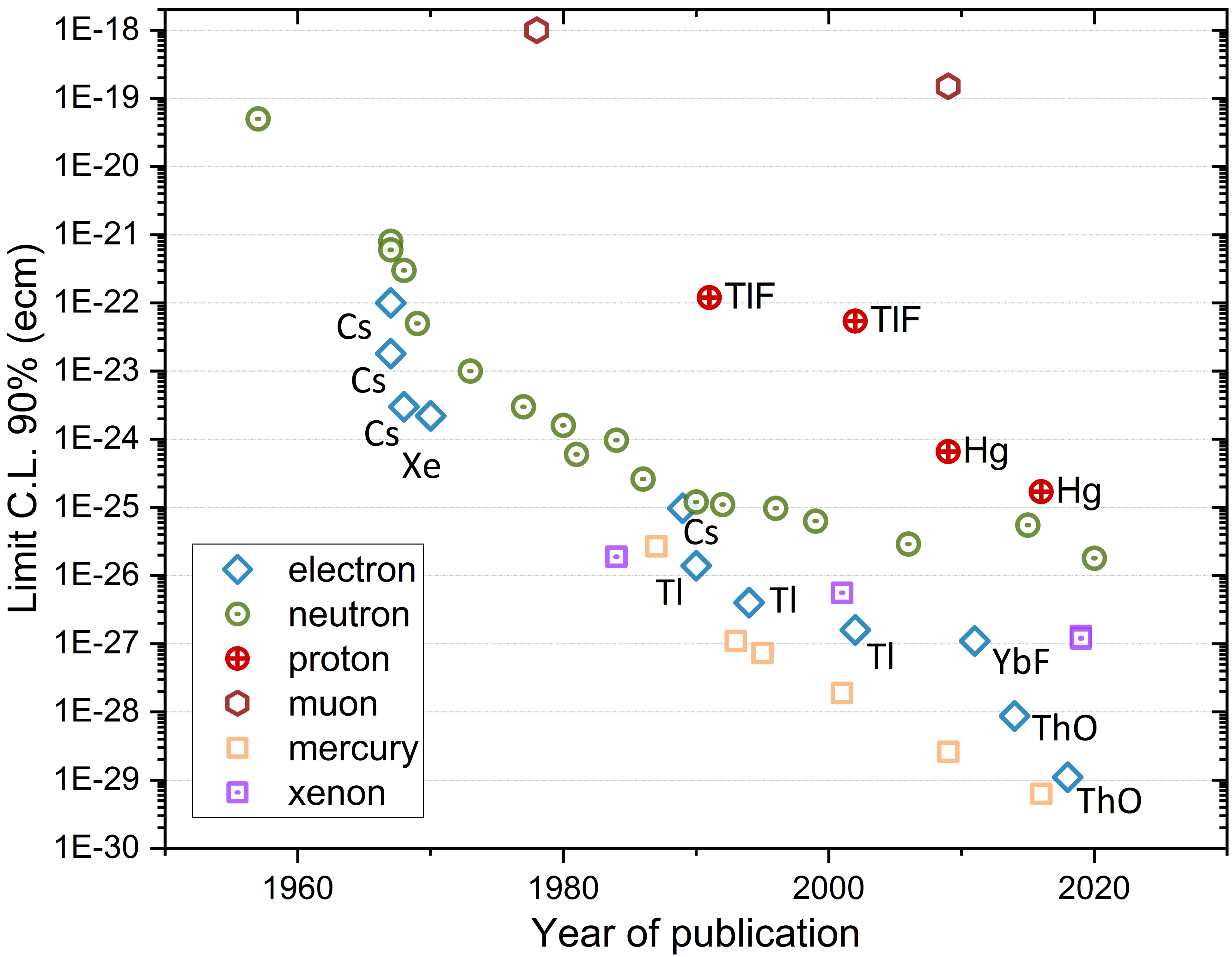


$$\Delta\omega = \omega_1 - \omega_2 = 4dE\,.$$

EDM experiments

Many efforts on detecting EDM in different systems

[Taken from Kirch & Schmidt-Wellenburg 20]



Shielding theorem

- (Non-relativistic) atomic Hamiltonian with external $\vec{E}$ and EDM:

$$\mathscr{H}_A = \mathscr{H}_N + \mathscr{H}_e + \Phi - \sum_k \left(e_k \vec{r}_k \cdot \vec{E}_{\rm ext} + \vec{d}_k \cdot \vec{E}(\vec{r}_k) \right),$$

where Φ : coulomb potential btw particles and $\vec{E} = \vec{E}_{\rm int} + \vec{E}_{\rm ext}$.

* $\vec{E}_{\rm int}(\vec{r}_k) = -\frac{\vec{\nabla}_k}{e_k}\Phi = -\frac{i}{e_k}\left[\vec{p}_k, \mathscr{H}_0\right]$ where $\mathscr{H}_0 = \mathscr{H}_N + \mathscr{H}_e + \Phi$.

- EDM without $\vec{E}_{\rm ext}$ induces mixing of (unperturbed) states as

$$|\Psi\rangle \simeq |0\rangle - \sum_{n\neq 0} \frac{\langle n | \sum_k \vec{d}_k \cdot \vec{E}_{\rm int}(\vec{r}_k) | 0\rangle}{E_0 - E_n} |n\rangle = \left(1 + \sum_k \frac{i}{e_k} \vec{d}_k \cdot \vec{p}_k \right) |0\rangle .$$

- This cancels the direct contribution to the atomic EDM:

$$\vec{d}_A = \langle \Psi | \sum_k \left(\vec{d}_k + e_k \vec{r}_k \right) | \Psi \rangle \simeq \sum_k \langle 0 | \left(\vec{d}_k - \sum_l \frac{i e_l}{e_k} \left[\vec{d}_k \cdot \vec{p}_k, \vec{r}_l \right] \right) | 0 \rangle = 0,$$

"Schiff shielding theorem"

[Purcell, Ramsey 50; Garwin, Lederman 59; Schiff 63]

Shielding theorem

- Two contributions to atomic EDM:

  (1) direct contribution from the constituent particle's EDM

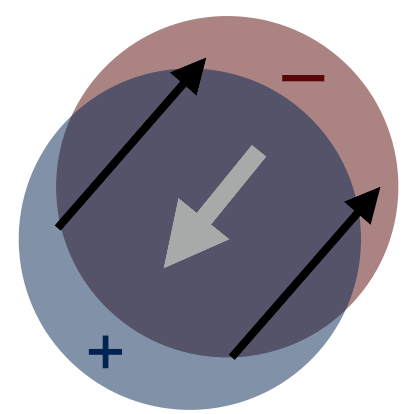
  (2) mixing of opposite parity wave functions through P,CP-odd interaction

  doesn't have to be EDM

$$\vec{d}_A \ni 2 \sum_{n \neq 0} \frac{\langle 0 | \sum_k e\vec{r}_k | n \rangle \langle n | \mathscr{H}_{\text{int}} | 0 \rangle}{E_n - E_0} .$$

  ➡ these two cancel for non-relativistic neutral point particle's EDM.

- This is a rearrangement due to the constitutions.



- Two ways out:

  (a) relativistic correction → paramagnetic atom (an unpaired electron)

  (b) finite size correction → diamagnetic atom (all electrons paired)

Paramagnetic atom/molecule

- Electron actually relativistic $v \sim Z\alpha \ \rightarrow d_e$ can induce d_A:

$$\vec{d}_A = d_e \sum_{i=1}^{Z} \left[\langle 0_e | \underline{\left(\gamma_0^{(i)} - 1\right)} \vec{\Sigma}^{(i)} | 0_e \rangle + 2 \sum_{n \neq 0} \frac{\langle 0_e | e\vec{r}_i | n_e \rangle}{E_0 - E_n} \langle n_e | \underline{\left(\gamma_0^{(i)} - 1\right)} \vec{\Sigma}^{(i)} \cdot \vec{E}_{\rm int} | 0_e \rangle \right]$$

relativistic correction to shielding

where $\vec{\Sigma} = \gamma_5 \gamma^0 \vec{\gamma}$.

$(\gamma_0 - 1)\vec{\Sigma} = -2 \begin{pmatrix} 0 & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$ in Dirac rep. → need an unpaired electron = paramagnetic atom.

- The latter (mixing of states) dominant,

[Sandars 65; ...]

and this is actually an enhancement: $d_A / d_e \sim Z^3 \alpha^2 \sim \mathcal{O}(10^2)$.

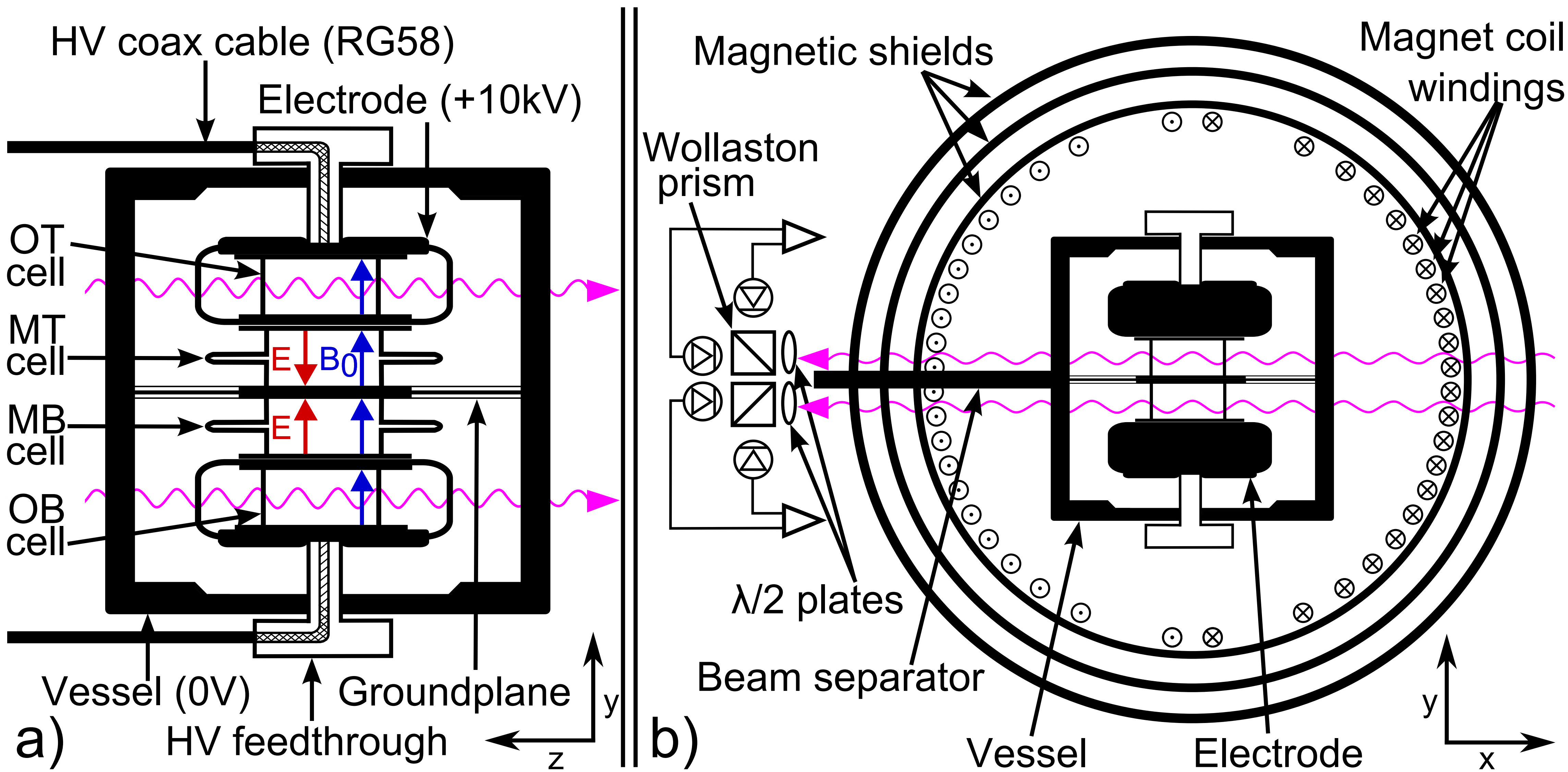
- CP-odd operator $C_S (G_F/\sqrt{2})\, \bar{e} i \gamma_5 e \times \bar{N} N$ also induces mixing of states.

➡ d_e and C_S degenerate.

$$d_e^{\rm equiv} = d_e + C_S \times 1.5 \times 10^{-20}\, e{\rm cm} \ \ \text{for} \ \ {\rm ThO}.$$

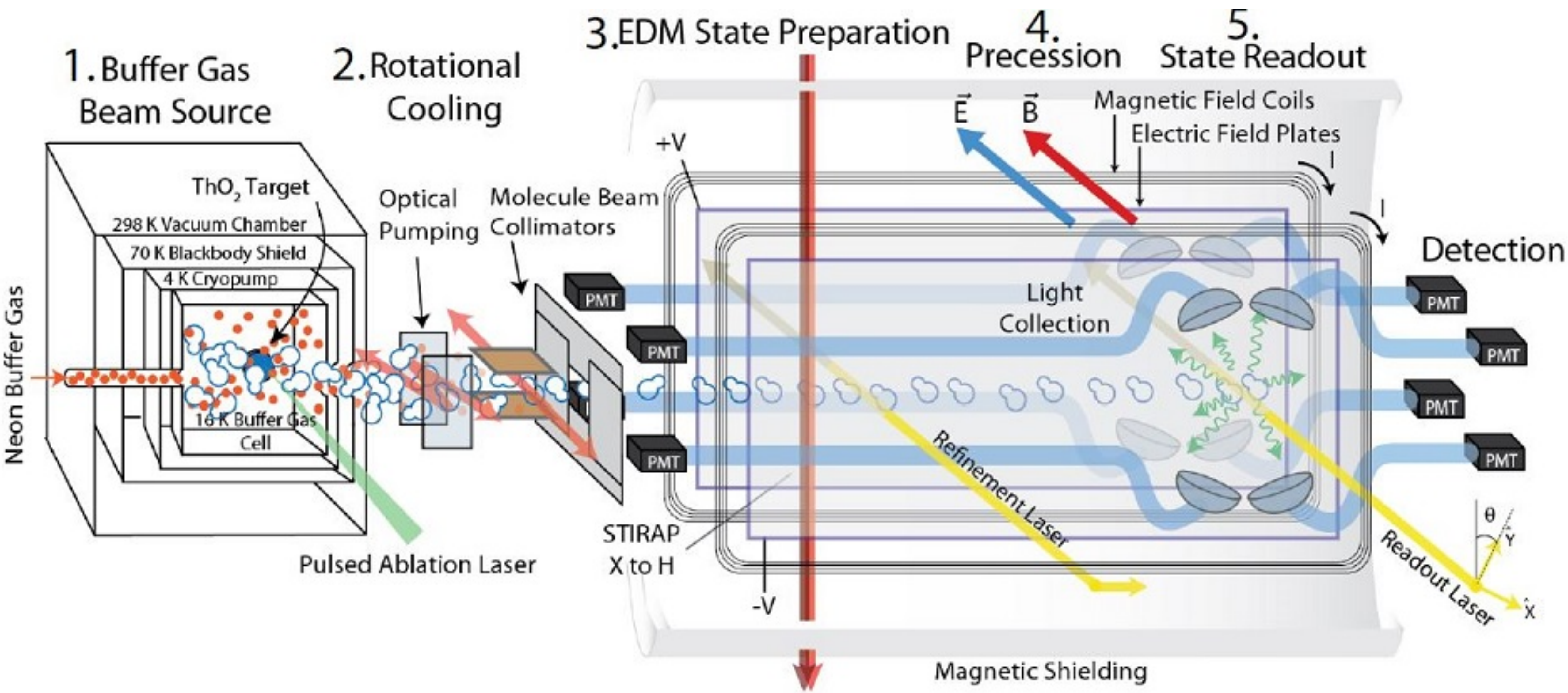
- Experimental constraint: $|d_e^{\rm equiv}| < 1.1 \times 10^{-29}\, e{\rm cm}$ [ACME 18].

^{199}Hg experiment



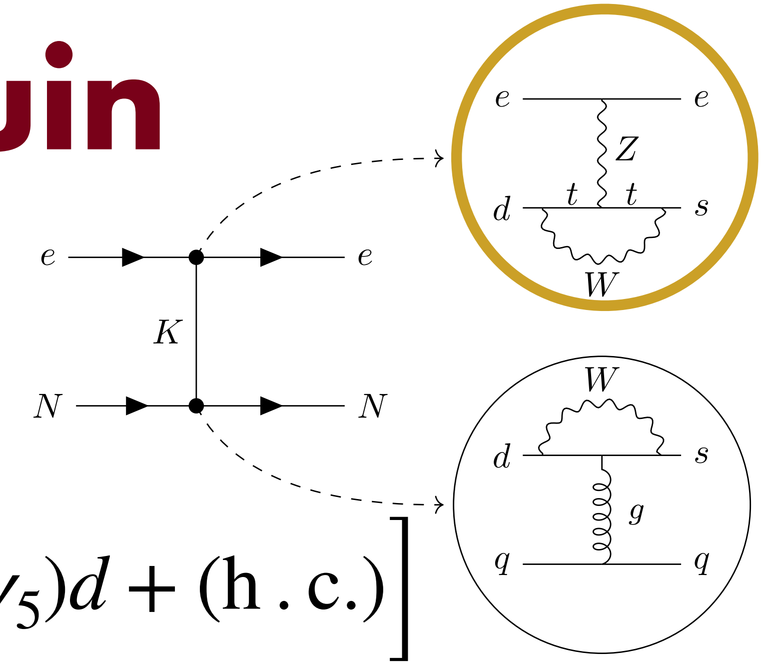
[Graner et.a. 16]

ACME ThO experiment



[ACME 18]

Electroweak penguin



- EW penguin (+ W box) induces

$$\mathscr{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi s_W^2} \left[\bar{e}\gamma^\mu(1-\gamma_5)e\right] \sum_{i=c,t} I(x_i) \left[V_{is}^* V_{id} \bar{s}\gamma_\mu(1-\gamma_5)d + (\mathrm{h.c.})\right]$$

where $x_i = m_i^2/m_W^2$ and $I(x) = \frac{3}{4}\left(\frac{x}{x-1}\right)^2 \log x + \frac{x}{4} - \frac{3}{4}\frac{x}{x-1}$. [Inami, Lim 81]

* Essentially no G_F suppression for top, $m_t^2/m_W^2 \sim \mathcal{O}(1)$.

- Below QCD scale:

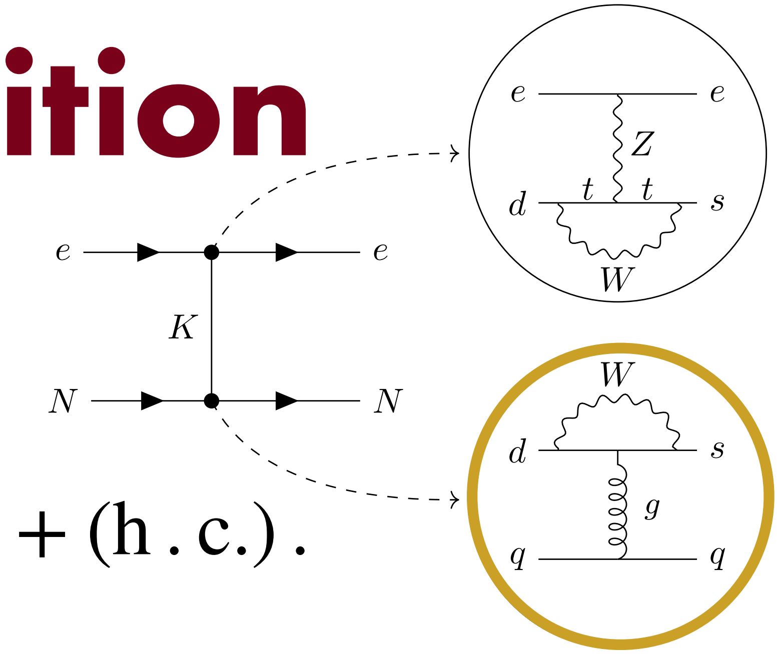
$$V_{is}^* V_{id} \bar{s}\gamma_\mu(1-\gamma_5)d + (\mathrm{h.c.}) \to \sqrt{2} f_K \partial_\mu \left(\mathrm{Re}[V_{is}^* V_{id}] K_L + \mathrm{Im}[V_{is}^* V_{id}] K_S\right).$$

$$\mathscr{L}_{\text{eff}} = \frac{\alpha}{2\pi s_W^2} G_F f_K m_e I(x_t) \mathrm{Im}[V_{ts}^* V_{td}] \times K_S \bar{e} i\gamma_5 e + \cdots,$$ with electron EoM used.

- This induces e.g. (short distance contribution to) $K_S \to \mu^+\mu^-$. [Isidori, Unterdorfer 03]

Not observed yet, $\mathrm{Br}(K_S \to \mu^+\mu^-)_{\text{short}} \sim 10^{-13}$ vs $\mathrm{Br}(K_S \to \mu^+\mu^-)_{\text{exp}} < 2.1 \times 10^{-10}$.

$\Delta I = 1/2$ weak transition



- Tree-level W exchange:

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^{*} V_{ud} \left[\bar{s}\gamma^{\mu}(1-\gamma_5)u\right] \left[\bar{u}\gamma_{\mu}(1-\gamma_5)d\right] + (\text{h.c.})\,.$$

- This contains **27** and **8** but **8** is enhanced below QCD scale: $\Delta I = 1/2$ rule.

$$\mathcal{L}_{\text{eff}} = -\,a\text{Tr}_F\left[\bar{B}\{\xi^{\dagger}h\xi, B\}\right] - b\text{Tr}_F\left[\bar{B}[\xi^{\dagger}h\xi, B]\right],$$

where B : baryon octet, $\xi = \exp(i\pi/f)$ with π: meson octet, $h_{ij} = \delta_i^2\delta_j^3$: spurion.

- This induces weak hyperon decay $\Sigma \to N\pi,\ \ \Lambda \to N\pi,\ \ \Xi \to \Lambda\pi\,.$

$a = 0.56 G_F [m_\pi^2] f_\pi\,,\ \ b = -\,1.42 G_F [m_\pi^2] f_\pi\,,$ from experiments [Bijnens+ 03].

- Focusing on Kaon part:

$$\mathcal{L}_{\text{eff}} = -\frac{\sqrt{2}K_S}{f_\pi}\left[(b-a)\bar{p}p + 2b\bar{n}n\right] + \cdots\,.$$

Nuclear electric field

- Nuclear electric field given by the charge distribution inside nuclei:

$$e\overrightarrow{E}_N(\vec{r}) = \frac{Ze^2}{4\pi}\int d^3 r_N \rho_q(\vec{r}_N)\,\overrightarrow{\nabla}\frac{1}{|\vec{r} - \overrightarrow{r_N}|}\,.$$

- We simply take the charge distribution as

$$\rho_q(r_N) = \frac{3}{4\pi R_N^3}\Theta\left(R_N - r_N\right), \quad R_N = \sqrt{\frac{5}{3}}r_c, \quad r_c = 5.45\,\mathrm{fm} \ \text{ for } {}^{199}\text{Hg}.$$

- The Woods-Saxon shape different only within $10\,\%$ in the final result.

Nuclear magnetic field

- ^{199}Hg has an unpaired outermost neutron with $2p_{1/2}$ $(n=2, l=1, j=1/2)$.

  ➡ $\overrightarrow{B}_N$ dominantly provided by this neutron.

- As a result $\overrightarrow{B}_N$ is given by

$$e\overrightarrow{B}_N(\vec{r}) = \frac{2e\mu_n}{3}\psi_n^\dagger(\vec{r})\overrightarrow{\sigma}\psi_n(\vec{r}) + \frac{e\mu_n}{4\pi}\left[\vec{\nabla}\left(\vec{\nabla}\cdot\right) - \frac{\vec{\nabla}^2}{3}\right]\int d^3r_n \frac{\psi_n^\dagger(\vec{r}_n)\overrightarrow{\sigma}\psi_n(\vec{r}_n)}{|\vec{r}_n - \vec{r}|}$$

$$= e\mu_n\frac{\left|R(r)\right|^2}{4\pi}\chi^\dagger\left[\left(\overrightarrow{n}\cdot\overrightarrow{\sigma}\right)\overrightarrow{n} - \overrightarrow{\sigma}\right]\chi + \frac{e\mu_n}{4\pi}\int_0^\infty dr_n\, r_n^2\left|R(r_n)\right|^2\chi^\dagger\overrightarrow{g}(\vec{r}, r_n)\chi,$$

where $\mu_n \simeq -1.91\dfrac{e}{2m_p}$: neutron magnetic moment, ψ_n : neutron wave function,

R_n: neutron radial wave function, χ neutron spinor, $\overrightarrow{n} = \vec{r}/r$.

- We use the nuclear shell model to obtain ψ_n.

Fudge factor from E_N^2 to $\bar{N}N$

- E_N^2 and $\bar{N}N$ are both localized around nuclei, but not exactly the same.

- Relevant electron transition btw $s_{1/2}$ and $p_{1/2}$ states.

  matrix element:

$$\begin{cases} \int d^3 r \rho_N \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right) & \text{for} \quad \bar{N}N \bar{e} i \gamma_5 e, \\ \int d^3 r |\vec{E}_N|^2 \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right) & \text{for} \quad E_N^2 \bar{e} i \gamma_5 e\,. \end{cases}$$

- We compute the fudge factor κ by solving the Dirac equation and get

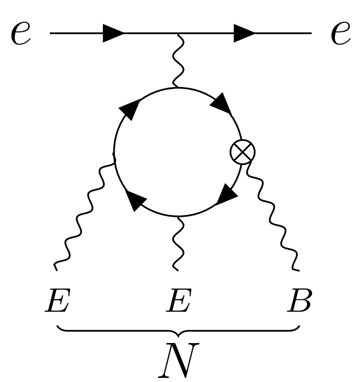
$$\kappa = \frac{\int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right)}{\int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right)} \simeq 0.66.$$

Schiff moment

- Schiff moment: $\mathcal{H}_{\rm int} = -4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)$.

$$\vec{d}_A = \sum_{i=1}^{Z}\langle\Psi|e\vec{r}_i|\Psi\rangle = -\sum_{n\neq 0}\frac{2}{E_0-E_n}\sum_{i=1}^{Z}\langle 0_e|e\vec{r}_i|n_e\rangle\langle n_e|4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_i\delta^{(3)}(\vec{r}_i)|0_e\rangle\,.$$

- E^3B with two E_N and one B_N induces effective EDM distribution:



$$= \int d^3r\left(\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|}\right)\cdot\frac{\vec{d}_N(\vec{r})}{e}\,,\quad \vec{d}_N \propto d_\mu\left(2\vec{E}_N(\vec{E}_N\cdot\vec{B}_N)+\vec{B}_N E_N^2\right).$$

- Difference between EDM and charge distribution gives Schiff moment:

$$\mathcal{H}_{\rm eff} = \int d^3r\left(\frac{\vec{d}_N}{e}-\rho_q\frac{\langle\vec{d}_N\rangle}{e}\right)\cdot\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|} = -4\pi\alpha\frac{\vec{S}}{e}\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)+\cdots\,.$$

- ^{199}Hg constraint: $|S_{^{199}\rm Hg}| < 3.1\times10^{-13}\,e{\rm fm}^3$. [Graner et.a. 16]

$$|d_\mu({\rm Hg})| < 6.4\times10^{-20}\,e{\rm cm}$$

[YE, Gao, Pospelov 21]

Magnetic quadrupole moment

- Magnetic quadrupole moment (MQM) also violates P and CP:

$$\mathscr{H}_{\rm eff} = -\frac{M}{6}\nabla_j B_i I_{ij}, \quad I_{ij} \equiv \frac{3}{2I(I-1)}\left(I_i I_j + I_j I_i - \frac{2}{3}\delta_{ij} I(I+1)\right).$$

- The E^3B operator converts EQM Q to MQM as

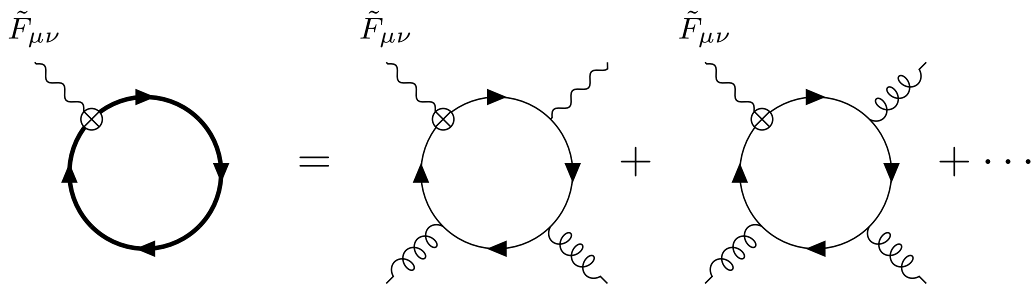
$$\frac{M}{e} \simeq -\frac{Z^2\alpha^3 d_\mu/e}{5\pi m_\mu^2 R_N^3}\frac{Q}{e} \simeq 1.1\times 10^{-4}\,\mathrm{fm}\left(\frac{Q/e}{300\,\mathrm{fm}^2}\right)\times d_\mu/e\,.$$

- Q can be large in nuclei with $I \geq 1$ and large deformation.

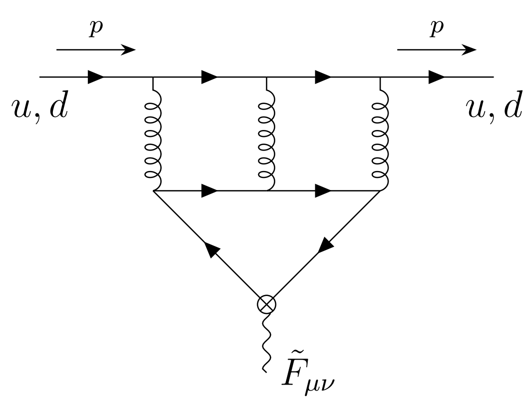
➡ can be an interesting observable in future.

Heavy quark EDMs

- The same technique can be used to constrain charm/bottom quark EDMs.



$$\Rightarrow \quad GGG\tilde{F}, \quad GGF\tilde{F}\,.$$



$$\Rightarrow \quad d_u, \quad d_d\,.$$

- These CP-odd operators source C_S and neutron EDM.

$$\begin{cases} |d_c| < 1.3 \times 10^{-20}\, e\,\text{cm} \quad |d_b| < 7.6 \times 10^{-19}\, e\,\text{cm} \ \text{ from } C_S, \\ |d_c| < 6 \times 10^{-22}\, e\,\text{cm}, \quad |d_b| < 2 \times 10^{-20}\, e\,\text{cm} \ \text{ from } d_n. \end{cases}$$

[YE, Gao, Pospelov 22]

- Constraints from d_n stronger but with more hadronic uncertainties.

* Top quark EDM has a larger contribution through other diagrams with intermediate Higgs. [Cirigliano+ 16]