The quantum nature of the "minimal" $\mathrm{SO}(10)$ GUT

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Motivation

- Grand Unified Theories (GUT): $\rightarrow$ SM interactions unify
  $\rightarrow$ predict proton decay

- Proton decay lifetime: usually significant uncertainty in prediction

Based on `2109.06784`, further work to appear soon.

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- "Minimal" $\mathrm{SO}(10)$ model: $45 + 126 + 10$ in Higgs sector

  $\rightarrow$ interesting: better control of proton lifetime uncertainties

  $\rightarrow$ model however pathological at tree-level

  $\rightarrow$ important to determine:

  Is the model saved at the *quantum level* (1-loop)?

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- Matter content: $\rightarrow$ fermions: $3 \times 16_F$

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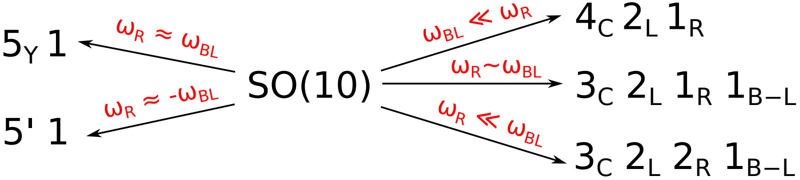
- Hope for robust proton decay prediction:

  no $(45_G \cdot 45_G \cdot S)/M_{Pl}$ operator modifying gauge coupling running

  $\rightarrow$ GUT scale robustly determined

Breaking at tree level

- GUT-breaking (SM-singlet) VEVs: ω_{BL}, $\omega_R \in 45$, $\sigma \in 126$
- What are the possibilities for intermediate symmetry G?



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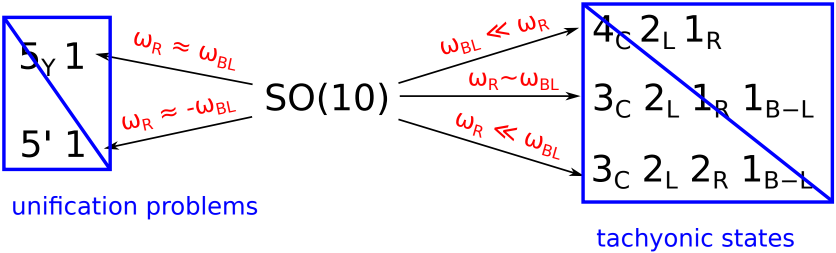

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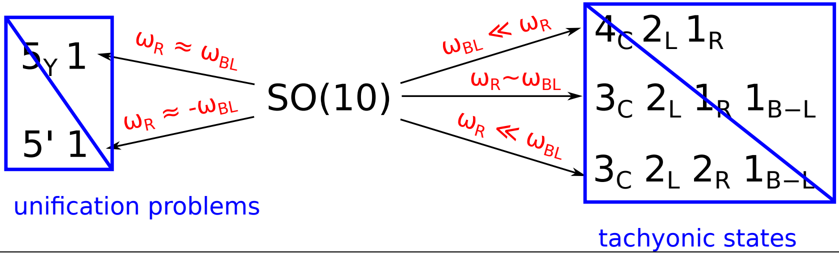
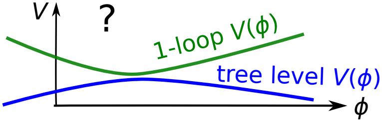


- Tachyonicity example at tree level: for $\omega_{BL} \ll \omega_R$

$$M^2_{(8,1,0)} = -2\, a_2\, \omega_R^2,$$
$$M^2_{(1,3,0)} = +4\, a_2\, \omega_R^2.$$

Take $|a_2| \ll 1$: stabilized at 1-loop?



Saved at 1-loop?

- Treatment at 1-loop: Coleman-Weinberg effective potential

$$V_1(\phi) = \frac{1}{64\pi^2}\mathrm{Tr}\left[\mathsf{M}_S^4(\phi)\left(\log\left[\frac{\mathsf{M}_S^2(\phi)}{\mu_R^2}\right] - \frac{3}{2}\right) + 3\mathsf{M}_G^4(\phi)\left(\log\left[\frac{\mathsf{M}_G^2(\phi)}{\mu_R^2}\right] - \frac{5}{6}\right)\right].$$

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- Analysis considerations:

  (1) non-tachyonicity (all states)

  (2) perturbativity (definition: degree of arbitrariness)

    (2a) corrections to masses δm^2 under control

    (2b) RGE (of scalar potential parameters) under control

  (3) unification of gauge couplings

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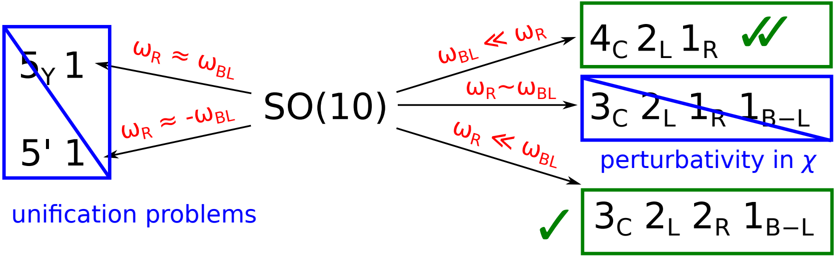
  - (3) unification of gauge couplings

- Observation: universal ratio $\chi := \frac{\omega_R \omega_{BL}}{|\sigma|^2}$

$$\text{e.g. } \tau = 2\beta_4'(3\omega_{BL}+2\omega_R) + a_2\,\chi(\omega_{BL}+\omega_R).$$

Breaking at 1-loop: Higgs model

- Perturbativity: $|\chi| \lesssim 1$: 2 potentially viable scenarios



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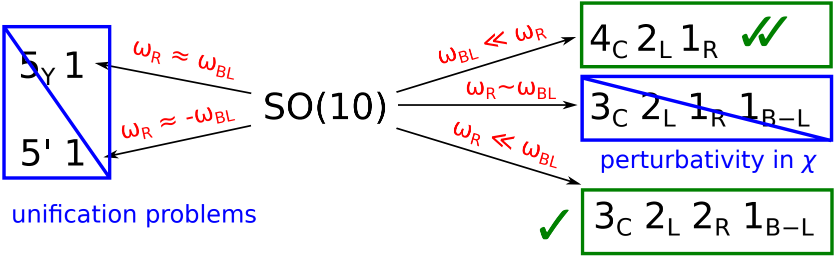
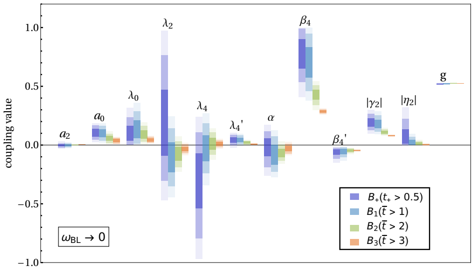


Viability: confirmed by numeric scan

✓✓: $|\omega_{BL}| \ll |\sigma| \ll |\omega_R|$
$(M_{GUT}, M_I) \sim (10^{15}, 10^{11})$ GeV,
better perturbativity

✓: $|\omega_R| \ll |\sigma| \ll |\omega_{BL}|$
$(M_{GUT}, M_I) \sim (10^{18}, 10^{8})$ GeV.



Next step: Yukawa sector

- Under further consideration: scenario ✓✓
- Doublets $(1,2,\pm 1/2)$: 2 in $\mathbf{126}$, 2 in $\mathbf{10}_{\mathbb{C}}$

$$\mathcal{L}_{\mathsf{Yuk}} = \mathbf{16}_{\mathsf{F}}\, \mathbf{16}_{\mathsf{F}}\, (Y_{10}\, \langle \mathbf{10} \rangle + \widetilde{Y}_{10}\, \langle \mathbf{10}^* \rangle + Y_{126}\, \langle \mathbf{126}^* \rangle\,)$$

all need EW VEVs for proper Yukawa fit

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- SM Higgs must be an admixture...

$$M^2_{(1,2,\pm 1/2)} = \begin{pmatrix} M^2_{126} & M^2_{\mathrm{mix}} \\ M^{2\dagger}_{\mathrm{mix}} & M^2_{10} \end{pmatrix} \sim \begin{pmatrix} M^2_{GUT} & |\sigma|^2 \\ |\sigma|^2 & M^2_{GUT} \end{pmatrix}$$

2×2 blocks $M^2_{10,126}$: one state in each must be tuned to σ-scale

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- Behavior of tuning M^2_{126}:
  - Tree level: cannot be tuned to $|\sigma|^2$ (another state becomes tachyonic first) $\rightarrow$ but can be tuned to size of 1-loop level
  - 1-loop level: numeric scan (preliminary) suggests block cannot be tuned to 2-loop estimated level $\rightarrow$ PROBLEM!
    If above is true, model not viable.

Summary

$\mathrm{SO}(10)$ GUT model with scalar sector $45 + 126 + 10_{\mathbb{C}}$:

- Interesting: "minimal", should give robust proton decay prediction
- Tricky: 1-loop is first consistent perturbative order
  - Technically challenging
  - Symmetry breaking does work in a small patch of parameter space
    $\rightarrow$ in that regard very predictive (requiring perturbativity)
  - Obtaining a good Higgs does not seem to work
    $\leftarrow$ SM Higgs component in 126 too small for Yukawa fit
- Ultimately unviable it seems

Thank you for your attention!

Backup: technical challenges

- A lot of particles: scalar mass matrix $M_S^2(\phi)$ in V_1 is 297×297 in Higgs model, 317×317 in full theory
- A lot of parameters: the scalar potential written schematically is

$$\begin{aligned} V(45,126) = \;& \mu^2\, 45^2 + a\, 45^4 + \\ & + \nu^2\, |126|^2 + \lambda\, |126|^4 + \eta\, 126^4 + \tau\, 45 \cdot |126|^2 + \\ & + (\alpha, \beta)\, 45^2 \cdot |126|^2 + \gamma\, 45^2 \cdot 126^2 + h.c., \\ V(45,126,10) = \;& V(45,126) + \xi^2\, 10^2 + h\, 10^4 \\ & + \kappa\, 10^2\, 45^2 + \zeta\, 45^2 \cdot 126 \cdot 10 + \rho 10^2\, |126|^2 + \\ & + \rho'\, 10^2\, 126^2 + \varphi\, |126|^2 \cdot 126 \cdot 10 + h.c. \end{aligned}$$

(possibly >1 independent contraction, for brevity 10^* was written as 10)

- Parameters in full theory:

$(15\, \mathbb{R} + 14\, \mathbb{C})$ dimensionless, $(5\, \mathbb{R} + 1\, \mathbb{C})$ massive