

B-meson lifetimes within the HQE

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In collaboration with A. Lenz, J. Müller and A. Rusov



Motivation

- ◇ Lifetime $\tau = \Gamma^{-1}$ is one of the fundamental properties of particles
- ◇ For heavy hadrons: systematic framework to compute $\Gamma(H_Q)$
 $m_Q \gg \Lambda_{QCD}$
- ◇ Focus on the B -system $m_b \sim 4.5 \text{ GeV} \gg 0.5 \text{ GeV} \sim \Lambda_{QCD}$
 - * Experimental precision very high $\mathcal{O}(\%)$ [HFLAV '21]
 - * Aim at competitive theoretical precision to
 - * Test the framework used
 - * Perform indirect NP searches

$$\underbrace{\frac{\tau(B_1)}{\tau(B_2)}}_{\text{exp.}} = 1 + \underbrace{[\Gamma^{\text{SM}}(B_2) - \Gamma^{\text{SM}}(B_1)] \tau^{\text{exp.}}(B_1)}_{\text{theory}} + \underbrace{[\Gamma^{\text{BSM}}(B_2) - \Gamma^{\text{BSM}}(B_1)] \tau^{\text{exp.}}(B_1)}_{\text{indirectly constrained}}$$

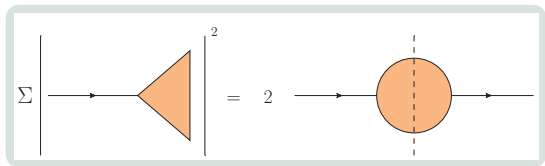
The total decay width of a B -meson

- Start from the definition

$$\Gamma(B) = \frac{1}{2m_B} \sum_n \int_{\text{PS}} (2\pi)^4 \delta^{(4)}(p_n - p_B) |\langle n | \mathcal{H}_{eff} | B \rangle|^2$$

- \mathcal{H}_{eff} - weak effective Hamiltonian describing b -quark decays
- Use **optical theorem** to rewrite [Shifman, Voloshin '85]

$$\Gamma(B) = \frac{1}{2m_B} \text{Im} \langle B | i \int d^4x T \{ \mathcal{H}_{eff}(x), \mathcal{H}_{eff}(0) \} | B \rangle$$



The heavy quark expansion (HQE)

◇ Heavy b -quark carries most of the hadron momentum $p_B^\mu = m_B v^\mu$

◇ Introduce parametrisation

$$p_b^\mu = m_b v^\mu + k^\mu$$

$$k \sim \Lambda_{QCD} \ll m_b$$

◇ Rescale heavy quark field

$$b(x) = e^{-im_b v \cdot x} b_v(x)$$

◇ Action of the covariant derivative

$$iD_\mu b(x) = e^{-im_b v \cdot x} (m_b v_\mu + iD_\mu) b_v(x)$$

$$D_\mu = \partial_\mu - iA_\mu^a(x)t^a$$

The HQE

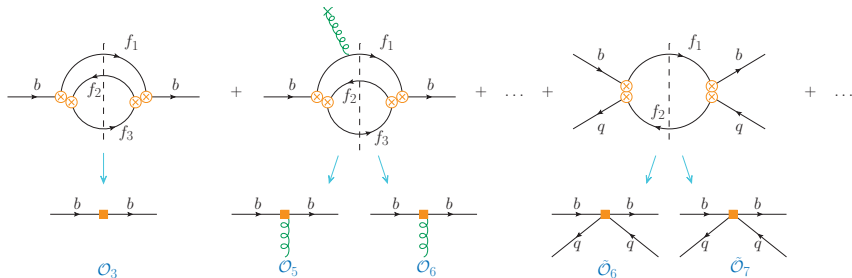
- ◇ Obtain systematic expansion

$$\Gamma(B) = \underbrace{\Gamma_3}_{\Gamma(b)} + \underbrace{\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \dots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \dots \right]}_{\delta\Gamma(B)}$$

- * $\Gamma_d, \tilde{\Gamma}_d$ - short distance coefficients
- * $\mathcal{O}_d, \tilde{\mathcal{O}}_d$ - local operators bilinear in the heavy quark field
- * $\Gamma(b)$ - total decay width of free b quark
- * $\delta\Gamma(B)$ - effects due to interaction with soft gluons and quarks

The HQE

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \dots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \dots \right]$$



Very advanced framework thanks to huge effort of big community

Status of HQE: perturbative side

$$\Gamma_d = \Gamma_d^{(0)} + \left(\frac{\alpha_s(m_b)}{4\pi} \right) \Gamma_d^{(1)} + \left(\frac{\alpha_s(m_b)}{4\pi} \right)^2 \Gamma_d^{(2)} + \dots$$

Semileptonic modes (SL)		Non-leptonic modes (NL)	
$\Gamma_3^{(3)}$	Fael, Schönwald, Steinhauser '20 Czakon, Czarnecki, Dowling '21	$\Gamma_3^{(2)}$	Czarnecki, Slusarczyk, Tkachov '05 *
$\Gamma_3^{(2)}$	Czarnecki, Melnikov, v. Ritbergen, Pak, Dowling, Bonciani, Ferroglia, Biswas, Brucherseifer, Caola '97-'13	$\Gamma_3^{(1)}$	Ho-Kim, Pham, Altarelli, Petrarca, Voloshin, Bagan, Ball, Braun, Gosdzinsky, Fiol, Lenz, Nierste, Ostermaier, Krinner, Rauh '84-'13
$\Gamma_5^{(1)}$	Alberti, Gambino, Nandi, Mannel, Pivovarov, Rosenthal '13-'15	$\Gamma_5^{(0)}$	Bigi, Uraltsev, Vainshtein, Blok, Shifman '92
$\Gamma_6^{(1)}$	Mannel, Moreno, Pivovarov '19, '21, '22	$\Gamma_6^{(0)}$	Lenz, MLP, Rusov, Mannel, Moreno, Pivovarov '20-'21
$\Gamma_7^{(0)}$	Dassinger, Mannel, Turczyk '06	$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste, Franco, Lubicz, Mescia, Tarantino, Rauh '02-'13
$\Gamma_8^{(0)}$	Mannel, Turczyk, Uraltsev '10	$\tilde{\Gamma}_7^{(0)}$	Gabbiani, Onishchenko, Petrov '03-'04

* Only partial result

Status of HQE: non-perturbative side

★ Fit to experimental data on semileptonic B -decays ★ HQET sum rules ★ Lattice QCD

	B_d, B^+	B_s
$\langle \mathcal{O}_5 \rangle$	Bernlochner et al. '22, ★ Bordone, Capdevila, Gambino, Schwanda et al. '13, '14, '21 ★ Ball, Braun, Neubert '93-'95 ★ Kronfeld, Simone, Gambino, Melis, Simula '00 -'17 ★	SU(3) _F -breaking for μ_π^2 Bigi, Mannel, Uraltsev '11 Spectroscopy relation for μ_G^2
$\langle \mathcal{O}_6 \rangle$	Bernlochner et al. '22, ★ Bordone et al. '13, '14, '21 ★ EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	Kirk, Lenz, Rauh '17 ★	King, Lenz, Rauh '21 ★
$\langle \tilde{\mathcal{O}}_7 \rangle$	Vacuum insertion approximation (VIA)	

- ◇ Some tension between different fits for ρ_D^3
- ◇ Fits to experimental semileptonic B_s -decays highly desirable

Belle II, BESIII, LHCb, super tau-charm factory?

Our setup

- ◇ The observables

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \dots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \dots \right]$$

$$\tau(B_{(s)}^{(+)})/\tau(B_d) = 1 + \left[\Gamma(B_d)^{\text{HQE}} - \Gamma(B_{(s)}^{(+)})^{\text{HQE}} \right] \tau(B_{(d)}^{(+)})^{\text{exp}}$$

- ◇ No two-quark contributions in $\tau(B^+)/\tau(B_d)$ in isospin limit
 - * Theoretically more clean
- ◇ For $\tau(B_s)/\tau(B_d)$ crucial role of $SU(3)_F$ breaking effects
 - * Two-quark dimension-six coefficient Γ_6 found to be large
[Lenz, MLP, Rusov '20; Mannel, Moreno, Pivovarov '20]
 - * Poor control on size of non-pert. input and of $SU(3)_F$ breaking

Result

◇ Scenario A

* Larger input for B_d

[Bordone et al. '21]

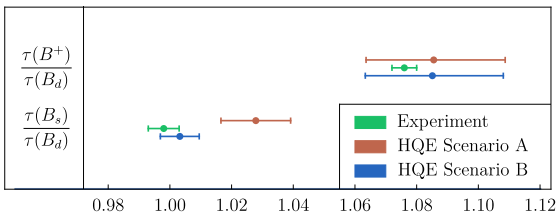
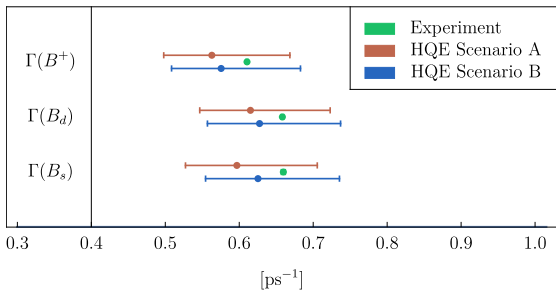
* Larger $SU(3)_F$ breaking

◇ Scenario B

* Smaller input for B_d

[Bernlochner et al. '22]

* Smaller $SU(3)_F$ breaking

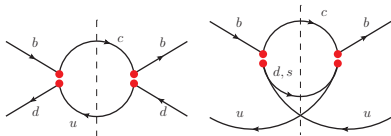


[Lenz, MLP, Rusov '22]

BSM effects in lifetimes and mixing

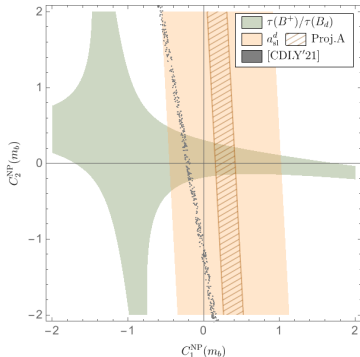
- ◇ Good agreement of HQE with data for $\tau(B^+)/\tau(B_d)$
- ◇ How large is space for NP in $b \rightarrow c\bar{u}d(s)$ decays ?
- ◇ Repeat computation with 20 additional NP operators, also for a_{sl}^d

$$\mathcal{H}_{eff} = \mathcal{H}_{eff}^{\text{SM}} + \mathcal{H}_{eff}^{\text{NP}}$$



Compare with study of decays like $\bar{B}_s \rightarrow D_s^+ \pi^-$

[Cai, Deng, Li, Yang '21]



[Lenz, Müller, MLP, Rusov '22]

Conclusion

- ◇ Up-to-date analysis of B -meson lifetimes (ratios) within HQE
- ◇ Very good agreement with data, but larger uncertainties
- ◇ Need better control on size of two-quark non-pert. input
 - * Accuracy of prediction for $\tau(B_s)/\tau(B_d)$ largely affected!
- ◇ Possibility to constrain coefficients of some NP operators

Thanks for the attention

Backup

Status of B -mesons lifetime ratios

	HFLAV	[Lenz, MLP, Rusov '22]
$\frac{\tau(B^+)}{\tau(B_d)}$	1.076(4)	1.085(23)
$\frac{\tau(B_s^0)}{\tau(B_d)}$	0.998(5)	$1.004(5) + 0.247 \underbrace{\rho_D^3(B_d)}_{\text{exp. fit}} \times \underbrace{\left[\frac{\rho_D^3(B_s)}{\rho_D^3(B_d)} - 1 \right]}_{\text{EOM}}$

- ◇ Good agreement of HQE with data for $\tau(B^+)/\tau(B_d)$
- ◇ Prediction for $\tau(B_s)/\tau(B_d)$ significantly affected by Darwin term
 - * Extract ρ_D^3 from fit to SL B -decays
e.g. [Bernlochner et al. '22; Bordone et al. '21]
 - * Estimate size of $SU(3)_F$ from EOM for gluon field
e.g. [Bigi, Mannel, Uraltsev '11]

The Darwin operator

- ◇ The Darwin operator can be rewritten as penguin operator

$$\mathcal{O}_{\rho_D^3} = \frac{1}{4m_B} \bar{b}_v [iD_\mu, [iD^\rho, iD^\mu]] v_\rho b_v = -\frac{g_s^2}{4m_B} (\bar{b}_v \gamma^\mu t^a b_v) \sum_q (\bar{q} \gamma_\mu t^a q) + \mathcal{O}\left(\frac{1}{m_b}\right)$$

- ◇ Apply Fierz-transformation and use VIA

$$\frac{\rho_D^3(B_s^0)}{\rho_D^3(B_d^0)} = \frac{f_{B_s}^2 m_{B_s}}{f_B^2 m_B}$$

- ◇ Obtain $SU(3)_F$ breaking effects of $\sim 50\%$!

Overestimate size of $SU(3)_F$ breaking effects?