CRC TRR 257: Project C1c

Non-Perturbative Calculations for B-mesons

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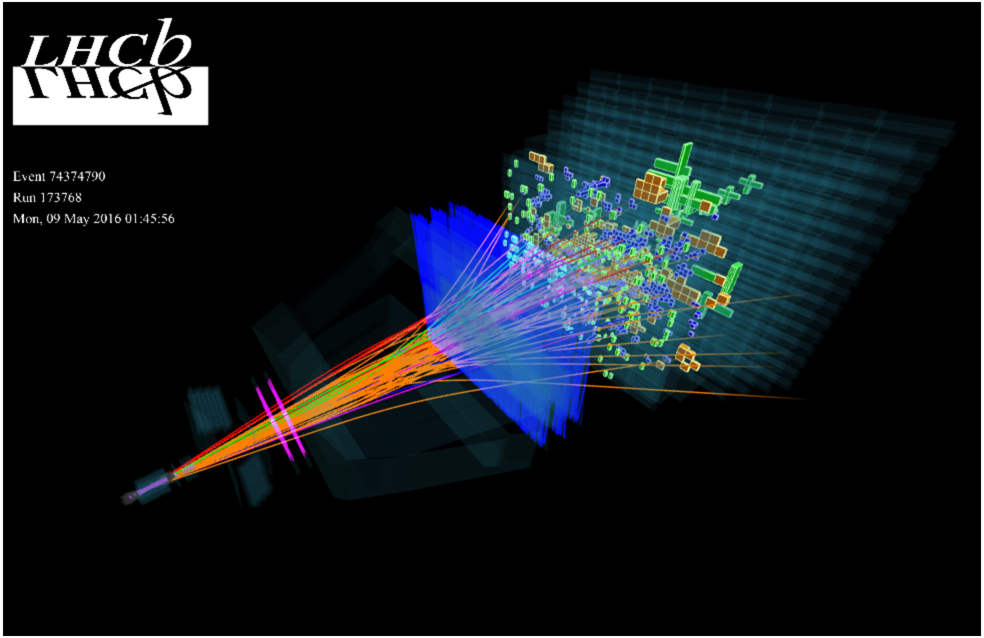
- Introduction
- Operator basis
- Gradient Flow
- Project Plan
- Summary



[https://cds.cern.ch/record/2151262]

- B-meson lifetimes and mixing are measured experimentally to high precision: [HFLAV '19]

$$\frac{\tau(B_s)}{\tau(B_d)} = 0.998 \pm 0.005, \qquad \frac{\tau(B^+)}{\tau(B_d)} = 1.076 \pm 0.004,$$
$$\Delta M_s = 17.741 \pm 0.050 \text{ ps}^{-1}, \qquad \Delta M_d = 0.5065 \pm 0.0019 \text{ ps}^{-1},$$
$$\Delta\Gamma_s = 0.082 \pm 0.005 \text{ ps}^{-1}, \qquad \Delta\Gamma_d = \text{not yet measured}$$

  ➥ Key observables for probing New Physics ➡ **high precision in theory needed!**

- For lifetimes and decay rates, we use the **Heavy Quark Expansion**
- Factorise observables into ➡ perturbative QCD contributions **[Project C1b, talk by A. Rusov]**
  ➡ **Non-Perturbative Matrix Elements**
- We will consider four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements of dimensions 6 and 7
- Two approaches: **QCD/HQET Sum Rules** and **Lattice QCD** (as part of RBC-UKQCD)
- **Gradient Flow:** A new method to match lattice calculations to the $\overline{\text{MS}}$ scheme using 'flowed' matrix elements and a perturbative matching matrix along an auxiliary dimension 'flow time'

- Mass difference of neutral mesons $\Delta M_q \, (q = d, s)$ governed by $\Delta B = 2$ four-quark operators
- In the SM, only dimension-6 $\mathcal{O}_1^q$ contributes

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_1^q \rangle = \langle \bar{B}_q | \mathcal{O}_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_1^q,$$

$$\mathcal{O}_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_2^q \rangle = \langle \bar{B}_q | \mathcal{O}_2^q | B_q \rangle = \frac{-5 M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_2^q,$$

$$\mathcal{O}_3^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1-\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_3^q \rangle = \langle \bar{B}_q | \mathcal{O}_3^q | B_q \rangle = \frac{M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_3^q,$$

$$\mathcal{O}_4^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \qquad \langle \mathcal{O}_4^q \rangle = \langle \bar{B}_q | \mathcal{O}_4^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{(m_b+m_q)^2} + \frac{1}{3} \right] f_{B_q}^2 M_{B_q}^2 B_4^q,$$

$$\mathcal{O}_5^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1+\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_5^q \rangle = \langle \bar{B}_q | \mathcal{O}_5^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{3(m_b+m_q)^2} + 1 \right] f_{B_q}^2 M_{B_q}^2 B_5^q.$$

- Matrix elements parameterised in terms of **decay constant** f_{B_q} and **bag parameters** B_i^q
- Five dimension-7 operators contributing to $\Delta\Gamma_q$: $R_0, \overset{(\sim)}{R}_2, \overset{(\sim)}{R}_3$ [Beneke, Buchalla, Dunietz '96]

- Matrix elements are calculated directly from lattice simulations

Decay constants well-known independently

$$\langle \mathcal{O}_i^q \rangle \quad ➡ \quad f_{B_q}\sqrt{B_i^q} \quad ➡ \quad B_i^q$$

- Higher precision in $SU(3)$-breaking ratios where some uncertainties cancel:

$$\frac{f_{B_s}}{f_{B_d}}, \quad \frac{B_1^s}{B_1^d}, \quad \xi = \frac{f_{B_s}\sqrt{B_1^s}}{f_{B_d}\sqrt{B_1^d}}$$

Use to determine $\left|\frac{V_{ts}}{V_{td}}\right|$

- [FLAG '21] reports on $\langle \mathcal{O}_1^q \rangle$ ➡ **tension between** most recent $2+1$ and $2+1+1$ calculations:

$$N_f = 2+1: \quad f_{B_s}\sqrt{\hat{B}_1^s} = 274(8)\,\mathrm{MeV}, \text{ [FNAL/MILC '16]}$$

$$N_f = 2+1+1: \quad f_{B_s}\sqrt{\hat{B}_1^s} = 256.1(5.7)\,\mathrm{MeV} \text{ [HPQCD '19A]}$$

- $\langle \mathcal{O}_{2-5}^{d,s} \rangle$ determined for $N_f = 2$ [ETM '13] and $N_f = 2+1$ [FNAL/MILC '11], [FNAL/MILC '16]
  - ➥ WIP by RBC-UKQCD + JLQCD at $N_f = 2+1$ [Boyle et al. '21]
- First lattice calculations for $\langle R_{2,3}^q \rangle$ and $\langle \widetilde{R}_{2,3} \rangle$ from [HPQCD '19B]
  - ➥ Suffers from large uncertainties e.g. from matching to continuum regularisation scheme

- In Vacuum Insertion Approximation (VIA), $B_i^q = 1$ ➡ write sum rules for $B_i^q - 1$
- $\langle \mathcal{O}_1^q \rangle$ calculated with HQET sum rules ➡ for ΔM_d [Grozin, Klein, Mannel, Pivovarov '16] [Kirk, Lenz, Rauh '17]
  ➡ for ΔM_s [King, Lenz, Rauh '19]
- Averages for $\langle \mathcal{O}_{1-5}^{d,s} \rangle$ combining lattice and sum rules found in [Luzio, Kirk, Lenz, Rauh '19]
- For $\langle R_{2,3}^q \rangle$ and $\langle \widetilde{R}_{2,3} \rangle$, [Mannel, Pecjak, Pivovarov '07] calculated condensate contributions
  - ➥ Very small deviations from VIA
  - ➥ Dominant 3-loop perturbative contributions missing
- Use HQET Sum Rules to determine perturbative part of dimension-7 matrix elements

- For lifetimes, we consider the dimension-6 $\Delta B = 0$ operators:

$$Q_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta, \qquad \langle Q_1^q \rangle = \langle B_q | Q_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \mathcal{B}_1^q,$$

$$Q_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1-\gamma_5) b^\beta, \qquad \langle Q_2^q \rangle = \langle B_q | Q_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 \mathcal{B}_2^q,$$

$$T_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma \gamma_\mu (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_1^q \rangle = \langle B_q | T_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \epsilon_1^q,$$

$$T_2^q = \bar{b}^\alpha (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_2^q \rangle = \langle B_q | T_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 \epsilon_2^q.$$

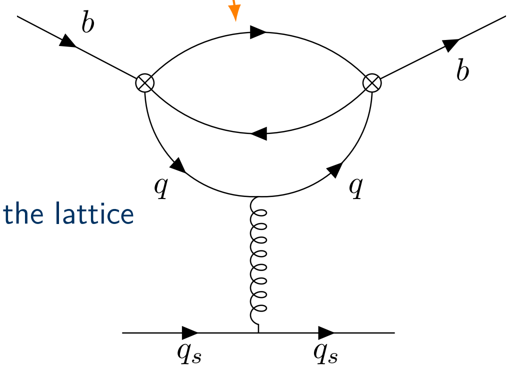
- In VIA, $\mathcal{B}_i^q = 1$ and $\epsilon_i^q = 0$
- Further dimension-7 four-quark operators, e.g. [King et al. '21]

$$P_1^q = m_q (\bar{b}^\alpha (1-\gamma_5) q^\alpha)(\bar{q}^\beta (1-\gamma_5) b^\beta), \qquad S_1^q = m_q (\bar{b}^\alpha (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta)(\bar{q}^\gamma (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta),$$

$$P_2^q = \frac{1}{m_b} (\bar{b}^\alpha \overleftarrow{D}_\nu \gamma_\mu (1-\gamma_5) D^\nu q^\alpha)(\bar{q}^\beta \gamma^\mu (1-\gamma_5) b^\beta), \; S_2^q = \frac{1}{m_b} (\bar{b}^\alpha \overleftarrow{D}_\nu \gamma_\mu (1-\gamma_5) (T^a)^{\alpha\beta} D^\nu q^\beta)(\bar{q}^\gamma \gamma^\mu (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta),$$

$$P_3^q = \frac{1}{m_b} (\bar{b}^\alpha \overleftarrow{D}_\nu (1-\gamma_5) D^\nu q^\alpha)(\bar{q}^\beta (1+\gamma_5) b^\beta), \qquad S_3^q = \frac{1}{m_b} (\bar{b}^\alpha \overleftarrow{D}_\nu (1-\gamma_5) (T^a)^{\alpha\beta} D^\nu q^\beta)(\bar{q}^\gamma (1+\gamma_5) (T^a)^{\gamma\delta} b^\delta)$$

- **Sum Rules:**
  - Subleading condensate contributions calculated [Baek, Lee, Liu, Song '97], [Cheng, Yang '98]
  - Matrix elements calculated recently ➡ for B_s mesons [King, Lenz, Rauh '21]
    ➡ for B_d, B^+ mesons [Kirk, Lenz, Rauh '17]
  - 'Eye' contractions also determined for the first time in [King, Lenz, Rauh '21]
  - Dimension-7 matrix elements to be calculated
- **Lattice:**
  - Early lattice studies 20 years ago [Pierro, Sachrajda '98] [Becirevic '01]
  - We aim to provide first $\Delta B = 0$ matrix element determinations on the lattice
  - Renormalisation of lattice matrix elements is non-trivial
  - Need a novel scheme to renormalise matrix elements...

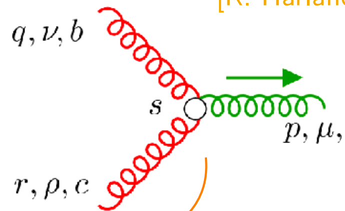


- Formulated by [Lüscher '10], [Lüscher '13] ➡ scale setting, RG β-function, **renormalisation**...
- Introduce auxiliary dimension, **flow time** t as a way to regularise the UV
- Extend gauge and fermion fields in flow time and express dependence with first-order differential equations:

$$\begin{aligned} \partial_t B_\mu(t,x) &= \mathcal{D}_\nu(t) G_{\nu\mu}(t,x), \quad & B_\mu(0,x) &= A_\mu(x), \\ \partial_t \chi(t,x) &= \mathcal{D}^2(t)\chi(t,x), & \chi(0,x) &= q(x). \end{aligned}$$

- Re-express effective Hamiltonian in terms of 'flowed' operators:

[R. Harlander]



$$\mathcal{H}_{\mathrm{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(t) \widetilde{\mathcal{O}}_n(t).$$

- Relate to regular operators in 'small-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(t) = \sum_m \zeta_{nm}(t) \mathcal{O}_m + O(t)$$

'flowed' MEs calculated on lattice replacing $A_\mu, q \to B_\mu, \chi$

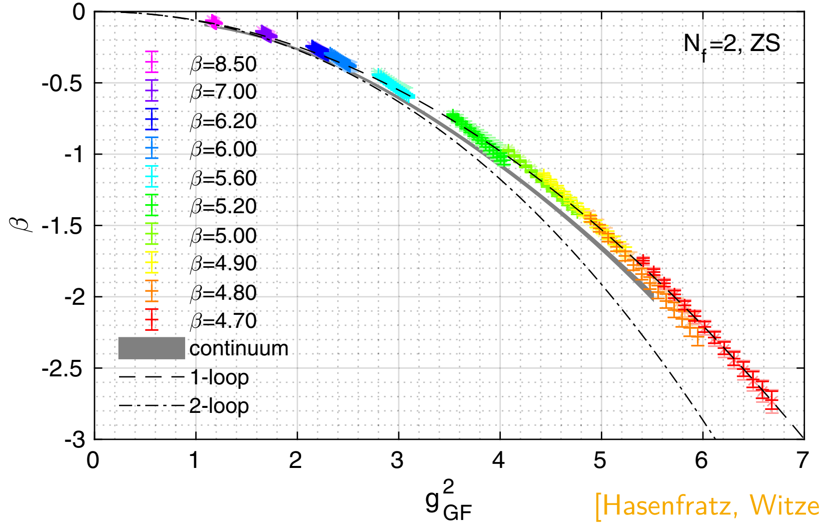
matching matrix calculated perturbatively

new Feynman diagrams

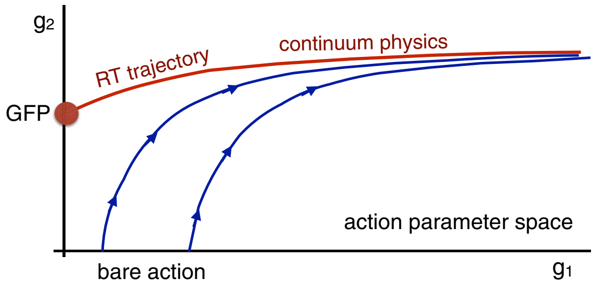
- RG β function:

$$\beta(g^2_{GF}) = \mu^2 \frac{dg^2_{GF}}{d\mu^2} = -t\frac{dg^2_{GF}}{dt}$$

- Define the gradient flow renormalised coupling g_{GF}
- Coloured data shows lattice predictions at bare couplings
- Grey band shows continuum limit



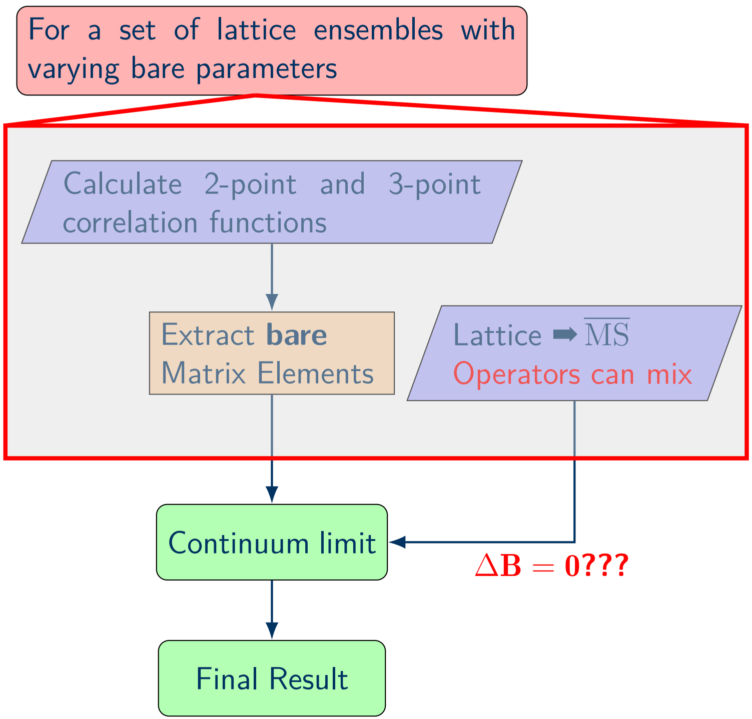
[Hasenfratz, Witzel '19]



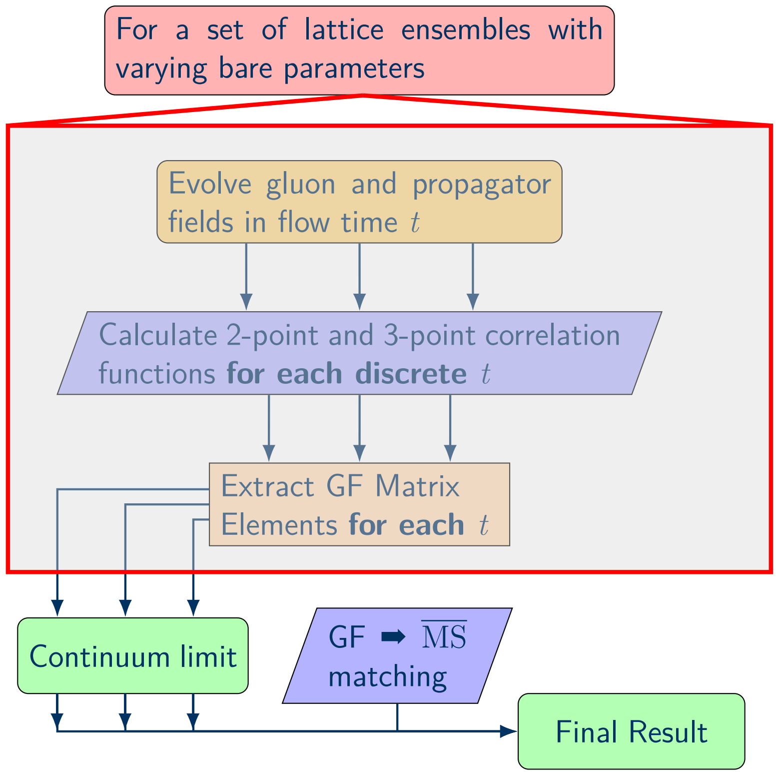
[Hasenfratz, Witzel '19]

- Need to remove UV fluctuations at small flow time
- Maximum flow time restricted to avoid finite volume effects
- RG flows of different bare lattice parameters approach 'renormalised trajectory' differently

- Standard Lattice Calculation



- **With Gradient Flow Renormalisation**



1) Sum rules for dimension-7 $\Delta B = 2$ operators for mixing (Lenz, Pivovarov)
2) Sum rules for dimension-7 $\Delta B = 0$ operators for lifetimes (Lenz, Pivovarov)
3) Gradient flow perturbative matching matrix calculation (Harlander, Lange)
4) Flowed matrix elements from lattice QCD (Harlander, Witzel, Black)
   - ➥ Use dimension-6 $\Delta B = 2$ operators as proof of principle
   - ➥ Dimension-7 $\Delta B = 2$ operators and dimension-6,-7 $\Delta B = 0$ operators
5) Extrapolation to zero flow time (Harlander, Witzel, Lange)
6) B-meson phenomenology in the SM and beyond (Lenz, Harlander, Nierste)
   - ➥ Analysis of SM predictions and comparisons with experiment
   - ➥ Determination of CKM matrix elements and unitarity triangle parameters
   - ➥ BSM physics in mixing and lifetimes

- Covered the plans for Project C1c of the next funding application of the CRC
- **First time** calculations using HQET sum rules of dimension-7 matrix elements
- **Pioneer** a **new Gradient Flow** renormalisation and matching scheme between lattice and $\overline{\mathrm{MS}}$
- **Validate** this GF scheme with well-known $\Delta B = 2$ matrix elements
- Provide **first lattice calculations** of $\Delta B = 0$ matrix elements
- Study **implications** of theoretical predictions on the SM and BSM theories

Thank you for your attention!