AutoEFT: Automating Effective Field Theories

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Outline

EFT operator bases

Some details on the explicit construction

Implementation of the algorithm

The Standard Model as EFT (SMEFT)

Lagrangian

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_i C_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)} + \dots$$

- $\mathcal{O}$s invariant under Lorentz and gauge symmetry

Operators

$$\mathcal{O}^{(5)} = (L_\alpha H)(L^\alpha H)$$

$$\mathcal{O}^{(6)} = (H^\dagger H)\, G^A_{\mu\nu} G^{A\,\mu\nu},\ (H^\dagger \tau^I H)\, W^I_{\mu\nu} B^{\mu\nu},\ \dots$$

$$\vdots$$

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$$\vdots$$

Redundancies

1. **Fierz identities:** $g_{\mu\nu}\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}$

2. **Equations of motion:** $i\not{D}\psi + J_{\psi} = 0$

3. **Integration by parts:** $X\, D_{\mu} Y \sim -D_{\mu} X\, Y$

4. **Flavour relations:** $(L^{f_1} H)(L^{f_2} H) = (L^{f_2} H)(L^{f_1} H)$

Brief History of SMEFT

1980 — Weinberg: $d = 5$ & $n_f = 1$
$\rightarrow$ 1 Operator (*LLHH*)

1986 — Buchmüller-Wyler: $d = 6$ & $n_f = 1$
$\rightarrow$ 80 Operators (22 redundant, 1 missing)

⋮

2010 — Grzadkowski *et al.*: $d = 6$ & $n_f = 1$
$\rightarrow$ 59 Operators

2020 — H.-L. Li *et al.*: $d = 8, 9$ & $n_f = 1, 3$

[Li,Ren,Shu,Xiao,Yu,Zheng, 2020]

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Operator Structure

Explicit index notation

$$\epsilon^{abc}\,(L^i\,Q_{j,a})\,(Q_{i,b}\,Q_c^j) = \epsilon^{abc}\,L^i_\alpha\,Q^\alpha_{j,a}\,Q^\beta_{i,b}\,Q^j_{c,\beta}$$

$$= \underbrace{\epsilon^{abc}}_{T_{\mathrm{SU}(3)}}\;\underbrace{\epsilon^{ik}\epsilon^{jl}}_{T_{\mathrm{SU}(2)}}\;\underbrace{\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}}_{T_{\mathrm{Lorentz}}}\;\underbrace{L_i^\alpha\,Q^\beta_{j,a}\,Q^\gamma_{k,b}\,Q^\delta_{l,c}}_{\prod_n \Phi_n}$$

operators of same *type* only differ in choice of T
e.g.: $T_{\mathrm{SU}(2)} = \epsilon^{ij}\epsilon^{kl}$

$\Rightarrow$ $T_{\mathrm{SU}(3)}$, $T_{\mathrm{SU}(2)}$ & T_{Lorentz} govern the symmetry

SMEFT Operators

General operator

$$\mathcal{O} = T^{\{g\}}_{\mathrm{SU}(3)} T^{\{h\}}_{\mathrm{SU}(2)} T^{\{l\}}_{\mathrm{Lorentz}} \times \prod_{i=1}^{N} (D^{n_i}\Phi_i)_{\{g\},\{h\},\{l\}}$$

Invariant tensors

$$T_{\mathrm{SU}(3)} \in \langle f^{ABC}, d^{ABC}, \delta^{AB}, (\lambda^A)_a^b, \epsilon_{abc}, \epsilon^{abc} \rangle$$

$$T_{\mathrm{SU}(2)} \in \langle \epsilon^{IJK}, \delta^{IJ}, (\tau^I)_i^j, \epsilon_{ij}, \epsilon^{ij} \rangle$$

$$T_{\mathrm{Lorentz}} \in \langle \sigma^{\mu\nu}_{\alpha\beta}, \bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}, \sigma^{\mu}_{\alpha\dot{\alpha}}, \bar{\sigma}^{\mu\,\dot{\alpha}\alpha}, \epsilon^{\alpha\beta}, \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \rangle$$

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Fundamental Representation

Gauge group: $SU(3)_C \times SU(2)_W \times U(1)_Y$

$$G_{abc} = \epsilon_{acd}(\lambda^A)^d_b G^A \qquad W_{ij} = \epsilon_{jk}(\tau^I)^k_i W^I$$

Lorentz group: $SL(2, \mathbb{C}) = SU(2)_l \times SU(2)_r$

$$\phi \in (0,0) \qquad \psi_\alpha \in (1/2, 0) \qquad F_{L\,\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0)$$

$$D_{\alpha\dot{\alpha}} = D_\mu \sigma^\mu_{\alpha\dot{\alpha}} \in (1/2, 1/2)$$

Explicit Contraction of Lorentz Indices

Integration by parts

$$-D_\mu\phi_1\,\phi_2\phi_3 D^\mu\phi_4 = \phi_1 D_\mu\phi_2\,\phi_3 D^\mu\phi_4 + \phi_1\phi_2 D_\mu\phi_3\,D^\mu\phi_4$$

Operators $\Leftrightarrow$ Tableaux

$$-\begin{array}{|c|c|}\hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} = \underbrace{-\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} + \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}}_{\text{semi-standard Young tableaux}}$$

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Operators ⇔ Tableaux

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⇒ 2 independent Lorentz invariants

Explicit Contraction of Gauge Group Indices

Littlewood–Richardson rule

$$Q : \boxed{j} \qquad Q : \boxed{k} \qquad Q : \boxed{l} \qquad L : \boxed{i} \qquad W : \boxed{m_1}\boxed{m_2}$$

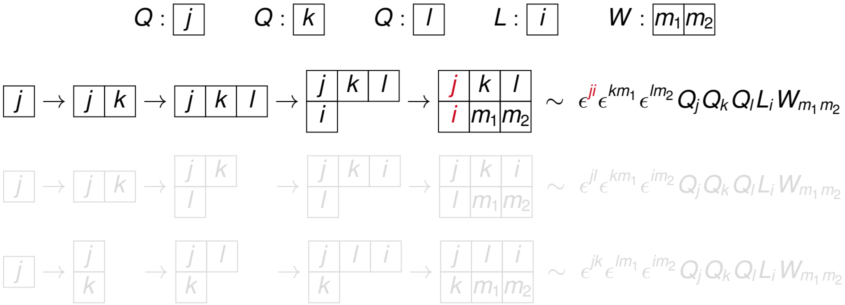
$$\sim \epsilon^{ji}\epsilon^{km_1}\epsilon^{lm_2} Q_j Q_k Q_l L_i W_{m_1 m_2}$$

$$\sim \epsilon^{jl}\epsilon^{km_1}\epsilon^{im_2} Q_j Q_k Q_l L_i W_{m_1 m_2}$$

$$\sim \epsilon^{jk}\epsilon^{lm_1}\epsilon^{im_2} Q_j Q_k Q_l L_i W_{m_1 m_2}$$

Explicit Contraction of Gauge Group Indices

Littlewood–Richardson rule



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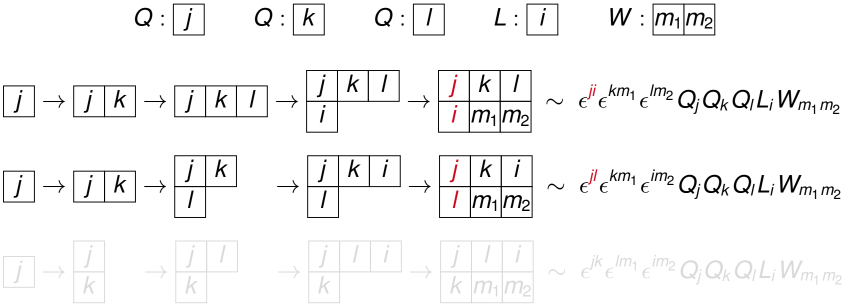

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$$\begin{array}{|c|}\hline j \\ \hline \end{array} \rightarrow \begin{array}{|c|c|}\hline j & k \\ \hline \end{array} \rightarrow \begin{array}{|c|c|c|}\hline j & k & l \\ \hline \end{array} \rightarrow \begin{array}{|c|c|c|}\hline j & k & l \\ \hline i \\ \cline{1-1} \end{array} \rightarrow \begin{array}{|c|c|c|}\hline j & k & l \\ \hline i & m_1 & m_2 \\ \hline \end{array} \sim \epsilon^{ji}\epsilon^{km_1}\epsilon^{lm_2} Q_j Q_k Q_l L_i W_{m_1 m_2}$$

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$\Rightarrow$ **3 independent SU(2) invariants**

Multiple Generations

Relations between generations

$$\mathcal{O}_{LLHH}^{f_1 f_2} = (L^{f_1} H)(L^{f_2} H) = \mathcal{O}_{LLHH}^{f_2 f_1}$$

$\Rightarrow$ 3 independent operators for $n_f = 2$:

$$\mathcal{O}_{LLHH}^{ee}, \quad \mathcal{O}_{LLHH}^{e\mu}, \quad \mathcal{O}_{LLHH}^{\mu\mu}$$

Higher rank tensors

$$\mathcal{O}^{f_1 f_2 f_3} = \mathcal{O}_{[3]}^{f_1 f_2 f_3} + \mathcal{O}_{[2,1]}^{f_1 f_2 f_3} + \mathcal{O}_{[1^3]}^{f_1 f_2 f_3} \qquad \mathcal{O}_{\lambda}^{f_1 f_2 f_3} = b_x^{\lambda}\, \mathcal{O}^{f_1 f_2 f_3}$$

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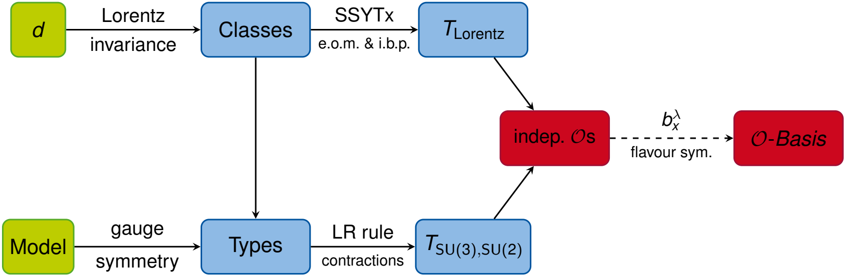
Automated Construction

AutoEFT



- implemented in `SageMath + FORM`

Custom Models

```yaml
name: SMEFT

# arbitrary SU(N) and
# U(1) groups

symmetries:
  SUN:
    SU3_C:
      N: 3
    SU2_W:
      N: 2
  U1:
    U1_Y
```

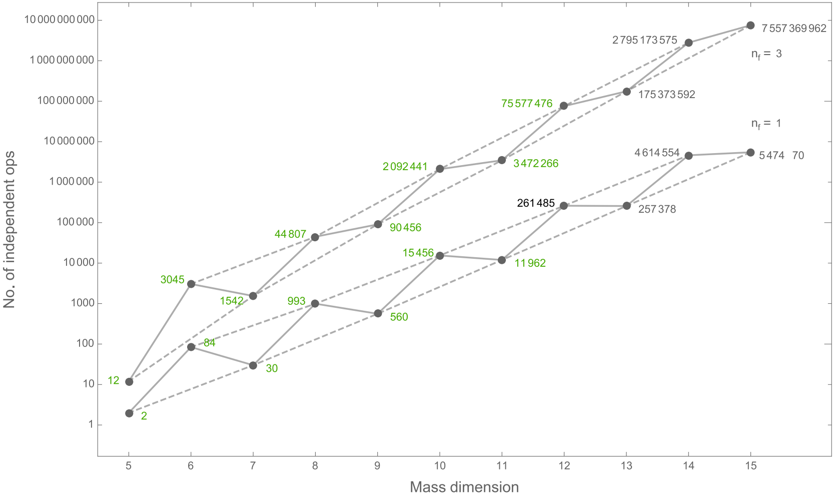
```yaml
fields:

# arbitrary fields in
# arbitrary representations

  G:
    reprs:
      Lorentz: -1
      SU3_C: [2,1]
  uC:
    reprs:
      Lorentz: -1/2
      SU3_C: [1,1]
      U1_Y: -2/3
    anticommute: True
    nf: 3
```

Comparison to Hilbert Series

[Henning,Lu,Melia,Murayama, 2017]



Summary

Operator Basis

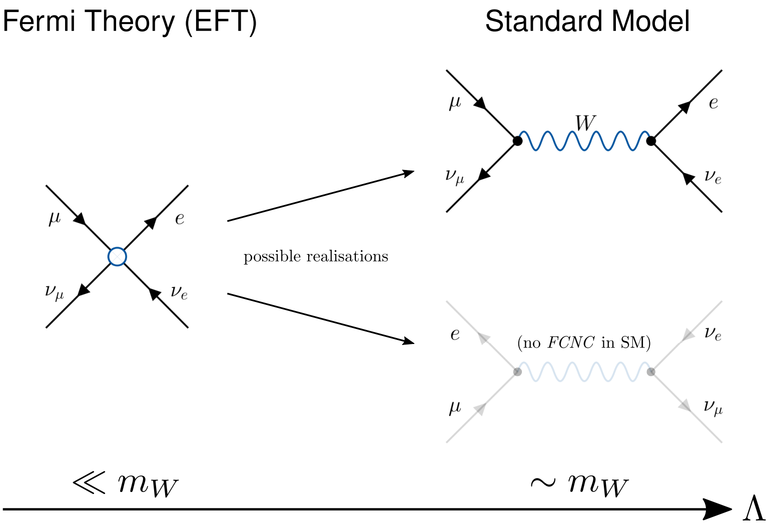
- Automated EFT construction
- Custom low energy models
- Arbitrary mass dimensions d
- $d = 9$ SMEFT in $\lesssim 1\,\text{min}$
- Validated up to $d = 12$

Work in progress

- Operator projection
- 1-loop matching using BSUOLEA [Summ, 2020]

Backup

New Physics in Effective Field Theories



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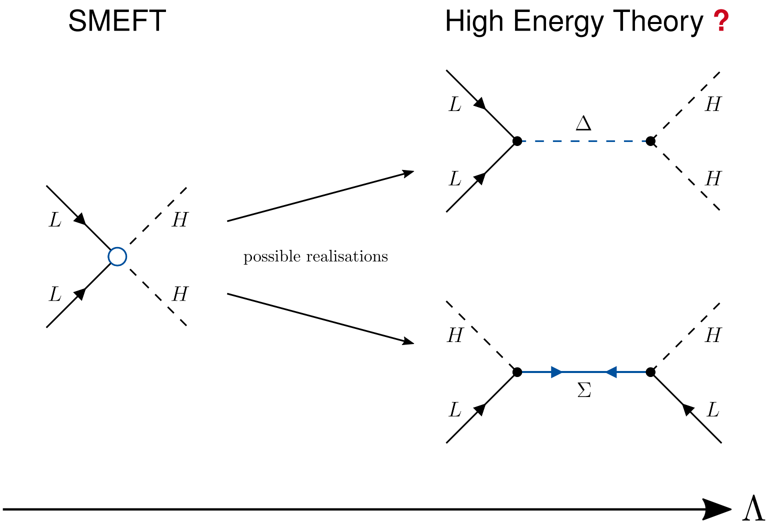

Redundancies

Fierz identities

$$g_{\mu\nu}\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}$$

$$\epsilon^{\alpha\beta}\delta^{\gamma}_{\kappa} + \epsilon^{\beta\gamma}\delta^{\alpha}_{\kappa} + \epsilon^{\gamma\alpha}\delta^{\beta}_{\kappa} = 0$$

$$\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}\delta^{\dot{\kappa}}_{\dot{\gamma}} + \tilde{\epsilon}_{\dot{\beta}\dot{\gamma}}\delta^{\dot{\kappa}}_{\dot{\alpha}} + \tilde{\epsilon}_{\dot{\gamma}\dot{\alpha}}\delta^{\dot{\kappa}}_{\dot{\beta}} = 0$$

Field strength tensor

$$[D_\mu, D_\nu] = -iF_{\mu\nu}$$

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Redundancies

Equations of motion

$$
\begin{aligned}
D^2\phi + J_\phi &= 0 \\
i\not{D}\psi + J_\psi &= 0 \\
D_\mu F^{\mu\nu} + J_A^\nu &= 0
\end{aligned}
$$

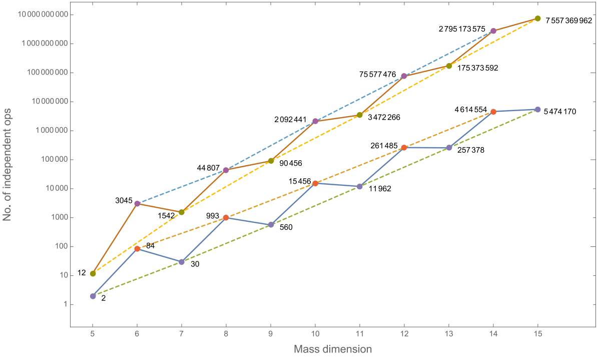
Integration by parts

$$D_\mu(XY) \sim 0 \implies X\,D_\mu Y \sim -D_\mu X\,Y$$

Hilbert Series

$$H(p, \phi_1, \ldots, \phi_n) = \int \mathrm{d}\mu_{\text{conf.}}\, \mathrm{d}\mu_{\text{gauge}} \sum_{i=1}^{\infty} p^i \chi^*_{[i;0]} \prod_j \mathrm{PE}[\phi_j \chi_j(q, \alpha, \beta)]$$

[Henning,Lu,Melia,Murayama, 2017]



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Building Blocks

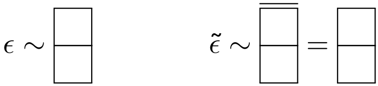
$$(D^{r-|h|}\Phi)_{\alpha^{(1)}\dots\alpha^{(r-h)}\dot\alpha^{(1)}\dots\dot\alpha^{(r+h)}} \equiv \begin{cases} D_{\alpha^{(1)}\dot\alpha^{(1)}}\dots D_{\alpha^{(r+h)}\dot\alpha^{(r+h)}}\Phi_{\alpha^{(r+h+1)}\dots\alpha^{(r-h)}}\,, & \text{h<0} \\ D_{\alpha^{(1)}\dot\alpha^{(1)}}\dots D_{\alpha^{(r-h)}\dot\alpha^{(r-h)}}\Phi_{\dot\alpha^{(r-h+1)}\dots\dot\alpha^{(r+h)}}\,, & \text{h>0} \end{cases}$$

$$\begin{aligned} D_{[\alpha\dot\alpha}D_{\beta]\dot\beta}\phi &= -\epsilon_{\alpha\beta}\tilde\epsilon_{\dot\alpha\dot\beta}D^2\phi + \frac{i}{2}\epsilon_{\alpha\beta}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}[D_\mu, D_\nu]\phi \\ D_{[\alpha\dot\alpha}\psi_{\beta]} &= -\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot\alpha} \\ D_{[\alpha\dot\alpha}F_{L\,\beta]\gamma} &= 2\epsilon_{\alpha\beta}\sigma^{\nu}_{\gamma\dot\alpha}D^\mu F_{\mu\nu} \end{aligned}$$

$$(D^{r-|h|}\Phi)^{(\dot\alpha^{(1)}\dots\dot\alpha^{(r+h)})}_{(\alpha^{(1)}\dots\alpha^{(r-h)})} \equiv (D^{r-|h|}\Phi)^{\dot\alpha^{r+h}}_{\alpha^{r-h}} \in \left(\frac{r-h}{2}, \frac{r+h}{2}\right)$$

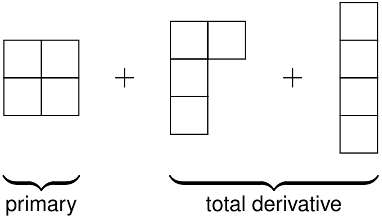
Lorentz Structure as $\mathrm{SU}(N)$ State

Young diagrams for $N = 4$ fields



$$\Downarrow$$

$$T_{\text{Lorentz}} = (\tilde{\epsilon}_{\dot{\alpha}_k \dot{\alpha}_l})^{\otimes \tilde{n}} (\epsilon^{\alpha_i \alpha_j})^{\otimes n} \overset{n=\tilde{n}=1}{\sim}$$



Field Definitions

$$q_L = \begin{pmatrix} Q \\ 0 \end{pmatrix} \,, \; u_R = \begin{pmatrix} 0 \\ u_{\mathbb{C}}^{\dagger} \end{pmatrix} \,, \; d_R = \begin{pmatrix} 0 \\ d_{\mathbb{C}}^{\dagger} \end{pmatrix} \,, \; l_L = \begin{pmatrix} L \\ 0 \end{pmatrix} \,, \; e_R = \begin{pmatrix} 0 \\ e_{\mathbb{C}}^{\dagger} \end{pmatrix}$$

$$F_{L/R} = \frac{1}{2}\left(F \pm i\tilde{F}\right)$$

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TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

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