Theory status of heavy meson lifetimes

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Introduction

Heavy meson lifetimes: experiment

- Lifetimes of heavy mesons are measured precisely at the experiment
- HFLAV and PDG

	B^+	B_d^0	B_s^0
τ [ps]	1.638 ± 0.004	1.519 ± 0.004	1.516 ± 0.006
$\tau(B_q)/\tau(B_d^0)$	1.076 ± 0.004	1	0.998 ± 0.005
	D^0	D^+	D_s^+
τ [ps]	0.4101 ± 0.0015	1.040 ± 0.007	0.504 ± 0.004
$\tau(D_q)/\tau(D^0)$	1	2.54 ± 0.02	1.20 ± 0.01

New measurement of $\tau(D^{+,0})$ by Belle II (ArXiv:2108.03216) not yet included

Heavy meson lifetimes: theory

- The total width of hadron H is given by $H = B, D$

$$\Gamma(H) = \frac{1}{2m_H} \sum_X \int_{\rm PS} (2\pi)^4 \delta^{(4)}(p_H - p_X)\ |\langle X(p_X)|\mathcal{H}_{\rm eff}|H(p_H)\rangle|^2$$

Optical Theorem

$$= \frac{1}{2m_H} \, {\rm Im}\, \langle H(p_H)|\, i \int d^4x \; T\left\{\mathcal{H}_{\rm eff}(x), \mathcal{H}_{\rm eff}(0)\right\} |H(p_H)\rangle$$

- The ratio of lifetimes $Q = b, c$

$$\frac{\tau(H_1)}{\tau(H_2)} = \frac{\Gamma_Q + \delta\Gamma_{H_2}}{\Gamma_Q + \delta\Gamma_{H_1}} = 1 + \left(\delta\Gamma_{H_2} - \delta\Gamma_{H_1}\right)\tau(H_1)$$

- May be sensitive to New Physics contributions

$$\frac{\tau(H_1)}{\tau(H_2)} = \frac{\Gamma_Q + \delta\Gamma_{H_2}}{\Gamma_Q + \delta\Gamma_{H_1}} = 1 + \left(\delta\Gamma^{\rm SM}_{H_2} - \delta\Gamma^{\rm SM}_{H_1}\right)\tau(H_1) + \left(\delta\Gamma^{\rm NP}_{H_2} - \delta\Gamma^{\rm NP}_{H_1}\right)\tau(H_1)$$

HQE: Bottom vs Charm

- Heavy Quark Expansion (HQE)
  - ▷ $p_Q = m_Q v + k, \qquad k \sim \Lambda$ $\qquad v = p_H/m_H$
  - ▷ $Q(x) = e^{-i m_Q (v \cdot x)} Q_v(x)$
- Expansion in powers of Λ/m_Q
- HQE in the bottom sector

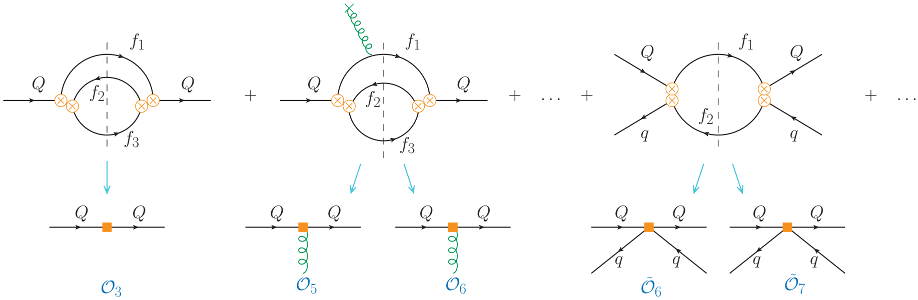
$$\frac{\Lambda}{m_b} \sim 0.12 \qquad \alpha_s(m_b) \sim 0.22, \quad \frac{\alpha_s(m_b)}{\pi} \sim 0.07$$

- HQE in the charm sector

$$\frac{\Lambda}{m_c} \sim 0.39 \qquad \alpha_s(m_c) \sim 0.38, \quad \frac{\alpha_s(m_c)}{\pi} \sim 0.12$$

$\Lambda = 0.5\,\mathrm{GeV},\ m_b = 4.2\,\mathrm{GeV},\ m_c = 1.27\,\mathrm{GeV}$

HQE: diagrams



$$\Gamma(H) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right]$$

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_i^{(1)} + \ldots$$

Theory status of B- and D-meson lifetimes

Theory status of the B-meson lifetimes

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$

	SL	NL
Γ_3	NNNLO [New]*	NNLO (partly)
Γ_5	NLO	LO
Γ_6	NLO	LO [New]**
Γ_7	LO	–
Γ_8	LO	–
$\tilde{\Gamma}_6$	NLO	NLO
$\tilde{\Gamma}_7$	LO	LO

	Source
$\langle \mathcal{O}_5 \rangle$	Fit to exp. data;
	Lattice QCD; Sum Rules
$\langle \mathcal{O}_6 \rangle$	Fit to exp. data;
	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET Sum Rules [New]*
$\langle \tilde{\mathcal{O}}_7 \rangle$	VIA

* [King, Lenz, Rauh, 2112.03691]

* [Fael, Schönwald, Steinhauser, 2011.11655]

** [Lenz, Piscopo, AR, 2004.09527], [Mannel, Moreno, Pivovarov, 2004.09485]

Theory status of the D-meson lifetimes

$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \ldots \right]$$

	SL	NL
Γ_3	NNNLO [New]*	NLO
Γ_5	NLO	LO
Γ_6	NLO	LO [New]**
Γ_7	LO	–
Γ_8	LO	–
$\tilde{\Gamma}_6$	NLO	NLO
$\tilde{\Gamma}_7$	LO	LO

	Source
$\langle \mathcal{O}_5 \rangle$	Heavy quark symmetry;
	Spectroscopy relations
$\langle \mathcal{O}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET Sum Rules [New]⋆
$\langle \tilde{\mathcal{O}}_7 \rangle$	VIA

⋆ [King, Lenz, Rauh, 2112.03691]

* [Fael, Schönwald, Steinhauser, 2011.11655]

** [Lenz, Piscopo, AR, 2004.09527], [King, Lenz, Piscopo, Rauh, AR, Vlahos, 2109.13219]

The Darwin operator contribution

based on [Lenz, Piscopo, AR, ArXiv: 2004.09527]

Outline of calculation

- Using the optical theorem:

$$\Gamma_{\rm NL}(B) = \frac{1}{2m_B} {\rm Im} \langle B(p_B)| \, i \int d^4x \, T\left\{\mathcal{L}_{\rm eff}(x), \mathcal{L}_{\rm eff}(0)\right\} |B(p_B)\rangle$$

- Effective Lagrangian describing NL decay $b \to q_1 \bar{q}_2 q_3$

$q_1 = u, c$
$\bar{q}_2 = \bar{u}, \bar{c}$
$q_3 = d, s$

$$\mathcal{L}_{\rm eff}(x) = -\frac{4 G_F}{\sqrt{2}} V^*_{q_1 b} V_{q_2 q_3} \left[C_1 \, Q_1(x) + C_2 \, Q_2(x)\right] + {\rm h.c.}$$

$$Q_1 = (\bar{q}_1^i \Gamma_\mu b^i)(\bar{q}_3^j \Gamma^\mu q_2^j) \qquad Q_2 = (\bar{q}_1^i \Gamma_\mu b^j)(\bar{q}_3^j \Gamma^\mu q_2^i)$$

- Contribution of penguin operators $Q_{n>2}$ are neglected

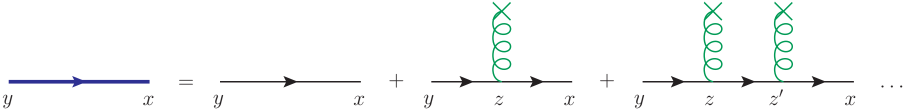
- Use the relation $Q_2 = \frac{Q_1}{N_c} + 2\,T$

T - colour-octet operator

Outline of calculation

- Expansion of the quark propagator in the external soft gluon field

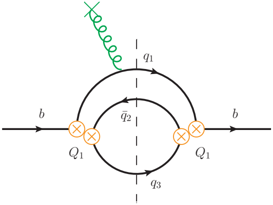
$$S_{ij}(x,y) = S^{(0)}(x-y)\,\delta_{ij} + \underbrace{S^{(1)\,a}(x,y)}_{\sim G_{\mu\nu},\, D_\rho G_{\mu\nu}}\; t^a_{ij} + \ldots$$



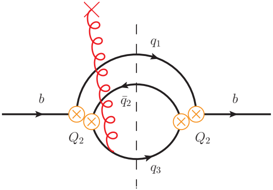
$$G_{\mu\nu} = G^a_{\mu\nu} t^a = -i\,[iD_\mu, iD_\nu] \qquad\qquad D_\rho G_{\mu\nu} = -[iD_\rho, [iD_\mu, iD_\nu]]$$

Diagrams

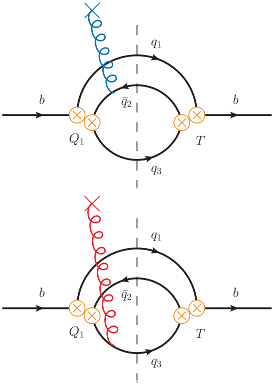
$Q_1 \otimes Q_1$



$Q_2 \otimes Q_2$



$Q_1 \otimes T$



$$Q_1 \otimes Q_2 = \frac{1}{N_c}(Q_1 \otimes Q_1) + 2\,(Q_1 \otimes T)$$

Computation of the diagrams

- Computation in $d = 4$, but keeping $m_{u,d,s}$ finite as a IR regulator
- Two-loop tensor integrals of rank $r = 1, 2, 3, 4$
- IBP-reduction to set of master integrals (using *LiteRed* package)
- Master integrals of the sunset type e.g. [Remidi, Tancredi (2016)]
- Expanding the matrix elements up to dimension-6 [Dassinger, Mannel, Turczyk (2007)]
- Non-perturbative parameters at dimension-5 and dimension-6

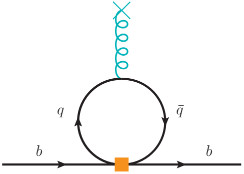
$$
\begin{aligned}
2m_B\,\mu_\pi^2(B) &= -\langle B(p_B)|\bar{b}_v(iD_\mu)(iD^\mu)b_v|B(p_B)\rangle \\
2m_B\,\mu_G^2(B) &= \langle B(p_B)|\bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})b_v|B(p_B)\rangle \\
2m_B\,\rho_D^3(B) &= \langle B(p_B)|\bar{b}_v(iD_\mu)(iv\cdot D)(iD^\mu)b_v|B(p_B)\rangle \\
2m_B\,\rho_{LS}^3(B) &= \langle B(p_B)|\bar{b}_v(iD_\mu)(iv\cdot D)(iD_\nu)(-i\sigma^{\mu\nu})\,b_v|B(p_B)\rangle
\end{aligned}
$$

Role of the four-quark operators

- At dimension-six, also four-quark operators arise

$$\tilde{\mathcal{O}}_{6,i}^{(q)} = (\bar{b}_v \Gamma_i \, q)(\bar{q} \, \Gamma_i \, b_v), \qquad i = 1, \dots 4$$

- One-loop matrix elements of the operators $\tilde{\mathcal{O}}_{6,i}^{(q)}$

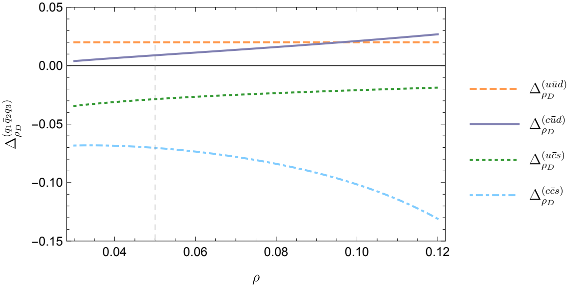


$$\langle \, \tilde{\mathcal{O}}_{6,i}^{(q)} \, \rangle^{(0)} \quad \sim \quad \left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m_q^2}\right) + a_i \right] \rho_D^3(B)$$

- All IR divergences cancel

Result

$$\Gamma^{(\rho_D)}_{\rm NL}(B) = \Gamma_0 \left(3\, C_1^2 \underbrace{\mathcal{C}^{(q_1 \bar{q}_2 q_3)}_{\rho_D,\,11}}_{\rm SL} + 2\, C_1 C_2 \underbrace{\mathcal{C}^{(q_1 \bar{q}_2 q_3)}_{\rho_D,\,12}}_{\rm new} + 3\, C_2^2 \underbrace{\mathcal{C}^{(q_1 \bar{q}_2 q_3)}_{\rho_D,\,22}}_{\rm new} \right) \frac{\rho_D^3}{m_b^3}$$



- Confirmed independently by using another approach

  [Mannel, Moreno, Pivovarov, 2004.09485]

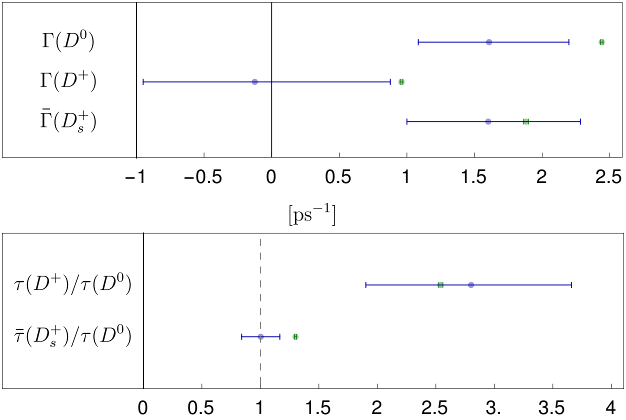
- Similar analysis also for the charm lifetimes

  [King, Lenz, Piscopo, Rauh, AR, Vlahos, 2109.13219]

Some results

Charm lifetimes and ratios

[King, Lenz, Piscopo, Rauh, AR, Vlahos, 2109.13219]



Confirmed also by [Gratrex, Melic, Nizandzic, 2204.11935]

Bottom lifetimes and ratios

[Lenz, Piscopo, AR (in progress)]

- Good agreement between theory and data for $\tau(B^+)/\tau(B^0)$

$$\left[\frac{\tau(B^+)}{\tau(B^0)}\right]^{\mathrm{HQE}} = 1.085 \pm 0.021$$

[Preliminary!]

$$\left[\frac{\tau(B^+)}{\tau(B^0)}\right]^{\mathrm{HFLAV}} = 1.076 \pm 0.004$$

- Also good agreement for $\Gamma(B^+), \Gamma(B_d^0), \Gamma(B_s^0)$
- Large effect of Darwin operator in $\tau(B_s^0)/\tau(B_d^0)$

$\rho_D^3(B) = 0.185\,\mathrm{GeV}^3$

$$\left[\frac{\tau(B_s^0)}{\tau(B_d^0)}\right]^{\mathrm{HQE}} = (1.004 \pm 0.005) + 0.0456 \times \underbrace{\left[\frac{\rho_D^3(B_s)}{\rho_D^3(B)} - 1\right]}_{\approx 46\%\ \text{(from E.o.M.)}}$$

[Preliminary!]

$$\left[\frac{\tau(B_s^0)}{\tau(B_d^0)}\right]^{\mathrm{HFLAV}} = 0.998 \pm 0.005$$

Future plans

Outlook

- Plan for project C1b (lifetimes section, perturbative side)

$$\Gamma(H) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \Gamma_7 \frac{\langle \mathcal{O}_7 \rangle}{m_Q^4} + ... + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + ... \right]$$

  - ▷ $\Gamma_5 = \Gamma_5^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_5^{(1)} + \ldots$ Siegen
  - ▷ $\Gamma_6 = \Gamma_6^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_6^{(1)} + \ldots$ Siegen
  - ▷ $\Gamma_7 = \Gamma_7^{(0)} + \ldots$ Siegen
  - ▷ $\tilde{\Gamma}_6 = \tilde{\Gamma}_6^{(0)} + \frac{\alpha_s}{4\pi} \tilde{\Gamma}_6^{(1)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \tilde{\Gamma}_6^{(2)} \ldots$ Karlsruhe
  - ▷ $\tilde{\Gamma}_7 = \tilde{\Gamma}_7^{(0)} + \frac{\alpha_s}{4\pi} \tilde{\Gamma}_7^{(1)} + \ldots$ Karlsruhe

- Non-perturbative side – project C1c [see talk by Matthew Black]

green - known orange - planned