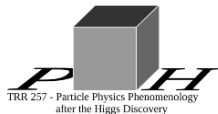


Mixed QCD-electroweak corrections to Higgs plus jet production at the LHC

Marco Bonetti

CRC TRR 257 Young Scientists Meeting



In collaboration with
E. Panzer, V. A. Smirnov, L. Tancredi
[2007.09813] [2203.17202]

- 1 Motivations & Overview
- 2 Process
- 3 Master Integrals
- 4 Results
- 5 Conclusions

Higgs boson at the LHC

[1602.00695] [1610.07922] [1802.00833]

Higgs production modes

ggH	VVH	WH	ZH	$t\bar{t}H$	Total
$44.1^{+11\%}_{-11\%}$	$3.78^{+2\%}_{-2\%}$	$1.37^{+2\%}_{-2\%}$	$0.88^{+5\%}_{-5\%}$	$0.51^{+9\%}_{-13\%}$	50.6

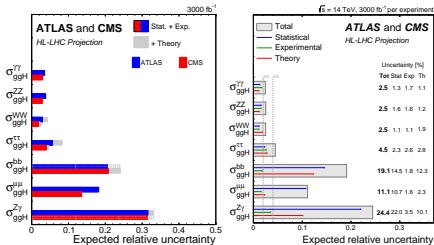
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HL-LHC projections



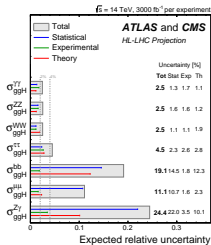
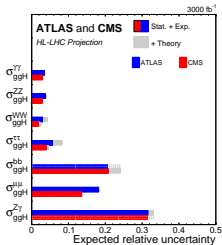
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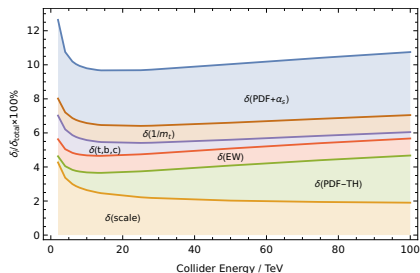
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HL-LHC projections



Theoretical uncertainties



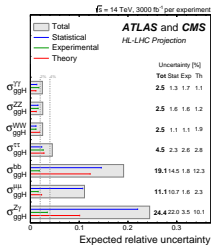
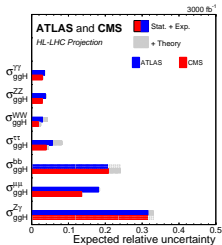
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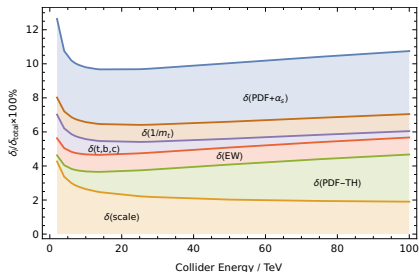
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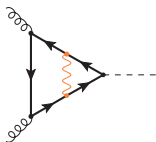
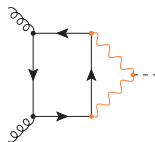


What is the source of $\delta(\text{EW})$?

QCD-EW contributions

[ph0404071] [ph0407249] [ph0610033]

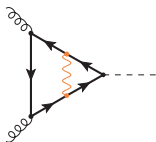
Exact LO Electroweak contributions

Yukawa coupling $\alpha_S \alpha Y_t$ Electroweak coupling $\alpha_S \alpha^2 v$ 

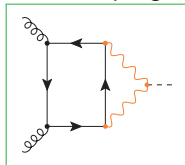
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Yukawa coupling $\alpha_S \alpha Y_t$ 

- Dominated by **top quark**
- $\sim 0.5\%$ of $\sigma_{\text{QCD}}^{\text{LO}}$

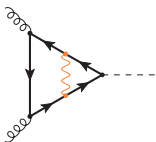
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- Dominated by **light quarks**
- $+5.3\%$ of $\sigma_{\text{QCD}}^{\text{LO}}$

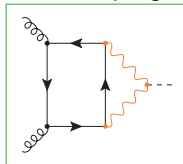
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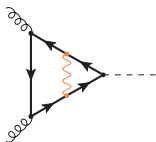
NLO QCD corrections in HEFT

$$\text{[Diagrams]} \xrightarrow[m_{W,Z} \gg m_H]{m_t \gg m_H} \text{[Vertex]} \sim C G_a^{\mu\nu} G_{\mu\nu}^a H$$

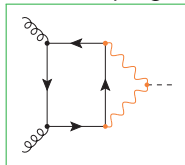
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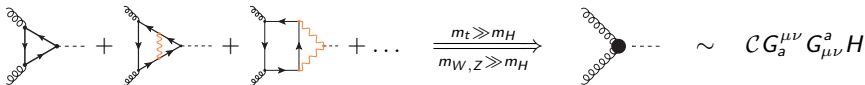
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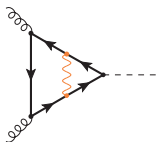


- Wrong mass relations
- QCD corrections might be large

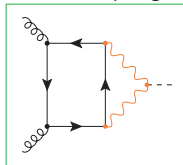
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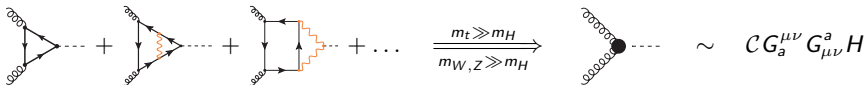
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NLO QCD corrections in HEFT



- Wrong mass relations
- QCD corrections might be large

Exact NLO computation required

QCD-EW contributions at the LHC

$$\sigma_{PP \rightarrow H+j}(\mu) = \int_0^1 \int_0^1 dx_1 dx_2 f_{a/P}(x_1, \mu) f_{b/P}(x_2, \mu) \bar{\sigma}_{ab \rightarrow H+j}$$

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We consider $\alpha^2 v$ contributions

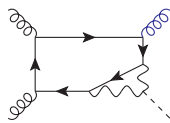
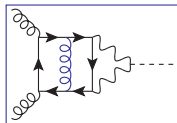
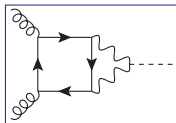
Partons

LO

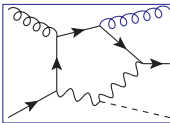
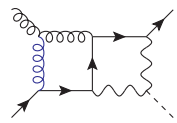
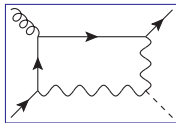
NLO virtual

NLO real

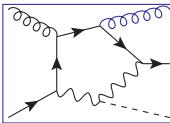
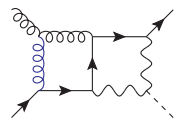
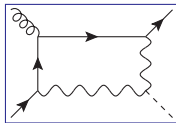
g g



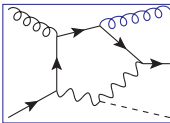
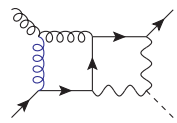
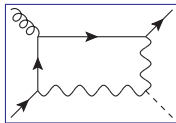
g q

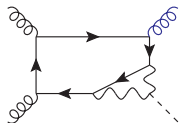


g \bar{q}

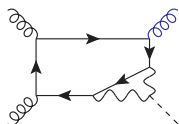


q \bar{q}



Tensor decomposition: $gg \rightarrow Hg$ 

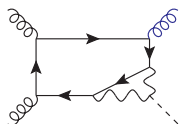
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Contributions from γ_5

Loop of massless quarks: sum over complete generations removes explicit γ_5 , rescaled couplings

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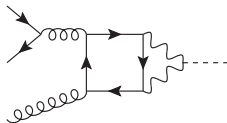


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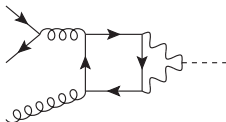
Loop of massless quarks: sum over complete generations removes explicit γ_5 , rescaled couplings

$$f^{c_1 c_2 c_3} \epsilon_{\lambda_1}^\mu(p_1) \epsilon_{\lambda_2}^\nu(p_2) \epsilon_{\lambda_3}^\rho(p_3) [\mathcal{F}_1 g_{\mu\nu} p_{2\rho} + \mathcal{F}_2 g_{\mu\rho} p_{2\nu} + \mathcal{F}_3 g_{\nu\rho} p_{2\mu} + \mathcal{F}_4 p_{3\mu} p_{1\nu} p_{2\rho}]$$

$$\mathcal{F}_j = 4 A_j(m_W^2) + \frac{2}{\cos^4 \theta_W} \left(\frac{5}{4} - \frac{7}{3} \sin^2 \theta_W + \frac{22}{9} \sin^4 \theta_W \right) A_j(m_Z^2)$$

Tensor decomposition $qg \rightarrow Hq$ 

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Contributions from γ_5

$\mathcal{F}_j^{\text{closed}}$ no γ_5 , rescaled couplings

$$T_{j_1 j_2}^{c_3} \bar{v}_{s_1}(p_1) \left[\tau_1^\mu \mathcal{F}_1^{\text{closed}} + \tau_2^\mu \mathcal{F}_2^{\text{closed}} \right] u_{s_2}(p_2) \epsilon_\mu^{\lambda_3}(p_3)$$

$$\mathcal{F}_j^{\text{closed}} = 4 A_j^{\text{closed}}(m_W^2) + \frac{2}{\cos^4 \theta_W} \left(\frac{5}{4} - \frac{7}{3} \sin^2 \theta_W + \frac{22}{9} \sin^4 \theta_W \right) A_j^{\text{closed}}(m_Z^2)$$

Tensor decomposition $qg \rightarrow Hq$ Contributions from γ_5 $\mathcal{F}_j^{\text{closed}}$ no γ_5 , rescaled couplings $\mathcal{F}_j^{\text{open}}$ “polarized” coupling

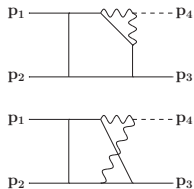
$$T_{j_1 j_2}^{c_3} \bar{v}_{s_1}(p_1) \left[\tau_1^\mu (\mathcal{F}_1^{\text{closed}} + \mathbb{P}_C \mathcal{F}_{C,1}^{\text{open}}) + \tau_2^\mu (\mathcal{F}_2^{\text{closed}} + \mathbb{P}_C \mathcal{F}_{C,2}^{\text{open}}) \right] u_{s_2}(p_2) \epsilon_\mu^{\lambda_3}(p_3)$$

$$\mathcal{F}_j^{\text{closed}} = 4 A_j^{\text{closed}}(m_W^2) + \frac{2}{\cos^4 \theta_W} \left(\frac{5}{4} - \frac{7}{3} \sin^2 \theta_W + \frac{22}{9} \sin^4 \theta_W \right) A_j^{\text{closed}}(m_Z^2)$$

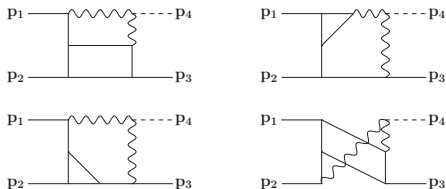
$$\mathcal{F}_{R,j}^{\text{open}} = 1 A_j^{\text{open}}(m_W^2) + \frac{2}{\cos^4 \theta_W} (T_q - Q_q \sin^2 \theta_W)^2 A_j^{\text{open}}(m_Z^2)$$

$$\mathcal{F}_{L,j}^{\text{open}} = \frac{2}{\cos^4 \theta_W} Q_q^2 \sin^4 \theta_W A_j^{\text{open}}(m_Z^2)$$

Reduction to MIs

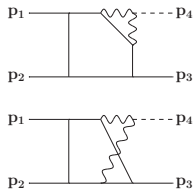
 $ggHg$ 

61 MIs

Additional for $qgH\bar{q}$ 

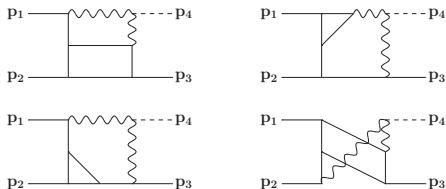
30 MIs

Reduction to MIs

 $ggHg$ 

61 MIs

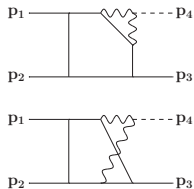
4 square roots

Additional for $qgH\bar{q}$ 

30 MIs

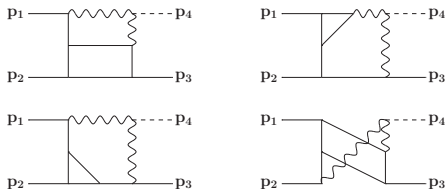
8 square roots

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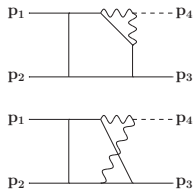
30 MIs

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Differential equations for MIs

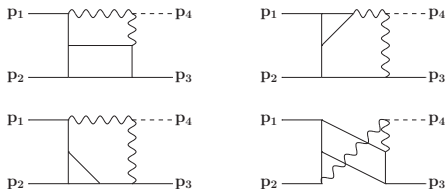
- Evolution in t , u , and m_V^2 , large-mass limit as boundary condition

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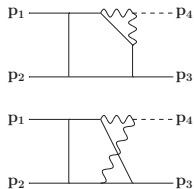
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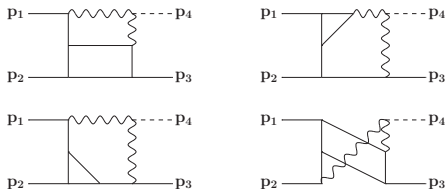
- Evolution in t , u , and m_V^2 , large-mass limit as boundary condition
- DEs for $ggHg$ in $\epsilon d \log$ form
- Single MIs contain at most 3 square roots and are rationalizable
- No global rationalization found

Reduction to MIs

 $ggHg$ 

61 MIs

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Differential equations for MIs

- Evolution in t , u , and m_V^2 , large-mass limit as boundary condition
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DEs uneffective for $ggHg$ & $qgH\bar{q}$

Direct integration & linear reducibility

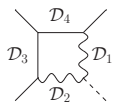
[Panzer,2014]

Direct integration over Feynman parameters

Direct integration & linear reducibility

[Panzer,2014]

Direct integration over Feynman parameters



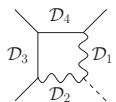
$$\propto \int_0^1 dx_2 \int_0^1 dx_1 \int_0^1 dx_3 \int_0^1 dx_4 \frac{\delta(1-X)}{[\sum x_j \mathcal{D}_j]^A}$$

$$\propto \int_0^1 dx_2 G(x_2 + \sqrt{\alpha x_2 + \beta} \dots; x_2) + \dots$$

Direct integration & linear reducibility

[Panzer,2014]

Direct integration over Feynman parameters



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Linear reducibility

There exists an integration order for the kernel f_0

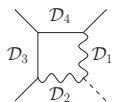
$$\int_0^{+\infty} dz_1 \cdots \int_0^{+\infty} dz_k f_0$$

such that each integral is a hyperlog of the next integration variable.

Direct integration & linear reducibility

[Panzer,2014]

Direct integration over Feynman parameters



$$\propto \int_0^1 dx_3 \int_0^1 dx_2 \int_0^1 dx_1 \int_0^1 dx_4 \frac{\delta(1-X)}{[\sum x_j \mathcal{D}_j]^A}$$

$$\propto \int_0^1 dx_3 G(\alpha + \sqrt{\beta} \dots; x_3) + \dots$$

Linear reducibility

There exists an integration order for the kernel f_0

$$\int_0^{+\infty} dz_1 \cdots \int_0^{+\infty} dz_k f_0$$

such that each integral is a hyperlog of the next integration variable.

- Integration over d logs: result as GPLs
- No integration variables under square roots: no rationalization needed

A quasi-finite basis

[Tarasov,1996][Lee,2010][von Manteuffel. . . ,2015]

- 2-loop MIs highly divergent: up to ϵ^{-4}
- Amplitudes well behaved: $ggHg: \epsilon^0$ $qgH\bar{q}: \epsilon^{-2}$

A quasi-finite basis

[Tarasov,1996][Lee,2010][von Manteuffel. . . ,2015]

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Quasi-finite basis

$$\mathcal{I}^{D+2}(a_1, \dots, a_7) = \frac{16}{s t u (D-4)(D-3)} \int \tilde{d}^D k_1 \tilde{d}^D k_1 \frac{G(k_1, k_2, p_1, p_2, p_3)}{\mathcal{D}_1^{a_1} \dots \mathcal{D}_7^{a_7}}$$

- **UV finiteness:** negative SDD by rising powers of (massive) propagators
- **IR finiteness:** Gram determinant cures soft & collinear divergences

A quasi-finite basis

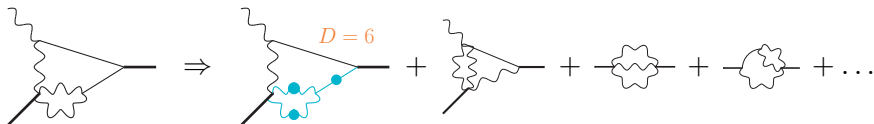
[Tarasov,1996][Lee,2010][von Manteuffel. . . ,2015]

- 2-loop MIs highly divergent: up to ϵ^{-4}
- Amplitudes well behaved: $ggHg: \epsilon^0$ $qgH\bar{q}: \epsilon^{-2}$

Quasi-finite basis

$$\mathcal{I}^{D+2}(a_1, \dots, a_7) = \frac{16}{stu(D-4)(D-3)} \int \tilde{d}^D k_1 \tilde{d}^D k_2 \frac{G(k_1, k_2, p_1, p_2, p_3)}{\mathcal{D}_1^{a_1} \dots \mathcal{D}_7^{a_7}}$$

- **UV finiteness:** negative SDD by rising powers of (massive) propagators
- **IR finiteness:** Gram determinant cures soft & collinear divergences



A quasi-finite basis

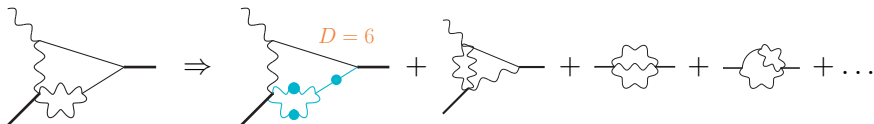
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- **UV finiteness:** negative SDD by rising powers of (massive) propagators
- **IR finiteness:** Gram determinant cures soft & collinear divergences



- MIs are shifted into finite integrals and divergent sub-graphs
- Good choices do not worsen the poles in the coefficients

Simplifying the amplitude

[Duhr... ,2019][Heller... ,2021]

~ 1 GiB

$$A = \frac{2y - x}{y^3 - x^2y} G_1 + \frac{x - 1}{y(y - x)} G_2 + \frac{-x^2 - xy + 2x - y}{y(x - y)(x + y)} G_3 + \dots$$

Simplifying the amplitude

[Duhr... ,2019][Heller... ,2021]

① Basis of algebraic prefactors

- Partial fraction decomposition (without extra poles)
- Basis of algebraic monomials
- Basis of algebraic prefactors

~ 1 GiB

↓

5526 (6789)

$$\begin{aligned}
 A = & \left[\frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{2} \frac{1}{y(x-y)} \right] G_1 + \\
 & \left[\frac{1}{y(x-y)} - \frac{1}{x-y} - \frac{1}{y} \right] G_2 + \\
 & \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] G_3 + \dots
 \end{aligned}$$

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690 (3823)

$$A = \left[\frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{2} \frac{1}{y(x-y)} \right] (G_1 + G_3) +$$

$$\left[\frac{1}{y(x-y)} - \frac{1}{x-y} - \frac{1}{y} \right] (G_2 + G_3) + \dots$$

Simplifying the amplitude

[Duhr... ,2019][Heller... ,2021]

1 Basis of algebraic prefactors

- Partial fraction decomposition (without extra poles)
- Basis of algebraic monomials
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2 Linearly independent transcendental expressions

- Numerical checks for zeroes & implementation
- PSLQ reduction

~ 1 GiB

↓

5526 (6789)

↓

690 (3823)

↓

325 (983)

~ 1 MiB

$$\begin{aligned}
 A &= \left[\frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{2} \frac{1}{y(x-y)} \right] (G_1 + G_3) + \\
 &\quad \left[\frac{1}{y(x-y)} - \frac{1}{x-y} - \frac{1}{y} \right] (G_1 + G_3) + \dots \\
 &= \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (G_1 + G_3) + \dots
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Simplifying the amplitude

[Duhr... ,2019][Heller... ,2021]

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2 Linearly independent transcendental expressions

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3 Manipulation of GPLs

- GPLs into log, Li_2 , Li_3 ...

$$\begin{aligned}
 A = & \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (C_1 \log a_1 + C_2 \log a_2) + \\
 & \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (C_4 \text{Li}_2(b_1, b_2) + C_5 \log b_3 \log b_4) + \\
 & \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (C_6 \log^3 c_1 + C_7 \zeta(3)) + \dots
 \end{aligned}$$

~ 1 GiB

↓

5526 (6789)

↓

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$qgH\bar{q}$: finite remainder

$$\mathcal{F}^{\text{closed}} = \left(\frac{\alpha_S^0}{2\pi} \right) \mathcal{F}_1^{\text{closed}}$$

$$\mathcal{F}^{\text{open}} = \mathcal{F}_0^{\text{open}} + \left(\frac{\alpha_S^0}{2\pi} \right) [\mathcal{B} N_C + \mathcal{C} N_C^{-1}]$$

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① UV div. α_S renormalization only

$$\alpha_S^0 = \alpha_S \left(\frac{\mu_R^2}{\mu_0^2}\right)^\epsilon S_\epsilon^{-1} \left[1 + \alpha_S \frac{\beta_0}{\epsilon} + \mathcal{O}(\alpha_S^2)\right]$$

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- 2 IR div. described by Catani's operator, removed by real corrections

$$\tilde{\mathcal{F}}_1^{\text{open}} = I_{q\bar{q}g}^{(1)} \tilde{\mathcal{F}}_0^{\text{open}} + \mathcal{F}_{1,\text{fin}}$$

$$I_{q\bar{q}g}^{(1)} = \frac{-e^{\epsilon\gamma}}{4\Gamma(1-\epsilon)} \left\{ \mathcal{V}_q \frac{4C_F - 2C_A}{C_F} \left(-\frac{\mu_R^2}{s} \right)^\epsilon + \left(\mathcal{V}_g + \frac{C_A}{C_F} \mathcal{V}_q \right) \left[\left(-\frac{\mu_R^2}{t} \right)^\epsilon + \left(-\frac{\mu_R^2}{u} \right)^\epsilon \right] \right\}$$

$$\mathcal{V}_q = \frac{C_F}{\epsilon^2} + \frac{3C_F}{2\epsilon} \quad \mathcal{V}_g = \frac{C_A}{\epsilon^2} + \frac{11C_A - 2N_f}{6\epsilon}$$

$qgH\bar{q}$: finite remainder

$$\mathcal{F}_{\text{fin}}^{\text{closed}} = \left(\frac{\alpha_S}{2\pi}\right) \overline{\mathcal{F}}_1^{\text{closed}} \quad \mathcal{F}_{\text{fin}}^{\text{open}} = \tilde{\mathcal{F}}_0^{\text{open}} + \left(\frac{\alpha_S}{2\pi}\right) [\overline{B} N_C + \overline{C} N_C^{-1} + \overline{D} N_f]$$

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Helicity amplitudes

- Physical (and simpler) result

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ggHg

$$\mathcal{A}_{++++}^{ggHg} = \frac{m_H^2}{\sqrt{2}\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \frac{su}{m_H^2} \left(\mathcal{F}_1 + \frac{t}{u} \mathcal{F}_2 + \frac{t}{s} \mathcal{F}_3 + \frac{t}{2} \mathcal{F}_4 \right)$$

$$\mathcal{A}_{+++-}^{ggHg} = \frac{[12]^3}{\sqrt{2}m_H^2[13][23]} \frac{um_H^2}{s} \left(\mathcal{F}_1 + \frac{t}{2} \mathcal{F}_4 \right)$$

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$$\mathcal{A}_{RL+}^{qgH\bar{q}} = \left\{ \frac{1}{2} \left[4\mathcal{F}_{1,m_W}^{\text{closed}} + \frac{2}{\cos^4 \theta_W} \left(\frac{5}{4} - \frac{7}{3} \sin^2 \theta_W + \frac{22}{9} \sin^4 \theta_W \right) \mathcal{F}_{1,m_Z}^{\text{closed}} \right] + \left[\mathcal{F}_{1,m_W}^{\text{open}} + \frac{2}{\cos^4 \theta_W} (T_q - Q_q \sin^2 \theta_W)^2 \mathcal{F}_{1,m_Z}^{\text{open}} \right] \right\} \frac{s}{\sqrt{2}} \frac{[23]^2}{[12]}$$

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qgHq̄

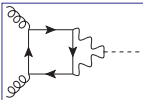
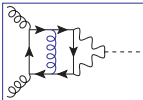
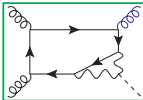
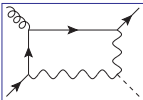
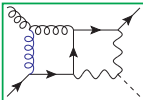
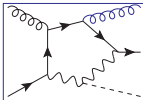
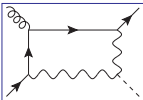
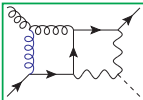
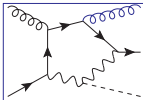
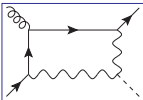
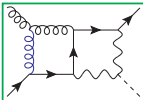
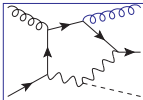
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- $\mathcal{F}_{2,m_V}(t, u) = \mathcal{F}_{1,m_V}(u, t)$

Conclusions & Outlook

Complete analytic results

Partons	LO	NLO virtual	NLO real
$g \quad g$			
$g \quad q$			
$g \quad \bar{q}$			
$q \quad \bar{q}$			

Conclusions & Outlook

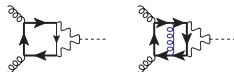
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The next steps

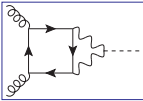
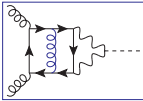
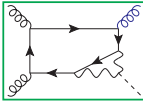
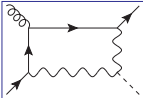
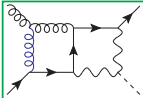
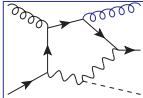
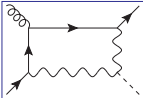
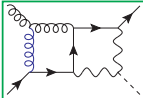
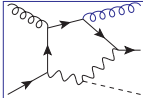
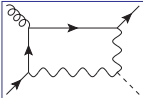
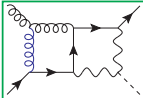
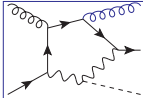
- Full $\sigma_{PP \rightarrow H+X}^{(\alpha_S^3 \alpha^2)}$ evaluation
- Top quark inclusion

$\sigma_{gg \rightarrow H+X}^{(\alpha_S^2 \alpha^2 + \alpha_S^3 \alpha^2)}$: [Becchetti..., 2020]



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The next steps

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- Top quark inclusion



New challenges

- Expression optimization
- Non-vanishing γ_5 contributions & masses

Thank you for your attention

