

Predictions for $gg \rightarrow hh$ at full NLO QCD comparing non-linear and linear EFT frameworks and truncation effects

Young Scientists Meeting of the CRC TRR 257

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INSTITUTE FOR THEORETICAL PHYSICS

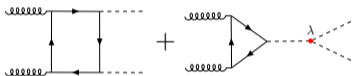


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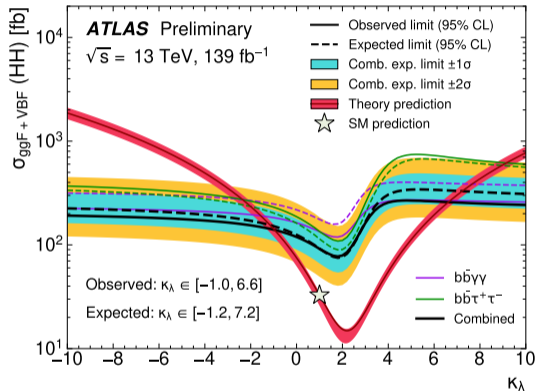
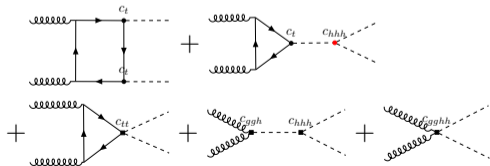
Why study hh production?

- Higgs potential largely unknown

⇒ Trilinear Higgs coupling accessible in hh production



- However, BSM deviations should enter in systematic way!



ATLAS-CONF-2021-052



Two distinct EFT systematics: HEFT vs. SMEFT

- HEFT:**
- BSM: can be strongly coupling New Physics
 - non-linear theory (EW χ L), chiral counting of operators $d_\chi(\partial, \bar{\psi}\psi, g, y) = 1$
 - light Higgs is EW gauge singlet $h(x)$, Goldstones have non-trivial transformation properties
 - expansion in $\frac{f^2}{\Lambda^2} \sim \frac{1}{16\pi^2}$ (\Rightarrow loop counting):

$$\mathcal{L}_{\text{HEFT}} \sim \mathcal{L}_{d_\chi=2} + \sum_{L=1} \sum_i \left(\frac{1}{16\pi^2} \right)^L c_i \mathcal{O}_i^{(d_\chi=2+2L)}$$

- SMEFT:**
- BSM: lightly coupling New Physics
 - light Higgs contained in EW doublet field $\phi(x)$
 - canonical counting (expansion in $\frac{1}{\Lambda}$):

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{n=1} \sum_i \frac{C_i}{\Lambda^{2n}} \mathcal{O}_i^{(4+2n)}$$

Relevant Lagrangian terms for hh

HEFT:

$$\mathcal{L}_{HEFT} \supset -m_t \left(c_t \frac{h}{v} + c_{tt} \frac{h^2}{v^2} \right) \bar{t}t - C_{hhh} \frac{m_h^2}{2v} h^3 + \frac{\alpha_s}{8\pi} \left(C_{ggh} \frac{h}{v} + C_{gghh} \frac{h^2}{v^2} \right) G_{\mu\nu}^a G^{a\mu\nu}$$

SMEFT:

$$\begin{aligned} \mathcal{L}_{SMEFT}^{(Warsaw)} \supset & \frac{C_{H\Box}}{\Lambda^2} (\phi^\dagger \phi) \Box (\phi^\dagger \phi) + \frac{C_{HD}}{\Lambda^2} (\phi^\dagger D_\mu \phi) (\phi^\dagger D^\mu \phi) + \frac{C_H}{\Lambda^2} (\phi^\dagger \phi)^3 \\ & + \left(\frac{C_{uH}}{\Lambda^2} (\phi^\dagger \phi) \bar{q}_L \phi^c t_r + h.c. \right) \\ & + \frac{C_{HG}}{\Lambda^2} (\phi^\dagger \phi) G_{\mu\nu}^a G^{a\mu\nu} \end{aligned}$$

Naive translation SMEFT \leftrightarrow HEFT after field redefinition up to $\mathcal{O}\left(\frac{1}{\Lambda^2}\right)$ in Lagrangian
 ($C_{H,kin} = C_{H\Box} - 4C_{HD}$)

However, formally:

$$c_i \sim \mathcal{O}(1) \text{ possible} \quad \leftrightarrow \quad \frac{E^2}{\Lambda^2} C_i \ll 1$$

HEFT	Warsaw
C_{hhh}	$1 - 2 \frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3 \frac{v^2}{\Lambda^2} C_{H,kin}$
c_t	$1 + \frac{v^2}{\Lambda^2} C_{H,kin} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2}m_t} C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2}m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H,kin}$
C_{ggh}	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s} C_{HG}$
C_{gghh}	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s} C_{HG}$

SMEFT truncation

Dimension 6 operators in amplitude $\left(\frac{C'_i}{\Lambda^2} = c_i - c_{i,sm}\right)$:

$$\begin{aligned}
 \mathcal{M} = & \text{[Diagram 1: Box diagram with wavy lines on the left and dashed lines on the right. Top and bottom horizontal lines are labeled } 1 + \frac{C'_t}{\Lambda^2} \text{.] } + \text{ [Diagram 2: Triangle diagram with wavy lines on the left and dashed lines on the right. Top and bottom horizontal lines are labeled } 1 + \frac{C'_t}{\Lambda^2} \text{ and } 1 + \frac{C'_{hh}}{\Lambda^2} \text{.] } + \text{ [Diagram 3: Triangle diagram with wavy lines on the left and dashed lines on the right. Top and bottom horizontal lines are labeled } \frac{C'_{tt}}{\Lambda^2} \text{.] } \\
 & + \text{ [Diagram 4: Triangle diagram with wavy lines on the left and dashed lines on the right. Top and bottom horizontal lines are labeled } \frac{C'_{ggh}}{\Lambda^2} \text{ and } 1 + \frac{C'_{hh}}{\Lambda^2} \text{.] } + \text{ [Diagram 5: Triangle diagram with wavy lines on the left and dashed lines on the right. Top and bottom horizontal lines are labeled } \frac{C'_{gghh}}{\Lambda^2} \text{ and } 1 + \frac{C'_{hh}}{\Lambda^2} \text{.] } + \dots \\
 = & \mathcal{M}_{SM} + \underbrace{\frac{1}{\Lambda^2} \mathcal{M}_{si}}_{\text{dim6}} + \underbrace{\frac{1}{\Lambda^4} \mathcal{M}_{di}}_{\text{dim6}^2}
 \end{aligned}$$

⇒ Double operator insertion same order as (neglected) dimension 8 operators (and field redefinition!)

Several possibilities for SMEFT truncation of final result:

$$\sigma \simeq \left\{ \begin{array}{l} \sigma_{\text{SM}} + \sigma_{\text{SM} \times \text{dim6}} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} + \sigma_{\text{SM} \times \text{dim6}^2} \\ \sigma_{(\text{SM} + \text{dim6} + \text{dim6}^2) \times (\text{SM} + \text{dim6} + \text{dim6}^2)} \end{array} \right.$$

- (a) Truncation at leading order in $1/\Lambda$ of cross section (commonly used, if SM unsuppressed)
- (b) Truncation at leading order in $1/\Lambda$ of amplitude (commonly used, if SM suppressed) \Rightarrow investigate uncertainty
- (c) Truncate cross section at $\mathcal{O}(1/\Lambda^4)$ from all dim6 operator insertions (ambiguous definition)
- (d) Complete insertion, naive translation SMEFT \leftrightarrow HEFT

POWHEG code ggHH_SMEFT

built on NLO HEFT code with full m_t dependence ggHH

[Borowka, Greiner, Heinrich, Jones, Kerner, et al. '16]

[Heinrich, Jones, Kerner, Luisoni, Vryonidou '17]

[Heinrich, Jones, Kerner, Luisoni, Scyboz '19]

[Heinrich, Jones, Kerner, Scyboz '20]

- available at <http://powhegbox.mib.infn.it> as

User-Processes-V2/ggHH

- modified for SMEFT Warsaw input and truncation options (a)-(d):

- modified **GoSam** 1-loop files interfaced to POWHEG for reals

- HEFT virtuals available as function of 23 grids a_i



$$\begin{aligned}
 |\mathcal{M}_{NLO}|^2 = & a_1 \cdot c_t^4 + a_2 \cdot c_{tt}^2 + a_3 \cdot c_t^2 c_{hhh}^2 + a_4 \cdot c_{ggh}^2 c_{hhh}^2 + a_5 \cdot c_{gggh}^2 + a_6 \cdot c_{tt} c_t^2 + a_7 \cdot c_t^3 c_{hhh} \\
 & + a_8 \cdot c_{tt} c_t c_{hhh} + a_9 \cdot c_{tt} c_{ggh} c_{hhh} + a_{10} \cdot c_{tt} c_{gggh} + a_{11} \cdot c_t^2 c_{ggh} c_{hhh} + a_{12} \cdot c_t^2 c_{gggh} \\
 & + a_{13} \cdot c_t c_{hhh}^2 c_{ggh} + a_{14} \cdot c_t c_{hhh} c_{gggh} + a_{15} \cdot c_{ggh} c_{hhh} c_{gggh} + a_{16} \cdot c_t^3 c_{ggh} \\
 & + a_{17} \cdot c_t c_{tt} c_{ggh} + a_{18} \cdot c_t c_{ggh}^2 c_{hhh} + a_{19} \cdot c_t c_{ggh} c_{gggh} + a_{20} \cdot c_t^2 c_{ggh}^2 \\
 & + a_{21} \cdot c_{tt} c_{ggh}^2 + a_{22} \cdot c_{ggh}^3 c_{hhh} + a_{23} \cdot c_{ggh}^2 c_{gggh}
 \end{aligned}$$

⇒ virtual grids can be directly reused for SMEFT except for truncation (b), where additional 1-loop contributions are added analytically

POWHEG code ggHH_SMEFT

Usage of code (only new part of the input file is shown):

```
! Choose EFT parametrization
usesmeft 0 ! 0: use HEFT parametrization and ignore CHbox, CH, CuH, CHG (no truncation);
! 1: use SMEFT (Warsaw) parametrization and ignore chhh, ct, ctt, cggh, cgghh (with truncation);
! 2: use HEFT parametrization and ignore CHbox, CH, CuH, CHG (with truncation!, testing purpose/SILH-Lag. calculation)

! Values of the Higgs couplings w.r.t SM: HEFT parametrization
chhh 1.0 ! Trilinear Higgs self-coupling
ct 1.0 ! Top-Higgs Yukawa coupling
ctt 0.0 ! Two-top-two-Higgs (tthh) coupling
cggh 0.0 ! Effective gluon-gluon-Higgs coupling
cgghh 0.0 ! Effective two-gluon-two-Higgses coupling

! Values of the Higgs couplings using SMEFT (Warsaw) parametrization (Wilson coefficients enter as C/Lambda^2)
Lambda 1.0 ! EFT counting mass Scale (in TeV)
CHbox 0.0 ! Kinetic term of SU(2)_L singlet (with d'Alembert operator)
CHD 0.0 ! second Kinetic term
CH 0.0 ! Additional term to Higgs potential
CuH 0.0 ! Modified Yukawa term
CHG 0.0 ! Higgs-Gluon-Gluon operator

! Truncation options:
! 3: cross section based on |A_SM+A_dim6+A_dbldim6|^2
! 2: cross section based on |A_SM+A_dim6|^2+2*Re(A_SM x conj(A_dbldim6))
! 1: cross section based on |A_SM+A_dim6|^2
! 0: cross section based on |A_SM|^2+2*Re(A_SM*conj(A_dim6))
multiple-insertion 1
```

multiple-insertion 0, ..., 3 ↔ truncation option (a), ..., (d)

⇒ now available at <http://powhegbox.mib.infn.it> as /User-Processes-V2/ggHH_SMEFT

[Heinrich,JL,Scyboz

'22]



NLO HEFT cross section parametrised as function of coefficients A_i (similar to $|\mathcal{M}_{NLO}|^2$)

$$\begin{aligned} \frac{\sigma_{BSM}}{\sigma_{SM}} = & A_1 \cdot c_t^4 + A_2 \cdot c_{tt}^2 + A_3 \cdot c_t^2 c_{hhh}^2 + A_4 \cdot c_{ggh}^2 c_{hhh}^2 + A_5 \cdot c_{ggh}^2 + A_6 \cdot c_{tt} c_t^2 + A_7 \cdot c_t^3 c_{hhh} \\ & + A_8 \cdot c_{tt} c_t c_{hhh} + A_9 \cdot c_{tt} c_{ggh} c_{hhh} + A_{10} \cdot c_{tt} c_{ggh} + A_{11} \cdot c_t^2 c_{ggh} c_{hhh} + A_{12} \cdot c_t^2 c_{ggh} \\ & + A_{13} \cdot c_t c_{hhh}^2 c_{ggh} + A_{14} \cdot c_t c_{hhh} c_{ggh} + A_{15} \cdot c_{ggh} c_{hhh} c_{ggh} + A_{16} \cdot c_t^3 c_{ggh} \\ & + A_{17} \cdot c_t c_{tt} c_{ggh} + A_{18} \cdot c_t c_{ggh}^2 c_{hhh} + A_{19} \cdot c_t c_{ggh} c_{ggh} + A_{20} \cdot c_t^2 c_{ggh}^2 \\ & + A_{21} \cdot c_{tt} c_{ggh}^2 + A_{22} \cdot c_{ggh}^3 c_{hhh} + A_{23} \cdot c_{ggh}^2 c_{ggh} \end{aligned}$$

■ Translation:

HEFT	Warsaw
c_{hhh}	$1 - 2 \frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3 \frac{v^2}{\Lambda^2} C_{H, \text{kin}}$
c_t	$1 + \frac{v^2}{\Lambda^2} C_{H, \text{kin}} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2} m_t} C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2} m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H, \text{kin}}$
c_{ggh}	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s} C_{HG}$
c_{gghh}	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s} C_{HG}$

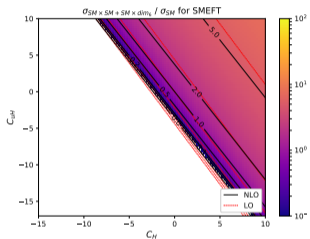
■ Truncation:

$$\sigma \simeq \begin{cases} \sigma_{SM} + \sigma_{SM \times \text{dim}6} & \text{(a)} \\ \sigma_{(\text{SM}+\text{dim}6) \times (\text{SM}+\text{dim}6)} & \text{(b)} \\ \sigma_{(\text{SM}+\text{dim}6) \times (\text{SM}+\text{dim}6)} + \sigma_{SM \times \text{dim}6^2} & \text{(c)} \\ \sigma_{(\text{SM}+\text{dim}6+\text{dim}6^2) \times (\text{SM}+\text{dim}6+\text{dim}6^2)} & \text{(d)} \end{cases}$$

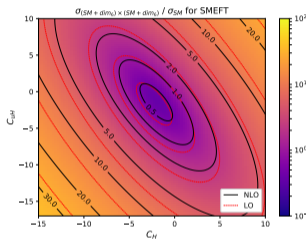


NLO cross section

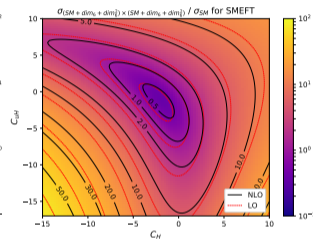
Generated at $\sqrt{s} = 13$ TeV with $\Lambda = 1$ TeV



option (a)



option (b)



option (d)

- large area of negative cross section for truncation (a)
- non-trivial shape for HEFT-like option (d)

- flat directions differ substantially

Consider benchmark points for characteristic m_{hh} shapes in HEFT

- benchmark 1: enhanced low m_{hh} region
- benchmark 3: enhanced low m_{hh} and second local maximum above $m_{hh} \simeq 2m_t$
- benchmark 6: close-by double peaks or shoulder left

benchmark (* = modified)	C_{hhh}	C_t	C_{tt}	C_{ggh}	C_{gggh}	$C_{H,\text{kin}}$	C_H	C_{uH}	C_{HG}	Λ
SM	1	1	0	0	0	0	0	0	0	1 TeV
1*	5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV
3	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*	13.5	2.64	12.6	0.0387	1 TeV
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV

(compare [[Capozi, Heinrich '19](#)], new benchmarks fulfilling current constraints by [Ludovic Scyboz](#))

⇒ SMEFT expansion based on $E^2 \frac{C_i}{\Lambda^2} \ll 1$ justified?

NLO cross section

Generated at $\sqrt{s} = 13$ TeV

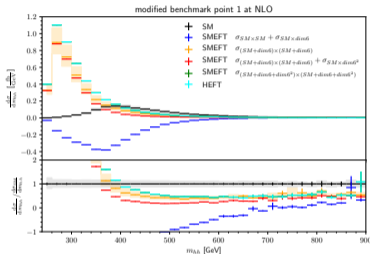
benchmark	σ_{NLO} [fb] option (b)	K-factor option (b)	ratio to SM option (b)	σ_{NLO} [fb] option (a)	σ_{NLO} [fb] HEFT
SM	27.94 ^{+13.7%} _{-12.8%}	1.67	1	-	-
$\Lambda = 1$ TeV					
1	74.29 ^{+19.8%} _{-15.6%}	2.13	2.66	-61.17	94.32
3	69.20 ^{+11.7%} _{-10.3%}	1.82	2.47	29.64	72.43
6	72.51 ^{+20.6%} _{-16.4%}	1.90	2.60	52.89	91.40
$\Lambda = 2$ TeV					
1	14.03 ^{+12.0%} _{-11.9%}	1.56	0.502	5.58	-
3	30.81 ^{+16.0%} _{-14.4%}	1.71	1.10	28.35	-
6	35.39 ^{+17.5%} _{-15.2%}	1.76	1.27	34.18	-



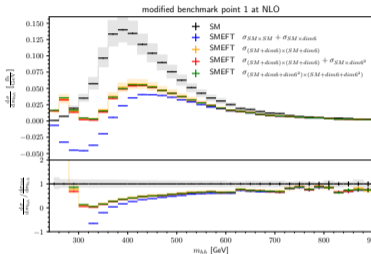
Invariant mass distributions at NLO QCD ($\sqrt{s} = 13 \text{ TeV}$)

HEFT benchmark 1:

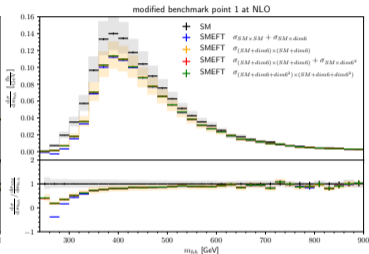
C_{hhh}	C_t	C_{tt}	C_{ggh}	C_{gggh}	$C_{H,kin}$	C_H	C_{UH}	C_{HG}
5.105	1.1	0	0	0	4.95	-6.81	3.28	0



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$



$\Lambda = 4 \text{ TeV}$

■ truncation (a): negative cross sections

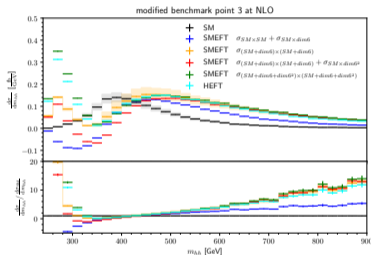
■ shape approaches SM for increasing Λ

⇒ valid HEFT point invalid in SMEFT after direct translation

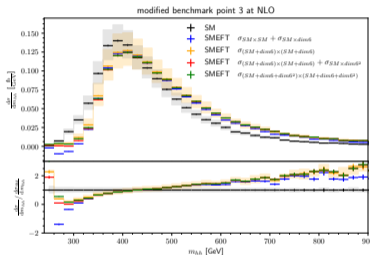
Invariant mass distributions at NLO QCD ($\sqrt{s} = 13 \text{ TeV}$)

HEFT benchmark 3:

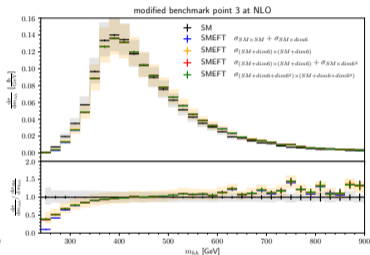
C_{hhh}	C_t	C_{tt}	C_{ggh}	C_{gggh}	$C_{H,kin}$	C_H	C_{UH}	C_{HG}
2.21	1.05	$-\frac{1}{3}$	0.5	0.25*	13.5	2.64	12.6	0.0387



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$



$\Lambda = 4 \text{ TeV}$

truncation (c): double operator insertion quite substantial

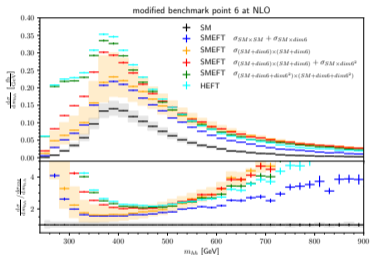
shape close but distinguishable from SM for increasing Λ



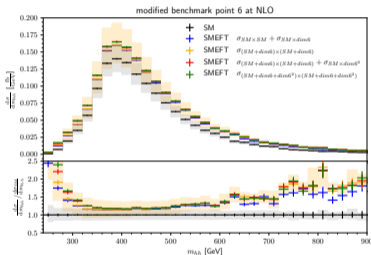
Invariant mass distributions at NLO QCD ($\sqrt{s} = 13$ TeV)

HEFT benchmark 6:

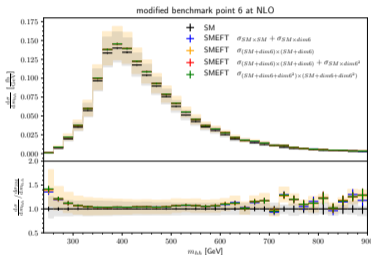
C_{hhh}	C_t	C_{tt}	C_{ggh}	C_{gggh}	$C_{H,kin}$	C_H	C_{uH}	C_{HG}
-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387



$\Lambda = 1$ TeV



$\Lambda = 2$ TeV



$\Lambda = 4$ TeV

no negative cross section

shape indistinguishable from SM for $\Lambda = 4$ TeV within scale uncertainties



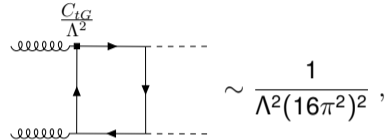
- NLO QCD code `ggHH_SMEFT` for SMEFT (and HEFT)
- comparison of HEFT and SMEFT and of different SMEFT truncation options
- naive translation from HEFT \rightarrow SMEFT can lead out of validity of $\frac{1}{\Lambda^2}$ expansion
- valid SMEFT points close to SM, often hardly distinguishable from SM within scale uncertainties
- Outlook: running Wilson coefficients and inclusion of loop-suppressed chromo-magnetic operator O_{tG} , 4-fermion operators, ...

Loop counting in SMEFT and chromo-magnetic operator

Following the procedure of *Loop counting matters in SMEFT* [Buchalla, Heinrich, Müller-Salditt, Pandler '22]

$$\mathcal{O}_{tG} \sim [\kappa^4] (\bar{q}_L \sigma^{\mu\nu} T^a t_R) \tilde{\phi} G_{\mu\nu}^a$$

with $d_\chi = 6 \Rightarrow$



with $C(6, d_\chi) = \frac{1}{\Lambda^2} \left(\frac{1}{16\pi^2} \right)^{(d_\chi-4)/2}$

\Rightarrow Enters with overall loop factor suppression $\frac{1}{16\pi^2}$ compared to

