Precision Test of the Muon-Higgs Coupling at a High-energy Muon Collider

Young Scientists Meeting

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based on 2108.05362 in collaboration with T.Han, W.Kilian, N.Kreher, Y.Ma, J.Reuter, T.Striegl and K.Xie
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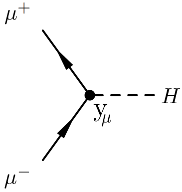
Outline

1. Introduction
2. Framework EFT and GBET
3. Results
4. Conclusion

Muon-Yukawa Coupling

- One of the fundamental parameters of the **minimal** Standard Model (SM) is the **muon Yukawa coupling** y_μ,

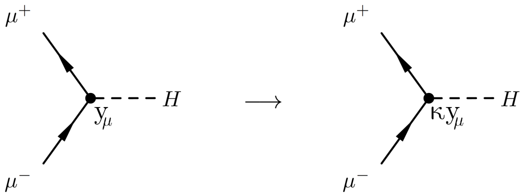


that is linked to the muon mass by the Higgs mechanism

$$m_\mu = v\frac{y_\mu}{\sqrt{2}} \quad .$$

- It is recently confirmed to have the predicted order of magnitude, but results are not yet at the 5σ level. [arXiv:2007.07830,2009.04363]
- The high-luminosity runs of the LHC in the late 2030s predict a measurement of the muon Yukawa coupling within an accuracy of $\mathcal{O}(10\%)$. [ATL-PHYS-PUB-2014-016]

Muon-Yukawa Coupling



- This leaves space for corrections κ caused by BSM physics. If $\kappa = 1$ that is the **minimal** SM case. $\Delta\kappa$ is the deviation from this value.
- The **delicate gauge cancellation** in the SM's high-energies manifests in **decreasing** energy dependence of the cross sections for **multiboson final states**.
- A modification κ of the muon Yukawa coupling **spoils such cancellations**, and thus might even cause an **increasing** energy dependence of the cross sections.

[e.g. arXiv:hep-ph/0106281]

Muon-Collider

- Due to technological development a high-luminosity, high-energy **muon collider** that could reach the multi-(tens of) TeV regime is considered recently:

$$\mathcal{L} = \left(\frac{\sqrt{s}}{10\,\mathrm{TeV}}\right)^2 10\ \mathrm{abarn}^{-1}$$

[arXiv:1901.06150,2001.04431,PoS(ICHEP2020)703, Nat.Phys.17, 289-292]

- Such a high-energy muon collider has potential for **BSM searches** from direct $\mu^+\mu^-$ annihilation as well as **precision measurements** for SM physics and beyond.

[e.g. arXiv:2006.16277,2008.12204,2102.08386]

Paradigm example for exploiting a high-energy muon collider

Direct measurement of the muon coupling associated with its mass generation.

Muon-Collider

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Experimental bound on $\Delta\kappa_\mu$ translates to a bound on the scale of new physics

$$\Lambda > 10\,\mathrm{TeV}\sqrt{\frac{g}{\Delta\kappa_\mu}}$$

Higgs Effective Field Theory

Non-Linear Model

- **Non-linear** representation in terms of Higgs H:

$$U = \exp\left(\frac{\mathrm{i}}{v}\,\phi^a\tau_a\right) \quad \text{with} \quad \phi^a\tau_a = \sqrt{2}\begin{pmatrix} \frac{\phi^0}{\sqrt{2}} & \phi^+ \\ \phi^- & -\frac{\phi^0}{\sqrt{2}} \end{pmatrix} \quad .$$

- **HEFT** Lagrangian contains the generalized Yukawa sector:

$$\begin{aligned}\mathcal{L}_{UH} =& \frac{v^2}{4}\,\mathrm{tr}\,[\mathrm{D}_\mu\, U^\dagger\, \mathrm{D}^\mu\, U] F_U(H) + \frac{1}{2}\partial_\mu H \partial^\mu H - V(H) \\ & - \frac{v}{2\sqrt{2}}\left[\sum_{n\geq 0} y_n \left(\frac{H}{v}\right)^n (\bar{\nu}_L,\, \bar{\mu}_L) U (1-\tau_3) \begin{pmatrix}\nu_R \\ \mu_R\end{pmatrix} + \mathrm{h.c.}\right]\end{aligned}$$

- Resulting parameterization of the muon Yukawa deviation:

$$m_\mu = \frac{v}{\sqrt{2}} y_0$$

$$\kappa = \frac{v}{\sqrt{2} m_\mu} y_1$$

- **Linear** Higgs H representation as a doublet:

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi^+ \\ v + H + \mathrm{i}\,\phi^0 \end{pmatrix}$$

- **SMEFT** Lagrangian contains the generalized Yukawa sector:

$$\mathcal{L}_\varphi = \left[-y_\mu \, \bar{\mu}_L \varphi \mu_R + \sum_{n=1}^{N} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} (\varphi^\dagger \varphi)^n \bar{\mu}_L \varphi \mu_R + \text{h.c.} \right]$$

- Resulting parameterization of the muon Yukawa deviation:

$$m_\mu = \frac{v}{\sqrt{2}} \left[y_\mu - \sum_{n=1}^{N} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \frac{v^{2n}}{2^n} \right]$$

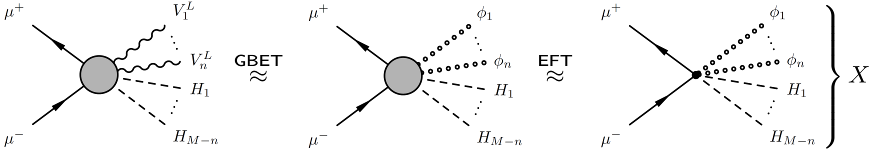
$$\kappa = 1 - \frac{v}{\sqrt{2} m_\mu} \sum_{n=1}^{N} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \frac{n v^{2n}}{2^{n-1}}$$

Goldstone Boson Equivalence Theorem in EFT

Goldstone Boson Equivalence Theorem (GBET):

- For a multi boson final state X the **longitudinal polarizations** will dominate for high energies.
- These can be replaced approximately by the corresponding Goldstone boson.

EFT introduces contact terms that dominate for high energies.



The corresponding cross section is simple

$$\sigma_X \approx \frac{1}{4}\left(\frac{\pi}{2(2\pi)^4}\right)^{M-1} \frac{s^{M-2}}{\Gamma(M)\Gamma(M-1)}|C_X|^2 \left(\prod_{j\in J_X}\frac{1}{n_j!}\right) .$$

8/16

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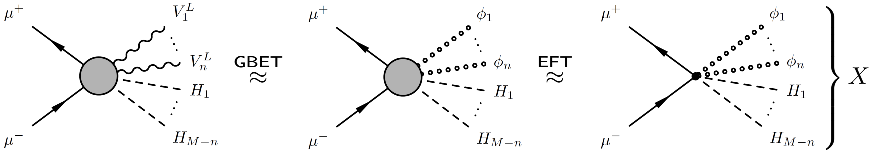


Ratios are even simpler

$$R = \frac{\sigma_X}{\sigma_Y} \approx \frac{|C_X|^2 \left(\prod_{j\in J_X} \frac{1}{n_j!}\right)}{|C_Y|^2 \left(\prod_{j\in J_Y} \frac{1}{n_j!}\right)} \quad .$$

Benchmark Scenario

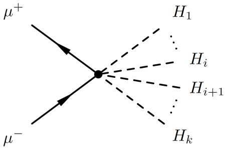
Extreme Case

Consider a model without muon Yukawa coupling.

$$\kappa = 0 \quad .$$

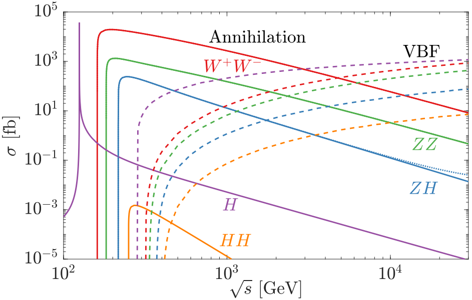
In this case:

- there are **no pure Higgs final states** at tree level.
- constraints on our effective "matched" description.



$$= -\mathrm{i}\frac{k!}{\sqrt{2}}\left[Y_\ell \delta_{k,1} - \sum_{n=n_k}^{M-1} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \binom{2n+1}{k} \frac{v^{2n+1-k}}{2^n}\right] \stackrel{!}{=} 0$$

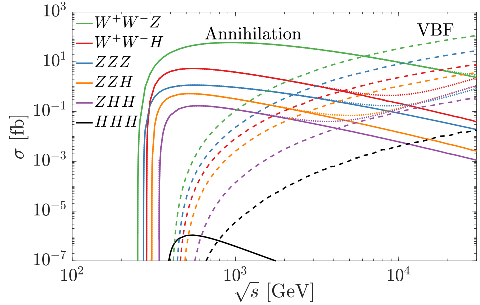
Multiplicity Two Final States



	SMEFT				HEFT	
$\mu^+\mu^- \to X$	dim_6	dim_8	$\text{dim}_{6,8}$	$\text{dim}_{6,8}^{\text{matched}}$	dim_∞	$\text{dim}_\infty^{\text{matched}}$
W^+W^-	1	1	1	1	1	1
ZZ	1/2	1/2	1/2	1/2	1/2	1/2
ZH	1	1/2	1	1	$R_{(2),1}^{\text{HEFT}}$	1
HH	9/2	25/2	$R_{(2),1}^{\text{SMEFT}}/2$	0	$2\,R_{(2),2}^{\text{HEFT}}$	0

[arXiv:2108.05362]

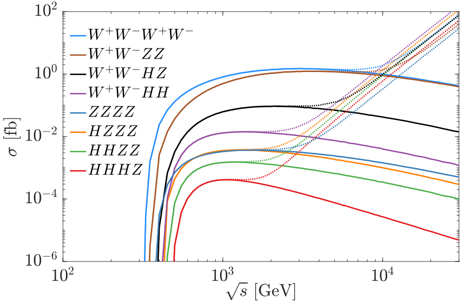
Final States With Multiplicity Three



	SMEFT				HEFT	
$\mu^+\mu^- \to X$	dim$_6$	dim$_8$	dim$_{6,8}$	dim$_{6,8}^{\text{matched}}$	dim$_\infty$	dim$_\infty^{\text{matched}}$
WWZ	1	1/9	$R_{(3),1}^{\text{SMEFT}}$	1/4	$R_{(3),1}^{\text{HEFT}}/9$	1/4
ZZZ	3/2	1/6	$3\,R_{(3),1}^{\text{SMEFT}}/2$	3/8	$R_{(3),1}^{\text{HEFT}}/6$	3/8
WWH	1	1	1	1	1	1
ZZH	1/2	1/2	1/2	1/2	1/2	1/2
ZHH	1/2	1/2	1/2	1/2	$2\,R_{(3),2}^{\text{HEFT}}$	1/2
HHH	3/2	25/6	$3\,R_{(3),2}^{\text{SMEFT}}/2$	75/8	$6\,R_{(3),3}^{\text{HEFT}}$	0

[arXiv:2108.05362]

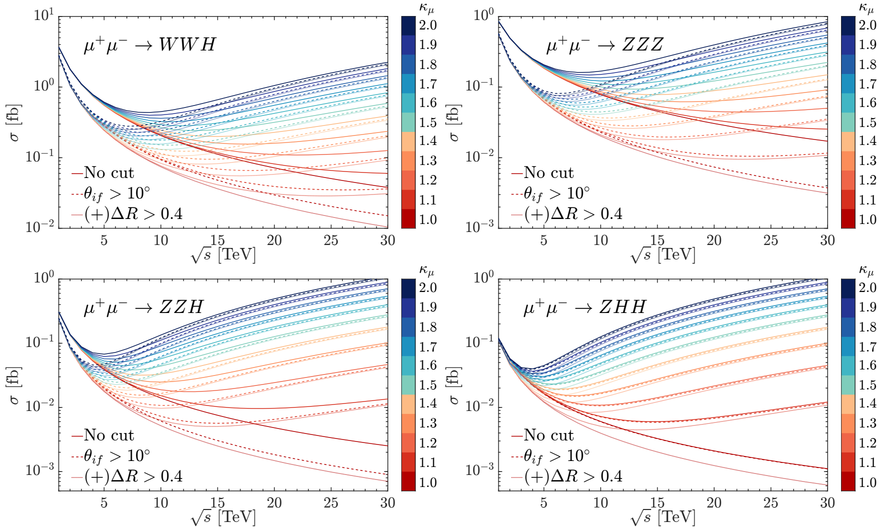
Final States With Multiplicity Four



	SMEFT				HEFT	
$\mu^+\mu^- \to X$	$\text{dim}_{6,8}$	dim_{10}	$\text{dim}_{6,8,10}$	$\text{dim}^{\text{matched}}_{6,8,10}$	dim_{∞}	$\text{dim}^{\text{matched}}_{\infty}$
$WWWW$	$2/9$	$2/25$	$2\,R^{\text{SMEFT}}_{(4),1}/9$	$1/2$	$R^{\text{HEFT}}_{(4),1}/18$	$1/2$
$WWZZ$	$1/9$	$1/25$	$R^{\text{SMEFT}}_{(4),1}/9$	$1/4$	$R^{\text{HEFT}}_{(4),1}/36$	$1/4$
$ZZZZ$	$1/12$	$3/100$	$R^{\text{SMEFT}}_{(4),1}/12$	$3/16$	$R^{\text{HEFT}}_{(4),1}/48$	$3/16$
$WWZH$	$2/9$	$2/25$	$2\,R^{\text{SMEFT}}_{(4),1}/9$	$1/2$	$R^{\text{HEFT}}_{(4),2}/8$	$1/2$
$WWHH$	1	1	1	1	1	1
$ZZZH$	$1/3$	$3/25$	$R^{\text{SMEFT}}_{(4),1}/3$	$3/4$	$R^{\text{HEFT}}_{(4),2}/12$	$3/4$
$ZZHH$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$
$ZHHH$	$1/3$	$1/3$	$1/3$	$1/3$	$3\,R^{\text{HEFT}}_{(4),3}$	$1/3$
$HHHH$	$25/12$	$49/12$	$25\,R^{\text{SMEFT}}_{(4),2}/12$	$1225/48$	$12\,R^{\text{HEFT}}_{(4),4}$	0

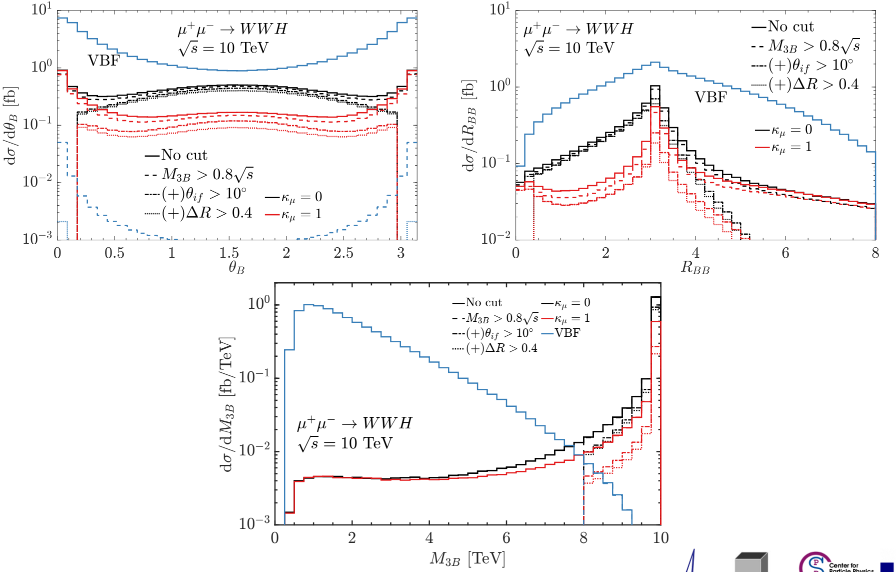
[arXiv:2108.05362]

κ-Scan



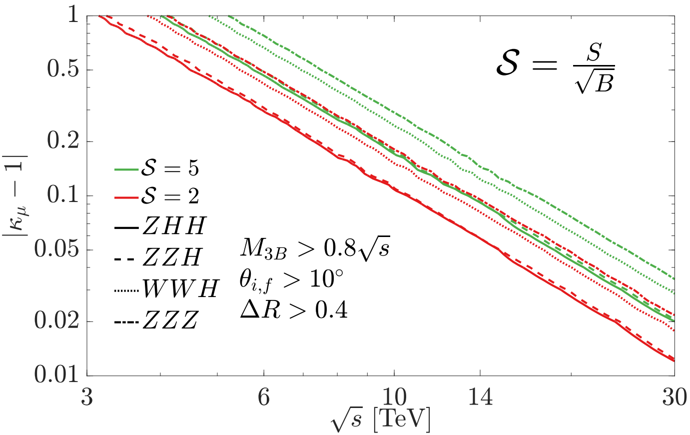
[arXiv:2108.05362]

Kinematic Distributions



[arXiv:2108.05362]

Sensitivity to the muon Yukawa coupling



[arXiv:2108.05362]

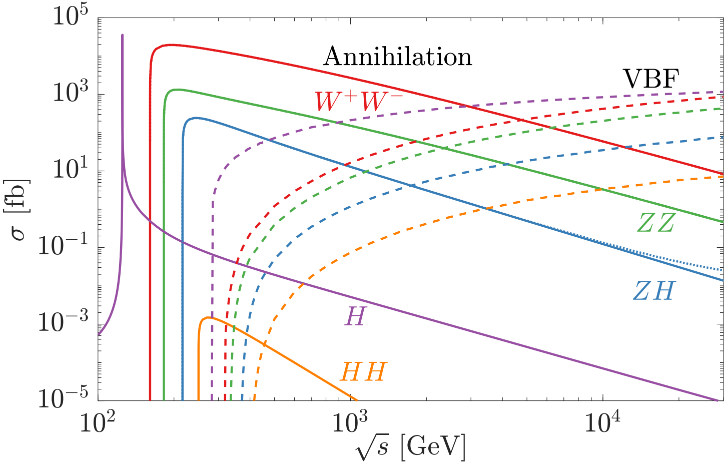
Conclusion

- The recent proposal for a multi-TeV muon collider motivated testing the muon-Higgs coupling at such a collider.
- The **sensitivity** to the anomalous muon-Higgs couplings **rises with the number of gauge bosons**:
  - two-boson final states have less sensitivity to the muon-Yukawa coupling
  - four-boson final states have lower production rates
  - Therefore optimal processes are **triboson productions**
- Isolation of signal from background (VBF) can be achieved by investigation of kinematical distributions like **invariant masses, their angular distributions** or ΔR **diboson seperation distances**.
- Testing the muon Yukawa coupling with a precision of $20\%(2\%)$ of a $10(30)\mathrm{TeV}$ collider allows for the search for new physics at $\Lambda \sim \mathbf{20} - \mathbf{70}\mathrm{TeV}$ ($\mathcal{S} = 5$).

Backup

Final States With Multiplicity Two

Backup



[arXiv:2108.05362]

Angular Distribution

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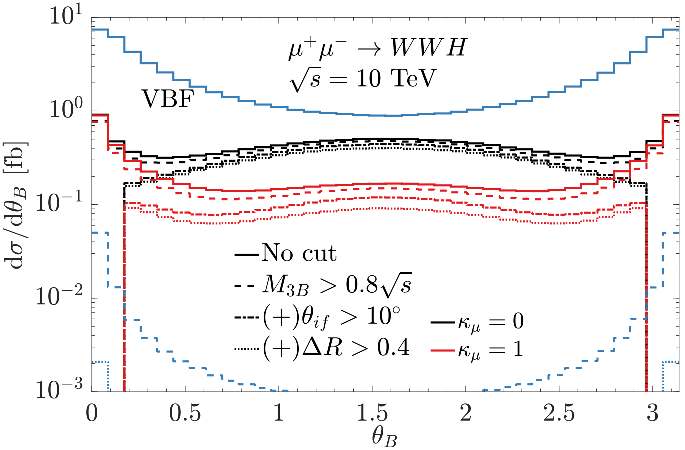

[arXiv:2108.05362]

Diboson Seperation Distance

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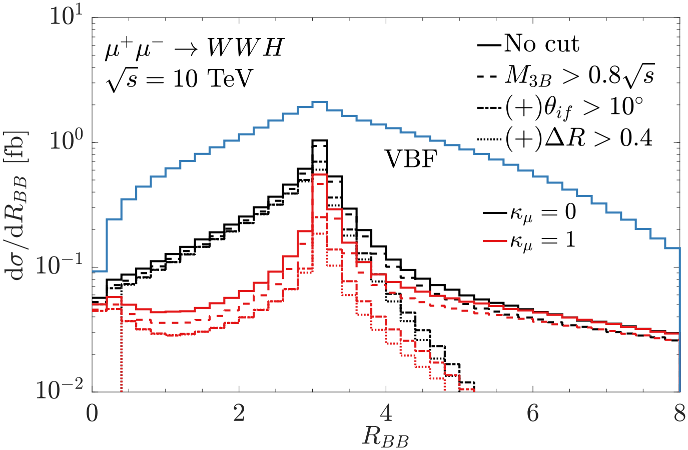

[arXiv:2108.05362]

Invariant Mass

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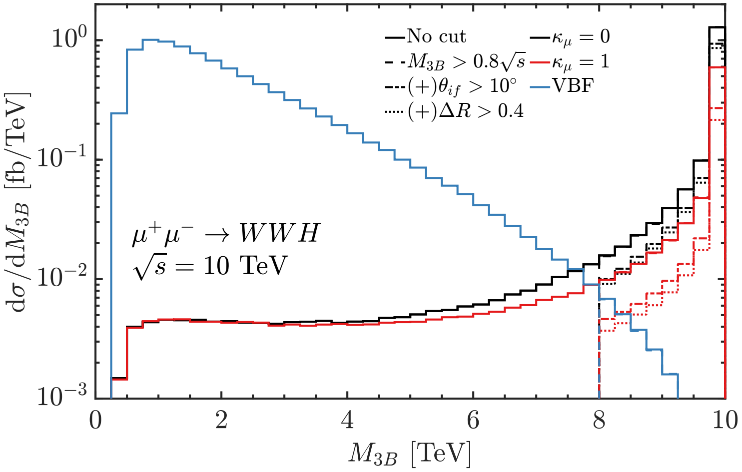


[arXiv:2108.05362]

Unitarity
Backup

Using the GBET the energy scale Λ_d where unitarity is violated by multiple emission of Goldstone bosons and Higgs [e.g. arXiv:hep-ph/0106281] is

$$\Lambda_d = 4\pi\kappa_d \left(\frac{v^{d-3}}{m_\mu}\right)^{1/(d-4)} \quad .$$

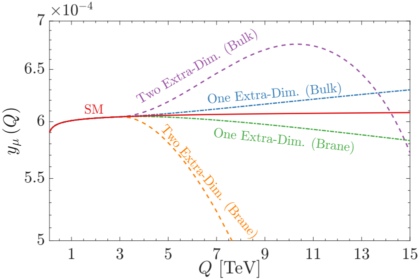
Therefore

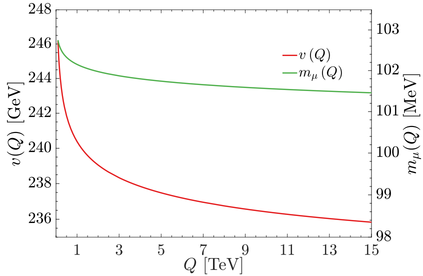
$$\kappa_d = \left(\frac{(d-5)!}{2^{d-5}(d-3)}\right)^{1/(2(d-4))} \quad .$$

This means:

- For $d = 6, 8, 10$, the numeric values of the unitarity bound are $95\,\mathrm{TeV}$, $17\,\mathrm{TeV}$, $11\,\mathrm{TeV}$.
- For $d \geq 8$, the values of these bounds are accessible at a future muon collider.

Running Couplings
Backup





[arXiv:2108.05362]

Multiplicity Four Final States vs VBF

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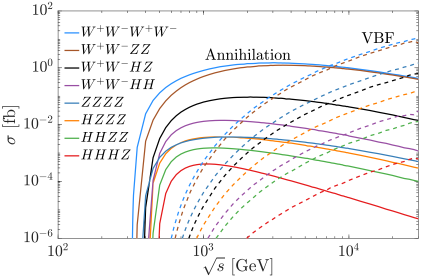

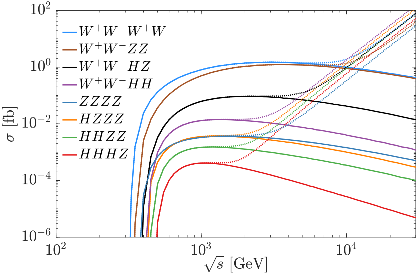

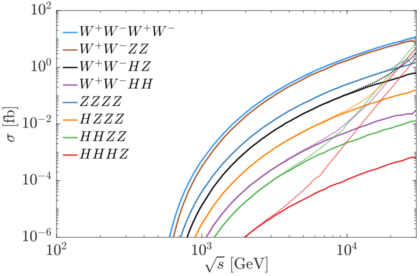

General Framework:

- Treating multiple boson final states is a taylored problem for usage of the **Goldstone boson equivalence theorem** (**GBET**).
- We parameterize variations of the muon Yukawa using an **effective field theory framework** (**EFT**).

The Higgs sector is treated twofold:

- The **non-linear** representation in terms of scalar Higgs H and

$$U = \exp\left(\frac{\mathrm{i}}{v}\phi^a\tau_a\right) \quad \text{with} \quad \phi^a\tau_a = \sqrt{2}\begin{pmatrix} \frac{\phi^0}{\sqrt{2}} & \phi^+ \\ \phi^- & -\frac{\phi^0}{\sqrt{2}} \end{pmatrix}$$

- The **linear** representation as a doublet

$$\varphi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\phi^+ \\ v + H + \mathrm{i}\,\phi^0 \end{pmatrix}$$