Precise predictions for vector-boson scattering

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in collaboration with

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Young Scientist Meeting CRC TRR 257, 10. June 2022

Outline

1. Introduction
2. Theoretical aspects
3. Details of calculation and tools
4. Quality of VBS approximation
5. Electroweak corrections to vector-boson scattering (VBS)
6. Polarised VBS
7. Conclusion

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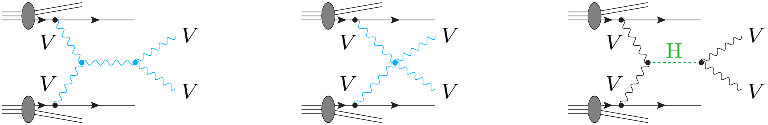
Status of collider physics

- Standard Model (SM) of particle physics describes practically all experiments at high-energy colliders.
- SM does not explain all experimental facts:
  mass spectrum of fundamental particles, dark matter
  baryon asymmetry of universe, gravitation
  ⇒ there is physics beyond the SM!
- aim of collider experiments: find physics beyond SM
  - search for direct signals of physics beyond SM (so far not successful)
  - explore the limits of the SM via precision tests of SM processes (perturbative corrections depend on physics beyond SM)
    ⇒ measure as many SM processes as precise as possible
  - Some hints for nearby physics beyond the SM
    $g-2$, flavour anomalies, CDF M_{W} measurement
- requirements for precision tests:
  - precise experiments (high luminosity, precise detectors, efficient triggers)
  - precise theoretical predictions for physical processes
    within the SM and its extensions

Vector-boson scattering (VBS)

Particularly interesting: scattering of vector bosons $VV \to VV$,
$V = \mathrm{W}, \mathrm{Z}, (\gamma)$

this talk: massive VBS with leptonically decaying bosons

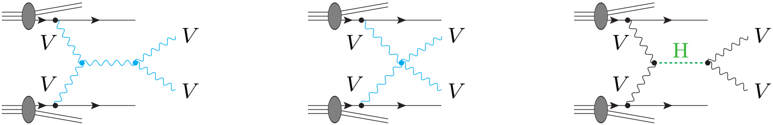


Physics issues of vector-boson scattering (VBS):

- subprocess of more complicated scattering process
  initial V's emitted from quarks, final V's decay
- VBS purely EW process $[\mathcal{O}(\alpha^4)$ for stable Vs]
- key process to test electroweak symmetry breaking
- involves triple and quartic gauge-boson couplings at tree level
- involves Higgs boson (couplings) at tree level
  crucial for unitarity of process

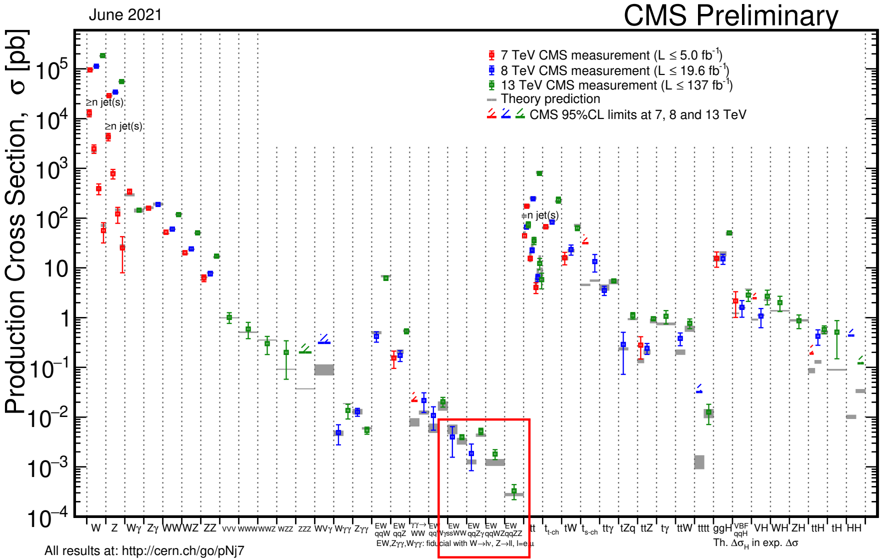
Vector-boson scattering (VBS)

Scattering of massive longitudinal vector bosons



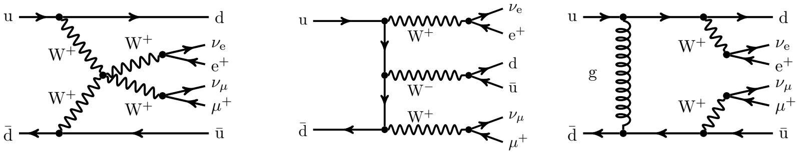
- For longitudinal vector bosons, individual diagrams grow with energy ($\propto E^4, E^2$) $\Rightarrow$ individual diagrams violate unitarity
- SM: gauge symmetry $\Rightarrow$ cancellation of unitarity-violating terms
- without Higgs boson: amplitude $\propto E^2$ $\Rightarrow$ Higgs boson required for unitarity
- no-loose theorem of LHC: (consequence of unitarity)
  - either Higgs boson exists with a sufficiently low mass
  - or there are new strong interactions between longitudinal gauge bosons
- (light) Higgs boson discovered at LHC in 2012
  VBS still allows for sensitive tests of physics beyond SM
  owing to gauge cancellations

CMS cross section measurements at LHC



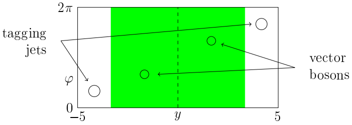
Experimental aspects of VBS

- Cross sections are small: $\mathcal{O}(1\,\mathrm{fb})$
- VBS is part of a more complicated physical process
  $\mathrm{pp} \to VV + 2\mathrm{j} \to 4\ell + 2\mathrm{j}$



  Non-VBS diagrams contribute to the same final state.

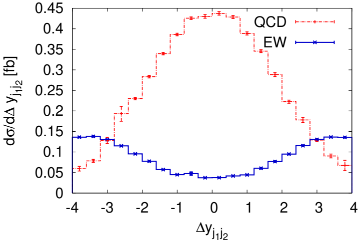
- Need to enhance contribution of VBS by appropriate methods
  (cuts, boosted decision trees, etc.)
  typical characteristics of VBS contributions:
  tagging jets in forward/backward direction
  vector-boson decay products between
  tagging jets
  typical VBS cuts: $M_\mathrm{jj} > 400\text{–}600\,\mathrm{GeV}$
  $\Delta y_\mathrm{jj} > 4\text{–}5,\ y_{\mathrm{j}_1} \times y_{\mathrm{j}_2} < 0$
  $y_{\mathrm{j_{min}}} < y_\ell < y_{\mathrm{j_{max}}}$

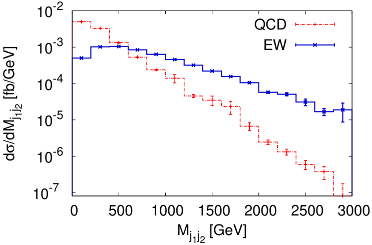


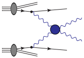
W^+W^+jj: QCD versus EW production

Jäger, Zanderighi '11

$\sqrt{s} = 7\,\mathrm{TeV}$, NLO QCD, basic cuts: $p_{\mathrm{T},j} > 20\,\mathrm{GeV}$

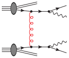






EW production:

- dominant for large $M_{\mathrm{j_1j_2}}$ and
- large rapidity separation $\Delta y_{\mathrm{j_1j_2}}$
- $\sigma^{\text{inclusive}}_{\text{EW}} = 1.10\,\mathrm{fb}$
- $\sigma^{\text{VBFcuts}}_{\text{EW}} = 0.201\,\mathrm{fb}$



QCD production:

- prefers small $M_{\mathrm{j_1j_2}}$ and
- small rapidity separation $\Delta y_{\mathrm{j_1j_2}}$
- $\sigma^{\text{inclusive}}_{\text{QCD}} = 2.12\,\mathrm{fb}$ 192%
- $\sigma^{\text{VBFcuts}}_{\text{QCD}} = 0.0074\,\mathrm{fb}$ 3.7%

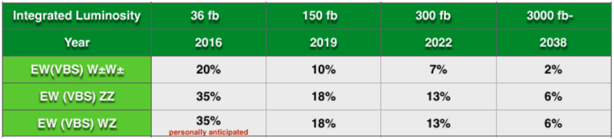
VBF cuts: $M_{\mathrm{j_1j_2}} > 600\,\mathrm{GeV}, \quad |\Delta y_{\mathrm{j_1j_2}}| > 4, \quad y_{\mathrm{j_1}} \times y_{\mathrm{j_2}} < 0$

VBS experiments

Existing experimental results (from run 2):

- VBS in $\mathrm{W^\pm W^\pm jj}$:	ATLAS	$\delta\sigma/\sigma = 0.20$,	6.5σ significance	1906.03203
	CMS	$\delta\sigma/\sigma = 0.11$,	$\gg 5\sigma$ significance	2005.01173
- VBS in WZjj:	ATLAS	$\delta\sigma/\sigma = 0.27$,	5.3σ significance	1812.09740
	CMS	$\delta\sigma/\sigma = 0.23$,	6.8σ significance	2005.01173
- VBS in ZZjj:	ATLAS	$\delta\sigma/\sigma = 0.11$,	5.5σ significance	2004.10612
	CMS	$\delta\sigma/\sigma = 0.35$,	4.0σ significance	2008.07013
- VBS in $\mathrm{W^+W^-jj}$:				
	CMS	$\delta\sigma/\sigma = 0.19$,	5.6σ significance	2205.05711

improvement of experimental precision



Integrated Luminosity	36 fb	150 fb	300 fb	3000 fb-
Year	2016	2019	2022	2038
EW(VBS) W±W±	20%	10%	7%	2%
EW (VBS) ZZ	35%	18%	13%	6%
EW (VBS) WZ	35% personally anticipated	18%	13%	6%

Jakob Salfeld-Nebgen in https://indico.cern.ce/event/711256 (2018)

must be matched by theoretical calculations

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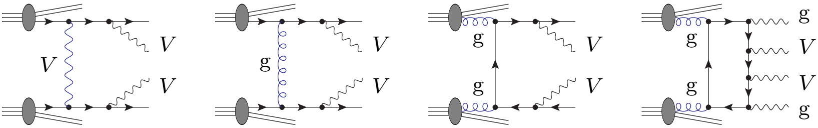
Irreducible background to VBS

Final state: $VV + 2\mathrm{j} \quad (4\ell + 2\mathrm{j})$



- full electroweak (EW) process $[\mathcal{O}(\alpha^4)$ for stable $V]$
  not separable from VBS in gauge-invariant way
- QCD process $[\mathcal{O}(\alpha_\mathrm{s}^2\alpha^2)$ for stable $V]$
  gauge-invariant contribution
- interferences between EW and QCD contributions $[\mathcal{O}(\alpha_\mathrm{s}\alpha^3)$ for stable $V]$
  appear only for channels with identical or weak-isospin partner quarks
- gg channels and $q\mathrm{g}$ channels for $Q_{VV} \le 1$
- loop-induced 4-gluon channels for neutral VV final states
- irreducible background can be suppressed by cuts on M_jj and $|\Delta y_\mathrm{jj}|$
  $\sigma_\mathrm{EW}^{\mathrm{W^+W^+}} \sim 10\,\sigma_\mathrm{QCD}^{\mathrm{W^+W^+}}, \qquad \sigma_\mathrm{EW}^{\mathrm{W^+Z}} \sim 0.25\,\sigma_\mathrm{QCD}^{\mathrm{W^+Z}}, \qquad \sigma_\mathrm{EW}^{\mathrm{ZZ}} \sim 0.5\,\sigma_\mathrm{QCD}^{\mathrm{ZZ}}$

Expansion in multiple couplings

LO: pure EW diagrams $\mathcal{O}(e^6)$ and diagrams with gluons $\mathcal{O}(e^4 g_\mathrm{s}^2)$

NLO: EW and QCD corrections to both types of diagrams

at level of cross section:

$$\begin{tikzcd}
\textbf{LO} & & \mathcal{O}(\alpha^6) \arrow[dl, "\mathrm{EW}"'] \arrow[dr, "\mathrm{QCD}"] & & \mathcal{O}(\alpha_\mathrm{s}\alpha^5) \arrow[dl, "\mathrm{EW}"'] \arrow[dr, "\mathrm{QCD}"] & & \mathcal{O}({\alpha_\mathrm{s}}^2\alpha^4) \arrow[dl, "\mathrm{EW}"'] \arrow[dr, "\mathrm{QCD}"] & \\
\textbf{NLO} & \mathcal{O}(\alpha^7) & & \mathcal{O}(\alpha_\mathrm{s}\alpha^6) & & \mathcal{O}({\alpha_\mathrm{s}}^2\alpha^5) & & \mathcal{O}({\alpha_\mathrm{s}}^3\alpha^4)
\end{tikzcd}$$

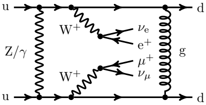
consequences:

- QCD and EW corrections cannot be separated in general
  ↪ distinction of EW signal and QCD background convention at NLO
  ↪ consider well-defined orders $\mathcal{O}(\alpha_\mathrm{s}^n \alpha^m)$
- QCD corrections to leading LO terms well defined $\mathcal{O}(\alpha_\mathrm{s}^3 \alpha^4)$
  EW corrections to LO EW process well defined $\mathcal{O}(\alpha^7)$

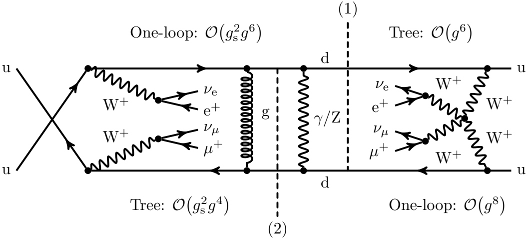
Mixing of virtual EW and QCD corrections

Virtual diagrams mix QCD and EW corrections:

- EW correction to LO QCD amplitude
- QCD correction to LO EW amplitude
- QED and QCD IR singularities



⇒ QCD and EW corrections mix at $\mathcal{O}\left(\alpha_s\alpha^6\right)$ and $\mathcal{O}\left(\alpha_s^2\alpha^5\right)$



- (1) QCD correction to LO EW cross section
- (1) EW correction to LO QCD amplitude interfered with EW amplitude
- (2) EW correction to LO EW amplitude interfered with QCD amplitude

⇒ separation into QCD and EW is not well-defined at NLO

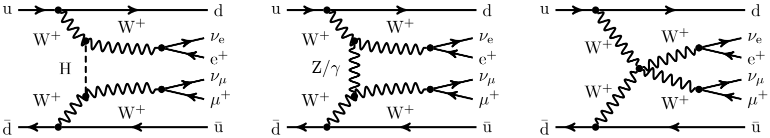
Contributions to full process $\mathrm{pp} \to 4\,\ell + 2\,\mathrm{j}$

- All off-shell and non-resonant contributions needed for realistic final state
- Many partonic processes contribute to VBS and its irreducible background:
  e.g. for $\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ $(\mathrm{W^+W^+})$: 12 qq channels
  - $\mathrm{uu} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{dd}$ (67%)
  - $\mathrm{uc} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{ds}$ (6%)
  - $\mathrm{cc} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{ss}$
  - $\mathrm{u\bar{d}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{d\bar{u}}$ (17%)
  - $\mathrm{u\bar{d}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{s\bar{c}}$
  - $\mathrm{u\bar{s}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{d\bar{c}}$ (8%)
  - $\mathrm{c\bar{d}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{s\bar{u}}$
  - $\mathrm{c\bar{s}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{d\bar{u}}$
  - $\mathrm{c\bar{s}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{s\bar{c}}$
  - $\mathrm{\bar{d}\bar{d}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{\bar{u}\bar{u}}$
  - $\mathrm{\bar{d}\bar{s}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{\bar{u}\bar{c}}$
  - $\mathrm{\bar{s}\bar{s}} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{\bar{c}\bar{c}}$
- partonic processes for leptonic final states (omitting bottom quarks)
  $qq' \to q''q'''V_1V_2$: $12/40/60$ processes for $V_1V_2 = \mathrm{W^\pm W^\pm}$, WZ, $\mathrm{ZZ/W^+W^-}$
  many channels involve diagrams of both orders e^6 and $e^4 g_\mathrm{s}^2$
  $qg \to q'\mathrm{g}V_1V_2$: $0/28/40$ processes for $V_1V_2 = \mathrm{W^\pm W^\pm}$, WZ, $\mathrm{ZZ/W^+W^-}$
  plus crossed processes
- large number of contributing Feynman diagrams
  typically 40000 loop diagrams for one partonic process for WZ scattering

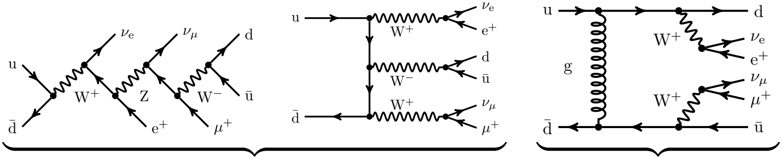
$\Rightarrow$ Full calculation requires automated tools.

Contributions to $\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ at LO

Vector-boson scattering (VBS) topologies: $\mathcal{O}\left(g^6\right)$ all t channel (u channel)



irreducible background to VBS:



EW background $\mathcal{O}(g^6)$, s channel

QCD background $\mathcal{O}\left(g_\mathrm{s}^2 g^4\right)$
t channel

t channel: incoming quarks/antiquarks connected to outgoing quarks/antiquarks
u channel: exchange identical quarks/antiquarks in final state
s channel: incoming quark and anti-quark connected, all boson propagators time like

VBS approximation

VBS approximation

Figy, Oleari, Zeppenfeld '03, Jäger, Oleari, Zeppenfeld '09

- Neglect interferences between t- and u-channel contributions and all s-channel contributions
  $\Rightarrow$ keep only squares of t- and u-channel contributions
- calculation simplifies considerably
  (~ 1000 loop diagrams per channel at $\mathcal{O}\big(\alpha_{\mathrm{s}}\alpha^6\big)$)
- only applicable to order α^6 and corresponding corrections for VBS cuts
  (tailored to VBS processes, not applicable to $\alpha_{\mathrm{s}}^2\alpha^4$)
- EW and QCD corrections to VBS uniquely defined
  (interferences neglected!)
- VBS approximation works within $\lesssim 1\%$ at LO
  Denner, Hošeková, Kallweit '12, Ballestrero et al. '18 (VBSCan)

Existing calculations for $VV+2$ jet production

- Full LO predictions: Ballestrero, Franzosi, Maina '10 (PHANTOM)

NLO QCD separately for EW ($\mathcal{O}(\alpha^6)$) and QCD-induced production ($\mathcal{O}(\alpha_s^2\alpha^4)$)

- NLO QCD corrections to EW production in VBS approximation:
  Jäger, Oleari, Zeppenfeld (+ Bozzi) '06, '07, '09 (VBFNLO);
  Denner, Hošeková, Kallweit '12
  PS matching: Jäger, Zanderighi '11, '13 + Karlberg '14 ($\mathrm{W^+W^+}$, $\mathrm{W^+W^-}$, ZZ)
  Rauch, Plätzer '16 ($\mathrm{W^+W^-}$), Jäger, Karlberg, Scheller '18 (WZ)
- NLO QCD corrections to QCD production:
  Melia, Melnikov, Röntsch, Zanderighi '10, '11 ($\mathrm{W^+W^+}$); Greiner et al. '12 ($\mathrm{W^+W^-}$);
  Campanario, Kerner, Ninh, Zeppenfeld '13, '14 (VBFNLO) ($\mathrm{W^+W^+}$, WZ, ZZ)
  PS matching: Melia, Nason, Röntsch, Zanderighi '11 ($\mathrm{W^+W^+}$)
- EW corrections for complete processes $\mathrm{pp}\to 4f+2\mathrm{j}$
  - NLO EW and QCD corrections for VBS into $\mathrm{W^\pm W^\pm}$, WZ, ZZ and $\mathrm{W^+W^-}$
    Biedermann et al.'16; Denner et al.'19, '20, '22
  - full NLO corrections to VBS into $\mathrm{W^\pm W^\pm}$ and ZZ
    Biedermann, Denner, Pellen '17; Denner, Franken, Pellen, Schmidt '21
  - NLO EW matched to EW PS and interfaced to QCD PS for $\mathrm{W^\pm W^\pm}$
    Chiesa, Denner, Lang, Pellen '19

Existing NLO calculations – state of the art

Calculations for VBS within the SM

- all processes known at NLO QCD accuracy matched to QCD PS
  - for both QCD-/EW-induced process
  - all available in VBFNLO
  - all available in POWHEG-BOX ⇒ parton-shower (PS) matching
  - often in VBS approximation (no int., s channel sometimes included)
  - possible to generate in MG5_AMC@NLO or SHERPA
- NLO EW corrections known for $\mathrm{W}^{\pm}\mathrm{W}^{\pm}$, WZ, ZZ, and $\mathrm{W}^{+}\mathrm{W}^{-}$ with leptonic decays
  NLO EW matched to EW PS and interfaced to QCD PS for $\mathrm{W}^{\pm}\mathrm{W}^{\pm}$
- full NLO computation only for $\mathrm{W}^{+}\mathrm{W}^{+}$ and ZZ with leptonic decays ($\mathrm{W}^{+}\mathrm{W}^{-}$ in progress)
- no NLO results for hadronically decaying vector bosons
- no NLO results for polarised vector bosons
- no NNLO results known

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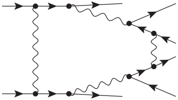
Details on virtual EW corrections

- All diagrams of considered order(s) for full process included
  virtual corrections involve up to 8-point functions of rank 4



- on-shell renormalisation scheme
- G_μ scheme for electromagnetic coupling:

$$\alpha_{G_\mu} = \frac{\sqrt{2}G_\mu M_\mathrm{W}^2}{\pi}\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)$$

  absorbs running of α to EW scale and some universal corrections $\propto m_\mathrm{t}^2$

- complex-mass scheme for gauge-boson and Higgs resonances

  Denner, Dittmaier, Roth, Wackeroth, Wieders '99, '05

  complex poles: $\mu_\mathrm{W}^2 = M_\mathrm{W}^2 - \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}, \ \mu_\mathrm{Z}^2 = M_\mathrm{Z}^2 - \mathrm{i}M_\mathrm{Z}\Gamma_\mathrm{Z}$
  ⇒ gauge-invariant amplitudes, universal treatment of resonances

- all matrix elements calculated with RECOLA Actis et al. '12, '16
  and COLLIER Denner et al. '16
  for WZ independent calculation with OPENLOOPS

  Cascioli, Maierhöfer, Pozzorini '11, Kallweit et al. '14

RECOLA

Generator for matrix elements at LO and NLO in the SM and beyond:

RECOLA: (Recursive computation of one-loop amplitudes)

RECOLA 1 Actis, Denner, Hofer, Scharf, Uccirati, arXiv:1605.01090

RECOLA 2 Denner, Lang, Uccirati, 1711.073881, https://recola.gitlab.io/recola2/

- full Standard Model (QCD + EW) and some extensions
    - tree level and one-loop amplitudes
    - Feynman rules for counter terms Denner '93
    - Feynman rules for rational terms (R_2) Garzelli, Malamos, Pittau '10
- complex-mass scheme for unstable particles
- mass- and dimensional regularisation supported for IR singularities
- renormalisation
    - EW sector: on-shell renormalisation $(\alpha(0),\ \alpha_{G_\mu},\ \alpha(M_\mathrm{Z}))$
    - α_s: $\overline{\mathrm{MS}}$ renormalisation (variable and fixed flavour schemes)
- selection of resonant contributions, e.g. $qg \to qg\mathrm{Z} \to qg\ell^+\ell^-$
- output: NLO amplitudes for specific helicities and colour structures
- output: colour- and spin-correlated amplitudes for dipole subtraction
- needed: external library for tensor integrals needed COLLIER
- input for RECOLA 2: model file

COLLIER

Library for one-loop integrals for scattering and decay processes:
COLLIER (Complex one loop library in extended regularizations)
Denner, Dittmaier, Hofer, arXiv:1604.06792, https://collier.hepforge.org/

- tensor integrals for arbitrary number of external momenta N
  (tested in physical processes up to $N = 10$)
- tensor-integral reduction à la Passarino Veltman
  and various expansion methods for exceptional phase-space points
  (to arbitrary order in expansion parameter)
- mass- and dimensional regularisation supported for IR singularities
- complex masses supported (unstable particles)
- cache-system to avoid recalculation of identical integrals
- output: coefficients $T^N_{0\cdots 0 i_1 \cdots i_k}$ or tensors $T^{N,\mu_1\cdots\mu_R}$
- error estimates for tensor coefficients and tensor integrals
- two independent implementations ⇒ checks during run possible
- complete set of one-loop scalar integrals for scattering processes

MoCaNLO and BBMC

Codes for the integration of multiparticle cross sections at LO and NLO:

MoCaNLO (Monte Carlo at NLO), Feger, Denner, Pellen, Pelliccioli, Schmidt, unpublished

BBMC (Boson Boson Monte Carlo)

Biedermann, Denner, Franken, Motz, Scharf, unpublished

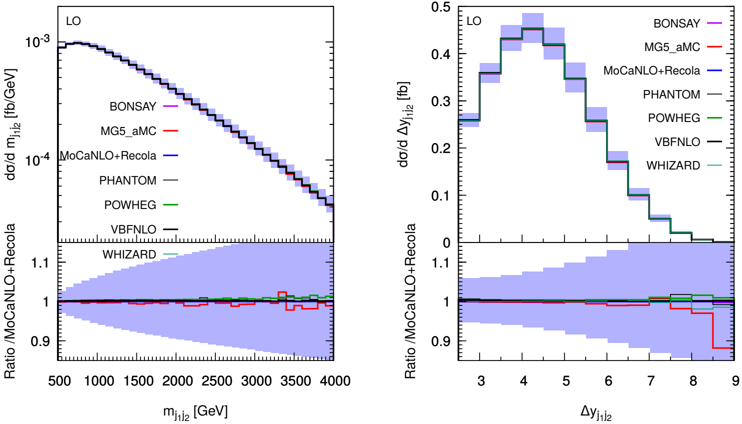
- generic Monte Carlo integration codes for arbitrary processes at NLO QCD and EW accuracy
- multi-channel integration based on generic phase-space mappings using channels for all Feynman diagrams
  Berends, Pittau, Kleiss '94; Denner, Dittmaier, Roth, Wackeroth '99
- subtraction of IR singularities using Catani–Seymour dipoles for QCD Catani, Seymour '97 and QED Dittmaier '00, '08
- α_{dipole} parameters to restrict subtraction to singular regions ($\Rightarrow$ checks)
  Nagy, Trócsányi '99
- photon-to-quark conversion function for final-state photon splitting into quark pairs Denner, Dittmaier, Pellen, Schwan '19
- quark-to-photon fragmentation function for photon–jet separation
  Glover, Morgan '93, Denner, Dittmaier, Gehrmann, Kurz '10

Outline

Vector-boson scattering approximation at LO

Comparison of codes with VBS approximation (Bonsay, Powheg, VBFNLO) and without (MoCaNLO+Recola, MG5_aMC, PHANTOM, Whizard)

$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ Ballestrero et al. '18 (VBSCan)



differences between codes below $\sim 1\%$ in fiducial region

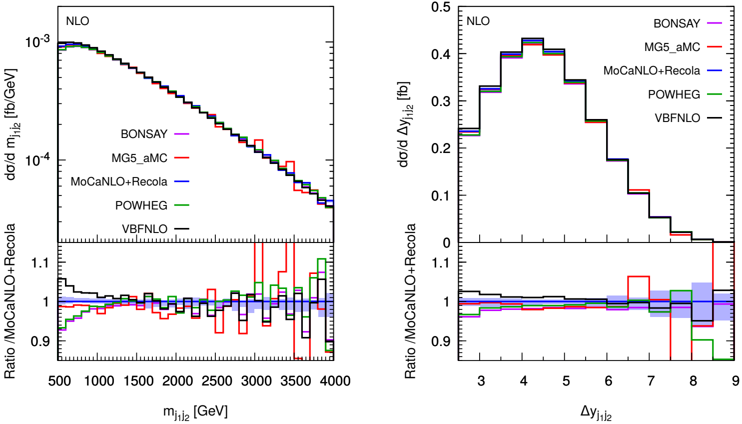
$\Rightarrow$ accuracy of VBS approximation below $\sim 1\%$ at LO

reminder: VBS approximation neglects all interferences and s-channel contributions

Vector-boson scattering approximation at NLO QCD

Comparison of codes with VBS approximation (BONSAY, POWHEG VBFNLO)
and without VBS approximation (MOCANLO+RECOLA, MG5_AMC)
$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$

Ballestrero et al. '18 (VBSCan)

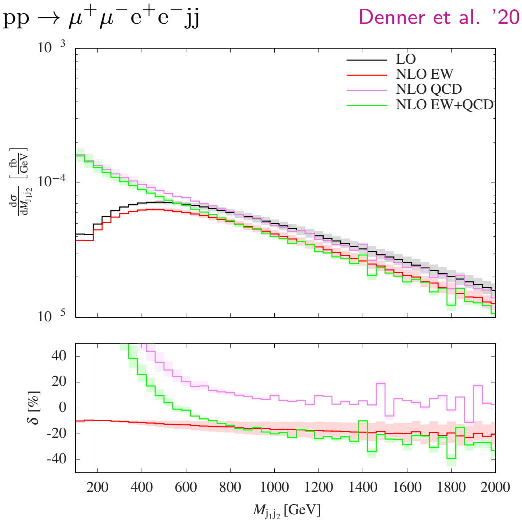


differences up to 10% outside the QCD scale uncertainty band

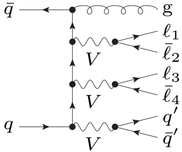
POWHEG, Bonsay: no s channel, on interference ⇒ reduction at small $M_{\mathrm{j_1j_2}}$, $\Delta y_{\mathrm{j_1j_2}}$

VBFNLO: no interference ⇒ enhancement at small $M_{\mathrm{j_1j_2}}$, $\Delta y_{\mathrm{j_1j_2}}$

NLO QCD and EW corrections to ZZ production in VBS



- Loose VBS cut:
  $M_{\mathrm{j_1j_2}} > 100\,\mathrm{GeV}$
  based on 1708.02812 (CMS)
- 24% NLO QCD corrections to fiducial cross section
- s-channel NLO contribution involving tri-boson production



Less suppression at NLO owing to extra gluon jet

⇒ include tri-boson contribution (s-channel) for loose VBS cuts

Outline

1. Introduction
2. Theoretical aspects
3. Details of calculation and tools
4. Quality of VBS approximation
5. Electroweak corrections to vector-boson scattering (VBS)
6. Polarised VBS
7. Conclusion

Corrections to fiducial cross sections

Large universal NLO EW corrections to VBS processes

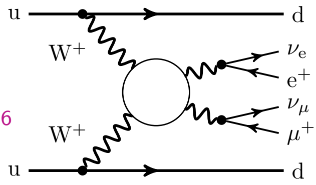
process	$\sigma_{\mathrm{LO}}^{\mathcal{O}(\alpha^6)}$ [fb]	$\Delta\sigma_{\mathrm{NLO,EW}}^{\mathcal{O}(\alpha^7)}$ [fb]	δ_{EW} [%]
Biedermann et al. '17			
$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ $(\mathrm{W}^+\mathrm{W}^+)$	$1.4178(2)$	$-0.2169(3)$	-15.3
Denner et al. '19			
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ (ZW^+)	$0.25511(1)$	$-0.04091(2)$	-16.0
Denner et al. '20			
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-\mathrm{jj}$ (ZZ)	$0.097681(2)$	$-0.015573(5)$	-15.9
Denner et al. '22			
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-\mathrm{jj}$ $(\mathrm{W}^+\mathrm{W}^-)$	$2.6988(3)$	$-0.307(1)$	-11.4

- surprisingly large EW corrections for fiducial cross section
- EW corrections similarly large for more inclusive setups
  $\Rightarrow$ intrinsic feature of VBS process
- σ^{LO} receives sizeable contributions involving large invariants $s_{ij} \gg M_\mathrm{W}$
  average partonic centre-of-mass energy $\langle\sqrt{\hat{s}}\rangle \sim 2\,\mathrm{TeV}$
  average four-lepton invariant mass $\langle M_{4\ell}\rangle \sim 400\,\mathrm{GeV}$

Source of large EW corrections

Double-pole approximation (DPA) for outgoing W bosons
effective vector-boson approximation (EVBA) for incoming W bosons

- DPA and EVBA reduce discussion to $V_1V_2 \to V_3V_4$
- DPA accurate for cross section within 1%
- EVBA crude approximation Kuss, Spiesberger '96 but sufficient to understand dominant effects



high-energy, logarithmic approximation for $V_1V_2 \to V_3V_4$ Denner, Pozzorini '00

$$d\sigma_{LL} = d\sigma_{LO}\left[1 - \frac{\alpha}{4\pi}4C_W^{EW}\log^2\left(\frac{Q^2}{M_W^2}\right) + \frac{\alpha}{4\pi}2b_W^{EW}\log\left(\frac{Q^2}{M_W^2}\right)\right]$$

$$C_W^{EW} = \frac{2}{s_w^2}, \quad b_W^{EW} = \frac{19}{6s_w^2} \qquad \text{for transverse W bosons,} \qquad Q \to M_{4\ell}$$

(double EW logs, collinear single EW logs, and single logs from parameter renormalisation included) (angular-dependent logarithms omitted, $\log\frac{t}{u}\log\frac{Q}{M_W}$)

large NLO EW corrections intrinsic feature of VBS

NLO EW corrections in logarithmic approximation

Simple formula for total cross section

$$\mathrm{d}\sigma_{\mathrm{LL}} = \mathrm{d}\sigma_{\mathrm{LO}}\left[1 - \frac{\alpha}{4\pi}4C_{\mathrm{W}}^{\mathrm{EW}}\log^2\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) + \frac{\alpha}{4\pi}2b_{\mathrm{W}}^{\mathrm{EW}}\log\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right)\right]$$

process	δ_{EW} [%]	$\delta_{\mathrm{EW}}^{\mathrm{log,int}}$ [%]	$\delta_{\mathrm{EW}}^{\mathrm{log,diff}}$ [%]	$\langle M_{4\ell}\rangle$ [GeV]
$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_{\mathrm{e}}\mathrm{jj}$	-16.0	-16.1	-15.0	390
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_{\mathrm{e}}\mathrm{jj}$	-16.0	-17.5	-16.4	413
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-\mathrm{jj}$	-15.9	-15.8	-14.8	385

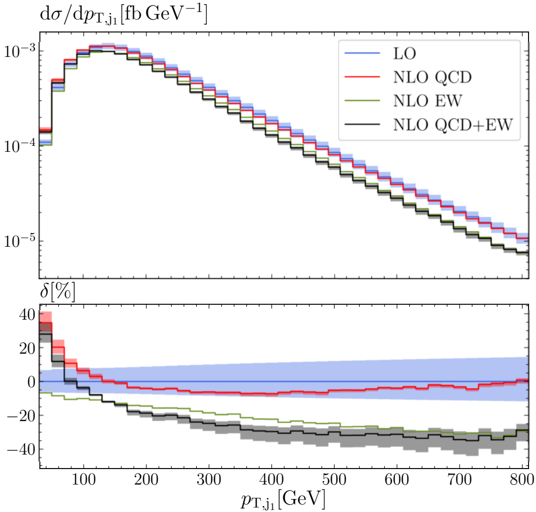
- surprisingly good agreement with complete calculation ($\delta_{\mathrm{EW}} = -16.0\%$)
- large EW corrections are due to large gauge couplings of vector bosons (C^{EW}) and large scale $Q \sim \langle M_{4\ell}\rangle \sim 400\,\mathrm{GeV}$
- angular-dependent logarithms different for different processes ~ 1–2% owing to cancellations

large NLO EW corrections intrinsic feature of VBS

Distributions for $\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ $(\mathrm{ZW^+jj})$

Distribution in transverse momentum of the leading jet

Denner et al. '19



- $\mathcal{O}(\alpha^7) \sim -30\%$ at $p_{\mathrm{T,j_1}} = 800\,\mathrm{GeV}$ (Sudakov logarithms) dominant correction larger than QCD scale uncertainty
- $\mathcal{O}(\alpha_\mathrm{s}\alpha^6) \lesssim 10\%$ for $p_{\mathrm{T,j_1}} > 100\,\mathrm{GeV}$ small QCD scale uncertainty owing to suitable dynamical scale $\mu = \sqrt{p_{\mathrm{T,j_1}} p_{\mathrm{T,j_2}}}$
- large correction for small $p_{\mathrm{T,j_1}}$ due to phase-space suppression at LO (all jets have small p_T) redistribution of events at NLO

Corrections to fiducial cross sections for W^+W^- scattering

Process $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu jj$

- EW corrections smaller than for other processes
- fiducial phase space contains Higgs resonance

setup	$\sigma_{LO}^{\mathcal{O}(\alpha^6)}$ [fb]	$\Delta\sigma_{NLO,EW}^{\mathcal{O}(\alpha^7)}$ [fb]	δ_{EW} [%]
VBS setup	$2.6988(3)$	$-0.307(1)$	-11.4
VBS setup, Higgs cut out VBS channels only	$1.6117(2)$	$-0.239(2)$	-14.8
VBS setup, Higgs cut out	$1.9750(2)$	$-0.260(2)$	-13.2
VBS setup, Higgs contribution	$0.7238(2)$	$-0.047(2)$	-6.5
Higgs setup	$1.5322(2)$	$-0.103(1)$	-6.7

- EW corrections to generic VBS contributions $\sim -15\%$
- EW corrections to Higgs resonance contribution $\sim -6.5\%$
- Higgs cut: $|M_{4\ell} - M_H| > 20\Gamma_H \approx 80\,\text{MeV}$, removes 98.4% of resonance
- Higgs contribution: $|M_{4\ell} - M_H| < 20\Gamma_H$
- Higgs setup: cuts inspired by CMS Higgs search CMS 1806.05246

Outline

Physical interest in polarised cross sections

Preliminaries

- All information about polarised cross sections is within angular distributions of final-state particles.
- Extracting polarised observables simplifies interpretation and theoretical analysis.

Polarised observables

- are important probes of Standard Model gauge and Higgs sectors,
- may provide discrimination power between SM and beyond-SM physics.

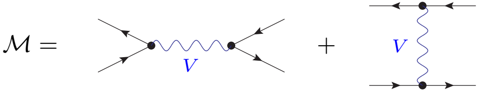
Longitudinal polarisation mode of vector bosons is

- a consequence of the EW Symmetry Breaking
- very sensitive to deviations from SM:
  unitarity of cross sections with longitudinally polarised vector bosons realized in SM via cancellation of different contributions.

⇒ Extract experimental results for cross sections with longitudinally polarised vector bosons.

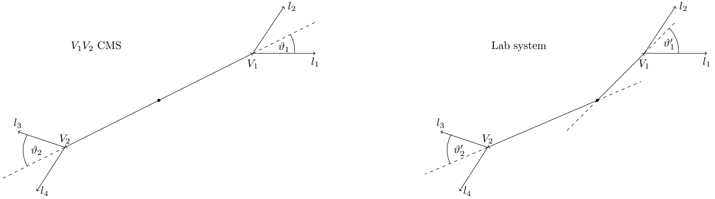
Ambiguities of polarised cross sections with vector bosons

- Unstable massive vector bosons appear only as virtual particles ⇒
  - no unique definition of vector-boson polarisations for off-shell bosons
  - diagrams without resonant vector bosons contribute to physical final state



- vector bosons are massive ⇒
  definition of polarisation depends on frame and on mass

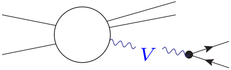
  natural choices for frame:
  - (di-boson-)centre-of-mass frame (same reference direction for V_1 and V_2)
  - laboratory frame.



Definition of polarised cross sections

Definitions of polarised cross sections in the literature:

- definition via projections on LO decay-angle distributions
  $|\mathcal{M}_-|^2 \propto (1-\cos\theta^*)^2$, $|\mathcal{M}_+|^2 \propto (1+\cos\theta^*)^2$, $|\mathcal{M}_\mathrm{L}|^2 \propto (\sin\theta^*)^2$ for W boson
  Baglio, Le Duc '18, '19, Bern et al. '11
  - tailored to inclusive LO predictions
  - assumes no cuts and negligible non-resonant background
  - only applicable for one polarised vector boson
  - results depend on cuts, background, and NLO corrections
- definition based on on-shell production and decay with spin correlations
  - neglects non-resonant contributions
  - implemented in Madgraph for LO Franzosi et al. '19
  - used for NNLO corrections to $\mathrm{W}+\mathrm{j}$ production
    Pellen, Poncelet, Popescu '21



Definition of polarisation based on pole approximation I

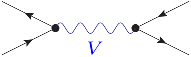
Idea: use pole approximation to extract resonant contributions in gauge-invariant way Ballestrero, Maina, Pellicciioli '17, '19

formulation developed by Denner, Pellicciioli '20

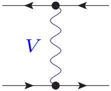
- not all diagrams involve required resonances
  resonant diagrams

non-resonant diagrams

$$\frac{R(k^2)}{k^2 - M^2 + \mathrm{i}M\Gamma} =$$



$$N(k^2) =$$



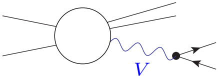
- split full matrix element into resonant part and non-resonant part using pole expansion (gauge-invariant)

$$\begin{aligned}\mathcal{A} &= \frac{R(k^2)}{k^2 - M^2 + \mathrm{i}M\Gamma} + N(k^2) \\ &= \frac{R(M^2)}{k^2 - M^2 + \mathrm{i}M\Gamma} + \frac{R(k^2) - R(M^2)}{k^2 - M^2} + N(k^2) = \mathcal{A}_{\mathrm{res}} + \mathcal{A}_{\mathrm{nonres}}\end{aligned}$$

- consider non-resonant part as irreducible background: no resonance
- define polarisation for on-shell residue $R(M^2)$

Definition of polarisation based on pole approximation II

Separate polarisation modes of resonant amplitude



split propagator numerator of resonant particle

$$\mathcal{A}_{\mathrm{res}} = \mathcal{P}_\mu \, \frac{-g^{\mu\nu}}{k^2 - M_{\mathrm{W}}^2 + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}} \, \mathcal{D}_\nu = \mathcal{P}_\mu \, \frac{\sum_\lambda \varepsilon_\lambda^{\mu\,*}(k) \varepsilon_\lambda^\nu(k)}{k^2 - M_{\mathrm{W}}^2 + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}} \, \mathcal{D}_\nu$$

$$= \sum_{\lambda=\mathrm{L},\pm} \frac{\mathcal{M}_\lambda^{\mathrm{prod}} \, \mathcal{M}_\lambda^{\mathrm{dec}}}{k^2 - M_{\mathrm{W}}^2 + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}} =: \sum_{\lambda=\mathrm{L},\pm} \mathcal{A}_\lambda \,,$$

$$\left|\mathcal{A}_{\mathrm{res}}\right|^2 = \sum_\lambda \left|\mathcal{A}_\lambda\right|^2 + \sum_{\lambda \neq \lambda'} \mathcal{A}_\lambda^* \, \mathcal{A}_{\lambda'}$$

- incoherent sum $\sum_\lambda \left|\mathcal{A}_\lambda\right|^2$: $\left|\mathcal{A}_\lambda\right|^2 \propto$ "polarised cross sections",

  "polarisation fractions": $f_\lambda = \frac{\left|\mathcal{A}_\lambda\right|^2}{\sum_\lambda \left|\mathcal{A}_\lambda\right|^2}$
- interferences $\sum_{\lambda\neq\lambda'} \mathcal{A}_\lambda^* \, \mathcal{A}_{\lambda'}$
  vanish for quantities fully inclusive in decay products, but not in general

Scope of method

- Method is applicable to arbitrary processes and multiple resonances at LO, NLO and beyond.
- needs pole approximation (or double-pole approximation) for all NLO contributions including subtraction terms! ⇒ technical complication
- results at LO for VBS for ss-WW, WZ, ZZ, os-WW

  Ballestrero, Maina, Pellicciolі '17, '19, '20 [PHANTOM]
- results at NLO QCD for
  - $\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}$ ($\mathrm{W}^+\mathrm{W}^-$ production) Denner, Pellicciolі '20 and
  - $\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_\mathrm{e}$ ($\mathrm{W}^+\mathrm{Z}$ production) Denner, Pellicciolі '20
- results at NLO EW for (neutral vector bosons)
  - $\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-$ (ZZ production) Denner, Pellicciolі '21
- generalisation in progress towards VBS at NLO QCD and NLO EW (involving charged resonances ⇒ more subtraction terms)

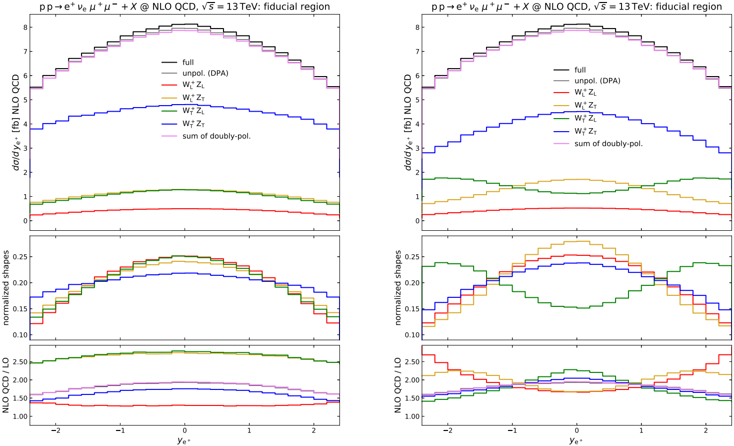
Method allows to separate

- polarised cross sections in arbitrary frames
- interference contributions between polarisations
- irreducible background.

Example results for distributions

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^+\mu^-$ $(\mathrm{W}^+\mathrm{Z})$: Distributions in the positron rapidity in the fiducial region for polarisations defined in the CM (left) and in the LAB (right) frame.

Denner, Pellicciolì '20



Distributions for pol. cross sections defined in different frames differ considerably!

Example results for polarisation fractions

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^+\mu^-$ $(\mathrm{W}^+\mathrm{Z})$:

	σ_{LO} [fb]	fraction$_{\mathrm{LO}}$	σ_{NLO} [fb]	fraction$_{\mathrm{NLO}}$	K-factor
full	$19.537(7)^{+4.1\%}_{-4.9\%}$	-	$35.27(1)^{+5.2\%}_{-4.2\%}$	-	1.81
unpolarized (DPA)	$19.125(6)^{+4.1\%}_{-5.0\%}$	100%	$34.63(1)^{+5.3\%}_{-4.2\%}$	100%	1.81
polarisations defined in the CM frame					
$\mathrm{W}^+_\mathrm{L}\mathrm{Z}_\mathrm{L}$ (DPA)	$1.508(1)^{+4.5\%}_{-5.3\%}$	7.9%	$1.968(1)^{+2.7\%}_{-2.2\%}$	5.7%	1.31
$\mathrm{W}^+_\mathrm{L}\mathrm{Z}_\mathrm{T}$ (DPA)	$2.018(1)^{+5.1\%}_{-6.0\%}$	10.6%	$5.354(1)^{+7.3\%}_{-5.9\%}$	15.5%	2.65
$\mathrm{W}^+_\mathrm{T}\mathrm{Z}_\mathrm{L}$ (DPA)	$1.902(1)^{+5.0\%}_{-5.9\%}$	9.9%	$5.097(2)^{+7.4\%}_{-5.9\%}$	14.7%	2.68
$\mathrm{W}^+_\mathrm{T}\mathrm{Z}_\mathrm{T}$ (DPA)	$13.555(5)^{+3.8\%}_{-4.7\%}$	70.9%	$21.992(9)^{+4.5\%}_{-3.6\%}$	63.5%	1.62
polarisations defined in the LAB frame					
$\mathrm{W}^+_\mathrm{L}\mathrm{Z}_\mathrm{L}$ (DPA)	$1.0824(4)^{+4.1\%}_{-4.9\%}$	5.7%	$2.063(1)^{+5.6\%}_{-4.5\%}$	6.0%	1.91
$\mathrm{W}^+_\mathrm{L}\mathrm{Z}_\mathrm{T}$ (DPA)	$3.165(1)^{+5.1\%}_{-6.0\%}$	16.5%	$6.108(2)^{+5.6\%}_{-4.5\%}$	17.6%	1.93
$\mathrm{W}^+_\mathrm{T}\mathrm{Z}_\mathrm{L}$ (DPA)	$4.381(2)^{+4.8\%}_{-5.7\%}$	22.9%	$7.409(4)^{+4.8\%}_{-3.8\%}$	21.4%	1.69
$\mathrm{W}^+_\mathrm{T}\mathrm{Z}_\mathrm{T}$ (DPA)	$10.526(4)^{+3.6\%}_{-4.4\%}$	55.0%	$18.964(7)^{+5.2\%}_{-4.2\%}$	54.8%	1.80

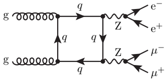
- irreducible background 2.1% at LO, 1.8% at NLO (full – unpolarised)
- interference effects below 1% at LO and NLO (unpolarised – sum polarised)
- polarisation fractions strongly affected by NLO QCD corrections and choice of reference frame
- LT and TL similar in CM frame (same reference axis) but different in LAB frame

Example results for polarisation fractions

$pp \to e^+e^-\mu^+\mu^-$ (ZZ):

mode	σ_{LO} [fb]	δ_{QCD}	δ_{EW}	δ_{gg}	σ_{NLO_+} [fb]
full	$11.1143(5)^{+5.6\%}_{-6.8\%}$	$+34.9\%$	-11.0%	$+15.6\%$	$15.505(6)^{+5.7\%}_{-4.4\%}$
unpol.	$11.0214(5)^{+5.6\%}_{-6.8\%}$	$+35.0\%$	-10.9%	$+15.7\%$	$15.416(5)^{+5.7\%}_{-4.4\%}$
$\mathrm{Z_LZ_L}$	$0.64302(5)^{+6.8\%}_{-8.1\%}$	$+35.7\%$	-10.2%	$+14.5\%$	$0.9002(6)^{+5.5\%}_{-4.3\%}$
$\mathrm{Z_LZ_T}$	$1.30468(9)^{+6.5\%}_{-7.7\%}$	$+45.3\%$	-9.9%	$+2.8\%$	$1.8016(9)^{+4.3\%}_{-3.5\%}$
$\mathrm{Z_TZ_L}$	$1.30854(9)^{+6.5\%}_{-7.7\%}$	$+44.3\%$	-9.9%	$+2.8\%$	$1.7933(9)^{+4.3\%}_{-3.4\%}$
$\mathrm{Z_TZ_T}$	$7.6425(3)^{+5.2\%}_{-6.4\%}$	$+31.2\%$	-11.2%	$+20.5\%$	$10.739(4)^{+6.2\%}_{-4.7\%}$

- small irreducible background (0.5%) and interferences (1.2%)
- sizeable QCD and EW corrections
- substantial contribution from loop-induced gg fusion for LL and TT
- polarisation fractions roughly conserved by NLO corrections owing to cancellations



Outline

Conclusion

Significant interest in VBS in theory and experimental community

- NLO QCD corrections matched to PS available for all VBS processes
  NLO QCD corrections at level of few percent for typical VBS cuts
- VBS approximation might not be sufficient at NLO   Ballestrero et al. '18
  NLO-QCD tri-boson contributions of $\mathcal{O}(20\%)$ for loose VBS cuts
- electroweak corrections for VBS
  - full NLO EW corrections known for

$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}\ (\mathrm{W}^+\mathrm{W}^+)$	Biedermann et al. '16, '17
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}\ (\mathrm{WZ})$	Denner et al. '19
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-\mathrm{jj}\ (\mathrm{ZZ})$	Denner et al. '20
$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{jj}\ (\mathrm{W}^+\mathrm{W}^-)$	Denner et al. '22

  - -16% EW corrections for fiducial VBS cross section
    intrinsic feature of VBS, reproducible by simple approximations
  - presence of Higgs resonance reduces EW corrections $\Rightarrow -7\%$
  - EW corrections in distributions even larger
    -40% for $p_{\mathrm{T,j_1}} = 800\,\mathrm{GeV}$
- NLO corrections to polarised cross sections in reach
  first results for polarised vector-boson pair production exist

Outlook

Significant theoretical progress in VBS in recent years!

Expected progress in theoretical predictions to VBS:

- NLO corrections for polarised VBS within reach
- predictions for VBS with semileptonic final states
- matching to EW parton showers
- predictions for VBS within extended models