Information Field Theory – Bayesian Inference for Fields

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MPA Garching

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# Bayesian Inference for Fields

Fields
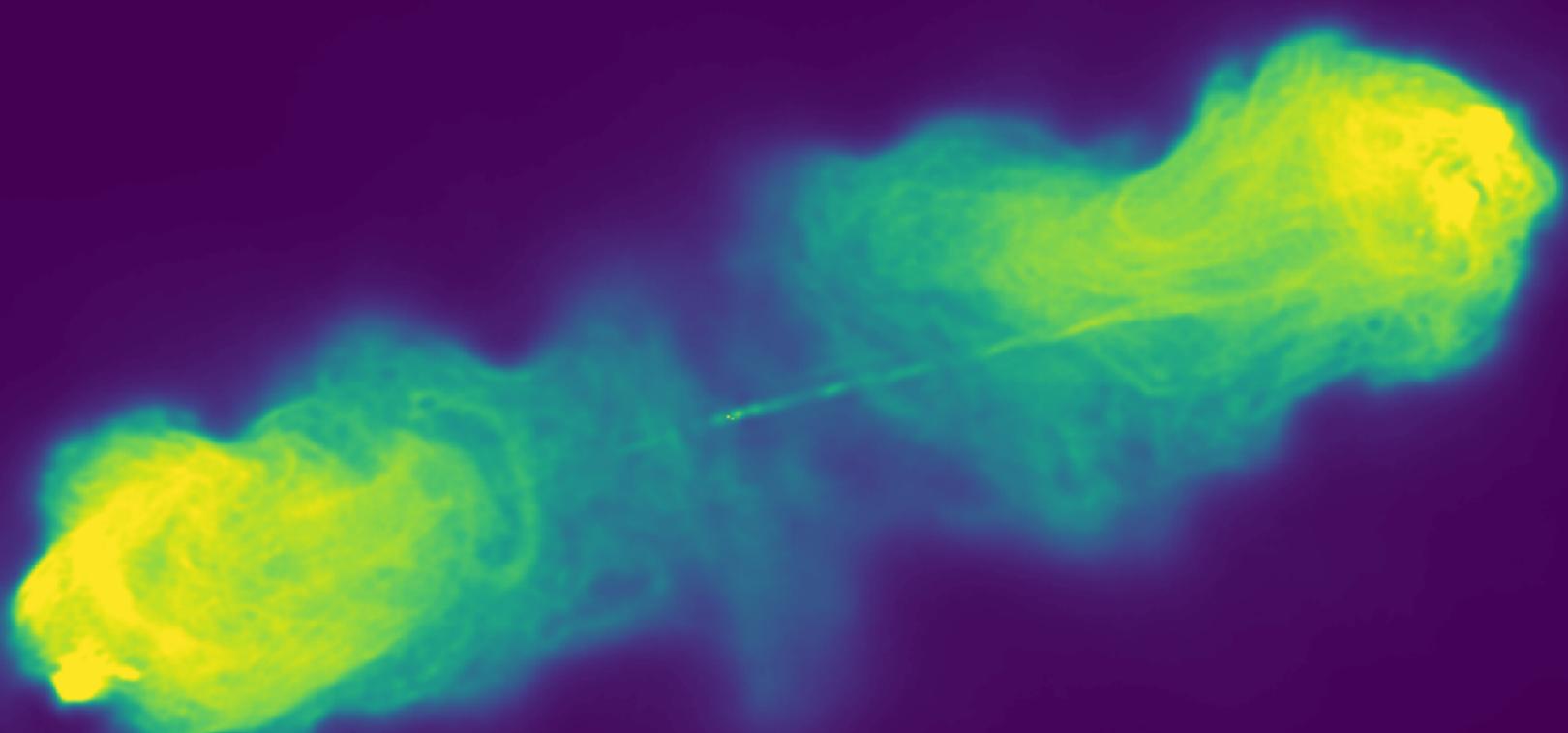
- Continuous quantities over space
- Temperature: 3D field
- Sky brightness: 2D field on sphere

Bayesian Inference
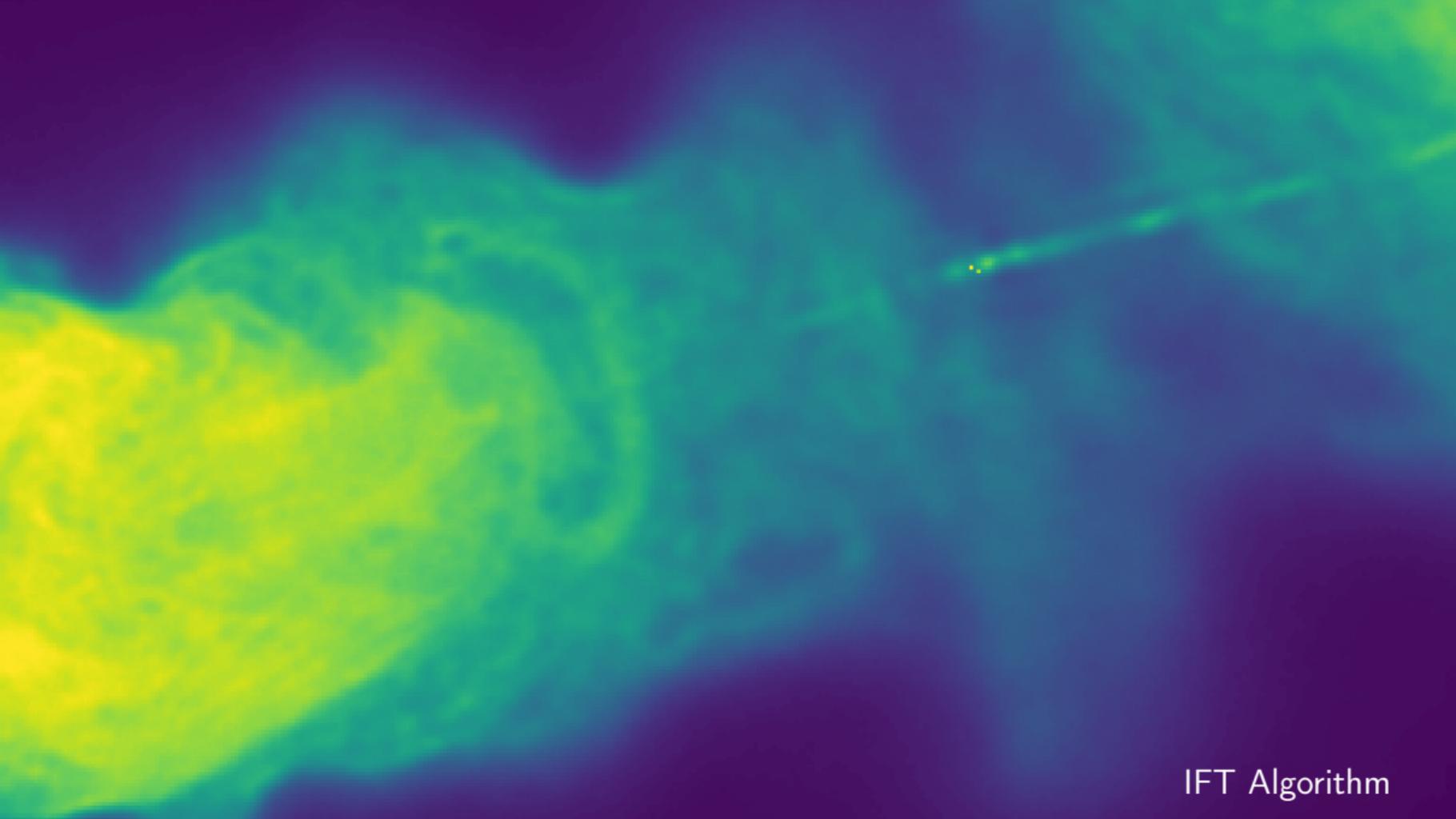
$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

- Data: d
- Signal: s
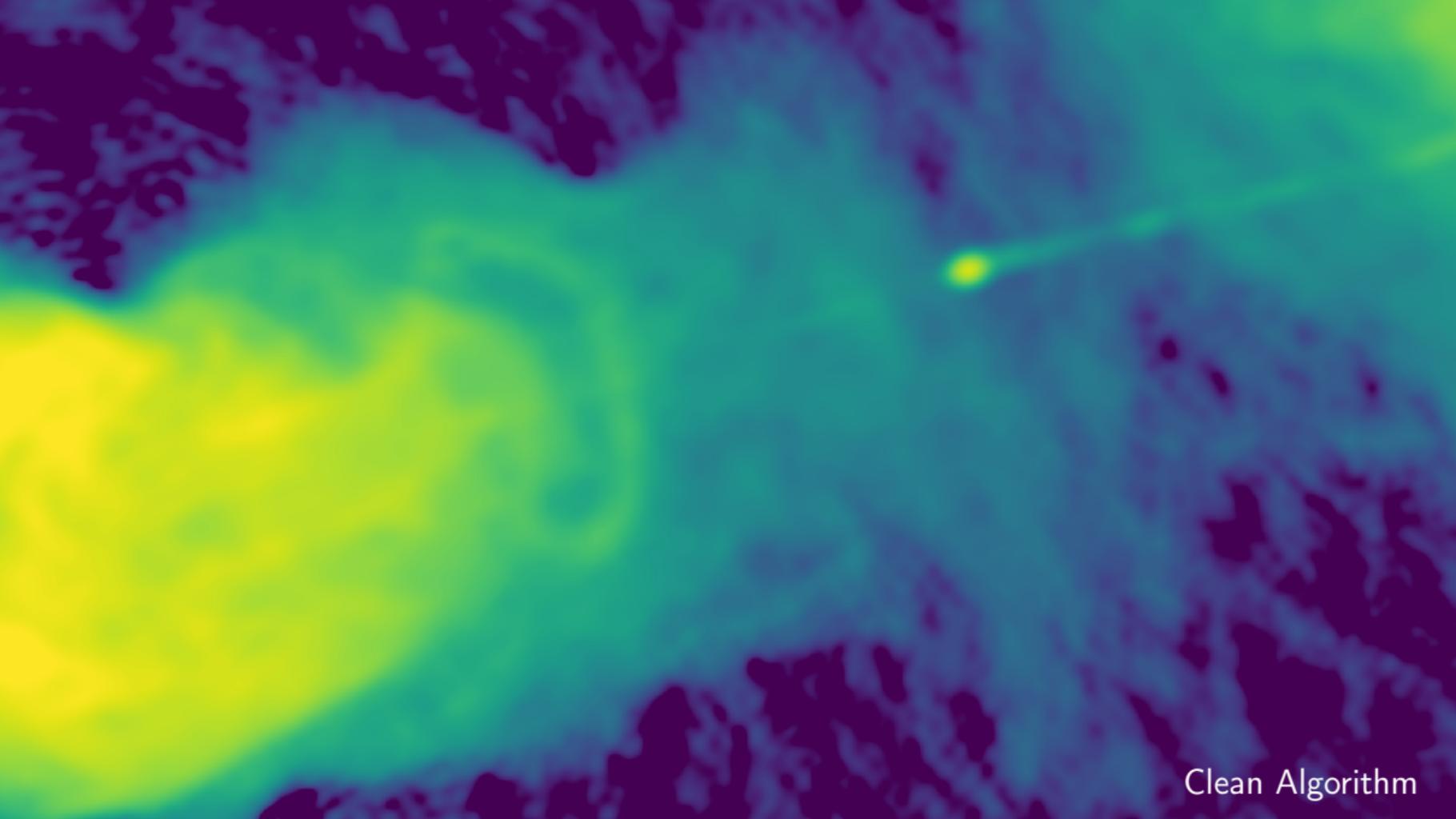
Determine posterior distribution $\mathcal{P}(s|d)$

IFT Algorithm
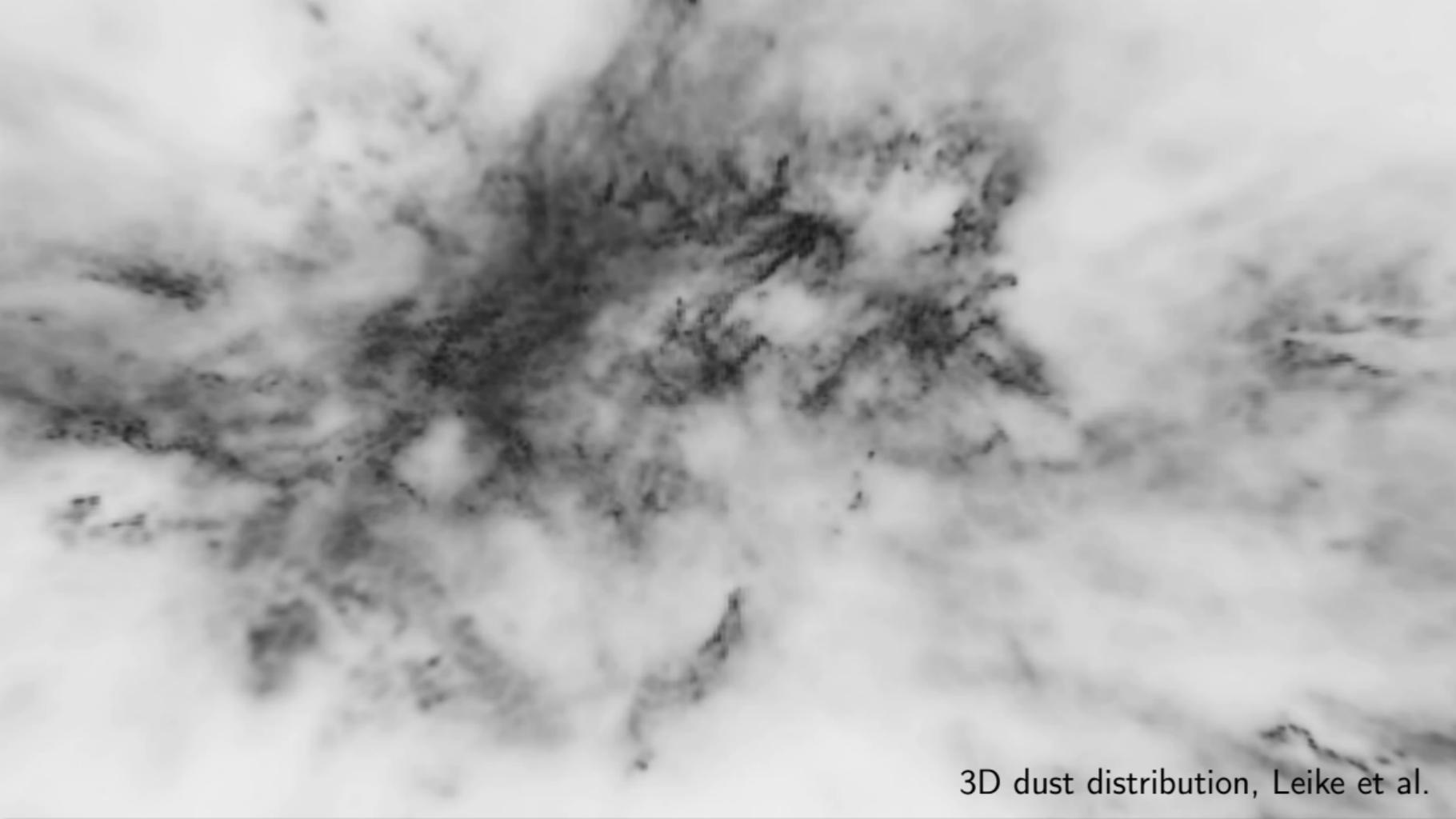
IFT Algorithm

Clean Algorithm
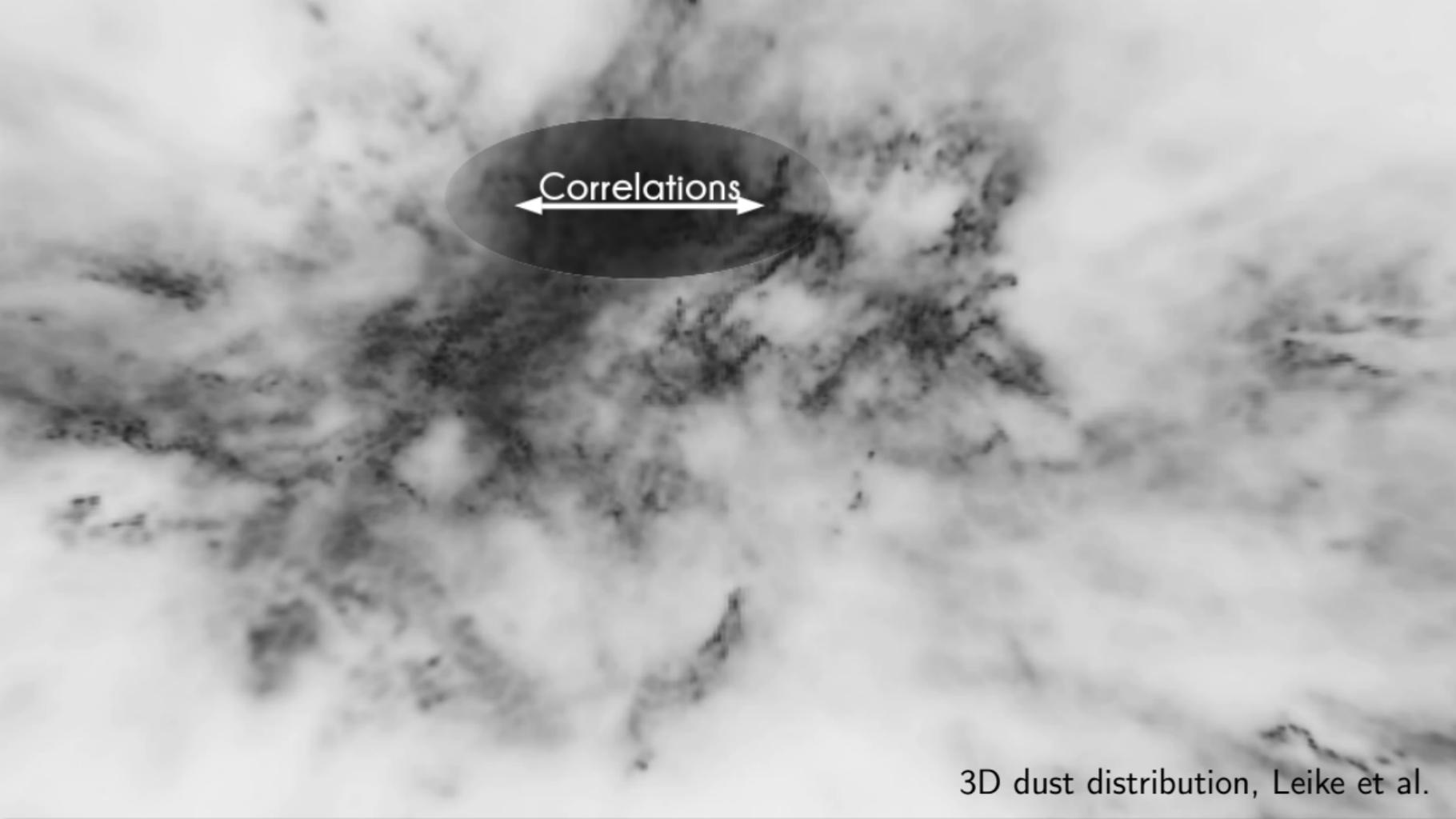
M87*, EHT Collaboration

M87*, Arras et al.

3D dust distribution, Leike et al.

Correlations

3D dust distribution, Leike et al.
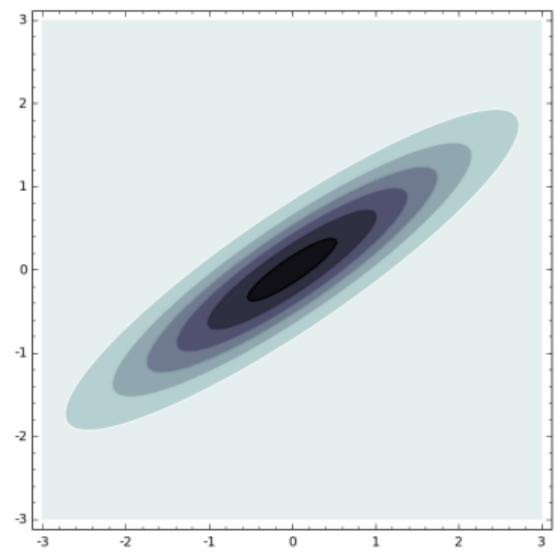
Correlations:

- Effectively reduces DOF
- Propagate information to locations not measured

$\Rightarrow$ Key concept for field inference
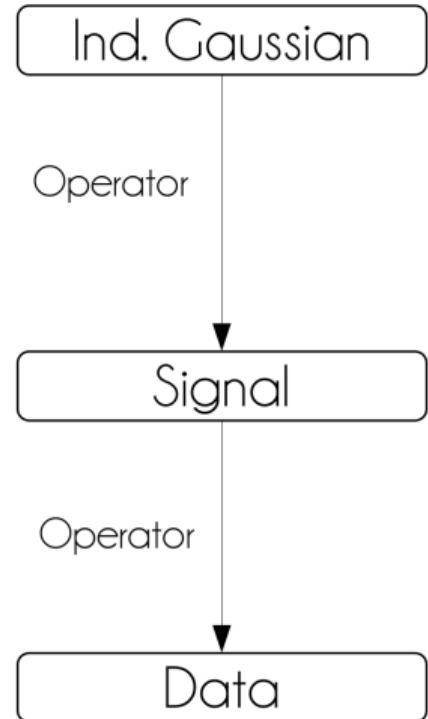
Prior Model

Prior Information:
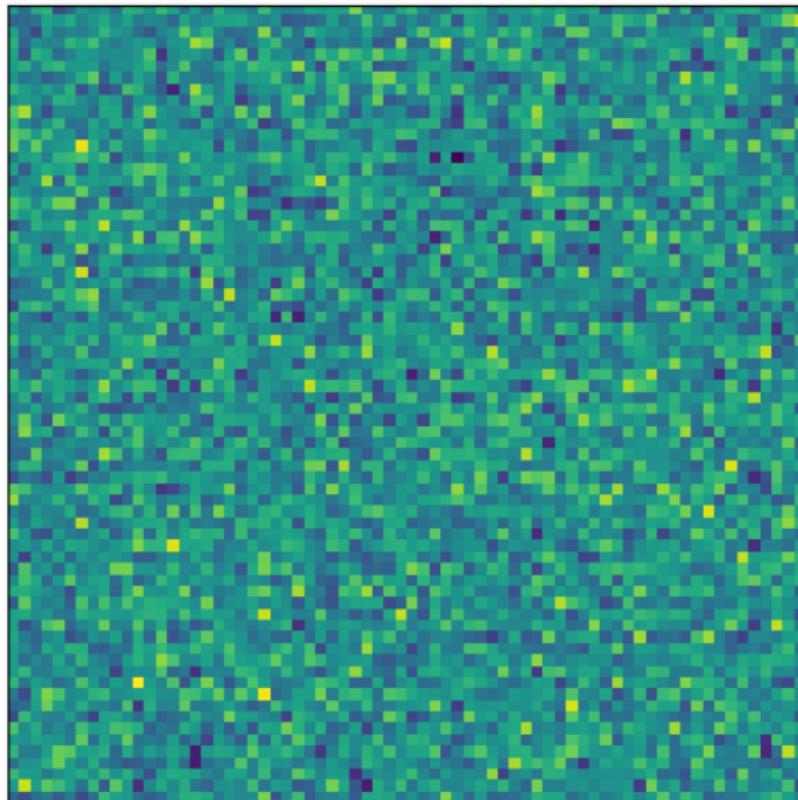
- Correlation
- Physical constraints on signal, e.g. positivity of flux
- Instrument response

$\Rightarrow$ Complicated prior distribution

Prior as a standardized generative model:

- Latent parameters ξ
- $\mathcal{P}(\xi) = \mathcal{G}(0, 1)$
- $\xi \mapsto s$

```
┌─────────────────┐
│  Ind. Gaussian  │
└─────────────────┘
        │
     Operator
        │
        ▼
┌─────────────────┐
│     Signal      │
└─────────────────┘
        │
     Operator
        │
        ▼
┌─────────────────┐
│      Data       │
└─────────────────┘
```
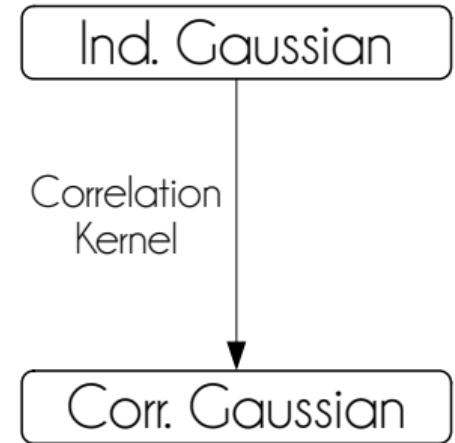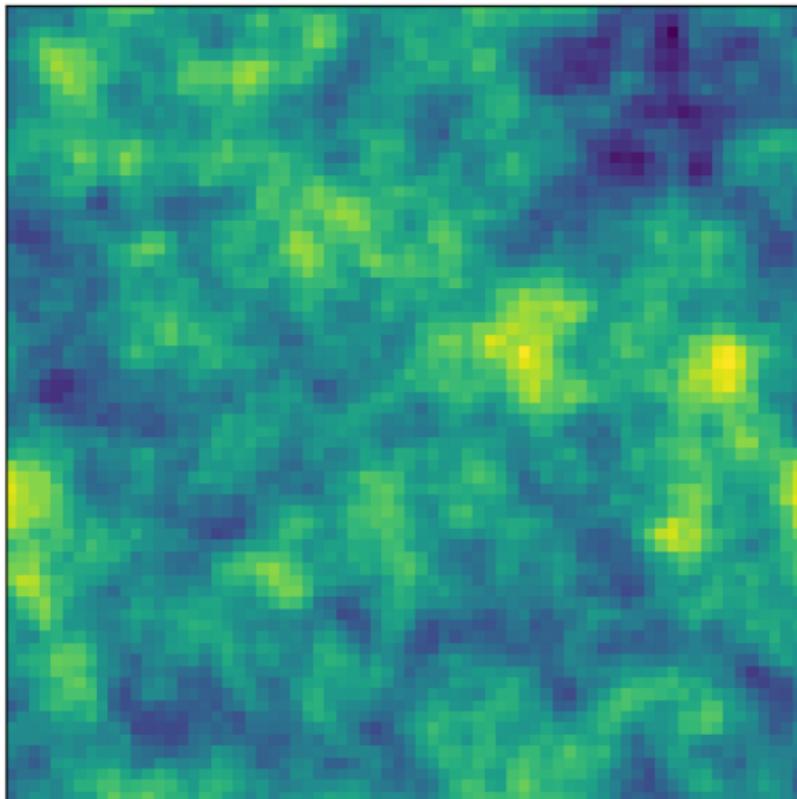
Generative Models

Ind. Gaussian

Generative Models

Ind. Gaussian

Correlation
Kernel

Corr. Gaussian

Generative Models


```
┌─────────────────┐
│  Ind. Gaussian  │
└─────────────────┘
         │
Correlation
  Kernel
         │
         ▼
┌─────────────────┐
│  Corr. Gaussian │
└─────────────────┘
         │
        Exp
         │
         ▼
┌─────────────────┐
│  Corr. Lognorm. │
└─────────────────┘
```
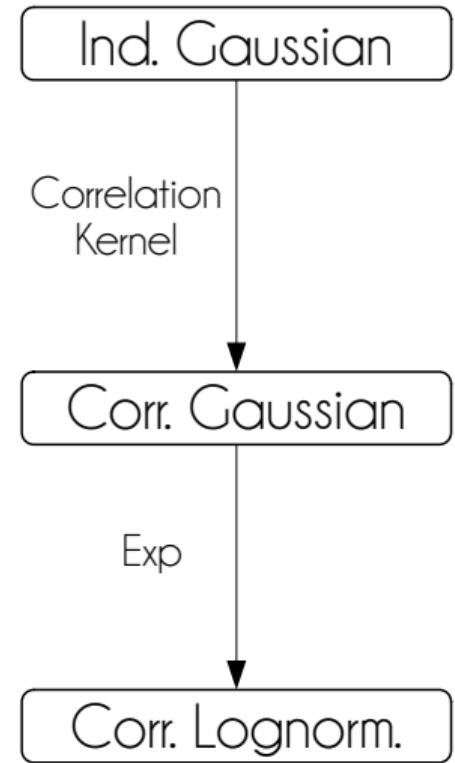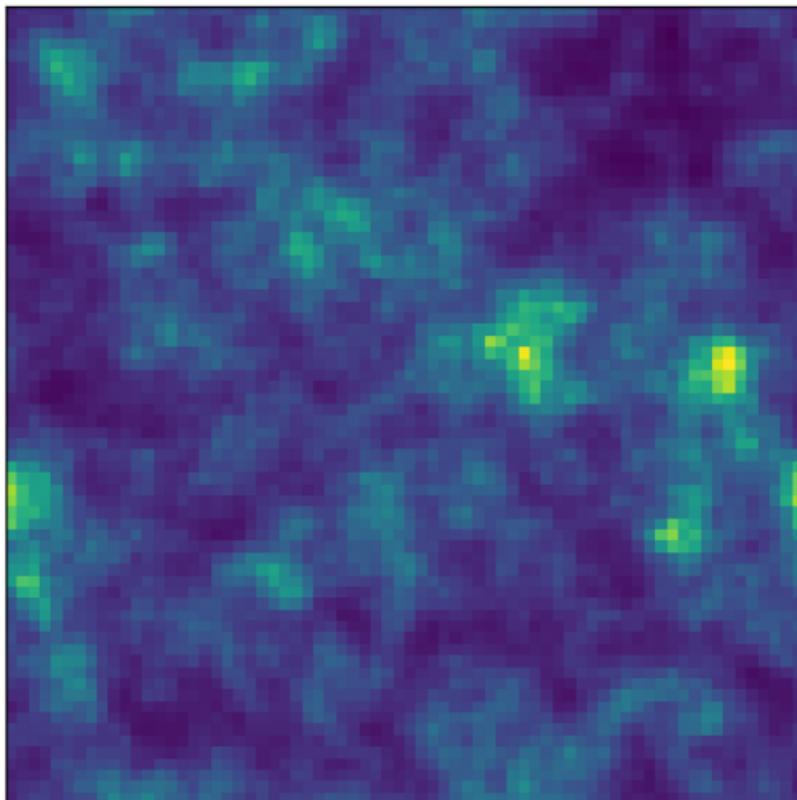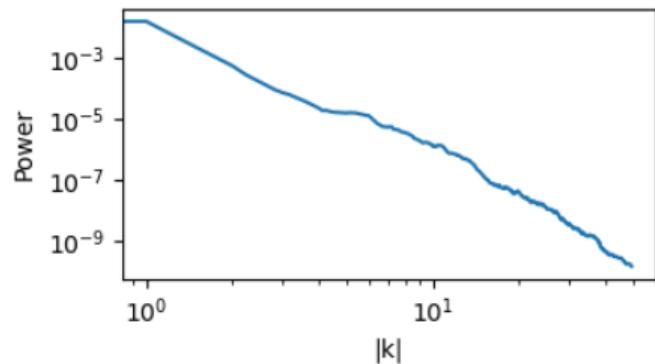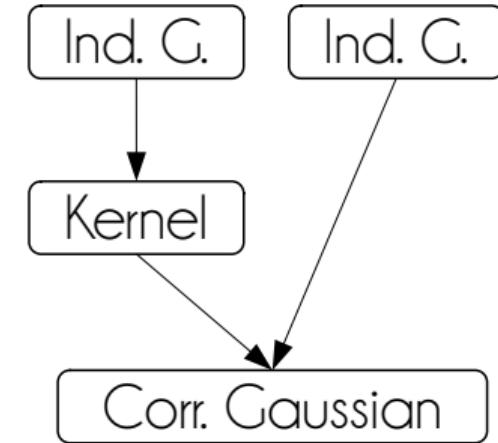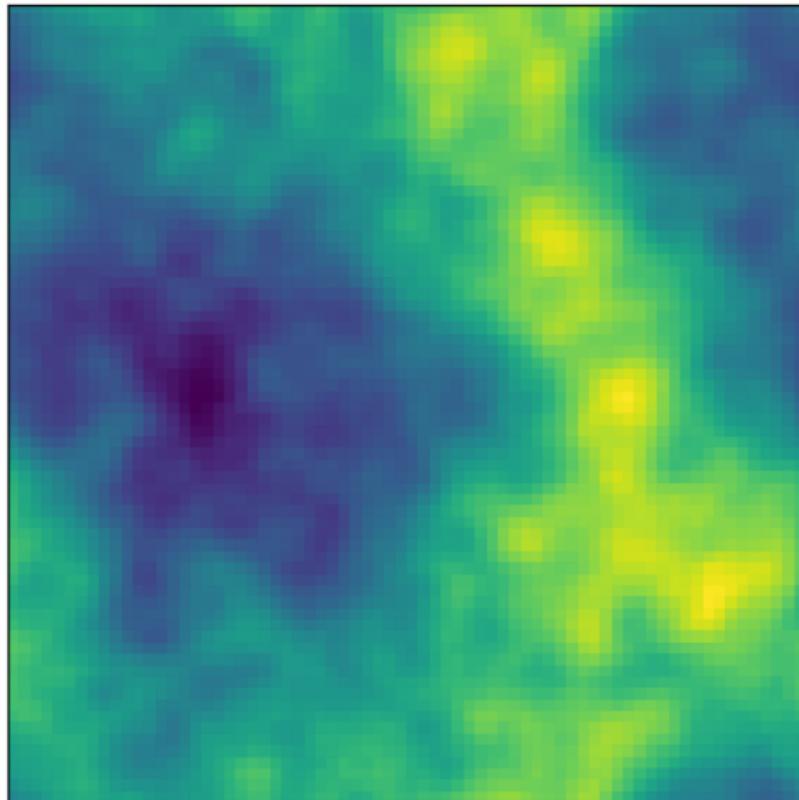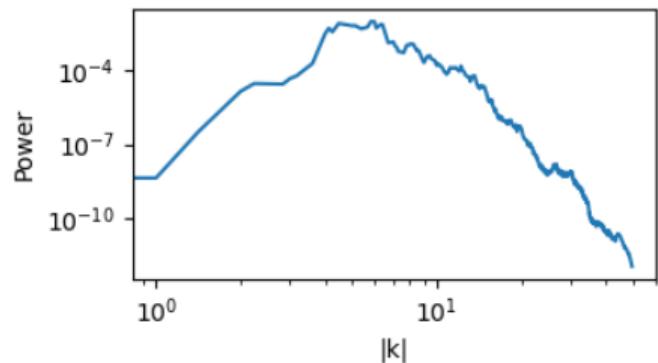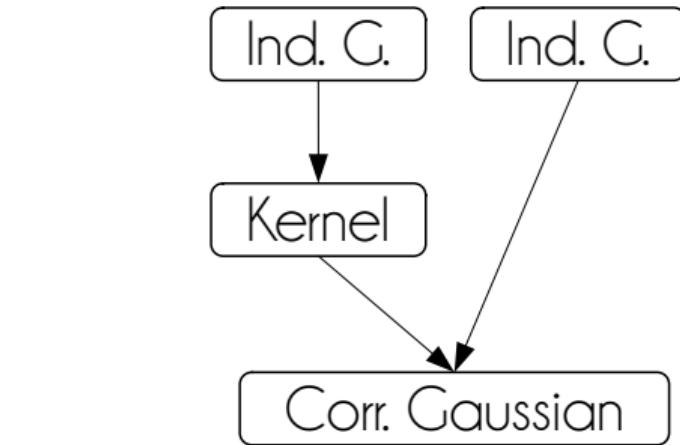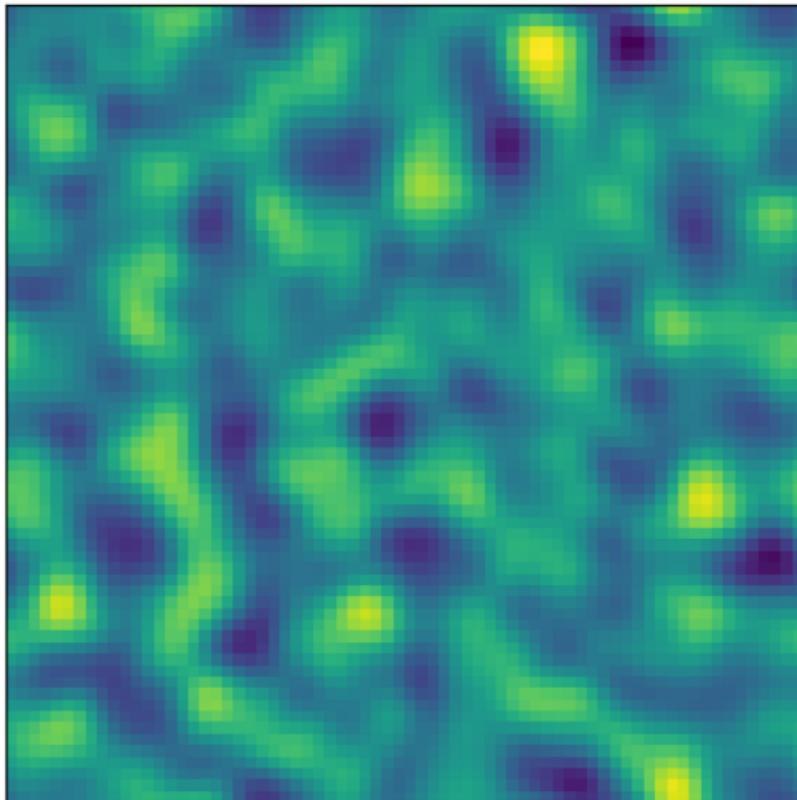
# Generative Models

Generative Models

Inference
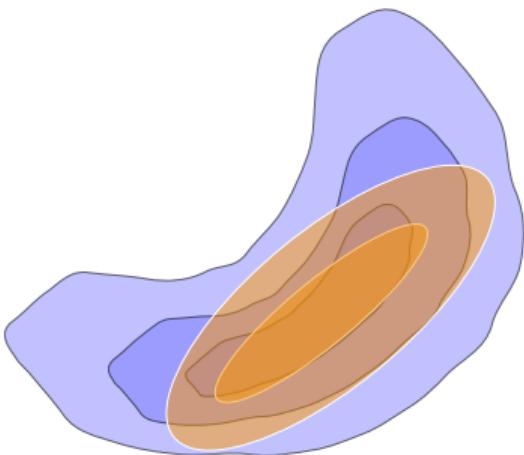
$$\mathcal{P}(\xi|d) = \frac{\mathcal{P}(d|s(\xi))\mathcal{P}(\xi)}{\mathcal{P}(d)} \tag{1}$$

How to obtain the posterior:

- Very low dimensions: Compute directly
- Medium dimensions: Sampling techniques, e.g. HMC
- High dimensions: Variational Inference

MGVI – Metric Gaussian Variational Inference

Knollmüller et. al 2019

- Gaussian Prior: $P(\xi) = \mathcal{G}(0, 1)$
- Non Gaussian Posterior: $P(\xi|d)$
- Gaussian approximation:
 Approximate $P(\xi|d)$ with $Q(\xi) = \mathcal{G}(\bar{\xi}, \Xi)$
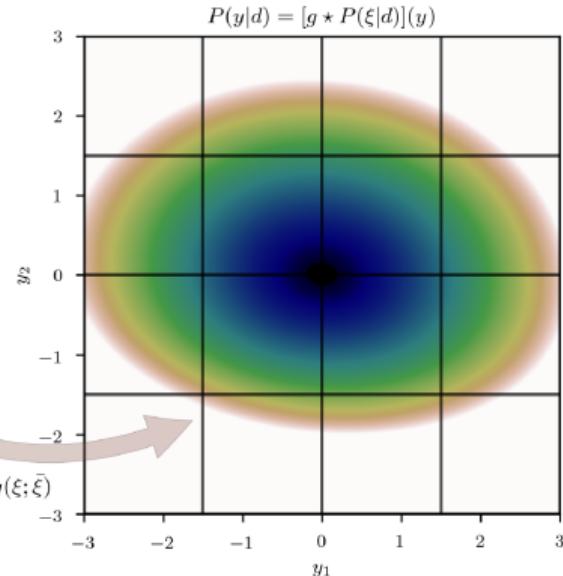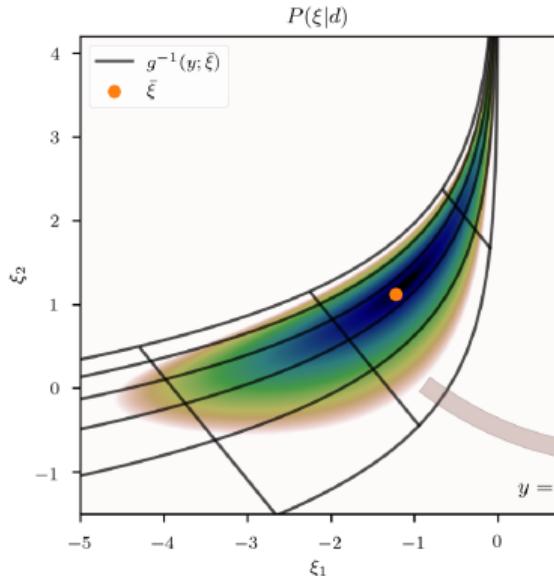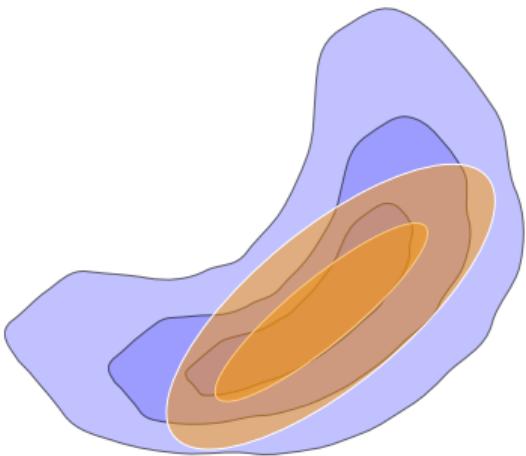
How to determine $\bar{\xi}$ and Ξ:

- $\bar{\xi} = \mathrm{armin}_{\bar{\xi}}(\mathcal{D}_{KL}(Q||P))$
- $\Xi^{-1}(\bar{\xi}) = \left\langle \frac{\partial^2 \mathcal{H}(d,\bar{\xi})}{\partial\bar{\xi}\partial\bar{\xi}^{\dagger}} \right\rangle_{P(d|\bar{\xi})}$
  with $\mathcal{H} = -\ln(P)$
- Linear scaling with dimensions of ξ

# GeoVI – Geometric Variational Inference

Frank et. al 2021

- Coordinate transformation in latent space
- Approximately transform the Posterior into a Gaussian
- Also linear scaling with number of dimensions

NIFTy – Numerical Information Field Theory

https://gitlab.mpcdf.mpg.de/ift/nifty

- Python package for bayesian field inference
- Correlation models
- Autodiff for generative models
- MGVI and GeoVI inference algorithms
- Several demos and extensive documentation