

Information Field Theory – Bayesian Inference for Fields

Jakob Roth

MPA Garching

September 15, 2022

Bayesian Inference for Fields

Fields

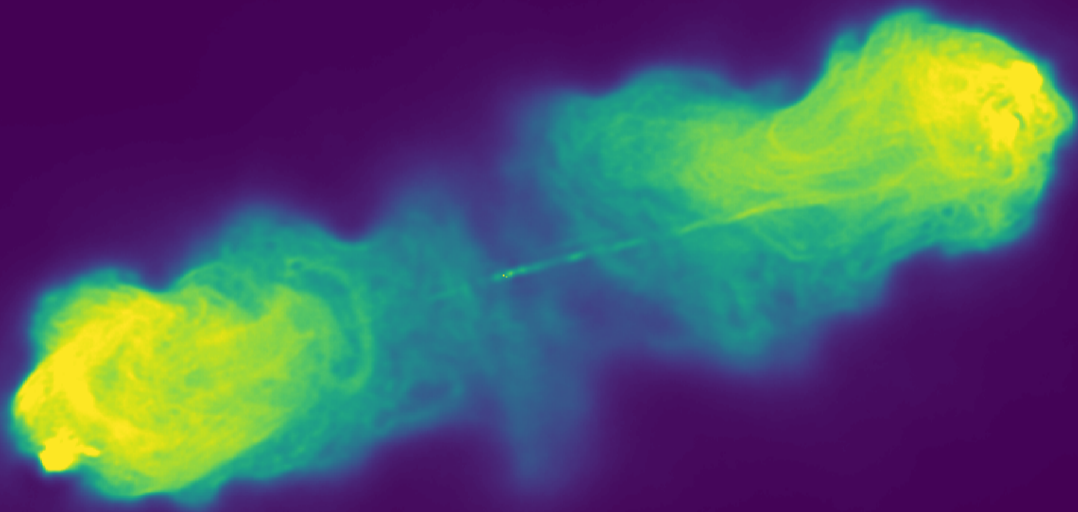
- Continuous quantities over space
- Temperature: 3D field
- Sky brightness: 2D field on sphere

Bayesian Inference

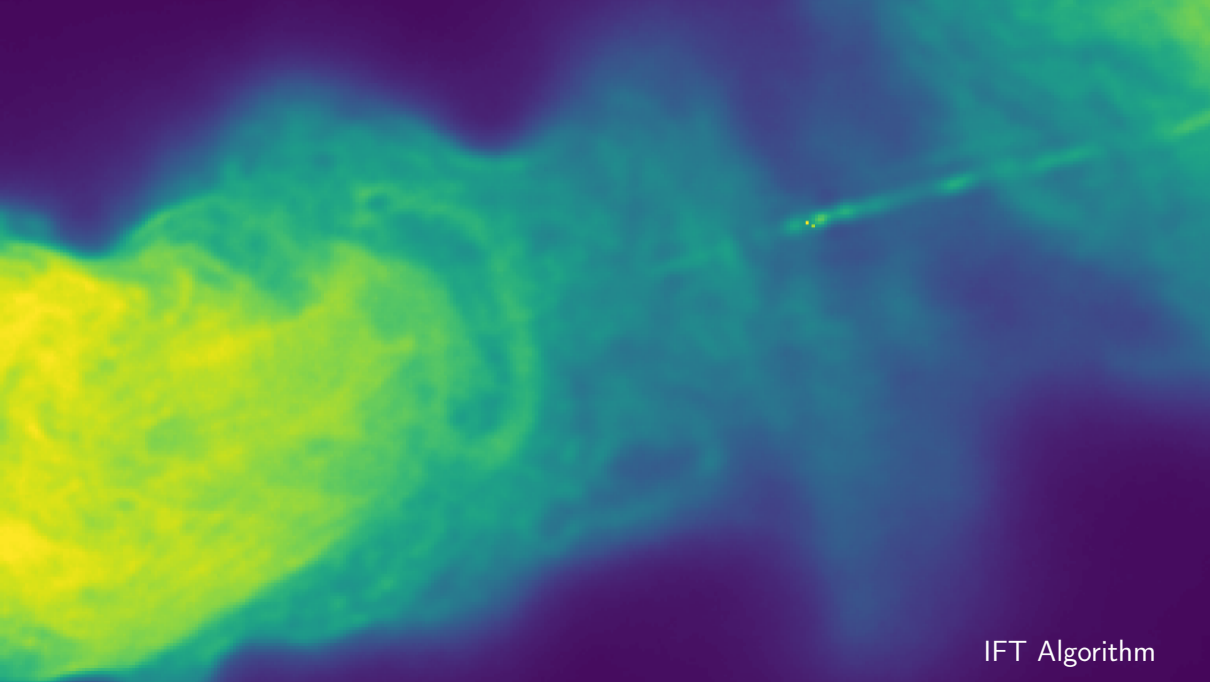
$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

- Data: d
- Signal: s

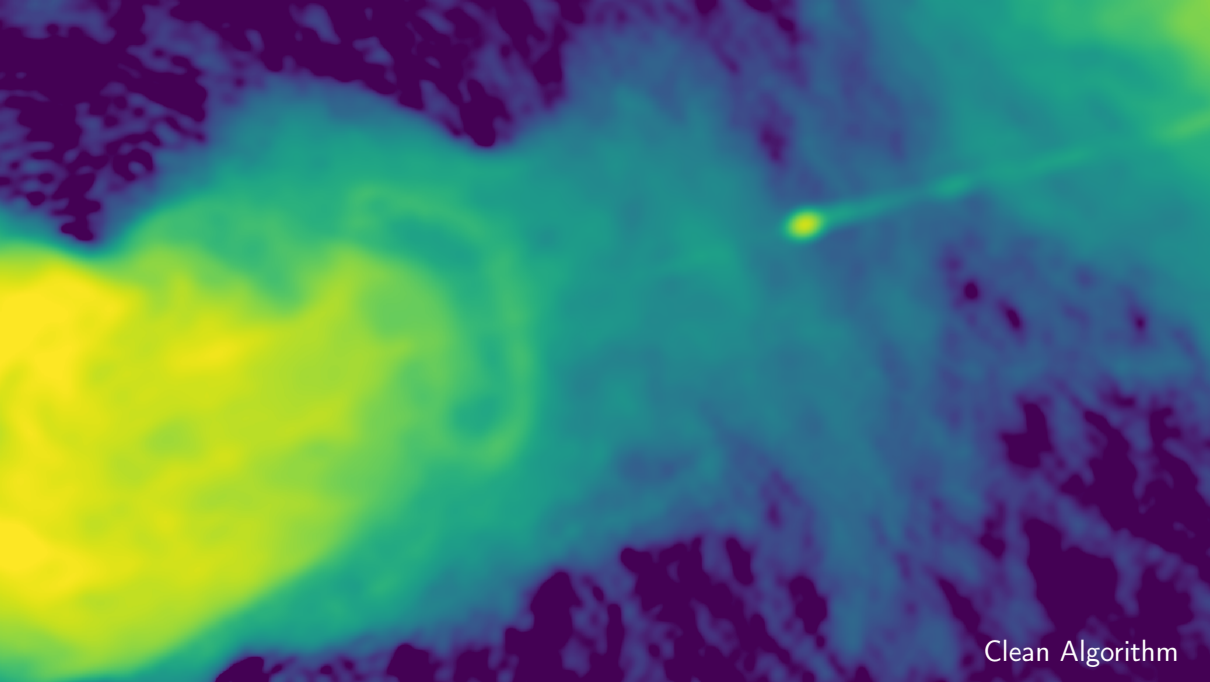
Determine posterior distribution $\mathcal{P}(s|d)$



IFT Algorithm



IFT Algorithm



Clean Algorithm



M87*, EHT Collaboration



M87*, Arras et al.



3D dust distribution, Leike et al.



Correlations

3D dust distribution, Leike et al.

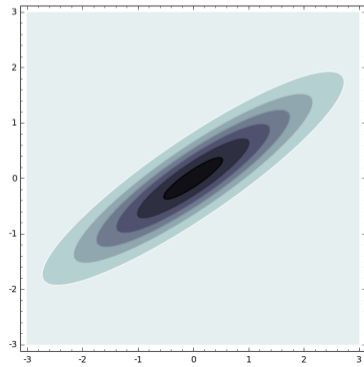


Correlations

Correlations:

- Effectively reduces DOF
- Propagate information to locations not measured

⇒ Key concept for field inference



Prior Model

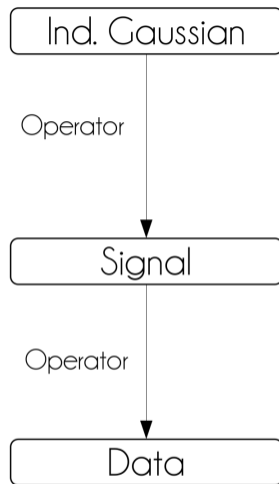
Prior Information:

- Correlation
- Physical constraints on signal, e.g. positivity of flux
- Instrument response

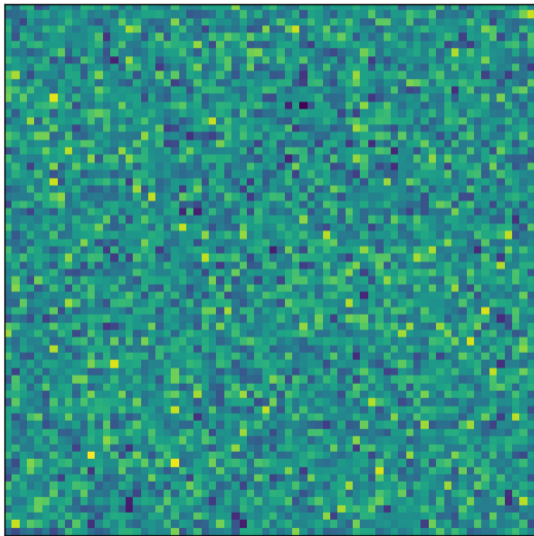
⇒ Complicated prior distribution

Prior as a standardized generative model:

- Latent parameters ξ
- $\mathcal{P}(\xi) = \mathcal{G}(0, 1)$
- $\xi \mapsto s$

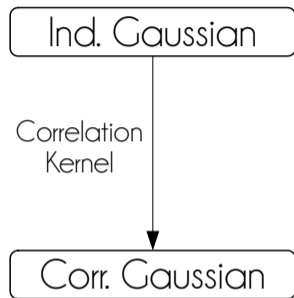
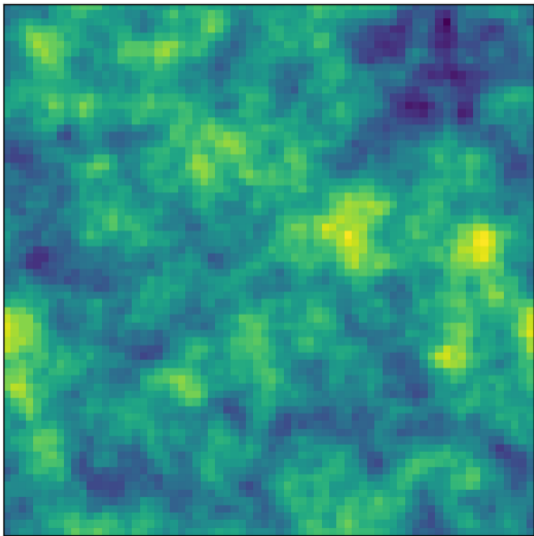


Generative Models

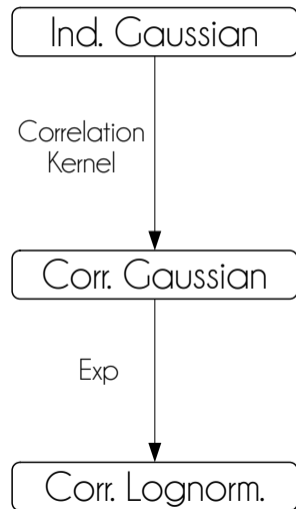
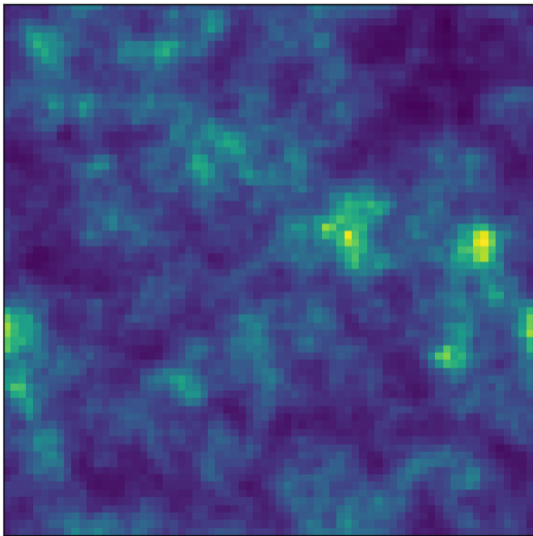


Ind. Gaussian

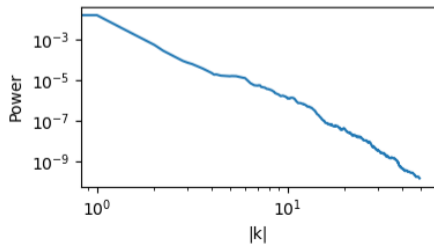
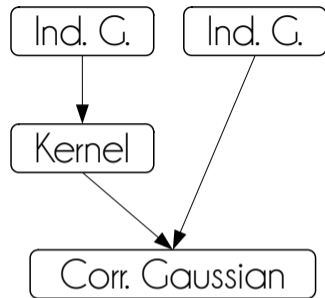
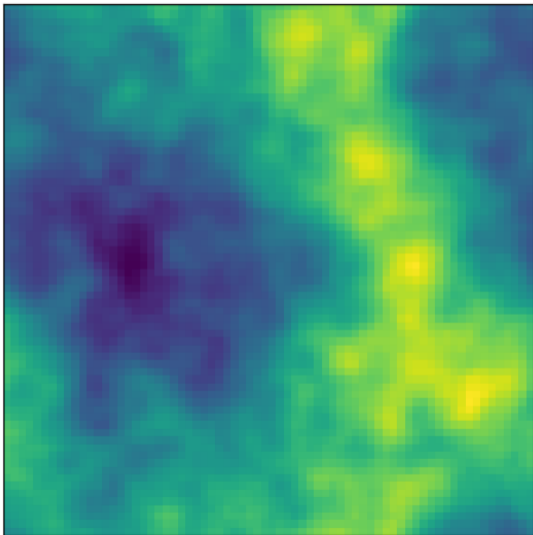
Generative Models



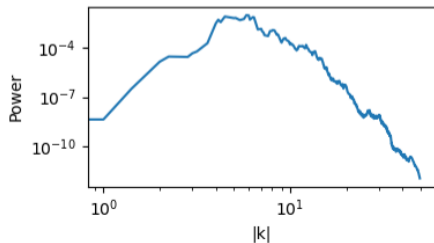
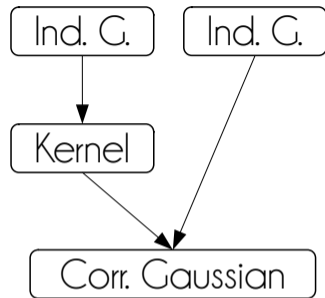
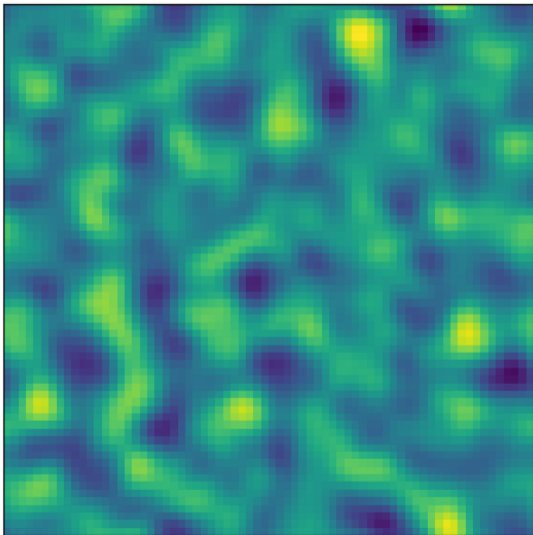
Generative Models



Generative Models



Generative Models



Inference

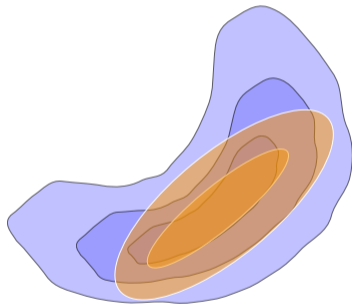
$$\mathcal{P}(\xi|d) = \frac{\mathcal{P}(d|s(\xi))\mathcal{P}(\xi)}{\mathcal{P}(d)} \quad (1)$$

How to obtain the posterior:

- Very low dimensions: Compute directly
- Medium dimensions: Sampling techniques, e.g. HMC
- High dimensions: Variational Inference

MGVI – Metric Gaussian Variational Inference

Knollmüller et. al 2019



- Gaussian Prior: $P(\xi) = \mathcal{G}(0, 1)$
- Non Gaussian Posterior: $P(\xi|d)$
- Gaussian approximation:
Approximate $P(\xi|d)$ with $Q(\xi) = \mathcal{G}(\bar{\xi}, \Xi)$

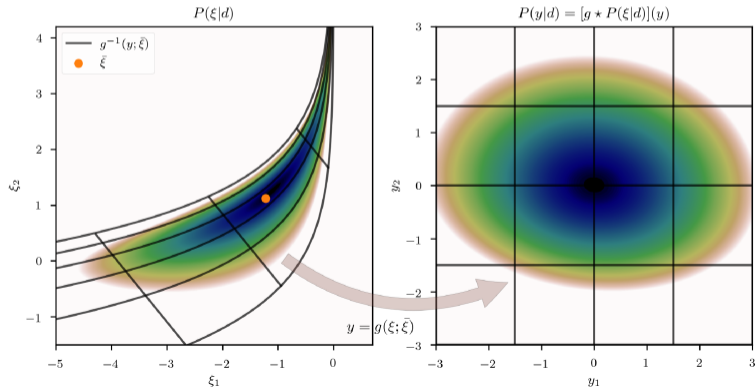
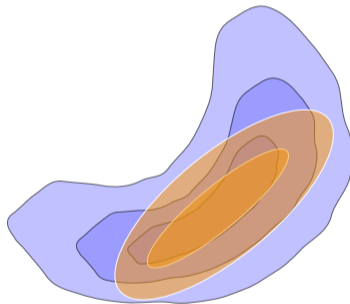
How to determine $\bar{\xi}$ and Ξ :

- $\bar{\xi} = \operatorname{armin}_{\bar{\xi}}(\mathcal{D}_{KL}(Q||P))$
- $\Xi^{-1}(\bar{\xi}) = \left\langle \frac{\partial^2 \mathcal{H}(d, \bar{\xi})}{\partial \bar{\xi} \partial \bar{\xi}^\dagger} \right\rangle_{P(d|\bar{\xi})}$
with $\mathcal{H} = -\ln(P)$
- Linear scaling with dimensions of ξ

GeoVI – Geometric Variational Inference

Frank et. al 2021

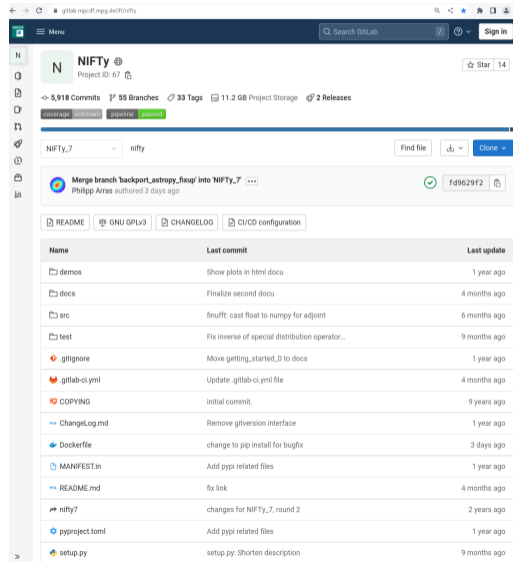
- Coordinate transformation in latent space
- Approximately transform the Posterior into a Gaussian
- Also linear scaling with number of dimensions



NIFTy – Numerical Information Field Theory

<https://gitlab.mpcdf.mpg.de/ift/nifty>

- Python package for bayesian field inference
- Correlation models
- Autodiff for generative models
- MGVI and GeoVI inference algorithms
- Several demos and extensive documentation



The screenshot shows the GitLab web interface for the NIFTy project. The page displays the repository name 'NIFTy', project ID '67', and statistics such as 5,918 commits, 55 branches, 33 tags, 11.2 GB project storage, and 2 releases. A merge request is visible, titled 'Merge branch 'backport_astropy_fixup' into 'NIFTy_7'', authored by Philipp Arras 3 days ago. Below the merge request, there are links for README, GNU GPL v3, CHANGELOG, and CI/CD configuration. A table lists the repository's files and folders, including 'demos', 'docs', 'src', 'test', and various configuration files like '.gitignore', '.gitlab-ci.yml', 'COPYING', 'ChangeLog.md', 'Dockerfile', 'MANIFEST.in', 'README.md', 'nifty7', 'pyproject.toml', and 'setup.py'. Each entry in the table shows the file name, the last commit message, and the time since the last update.

Name	Last commit	Last update
demos	Show plots in html docu	1 year ago
docs	Finalize second docu	4 months ago
src	finufft: cast float to numpy for adjoint	6 months ago
test	Fix inverse of special distribution operator...	9 months ago
.gitignore	Move getting_started_0 to docs	1 year ago
.gitlab-ci.yml	Update .gitlab-ci.yml file	4 months ago
COPYING	initial commit.	9 years ago
ChangeLog.md	Remove gitversion interface	1 year ago
Dockerfile	change to pip install for bugfix	3 days ago
MANIFEST.in	Add pypi related files	1 year ago
README.md	fix link	4 months ago
nifty7	changes for NIFTy_7, round 2	2 years ago
pyproject.toml	Add pypi related files	1 year ago
setup.py	setup.py: Shorten description	9 months ago