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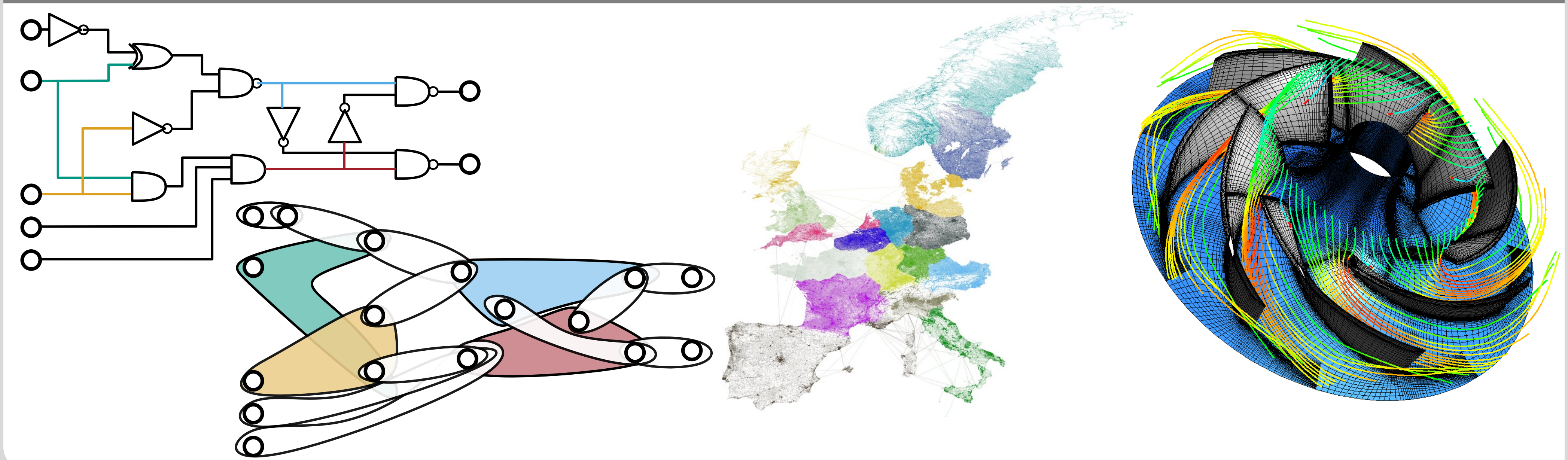
# High Quality Graph and Hypergraph Partitioning

**2nd BMBF Big Data All Hands Meeting · October 11, 2017**

**Yaroslav Akhremtsev, Peter Sanders, Sebastian Schlag, Christian Schulz**

Institute of Theoretical Informatics · Algorithmics Group

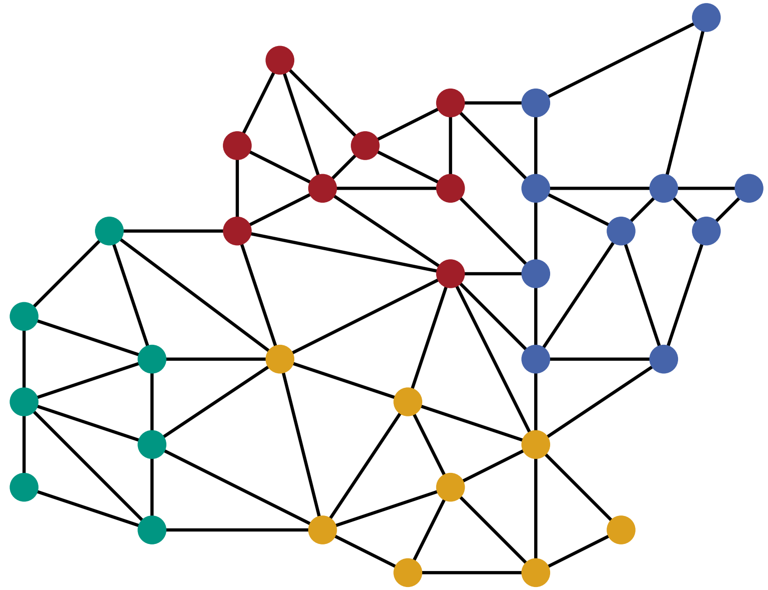


# Graphs and Hypergraphs

**Graph** $G = (V, E)$

vertices → $V$, edges → $E$

- models **relationships** between **objects**
- dyadic (**2-ary**) relationships



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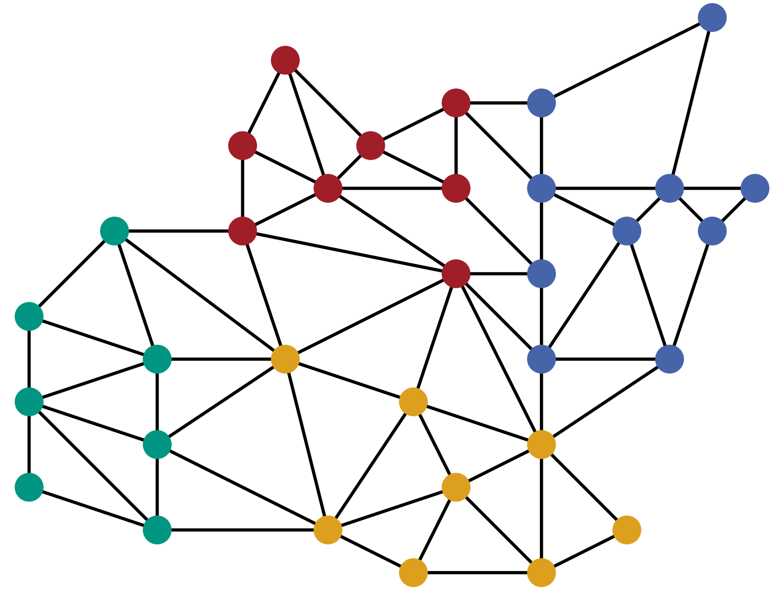
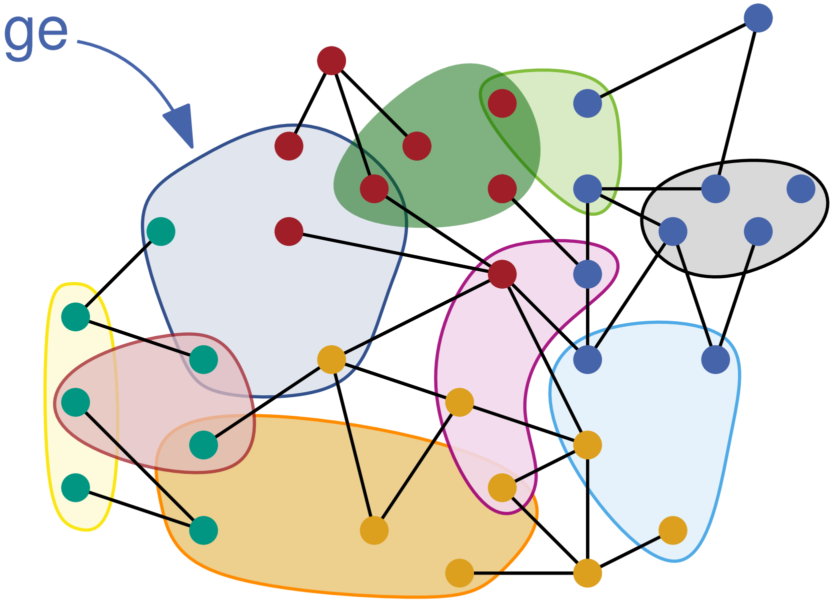


hyperedge



## Hypergraph $H = (V, E)$

- Generalization of a graph
  $\Rightarrow$ hyperedges connect $\geq 2$ nodes
- arbitrary (**d-ary**) relationships
- Edge set $E \subseteq \mathcal{P}(V) \setminus \emptyset$

# $\varepsilon$-Balanced Graph and Hypergraph Partitioning

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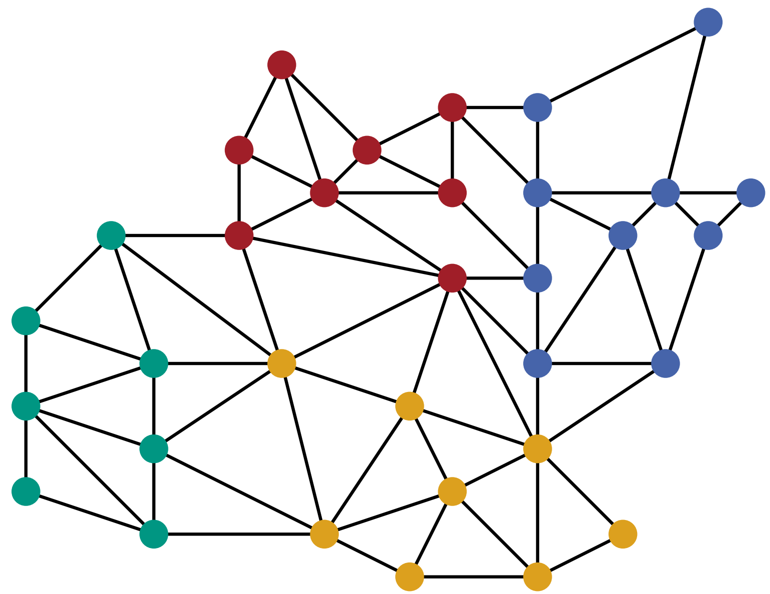
**Partition** (hyper)graph $G = (V, E, c : V \to \mathbb{R}_{>0}, \omega : E \to \mathbb{R}_{>0})$ into **k** disjoint blocks $V_1, \ldots, V_k$ s.t.

- blocks $V_i$ are **roughly equal-sized**:

$$c(V_i) \leq (1 + \varepsilon) \left\lceil \frac{c(V)}{k} \right\rceil$$

- **objective** function on edges is **minimized**



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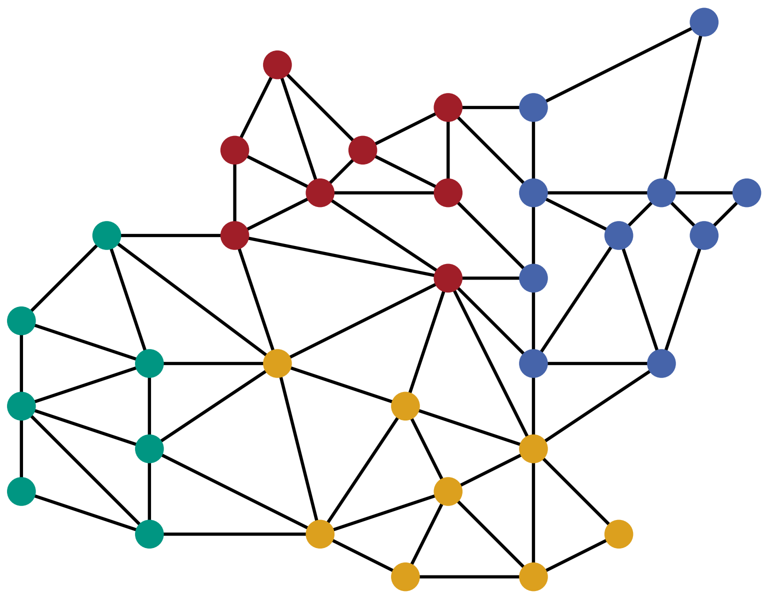

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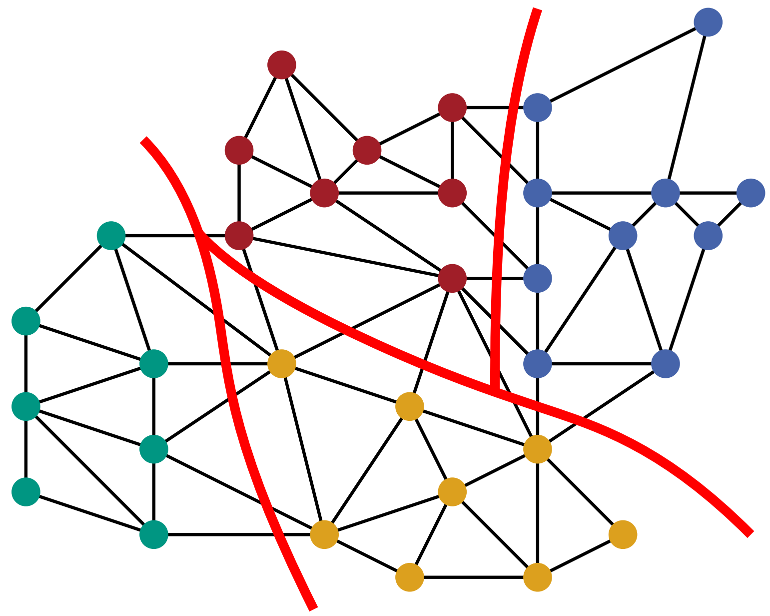

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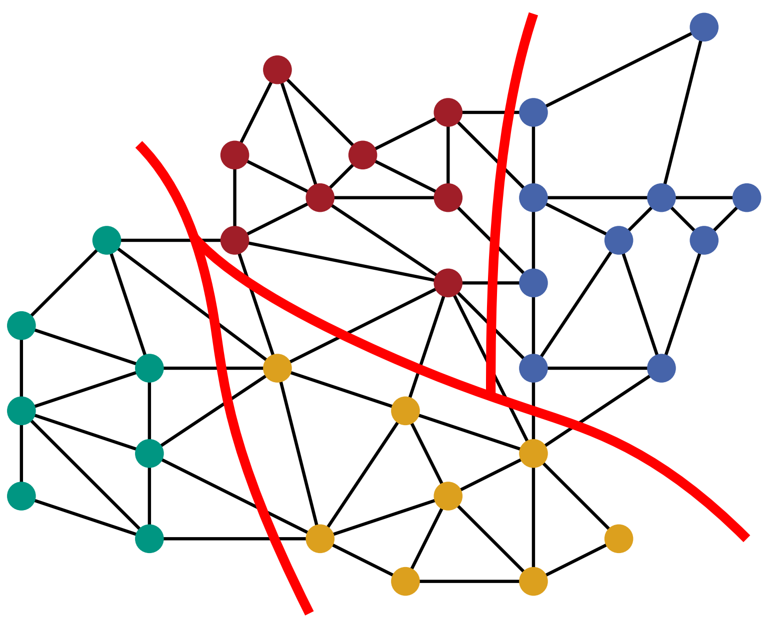


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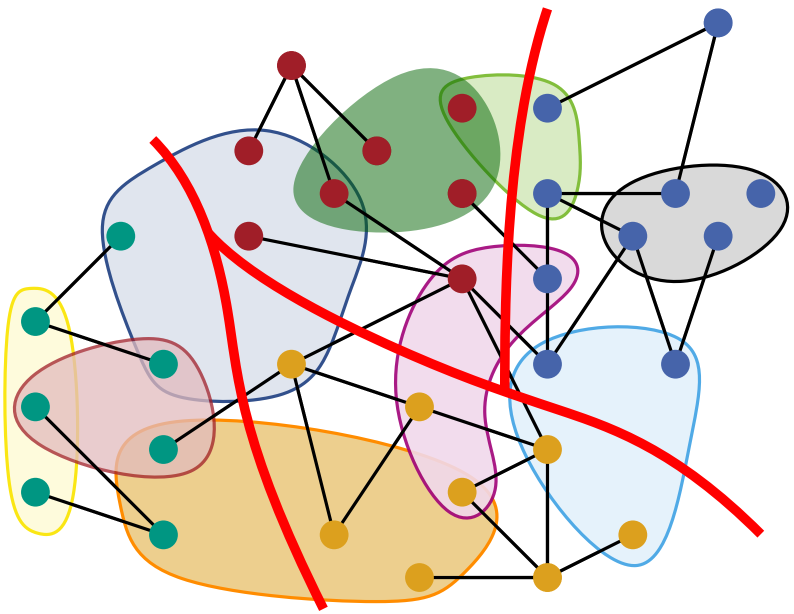

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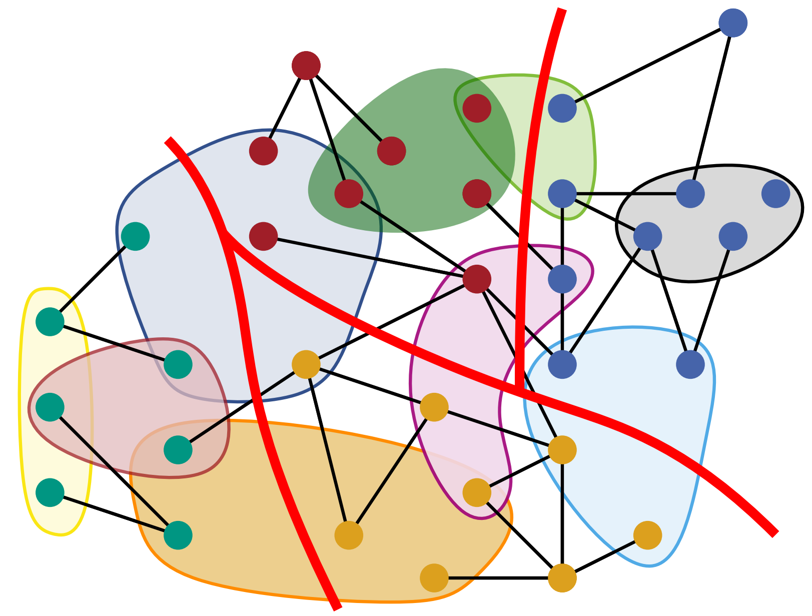

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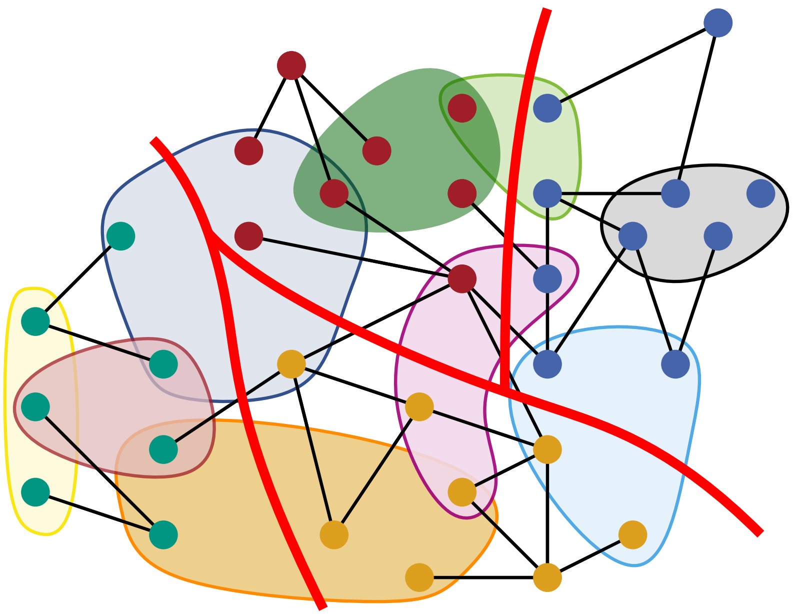


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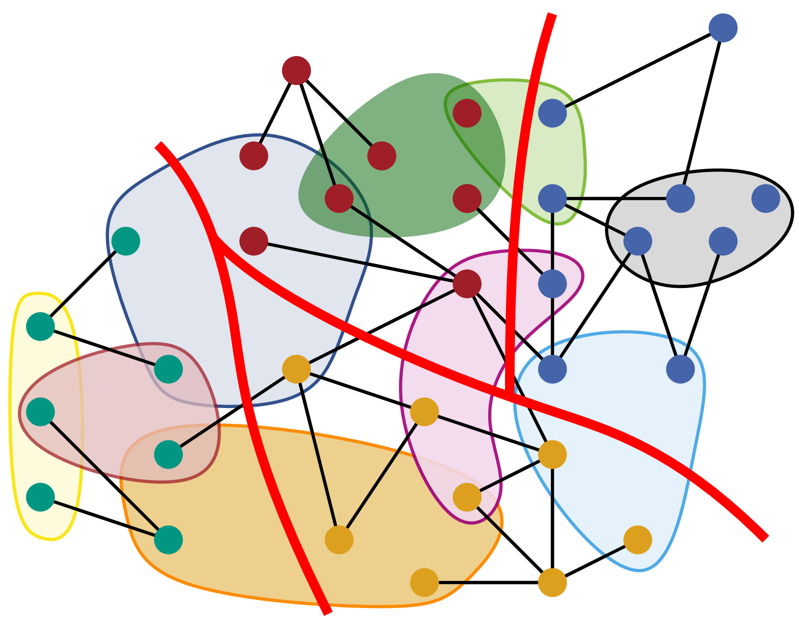
  - connectivity: $\sum_{e \in \text{cut}} (\lambda - 1)\, \omega(e)$



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- Hypergraphs:
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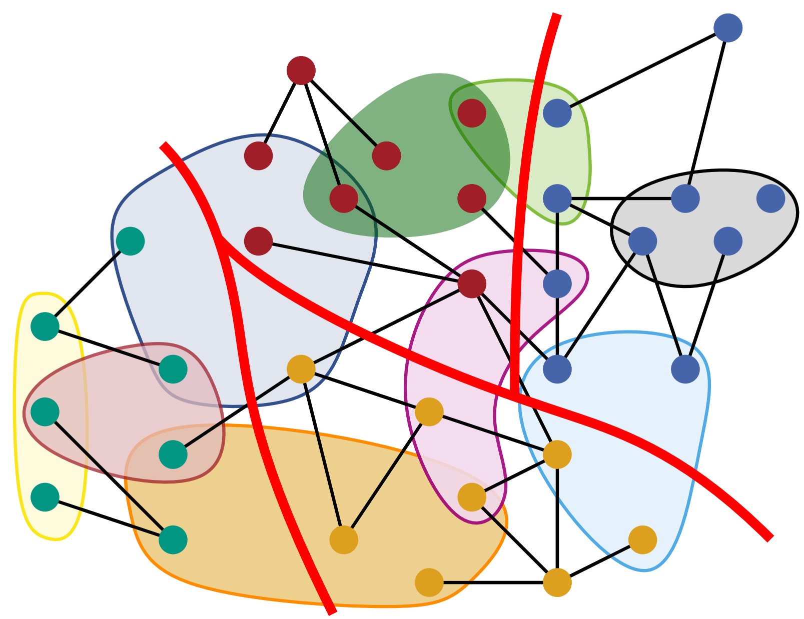
  - connectivity: $\sum_{e \in \text{cut}} (\lambda - 1)\, \omega(e)$

**# blocks** connected by $e$



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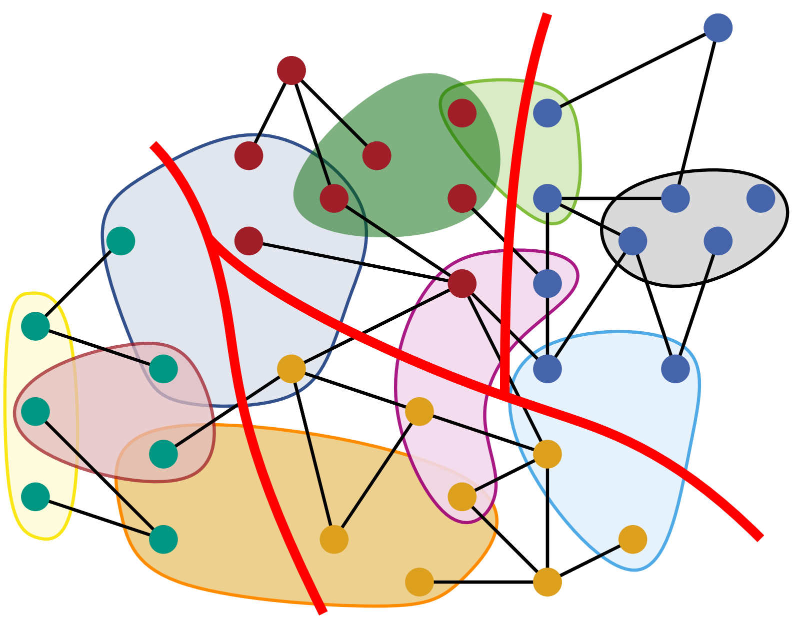
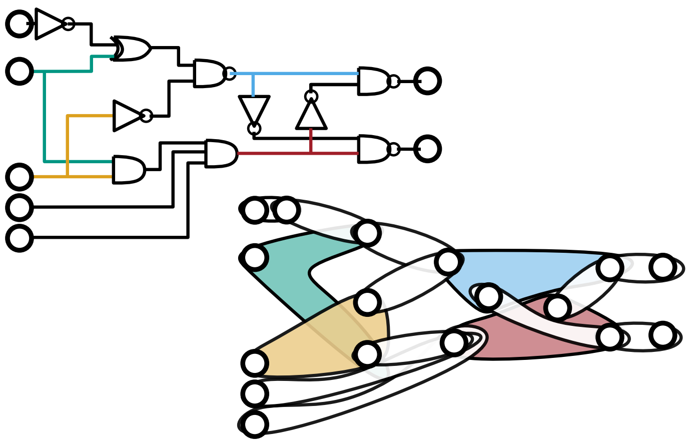
**# blocks** connected by $e$



# Applications



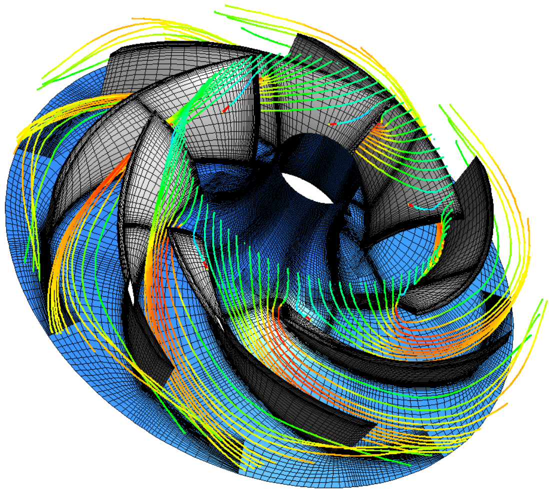
VLSI Design

Warehouse Optimization

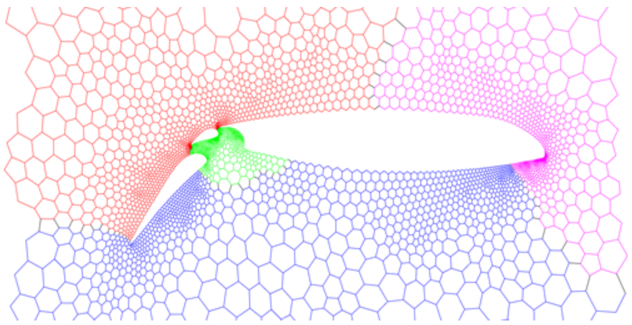
[Martin Grandjean, via Wikimedia Commons]

Complex Networks

Route Planning



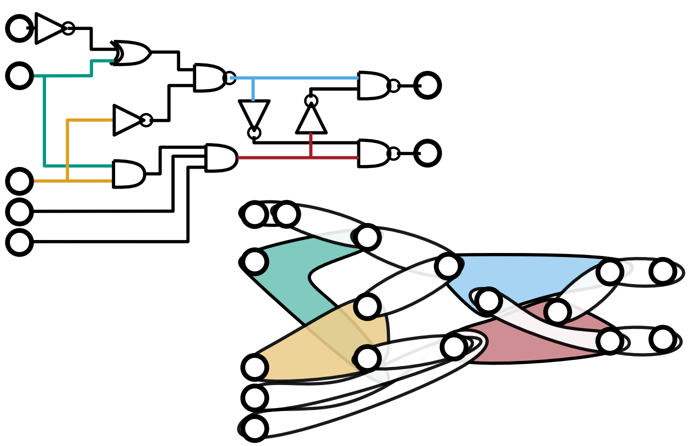
Simulation



$$\mathbf{R}^{n \times n} \ni Ax = b \in \mathbf{R}^n$$

Scientific Computing

# Applications



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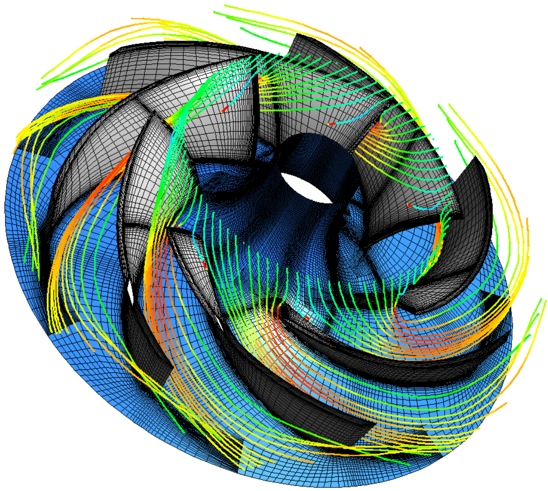

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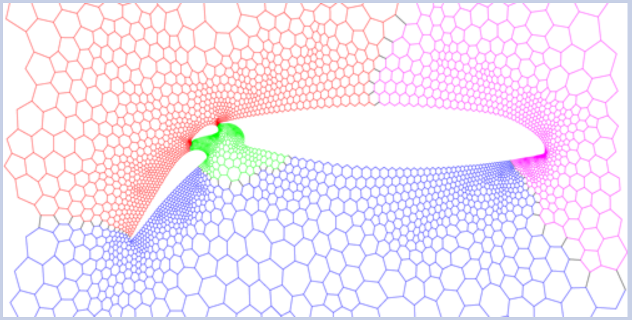

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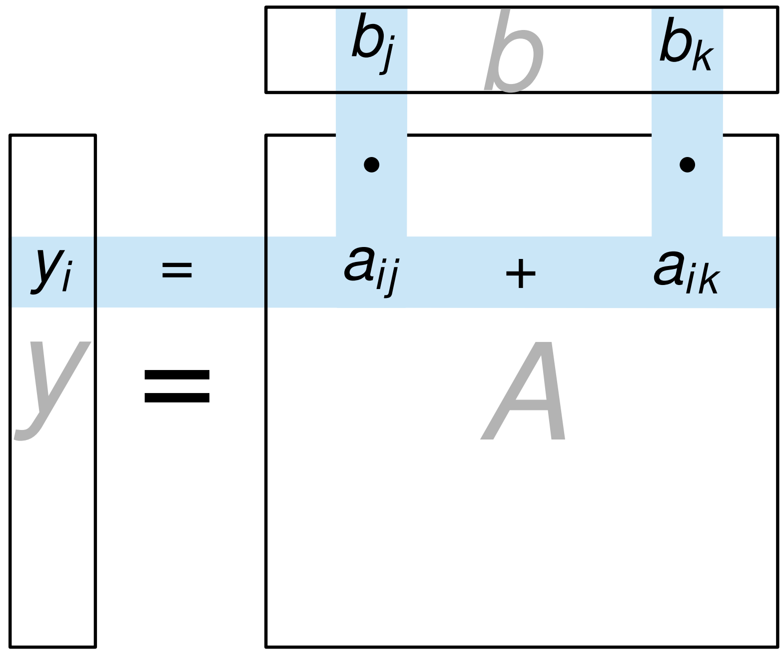
Scientific Computing

# Parallel Sparse-Matrix Vector Product (SpM$\times$V)

$$y = A\,b$$




**Setting:**

- repeated SpM$\times$V on supercomputer
- $A$ is large $\Rightarrow$ distribute on multiple nodes
- symmetric partitioning $\Rightarrow$ $y$ & $b$ divided conformally with $A$

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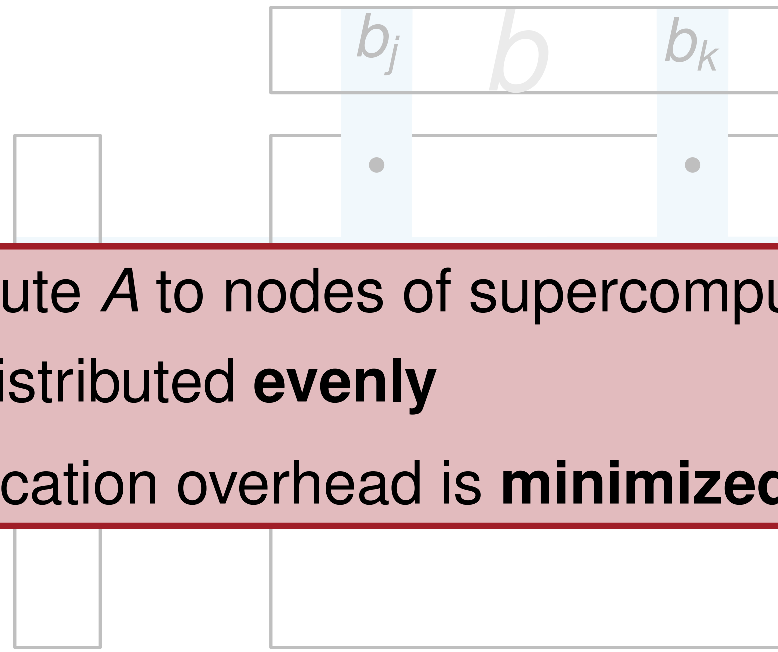


**Task:** distribute $A$ to nodes of supercomputer such that

- work is distributed **evenly**
- communication overhead is **minimized**

**Setting:**

- repeated SpM×V on supercomputer
- $A$ is large $\Rightarrow$ distribute on multiple nodes
- symmetric partitioning $\Rightarrow$ $y$ & $b$ divided conformally with $A$

# Naive Approach: Rowwise Decomposition

$A \in \mathbf{R}^{16 \times 16}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ | $b$ |
| 1 | $x$ | | | | | | | | | $x$ | | | | | | |
| 2 | | $x$ | | | $x$ | | | | | | | $x$ | | | | |
| 3 | | | $x$ | | | $x$ | | | | | | | | | | |
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| 6 | | | | | $x$ | $x$ | | | | | | | | | | |
| 7 | | $x$ | | | $x$ | | $x$ | | $x$ | | | | | | | |
| 8 | | | | $x$ | | | | $x$ | | | $x$ | | | | | |
| 9 | | | | | | | | | $x$ | | | $x$ | | | $x$ | |
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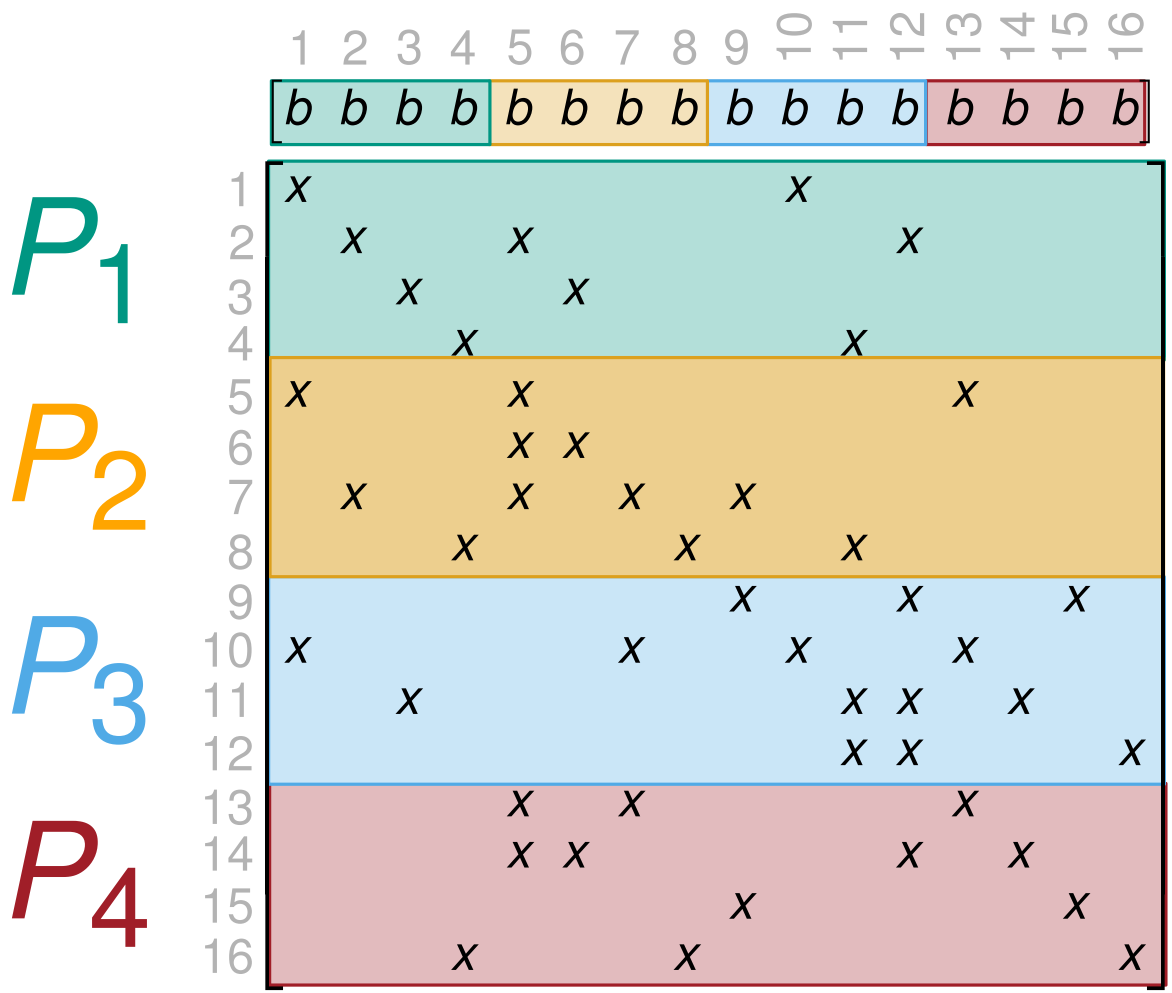

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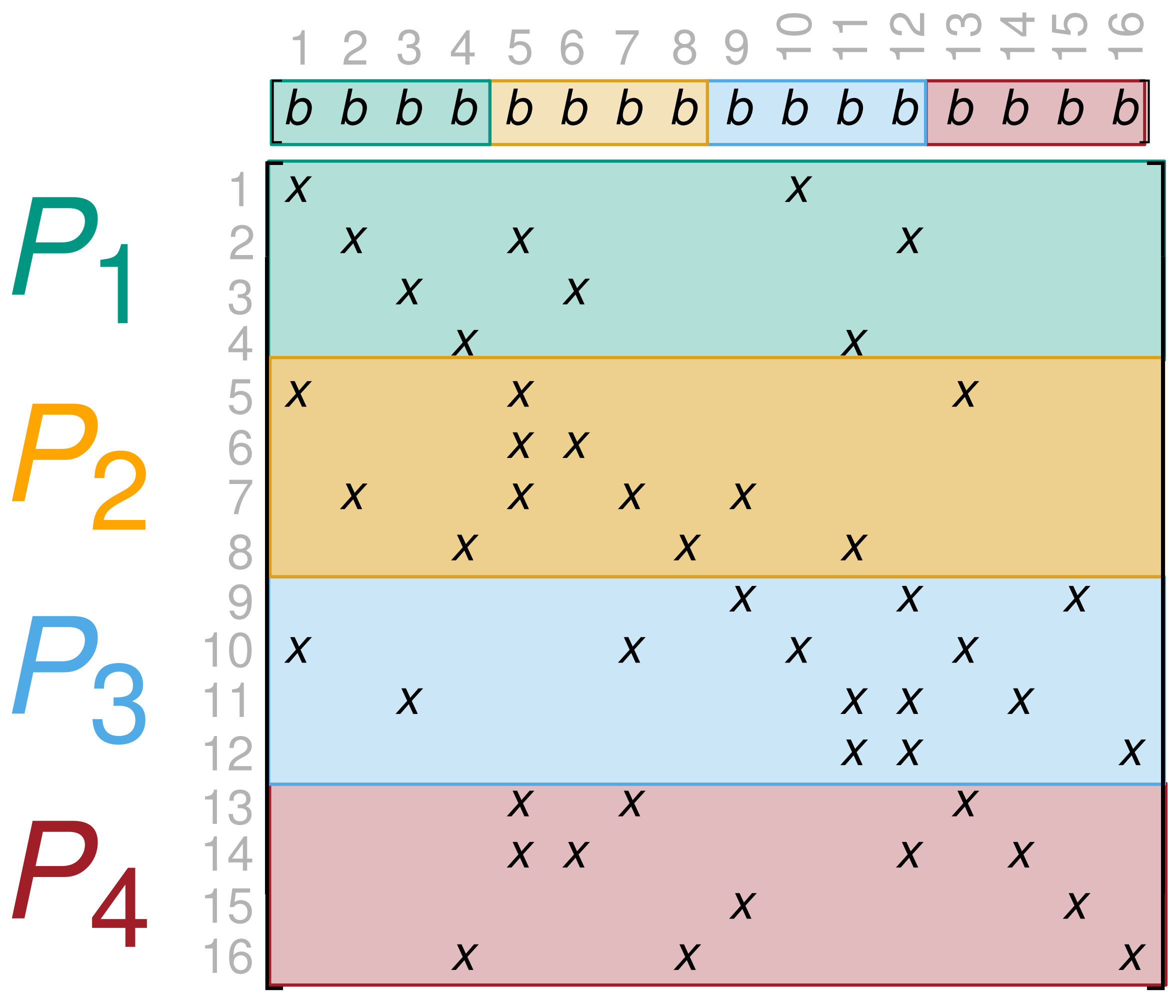


**Load Balancing?**

$\Rightarrow 9$

$\Rightarrow 12$

$\Rightarrow 14$

$\Rightarrow 12$

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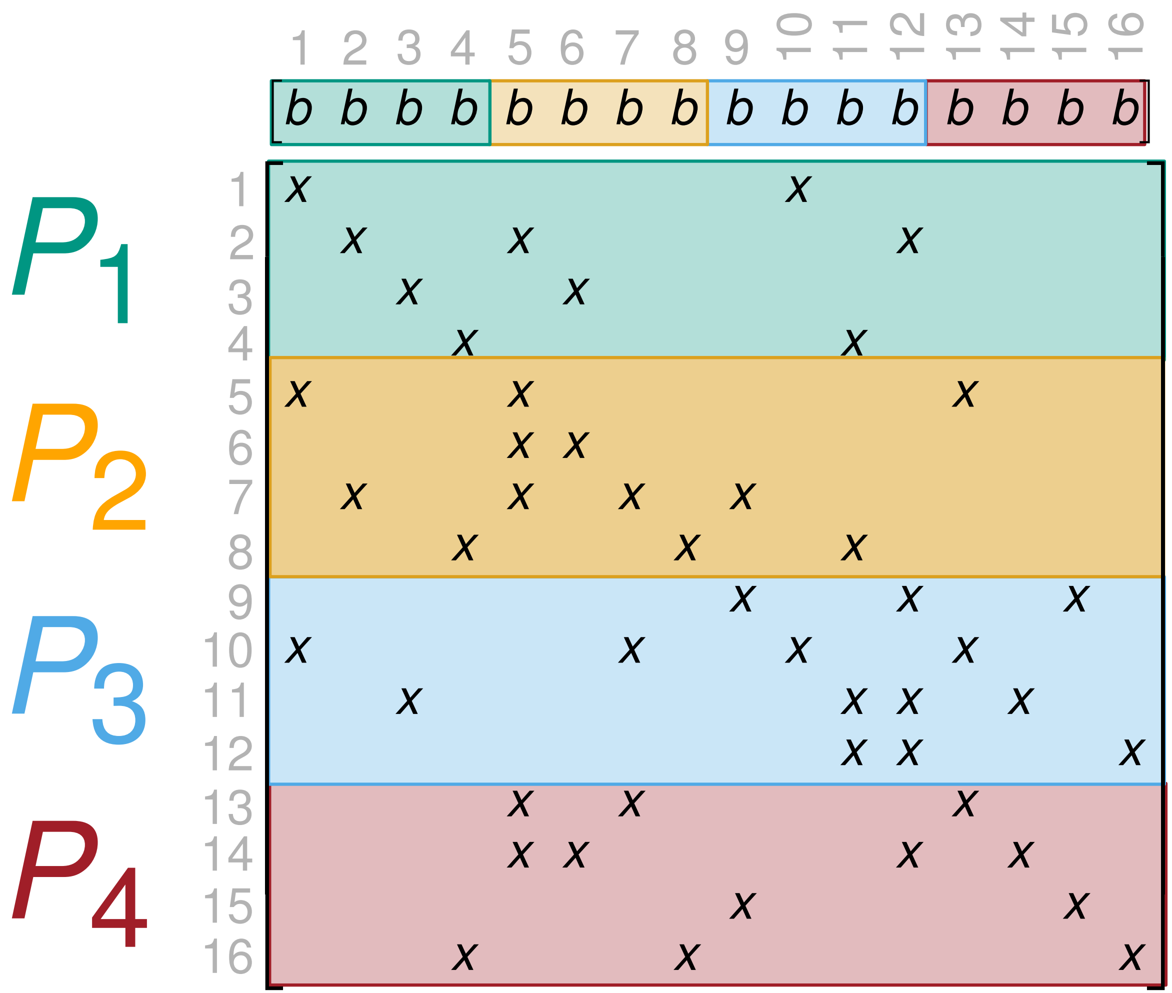


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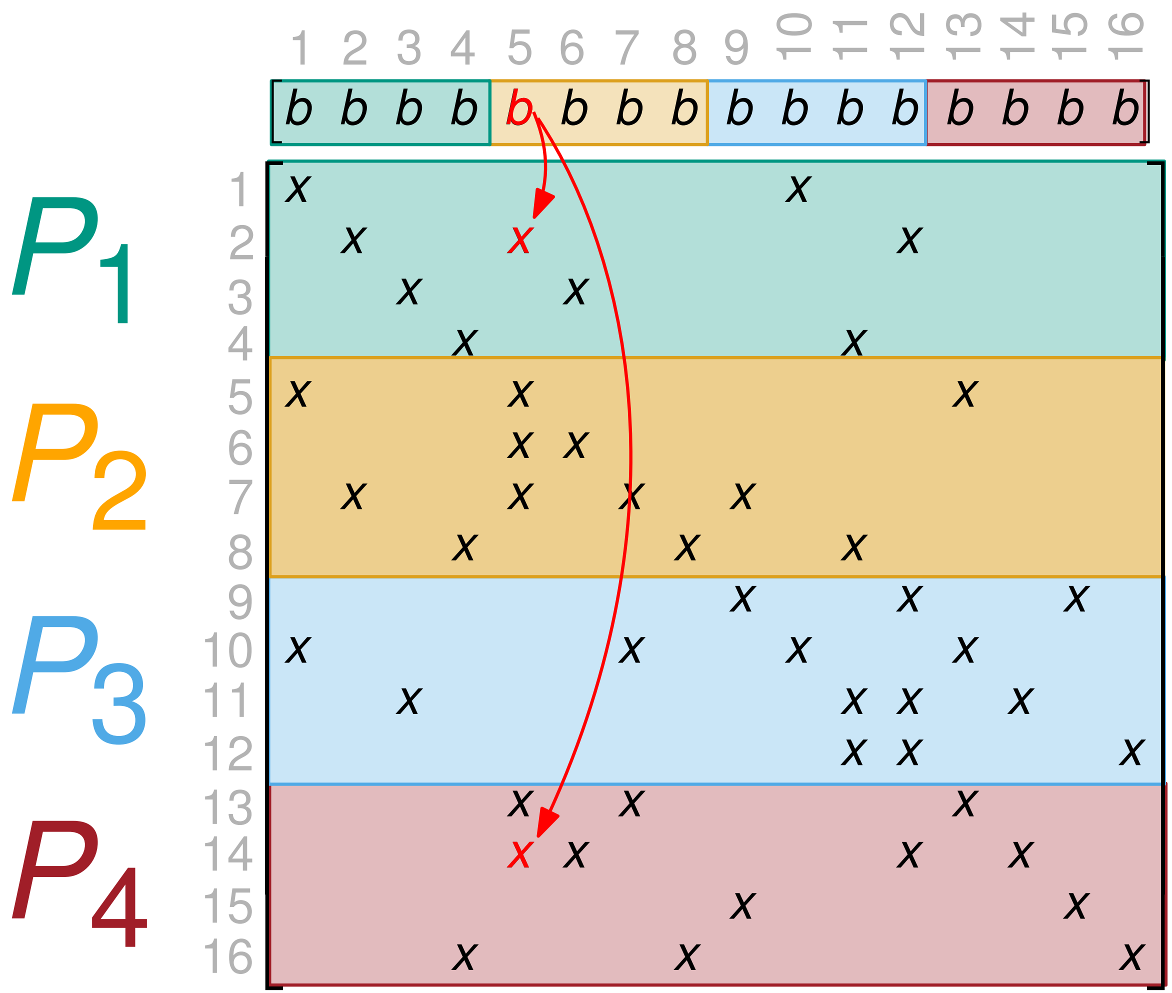


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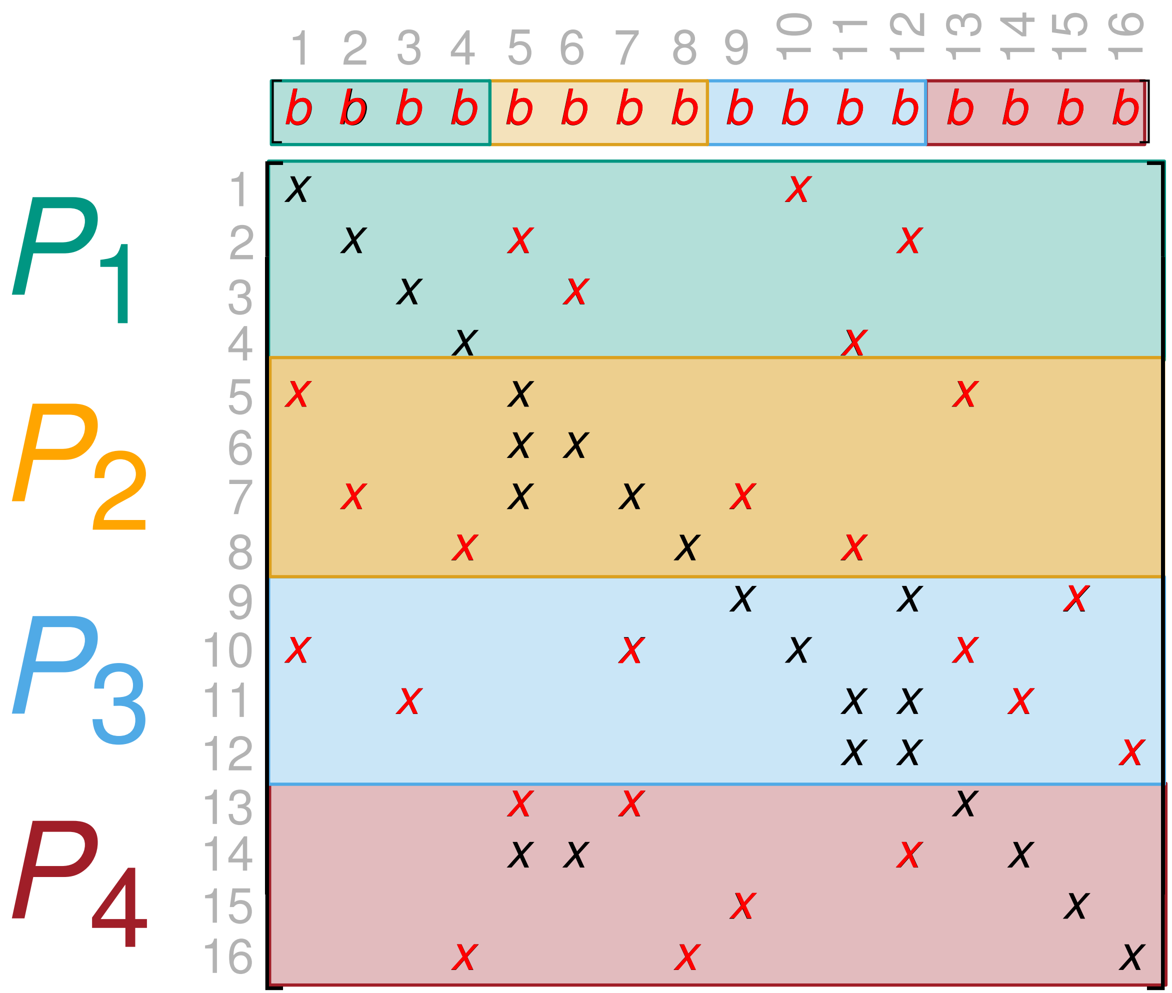


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**Commuication Volume?** $\Rightarrow$ 24 entries!

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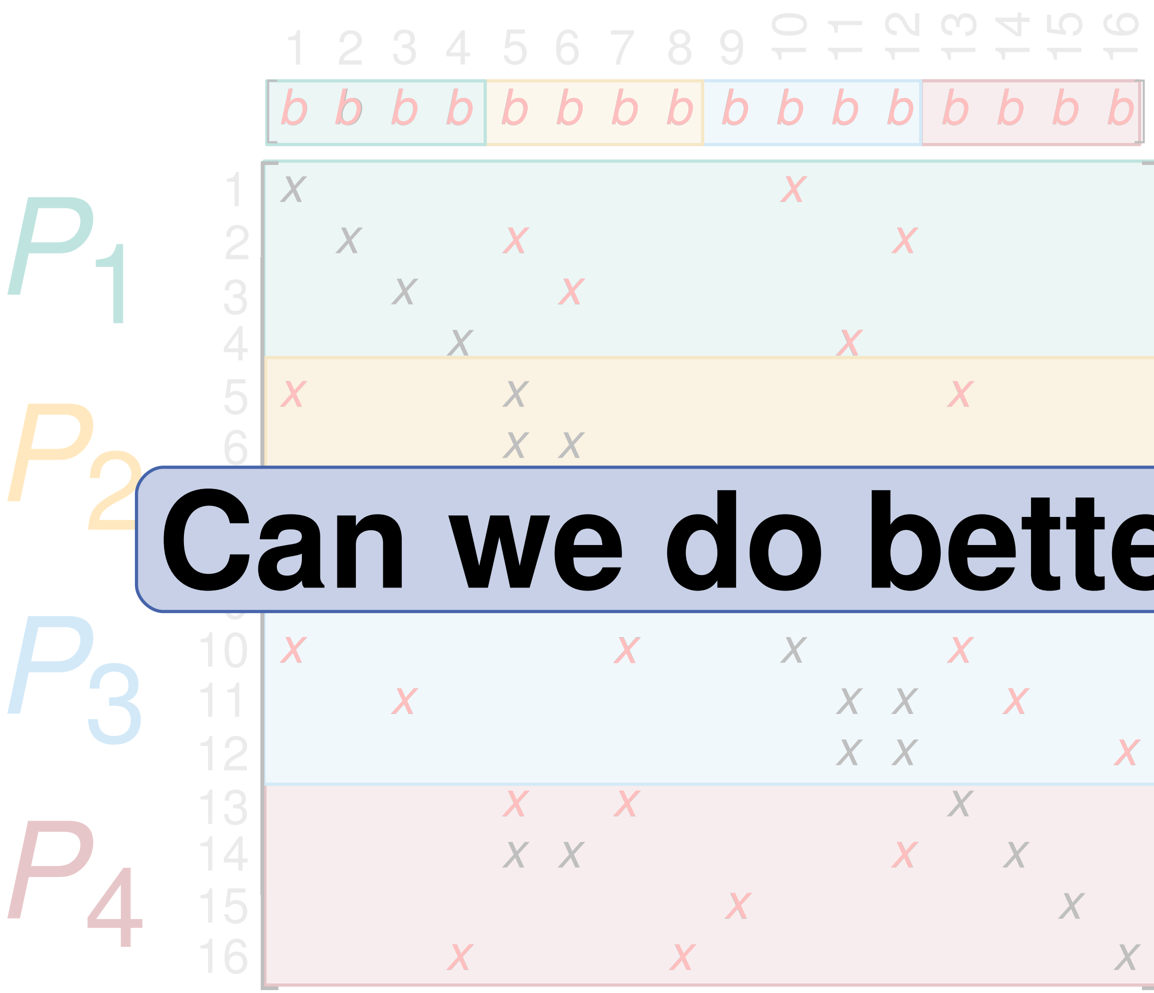


**Load Balancing?**

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**Can we do better?**

**Commuication Volume?** $\Rightarrow$ 24 entries!

# From SpM×V to Hypergraph Partitioning

$A \in \mathbb{R}^{16\times16} \Rightarrow H = (V_R, E_C)$

- One vertex per row:

  $\Rightarrow V_R = \{v_1, v_2, \ldots, v_{16}\}$

- One hyperedge per column:

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$$\begin{bmatrix} b & b & b & b & b & b & b & b & b & b & b & b & b & b & b & b \end{bmatrix}$$

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| 4 | | | | x | | | | | | | x | | | | | |
| 5 | x | | | | x | | | | | | | | x | | | |
| 6 | | | | | x | x | | | | | | | | | | |
| 7 | | x | | | x | | x | | x | | | | | | | |
| 8 | | | | x | | | | x | | | x | | | | | |
| 9 | | | | | | | | | x | | | x | | | x | |
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| 12 | | | | | | | | | | | x | x | | | | x |
| 13 | | | | | x | | x | | | | | | x | | | |
| 14 | | | | | x | x | | | | | | x | | x | | |
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$v_i \in V_R$ :

- task to compute inner product of row $i$ with $b$
- $\Rightarrow c(v_i) :=$ # nonzeros

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| $v_9$ | 9 | | | | | | | | | $x$ | | | $x$ | | | $x$ | |
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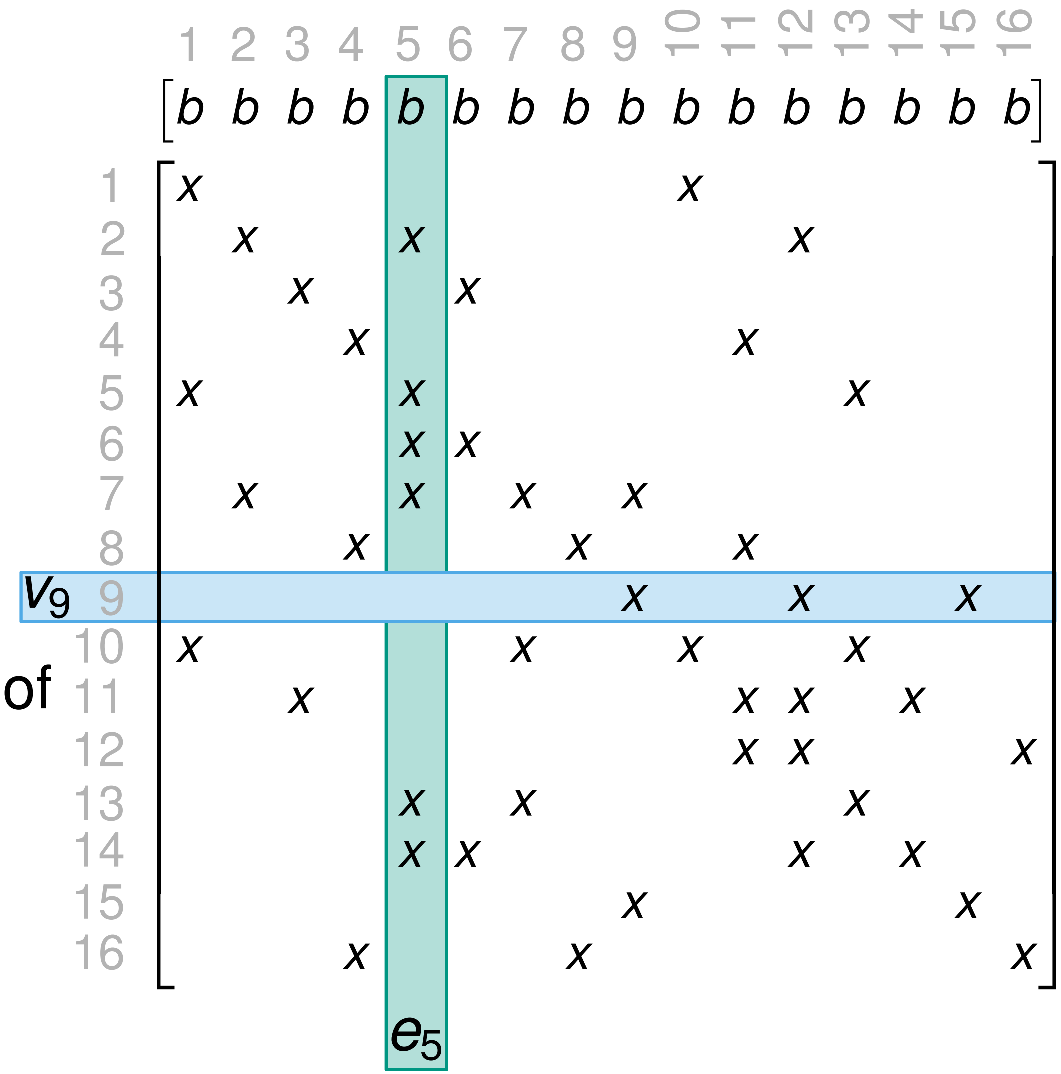
  $\Rightarrow V_R = \{v_1, v_2, \ldots, v_{16}\}$
- One hyperedge per column:

  $\Rightarrow E_C = \{e_1, e_2, \ldots, e_{16}\}$

$v_i \in V_R$ **:**

- task to compute inner product of row $i$ with $b$
- $\Rightarrow c(v_i) :=$ # nonzeros




$e_j \in E_C$: set of vertices that need $b_j$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# From SpM×V to Hypergraph Partitioning

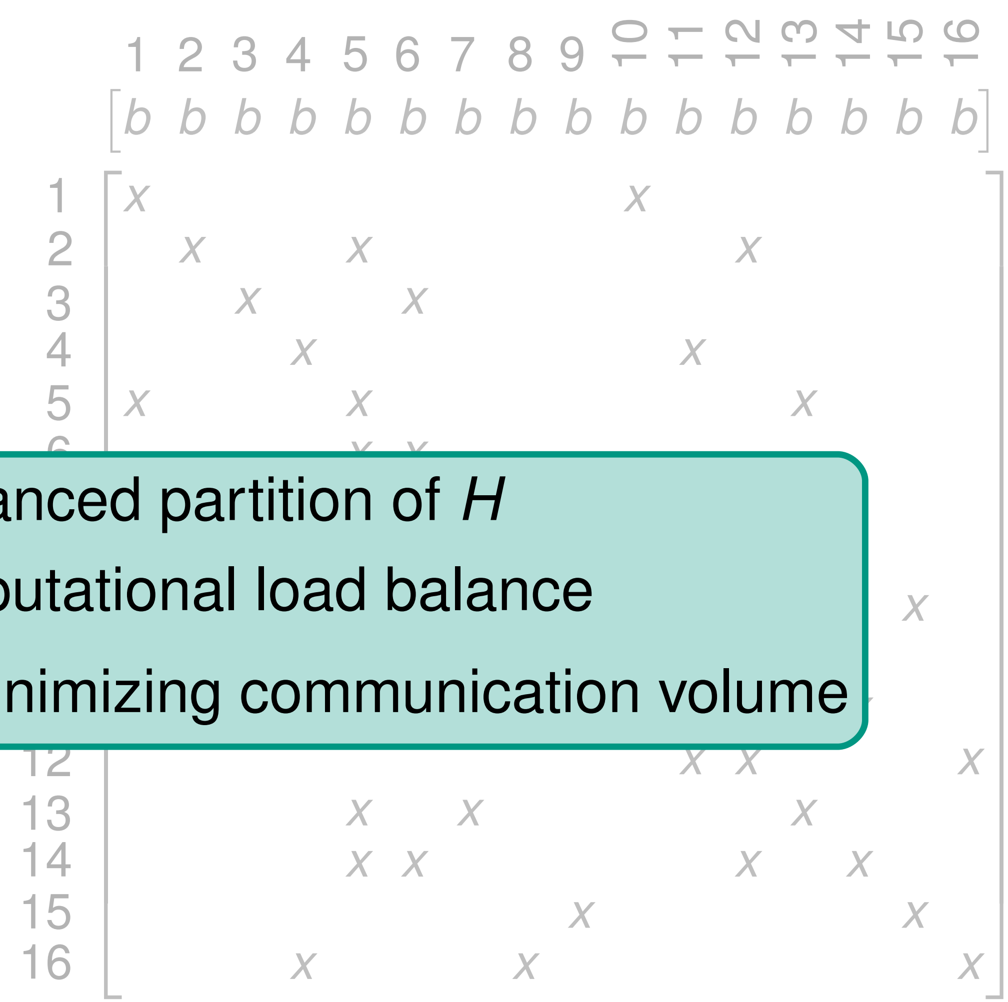
$A \in \mathbb{R}^{16\times16} \Rightarrow H = (V_R, E_C)$

- One vertex per row:

  $\Rightarrow V_R = \{v_1, v_2, \ldots, v_{16}\}$

- One hyperedge per column:

  $\Rightarrow E_C = \{e_1, e_2, \ldots, e_{16}\}$



**Solution:** $\varepsilon$-balanced partition of $H$

- balanced partition $\rightsquigarrow$ computational load balance
- small $(\lambda - 1)$-cutsize $\rightsquigarrow$ minimizing communication volume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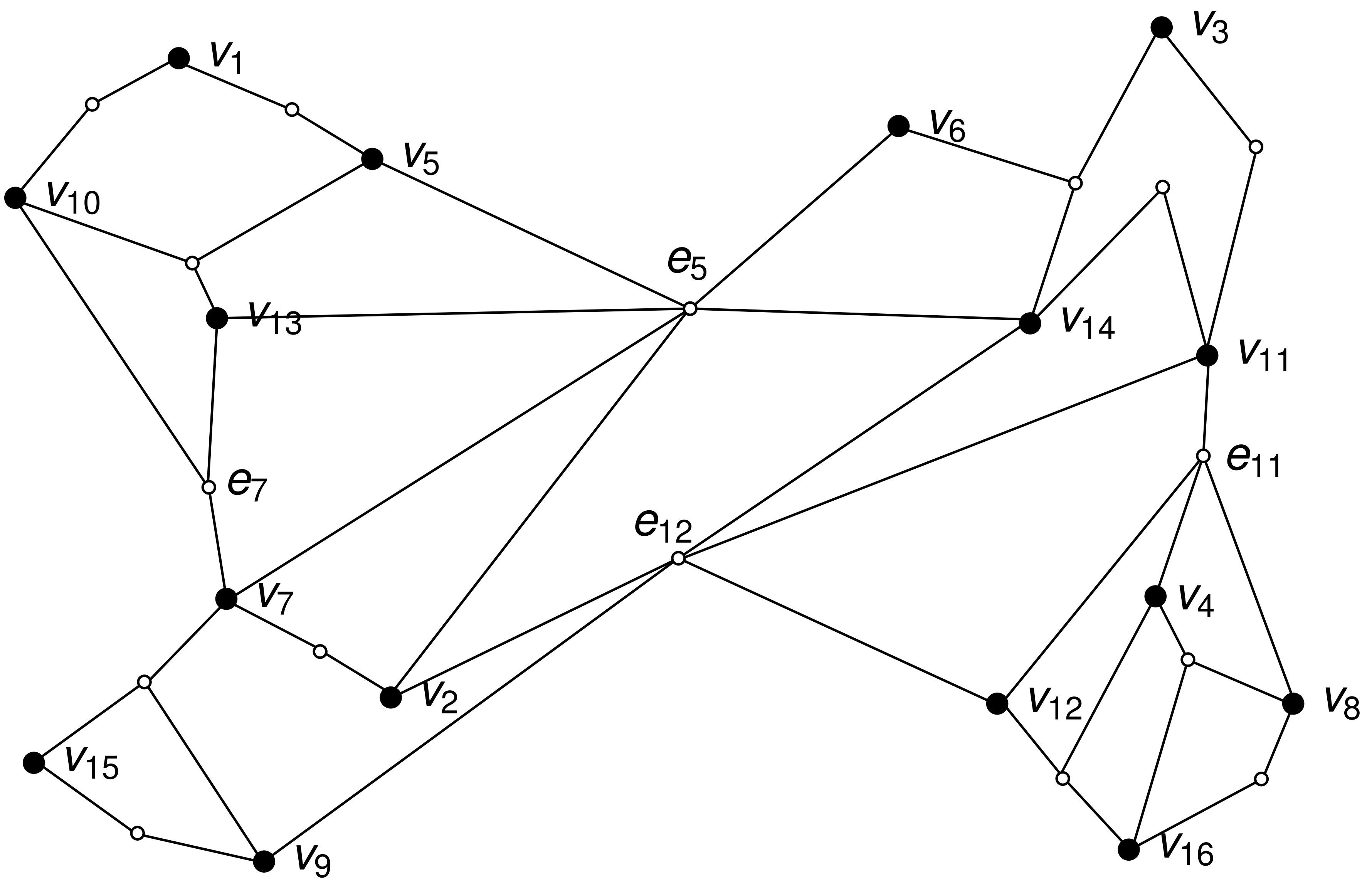
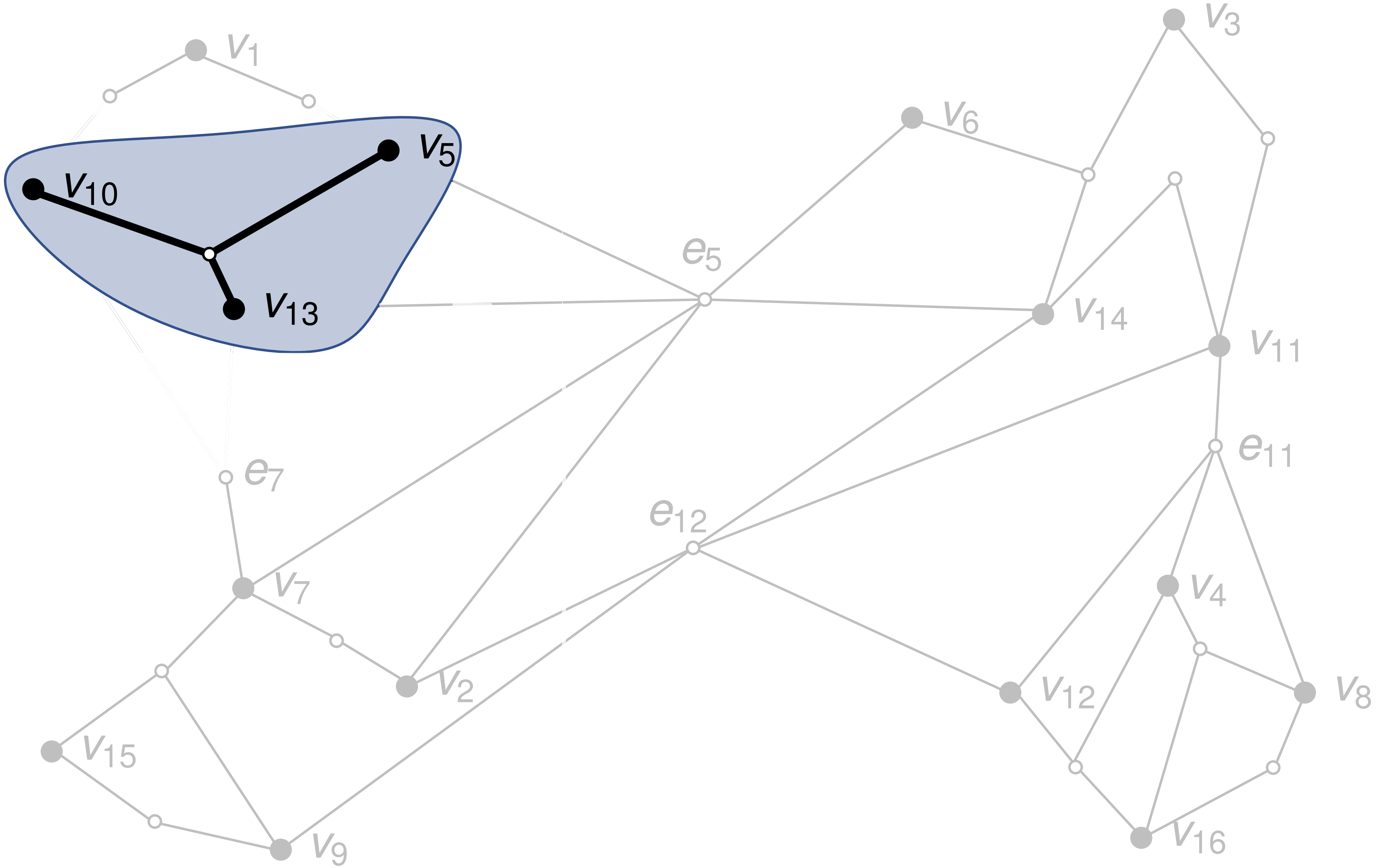
# From SpM×V to Hypergraph Partitioning



# From SpM×V to Hypergraph Partitioning



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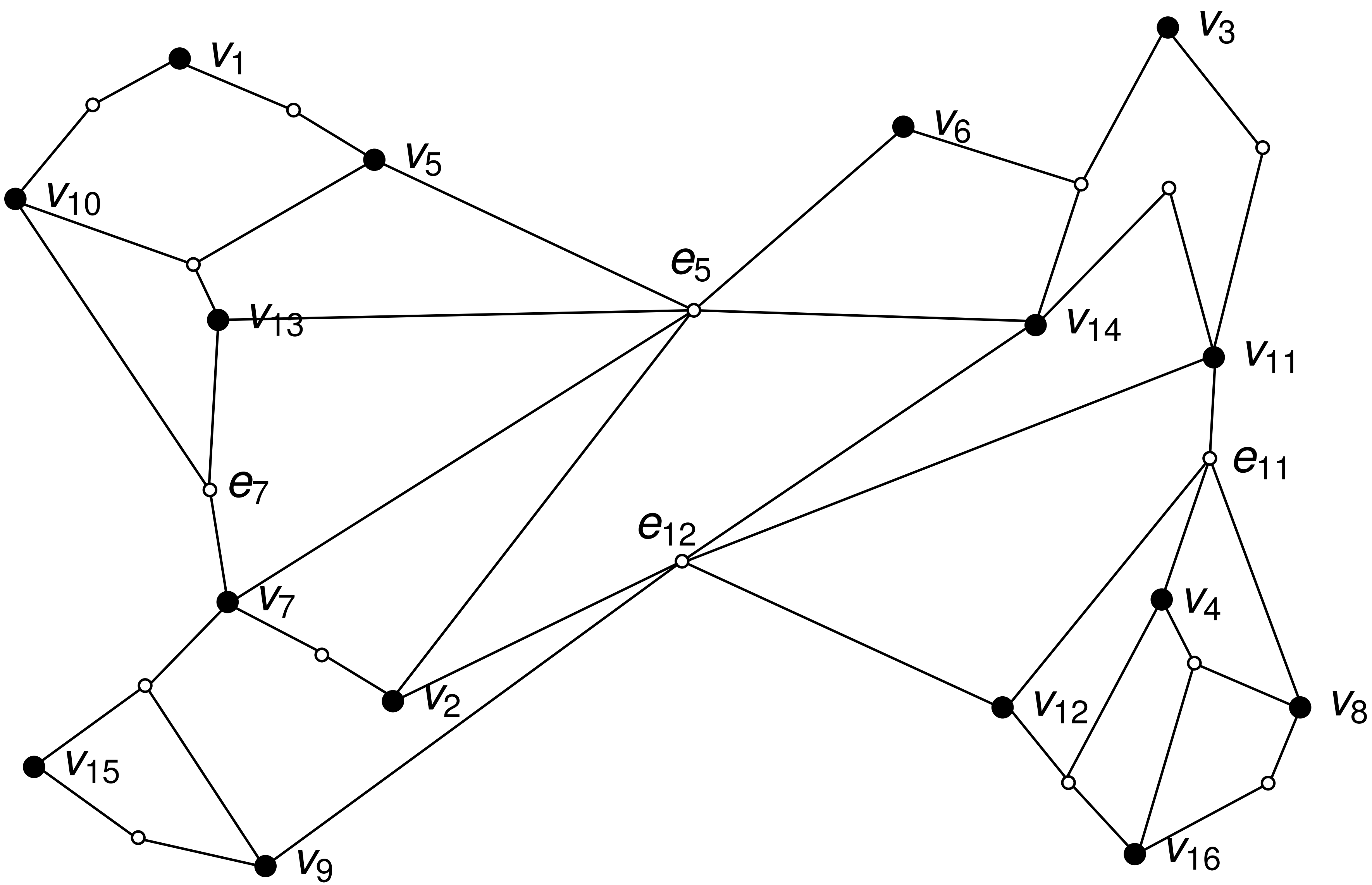

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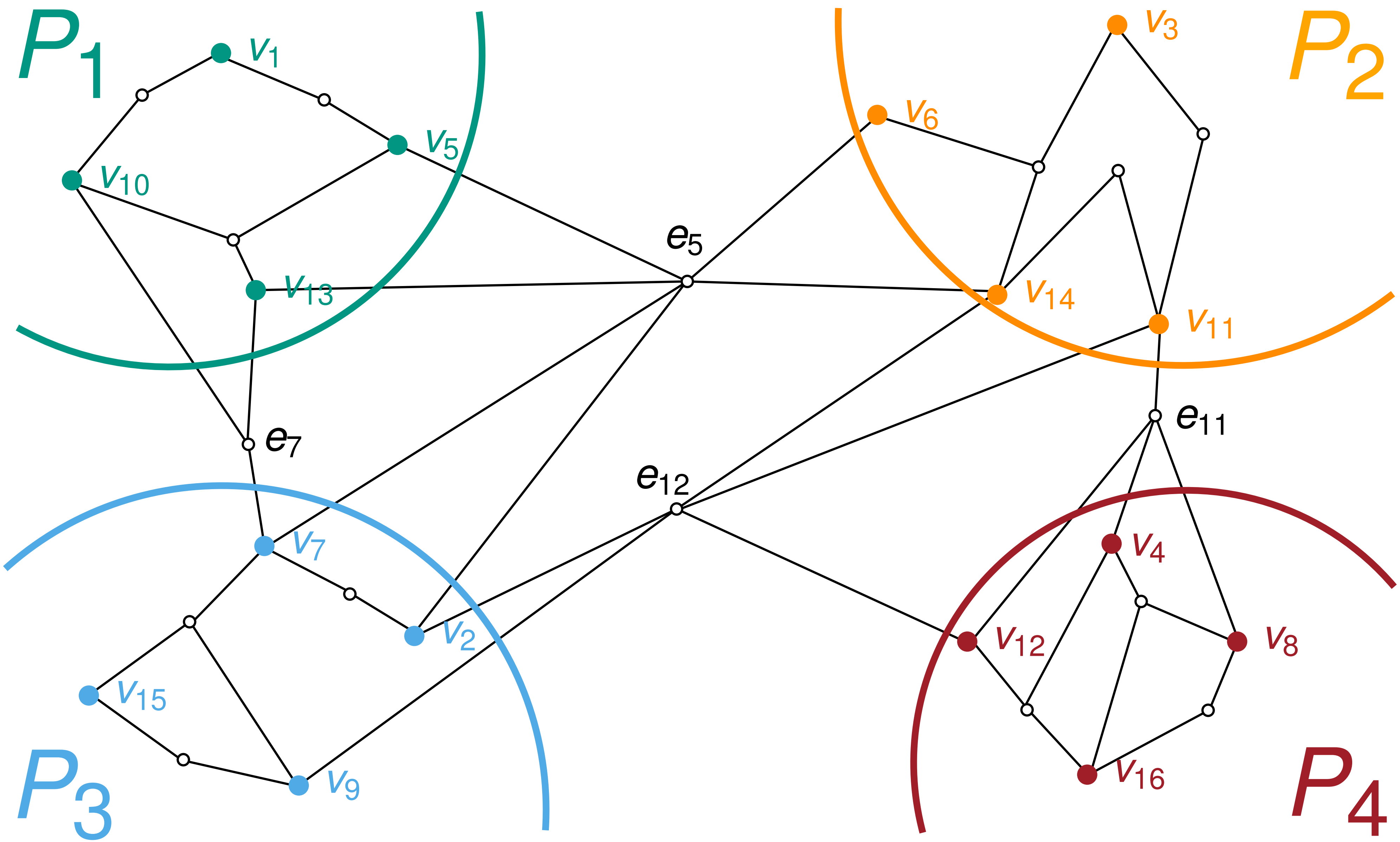

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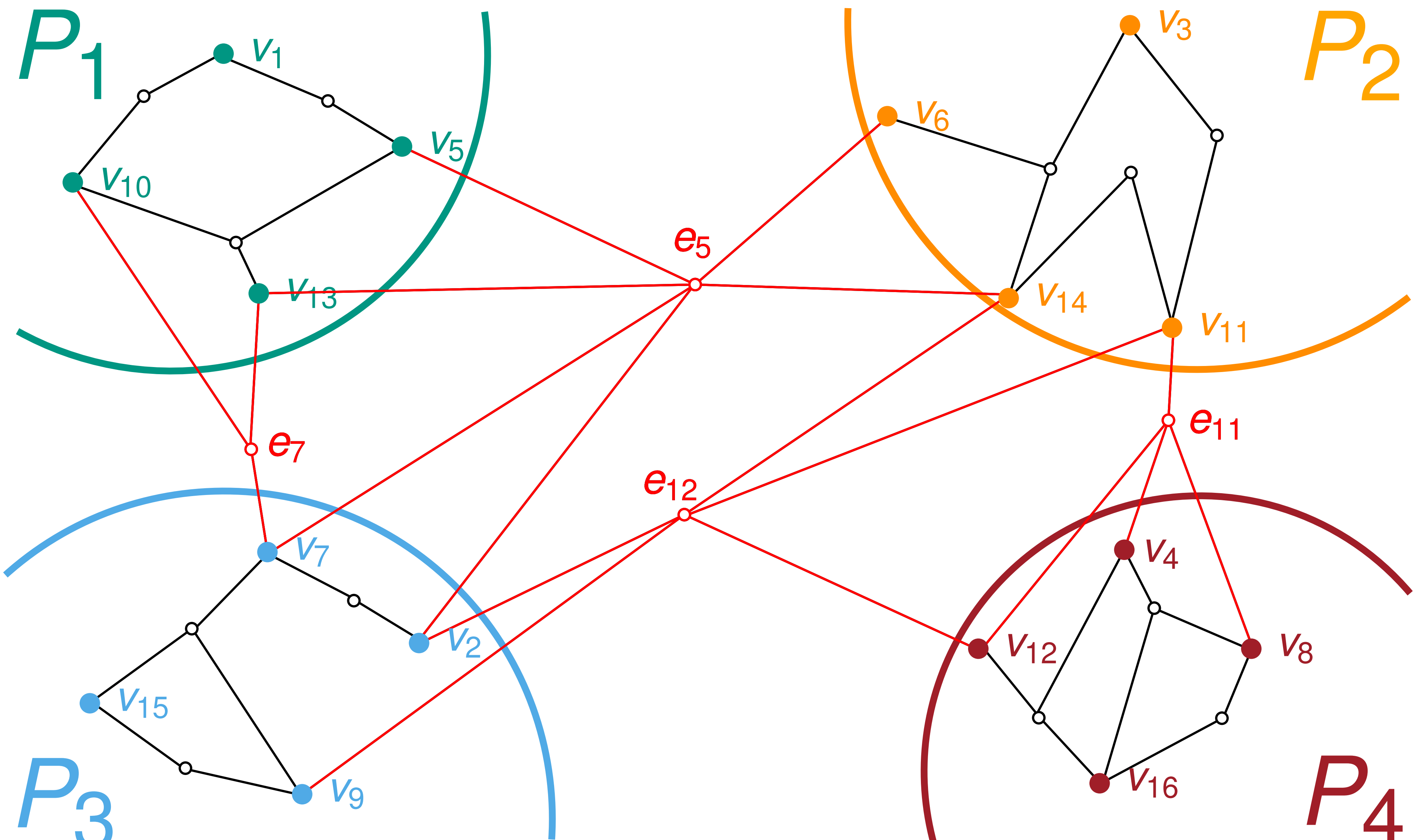
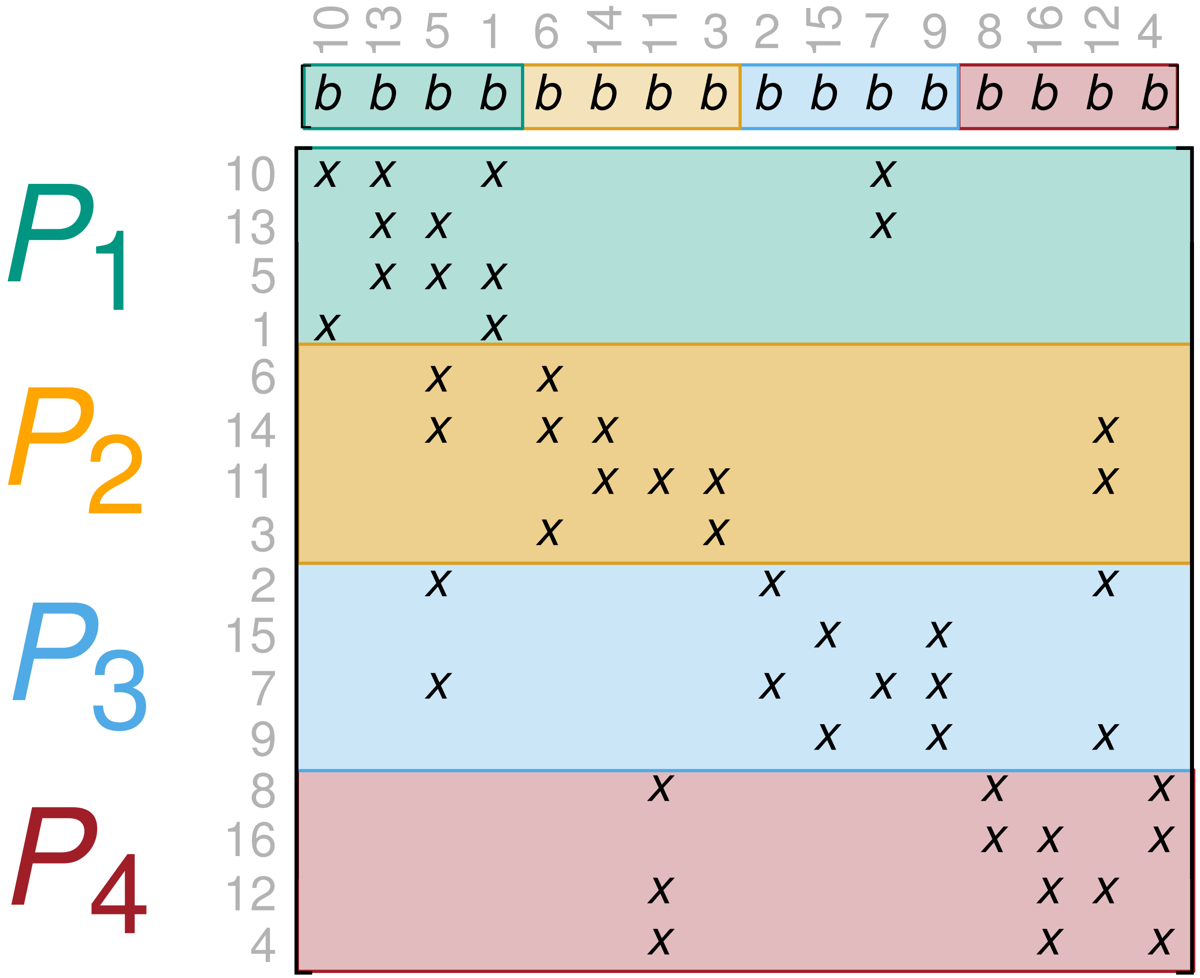


# From Hypergraph Partitioning to SpM×V





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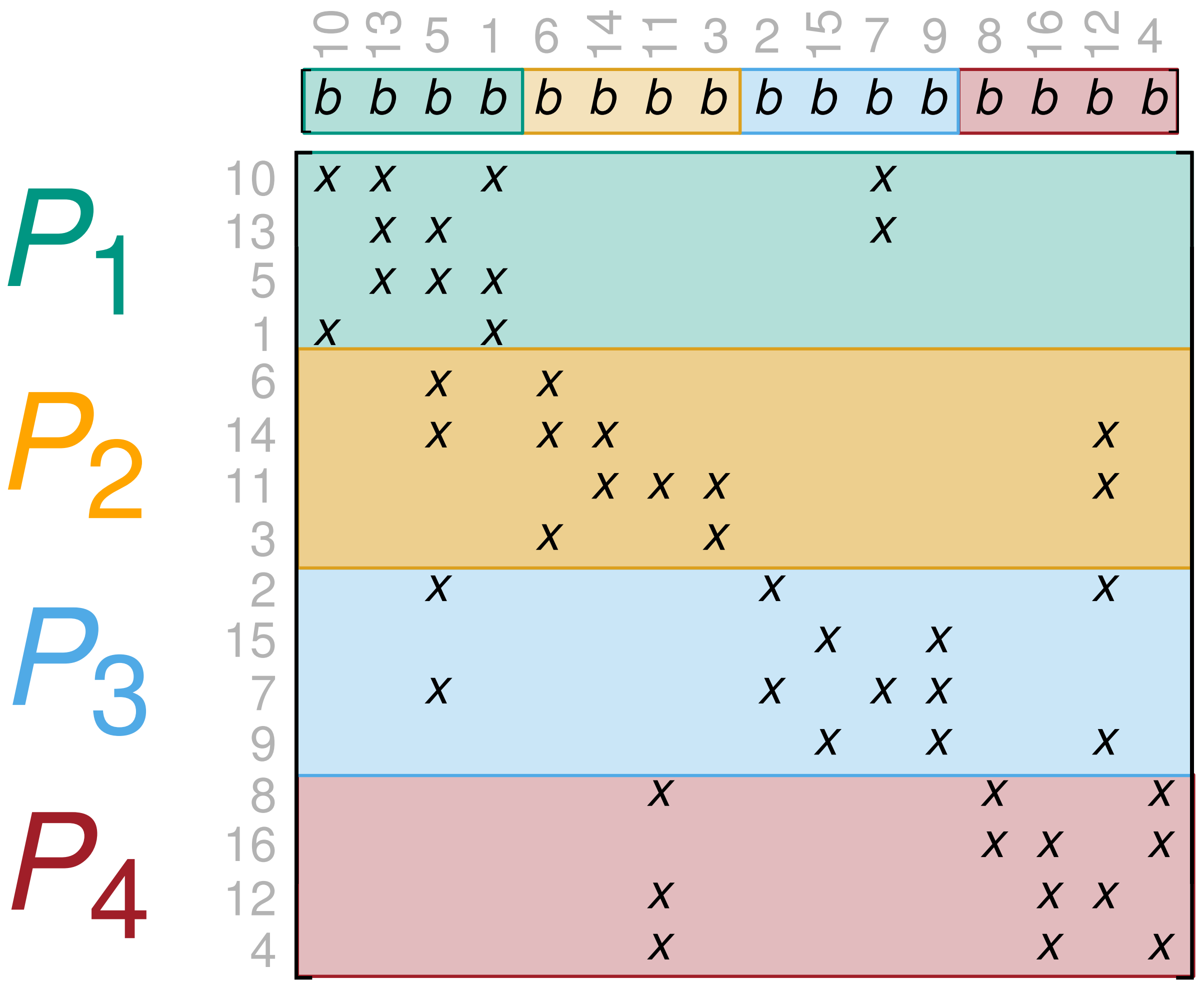
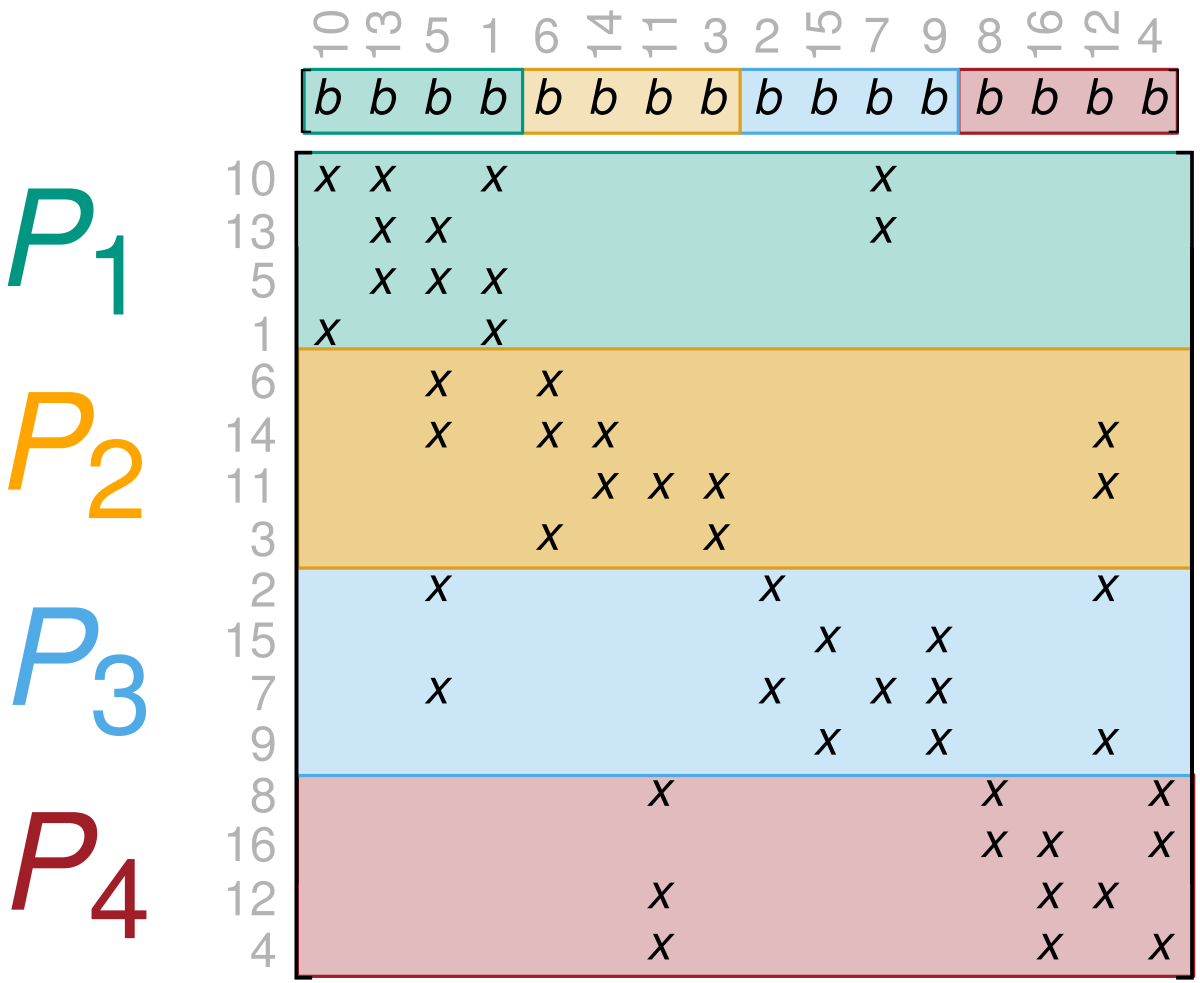
# From Hypergraph Partitioning to SpM×V




**Load Balancing?**

# From Hypergraph Partitioning to SpM×V




**Load Balancing?**

$\Rightarrow$ 12

$\Rightarrow$ 12

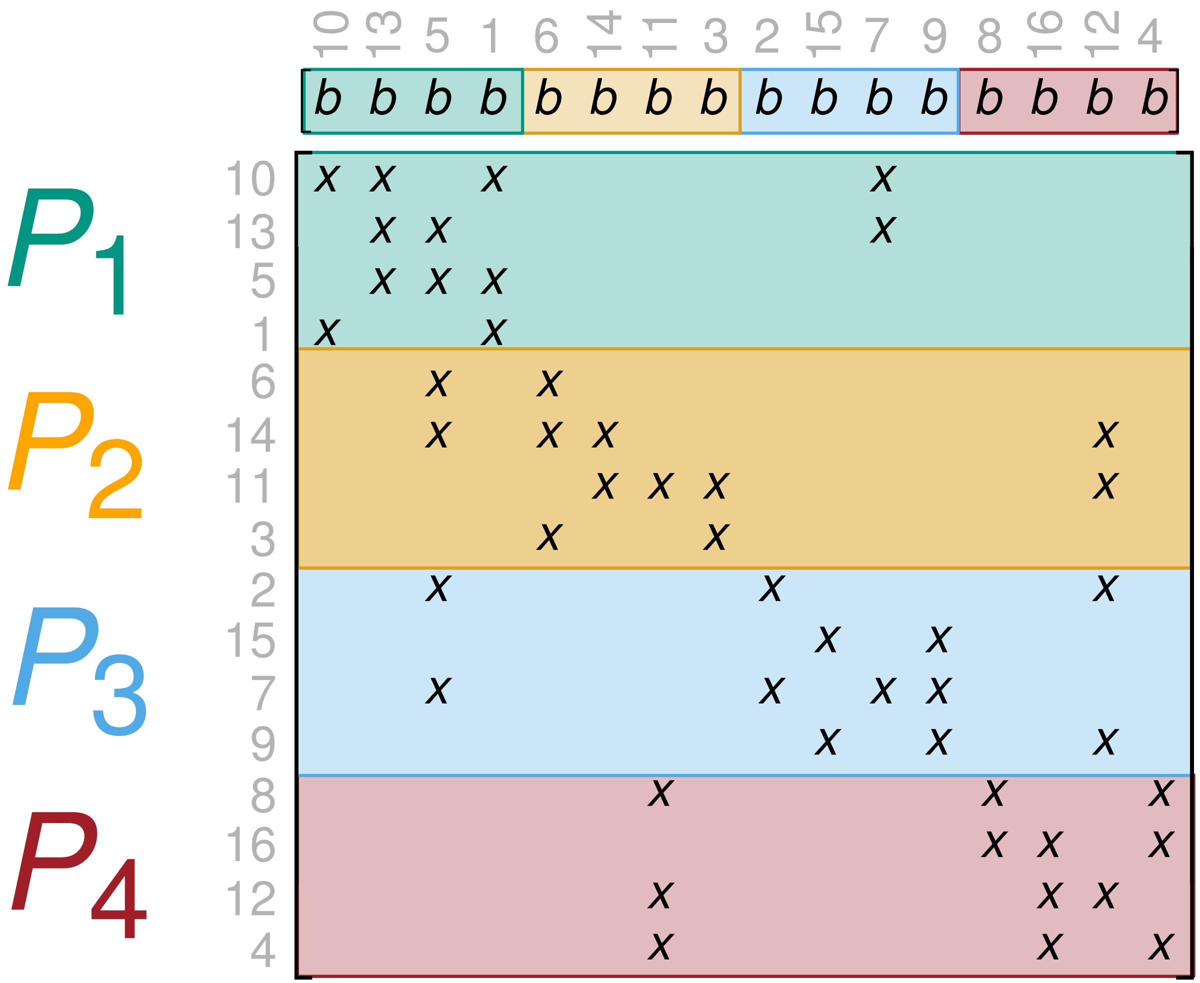
$\Rightarrow$ 12

$\Rightarrow$ 12

# From Hypergraph Partitioning to SpM×V



## Where are the cut-hyperedges?




**Load Balancing?**

$\Rightarrow$ 12

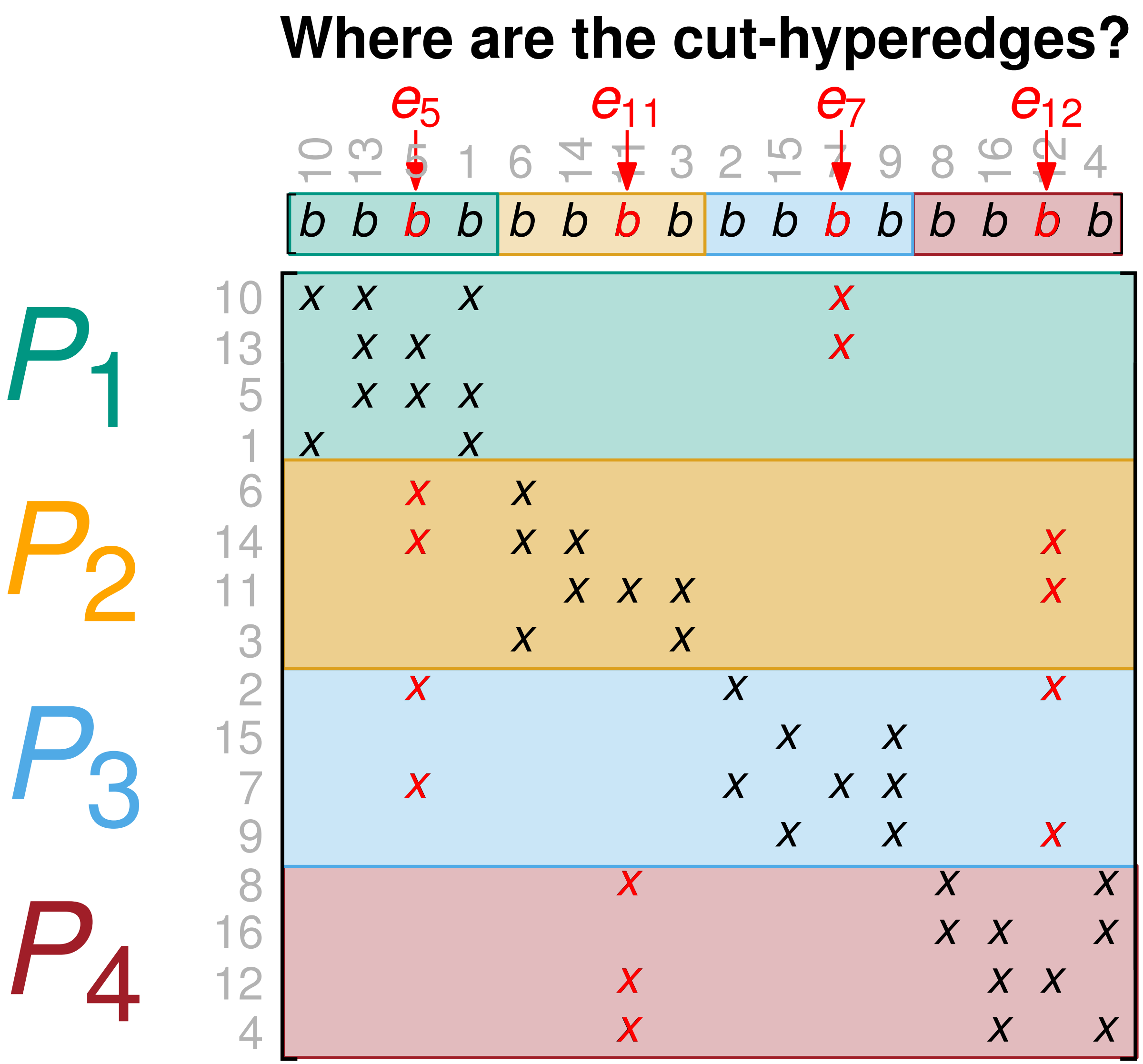
$\Rightarrow$ 12

$\Rightarrow$ 12

$\Rightarrow$ 12

**Commuication Volume?**

# From Hypergraph Partitioning to SpM×V




**Load Balancing?**

⇒ 12

⇒ 12

⇒ 12

⇒ 12

**Commuication Volume?** ⇒ 6 entries!

# How does (Hyper)Graph Partitioning work?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# How does

**Bad News:**

- Hypergraph Partitioning is $\mathbf{NP}$-hard
- even finding good approximate solutions for graphs is $\mathbf{NP}$-hard

# work?



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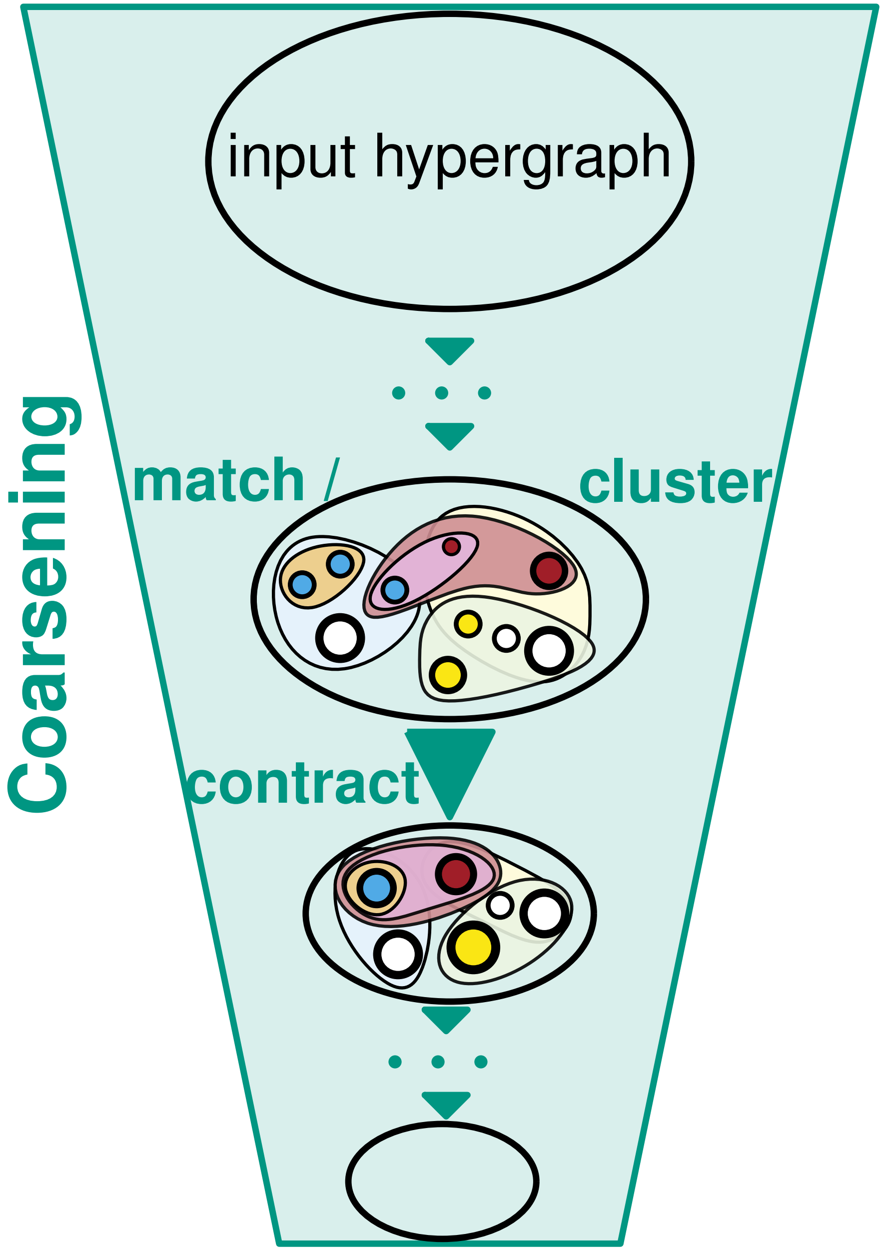
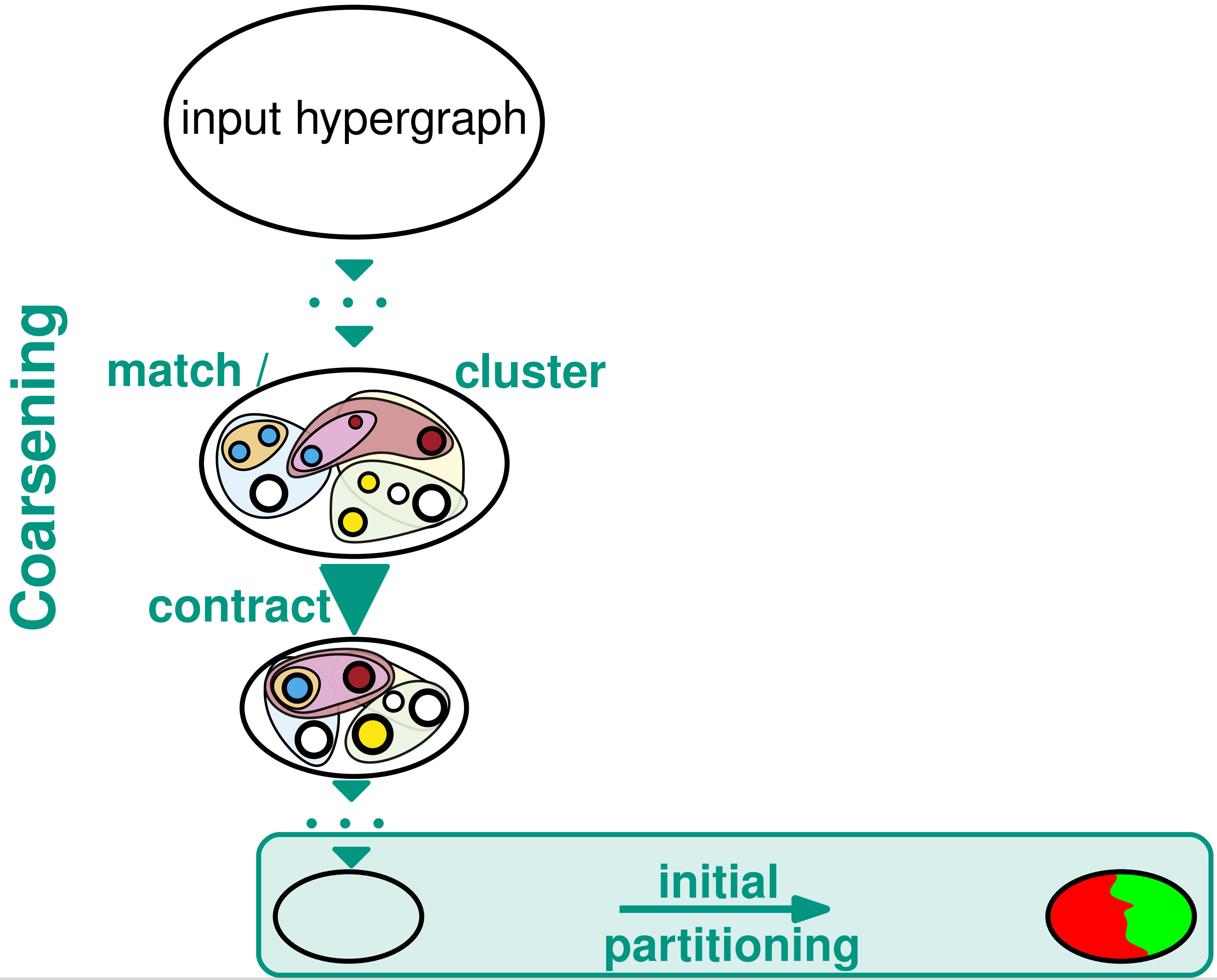
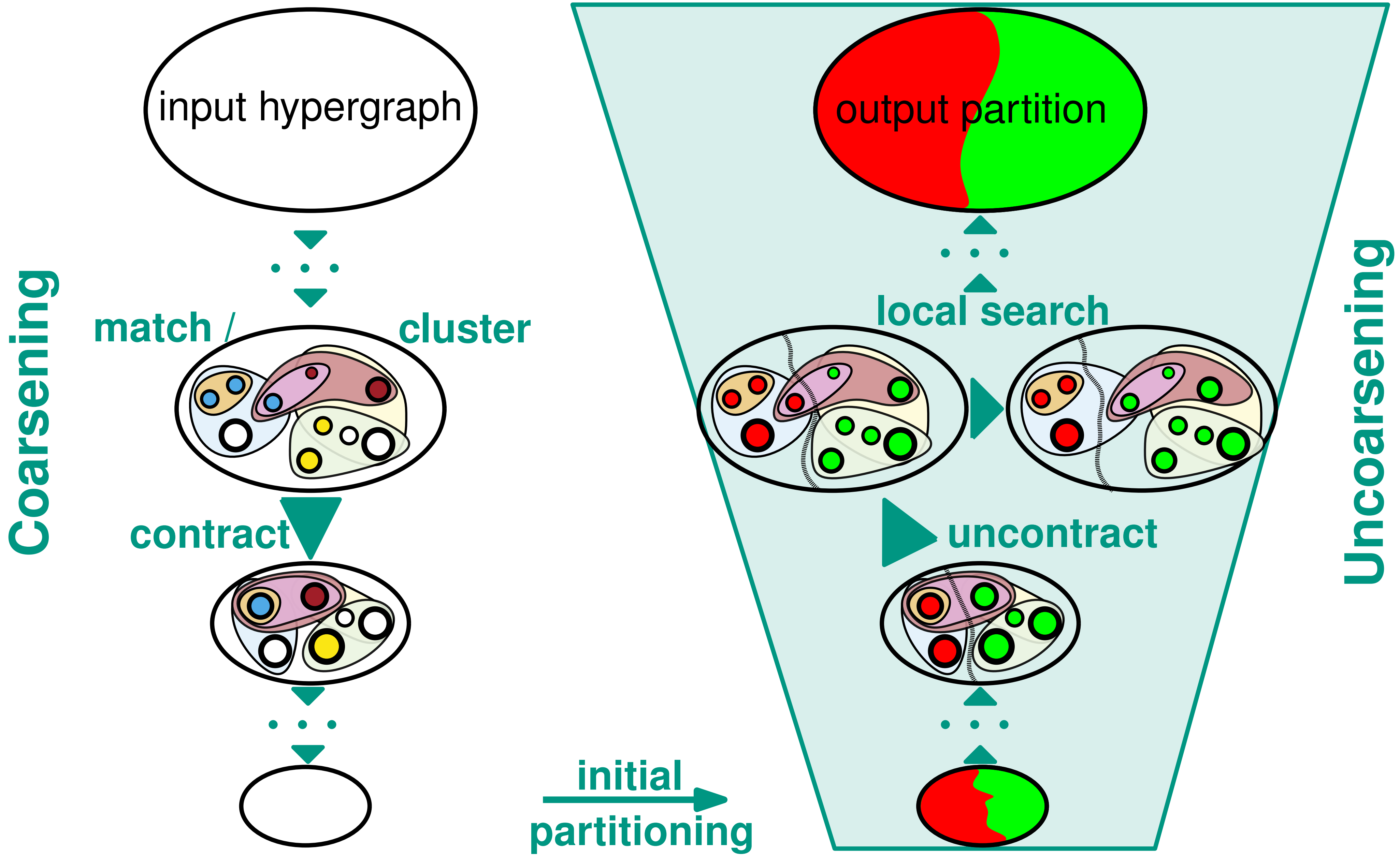
# Successful Heuristic: Multilevel Paradigm



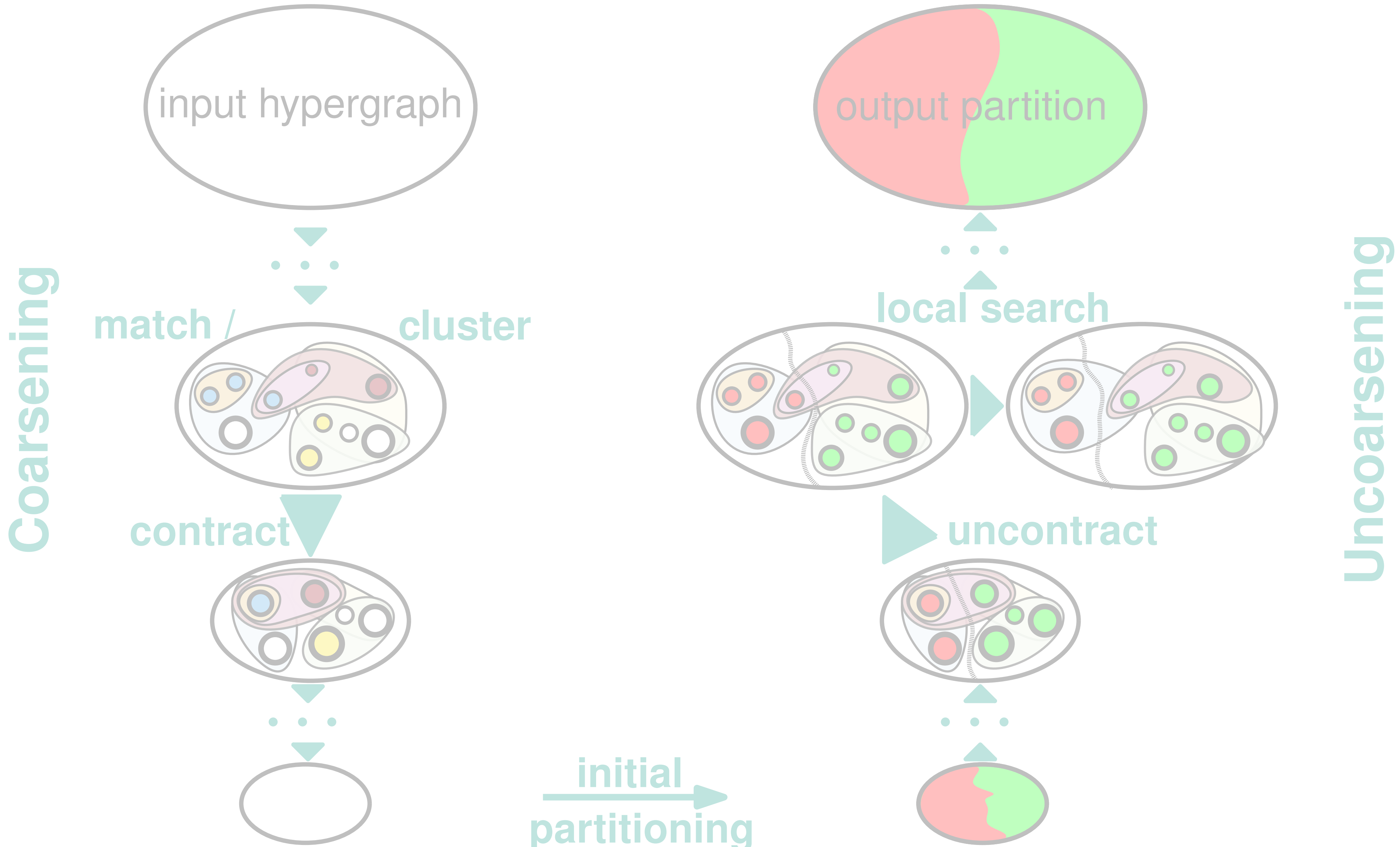
# Successful Heuristic: Multilevel Paradigm



# Successful Heuristic: Multilevel Paradigm



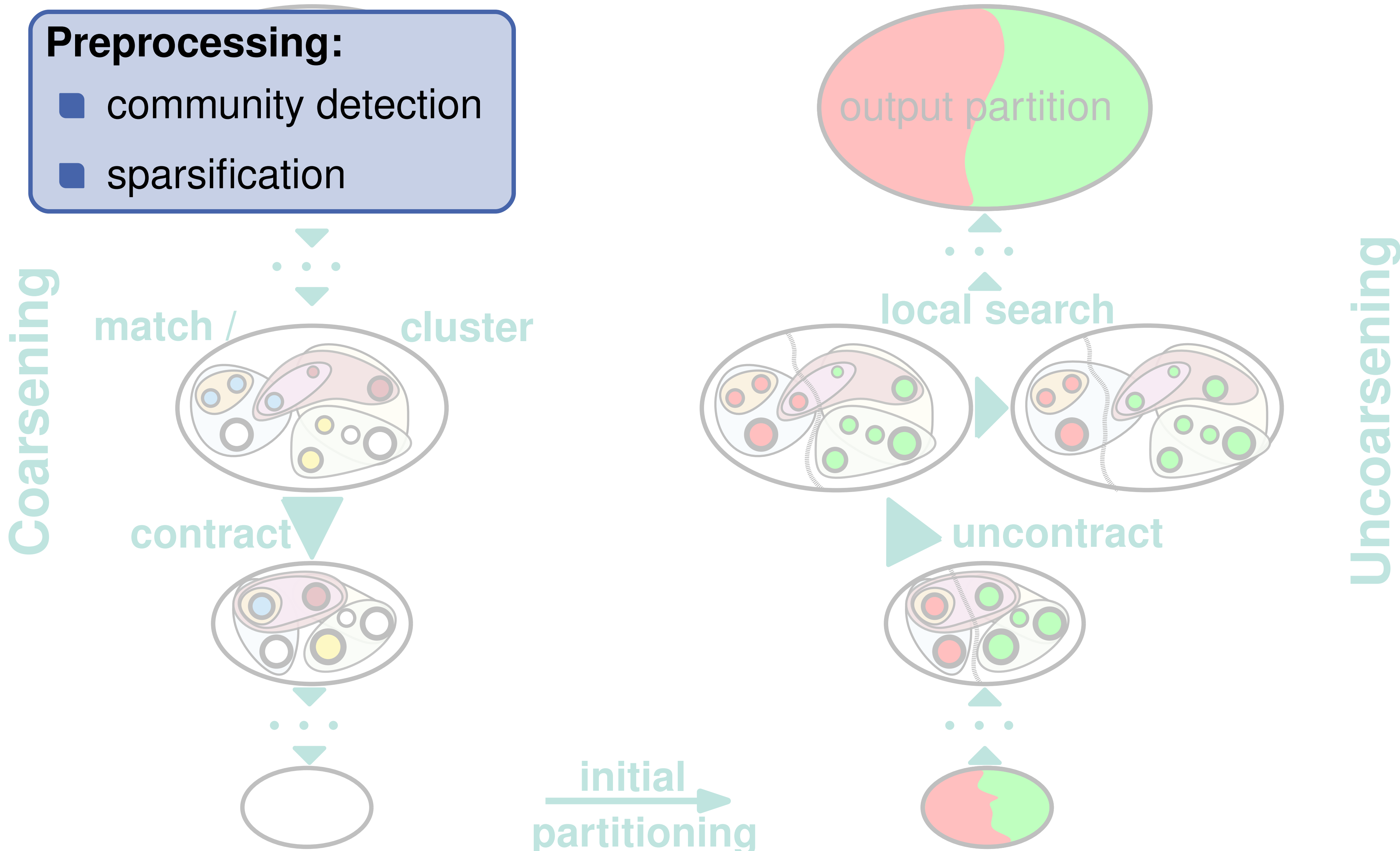
# Multilevel Paradigm - Algorithmic Ingredients



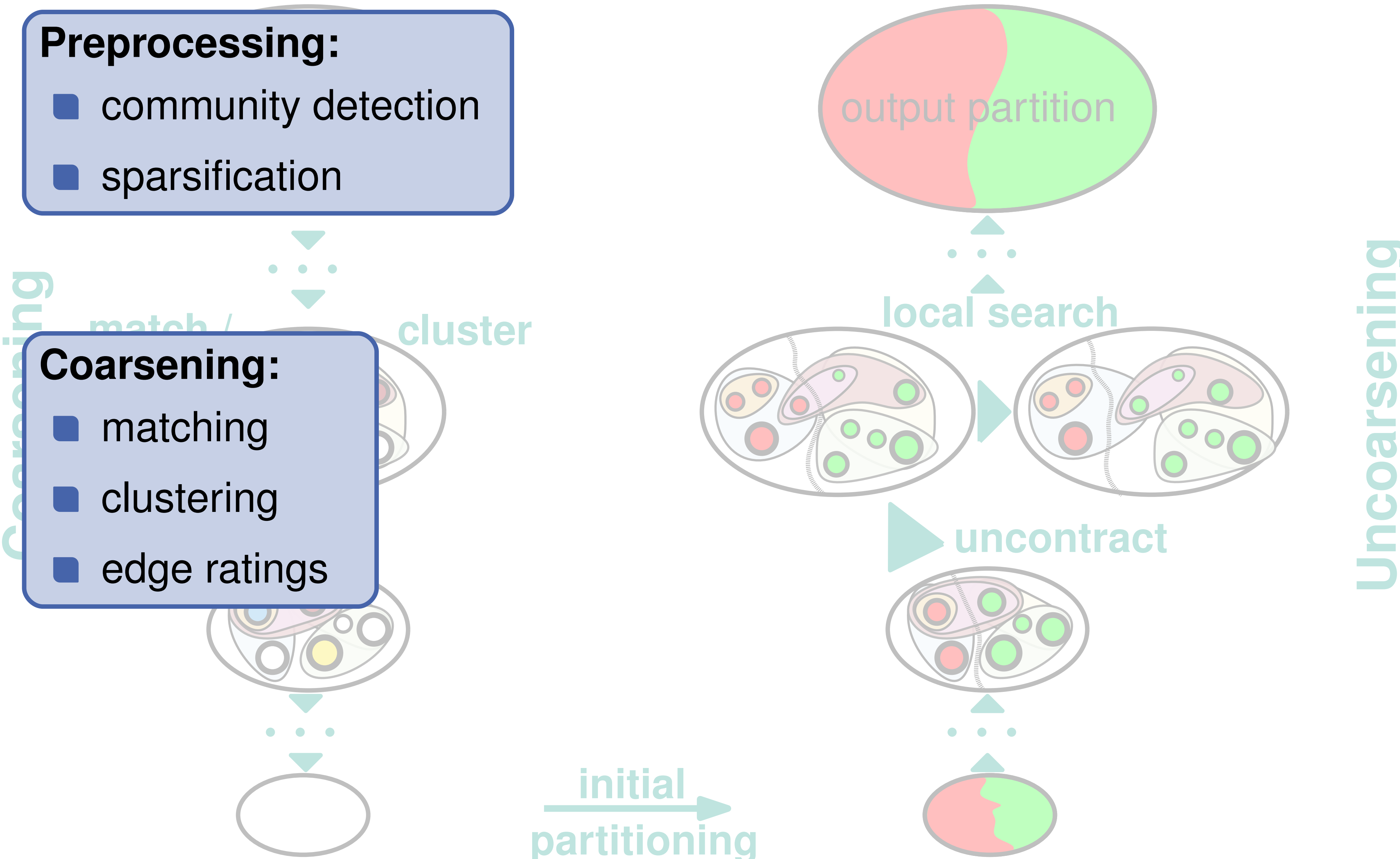
# Multilevel Paradigm - Algorithmic Ingredients

**Preprocessing:**

- community detection
- sparsification



# Multilevel Paradigm - Algorithmic Ingredients




**Preprocessing:**

- community detection
- sparsification

**Coarsening:**

- matching
- clustering
- edge ratings

# Multilevel Paradigm - Algorithmic Ingredients

**Preprocessing:**

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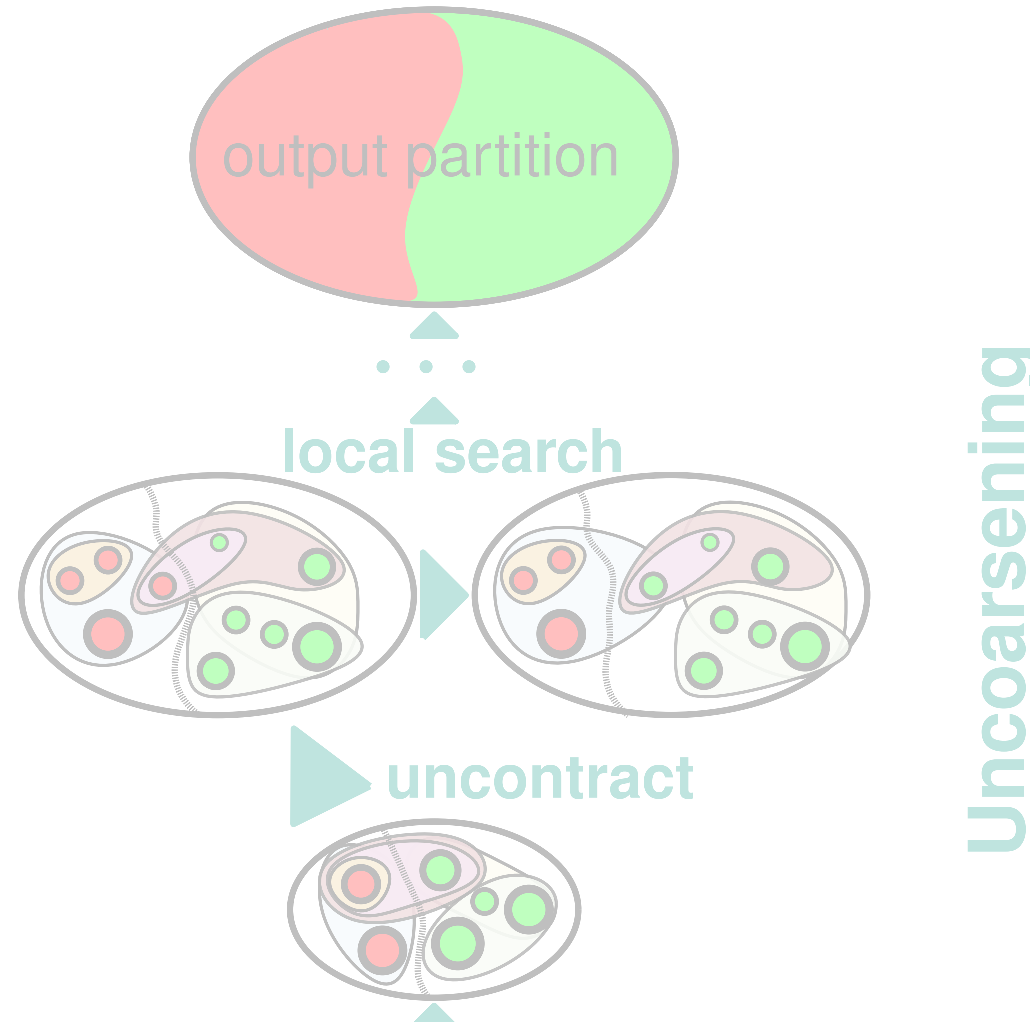
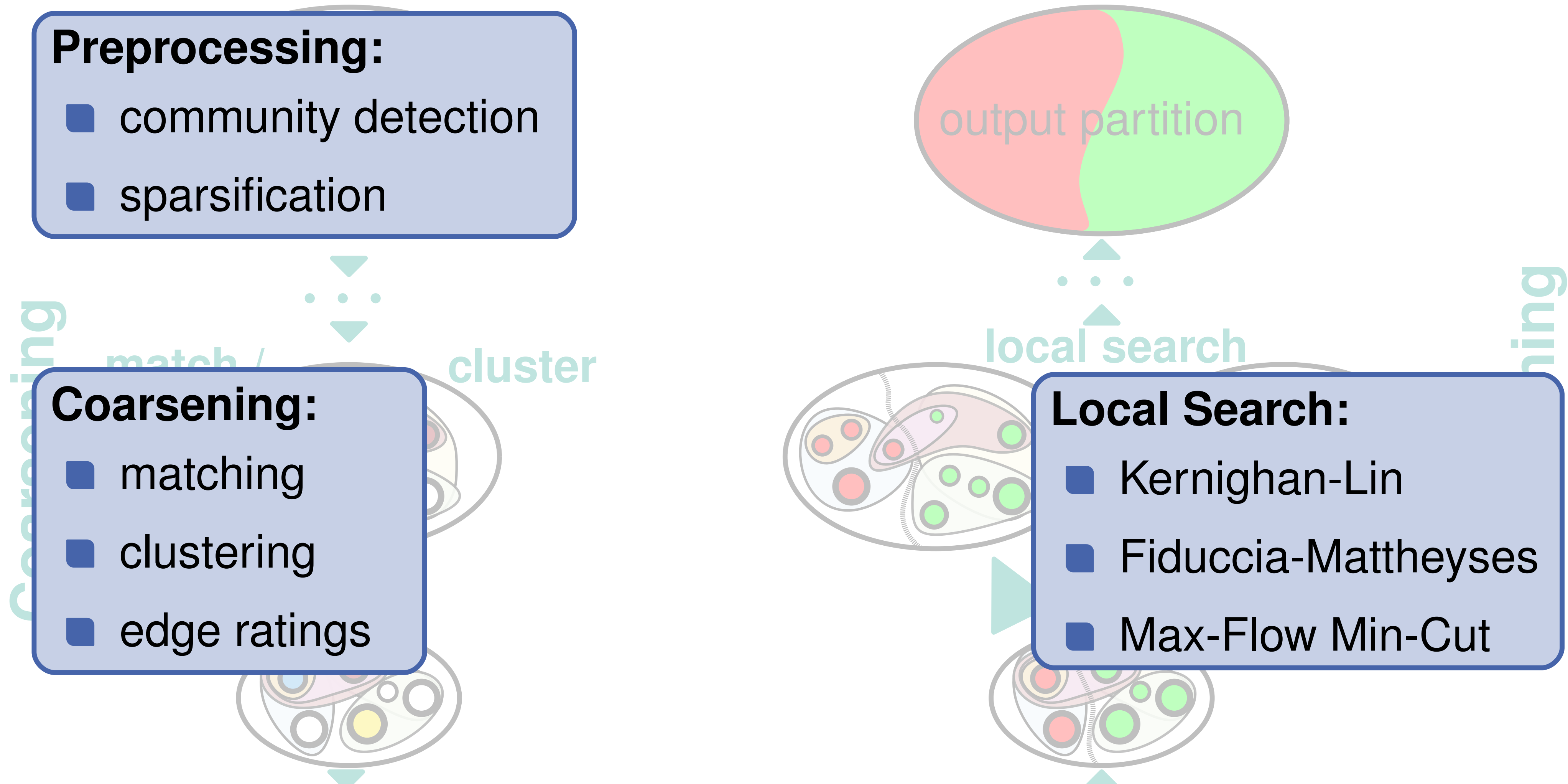



**Initial Partitioning:**

- portfolio of various algorithms $\rightsquigarrow$ diversification

# Multilevel Paradigm - Algorithmic Ingredients




**Preprocessing:**

- community detection
- sparsification

**Coarsening:**

- matching
- clustering
- edge ratings

**Local Search:**

- Kernighan-Lin
- Fiduccia-Mattheyses
- Max-Flow Min-Cut

**Initial Partitioning:**

- portfolio of various algorithms $\rightsquigarrow$ diversification

# Multilevel Paradigm - Algorithmic Ingredients

**Preprocessing:**
- community detection
- sparsification

**Metaheuristics:**
- Global Search
- Evolutionary Algorithms

**Coarsening:**
- matching
- clustering
- edge ratings

**Local Search:**
- Kernighan-Lin
- Fiduccia-Mattheyses
- Max-Flow Min-Cut

**Initial Partitioning:**
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# Multilevel Paradigm - Algorithmic Ingredients

**Preprocessing:**

- community detection
- sparsification

**Metaheuristics:**

- Global Search
- Evolutionary Algorithms

**Coarsening:**

- matching
- clustering
- edge ratings

**Parallelization:**

- shared memory
- distributed memory

**Local Search:**

- Kernighan-Lin
- Fiduccia-Mattheyses
- Max-Flow Min-Cut

**Initial Partitioning:**

- portfolio of various algorithms $\rightsquigarrow$ diversification

# Fiduccia-Mattheyses Algorithm

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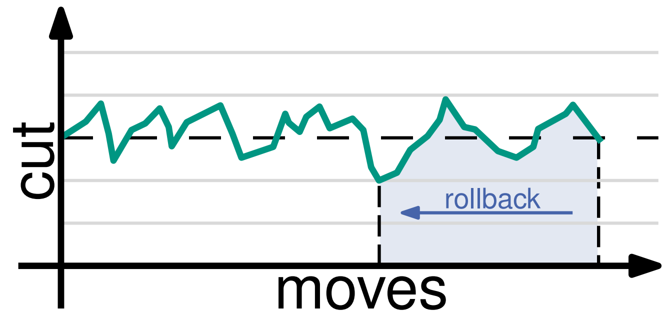
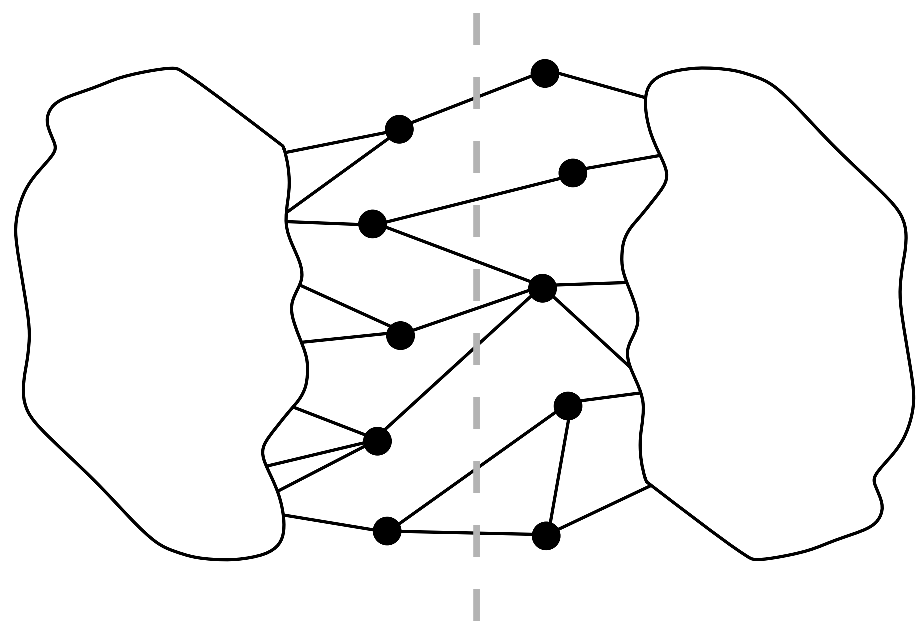
**Algorithm 1:** FM Local Search

**while** $\neg$ *done* **do**
    `find` best move
    `perform` best move
`rollback` to best solution




can worsen solution



- compute gain $g(v) = d_{\text{ext}}(v) - d_{\text{int}}(v)$
- alternate between blocks
- edge-cut: **7**

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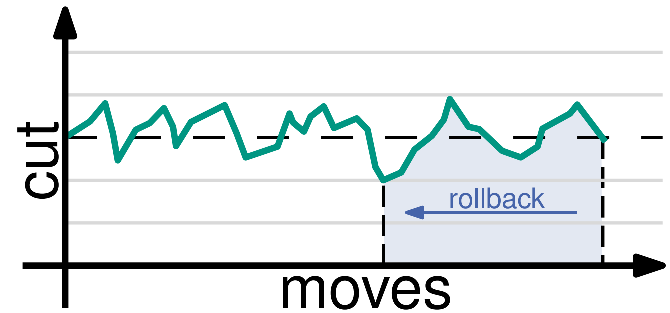


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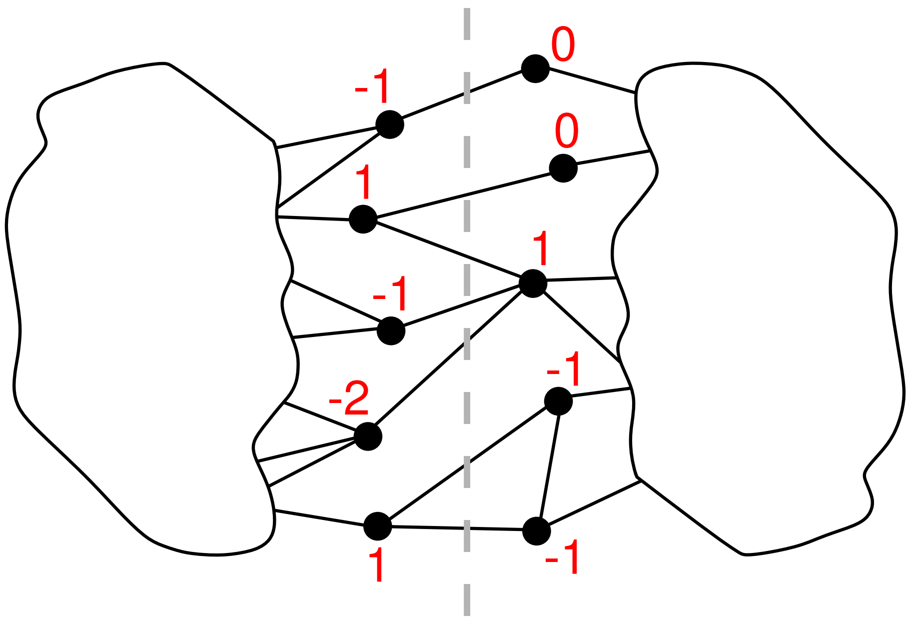




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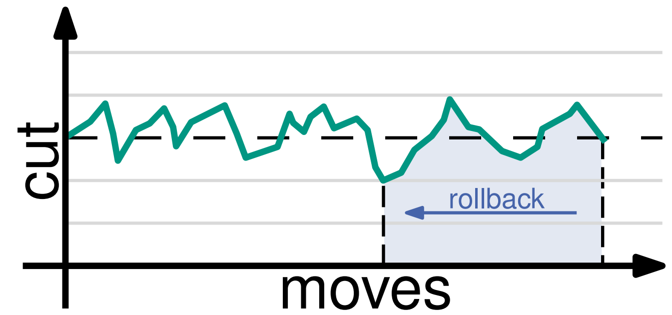


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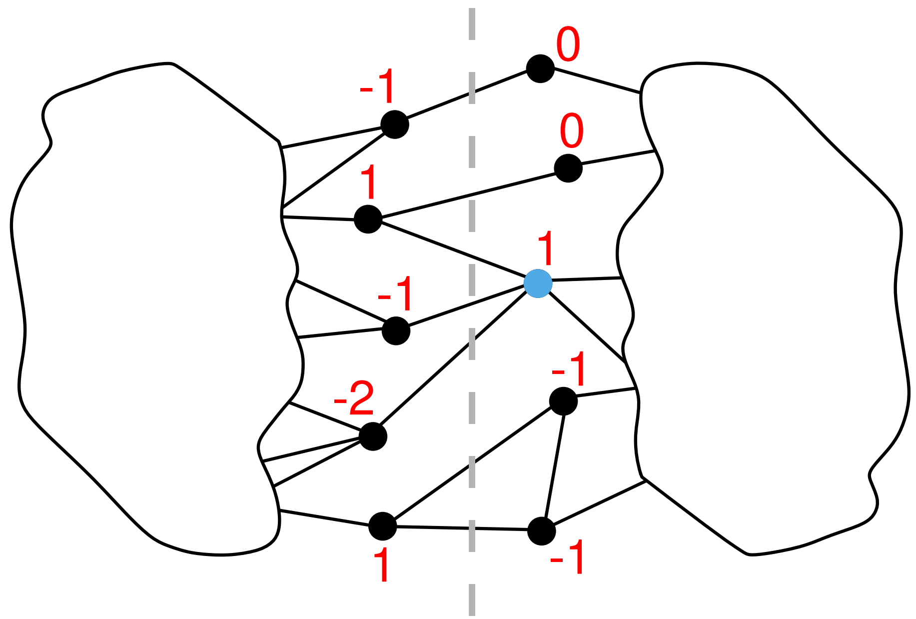

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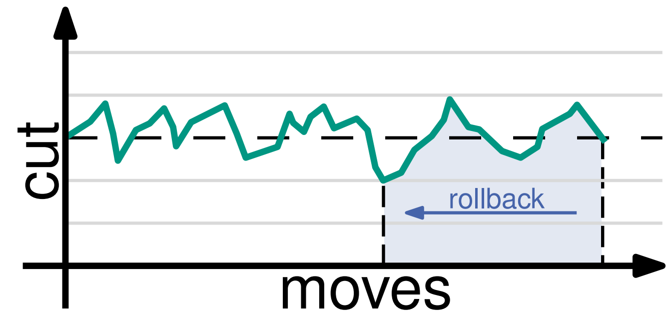
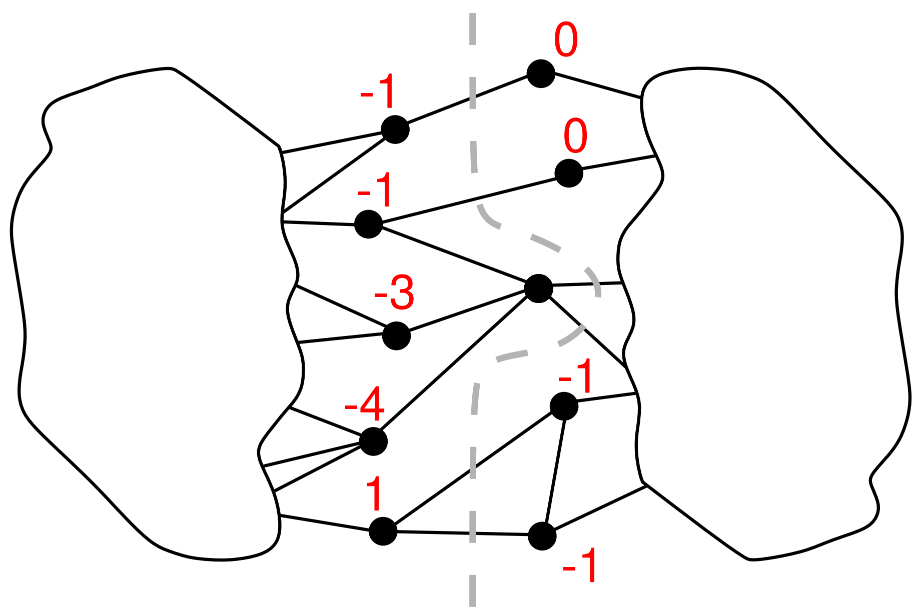



can worsen solution

- recalculate gain $g(v)$ of neighbors
- move each node at most once
- edge-cut: **7**, **6**





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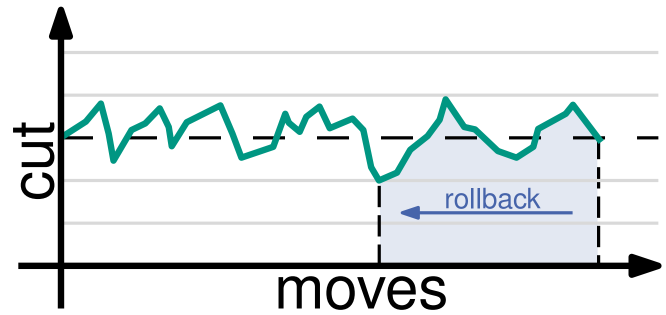


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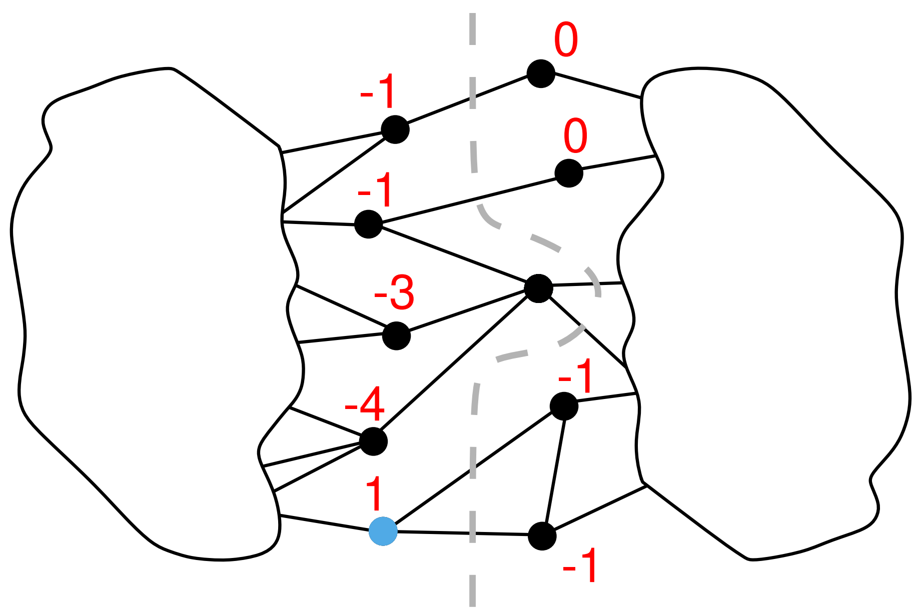
- edge-cut: **7**, **6**





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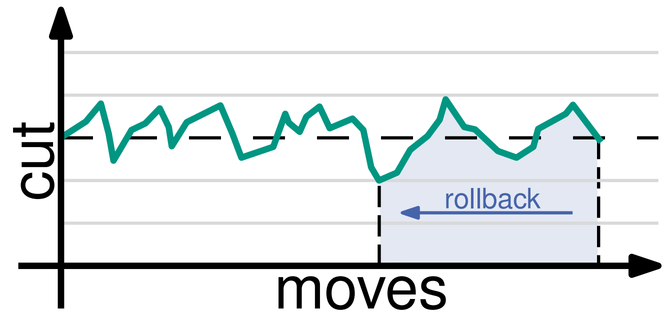
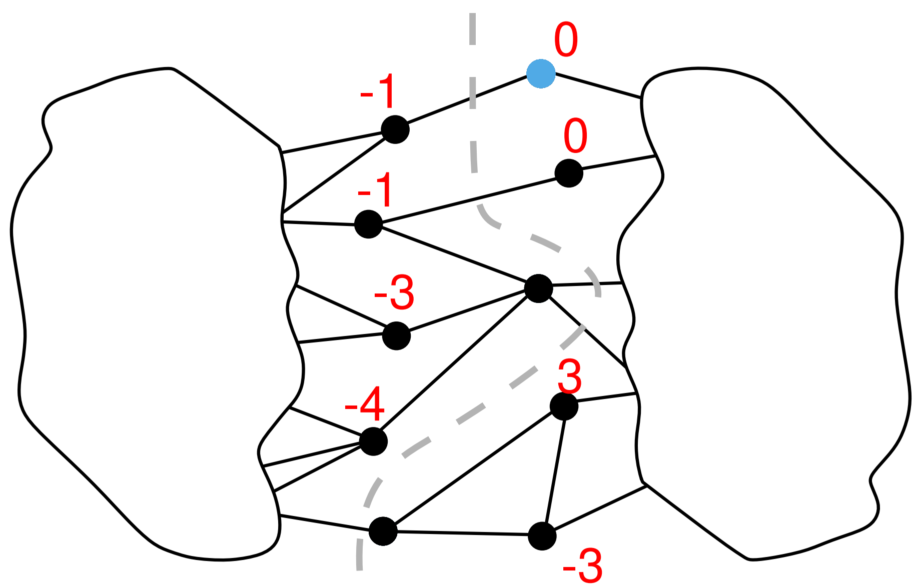



can worsen solution

- recalculate gain $g(v)$ of neighbors
- move each node at most once
- edge-cut: **7**, **6**,**5**



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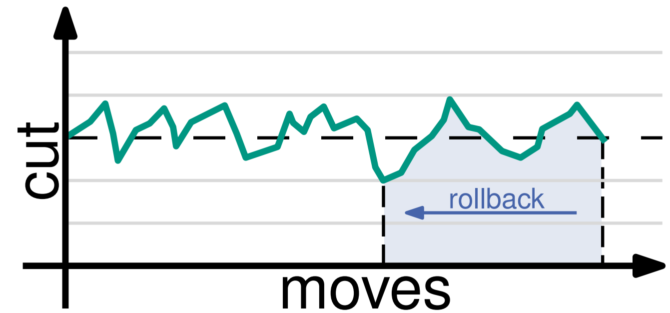
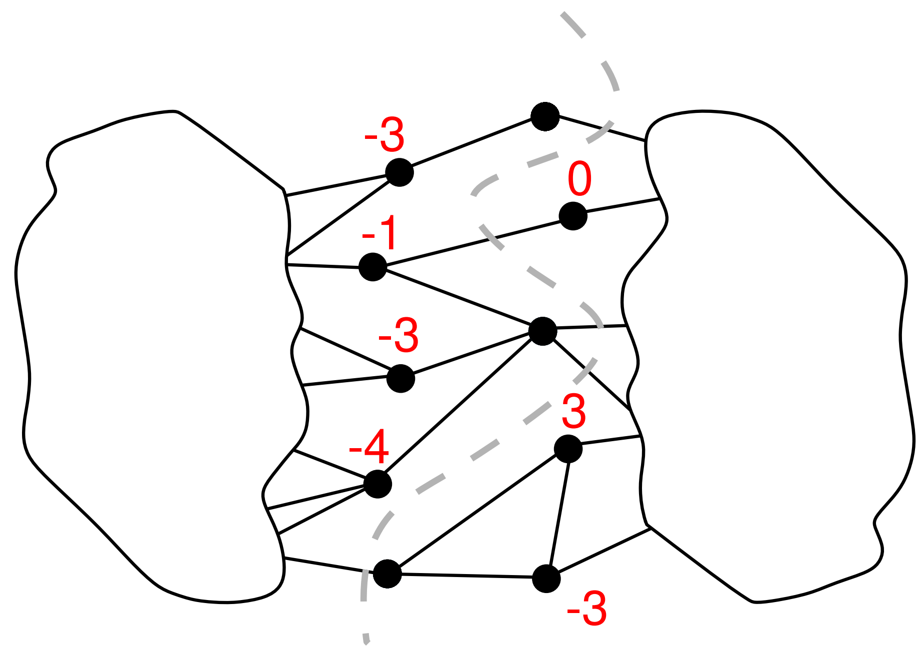



can worsen solution

- recalculate gain $g(v)$ of neighbors
- move each node at most once
- edge-cut: **7**, **6**,**5**,**5**



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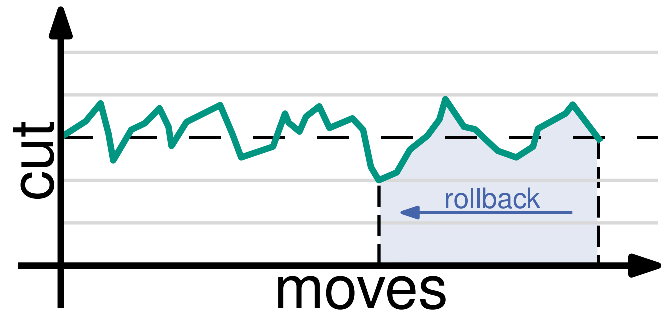


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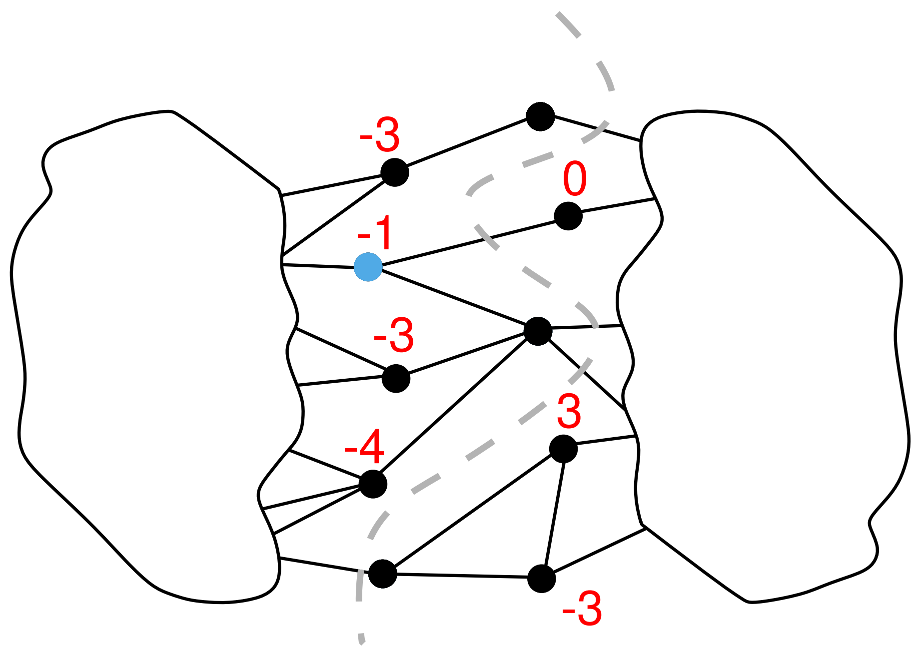




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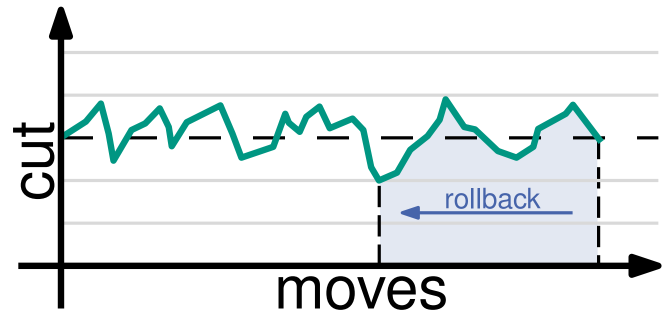
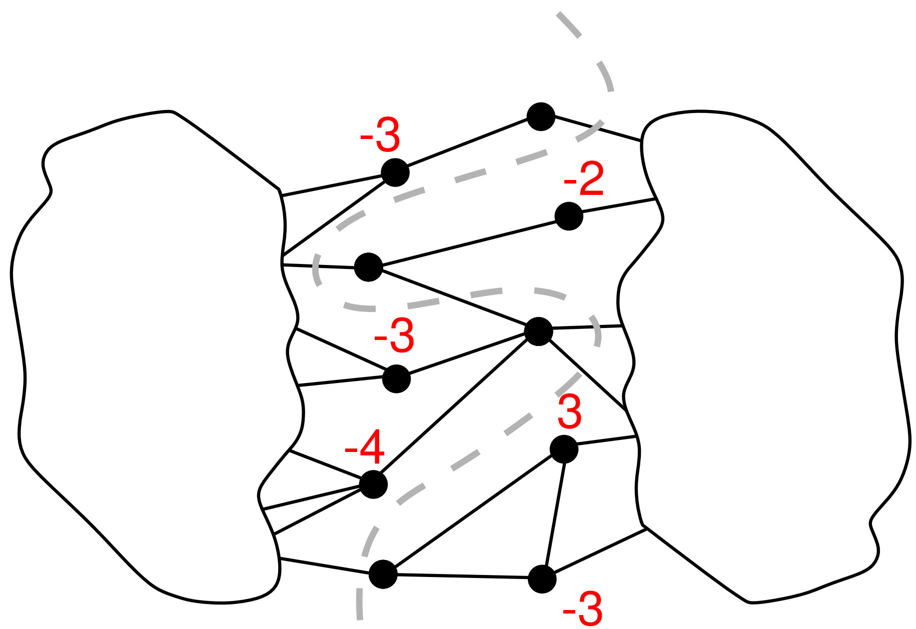



can worsen solution

- recalculate gain $g(v)$ of neighbors
- move each node at most once
- edge-cut: **7**, **6**,**5**,**5**,**6**



# Fiduccia-Mattheyses Algorithm

Karlsruhe Institute of Technology

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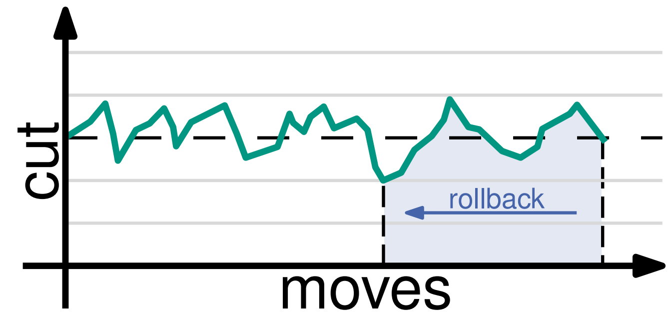
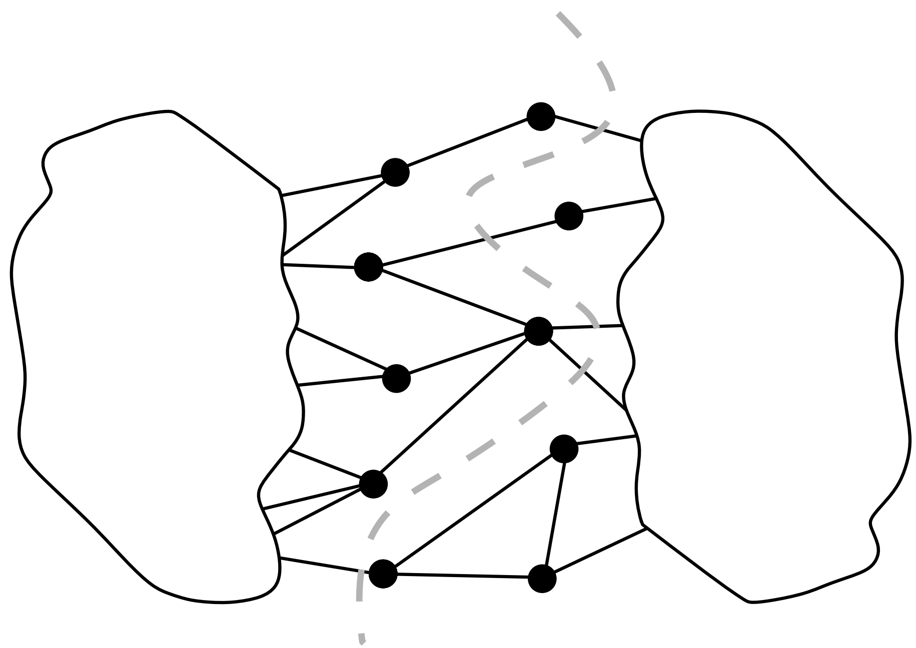


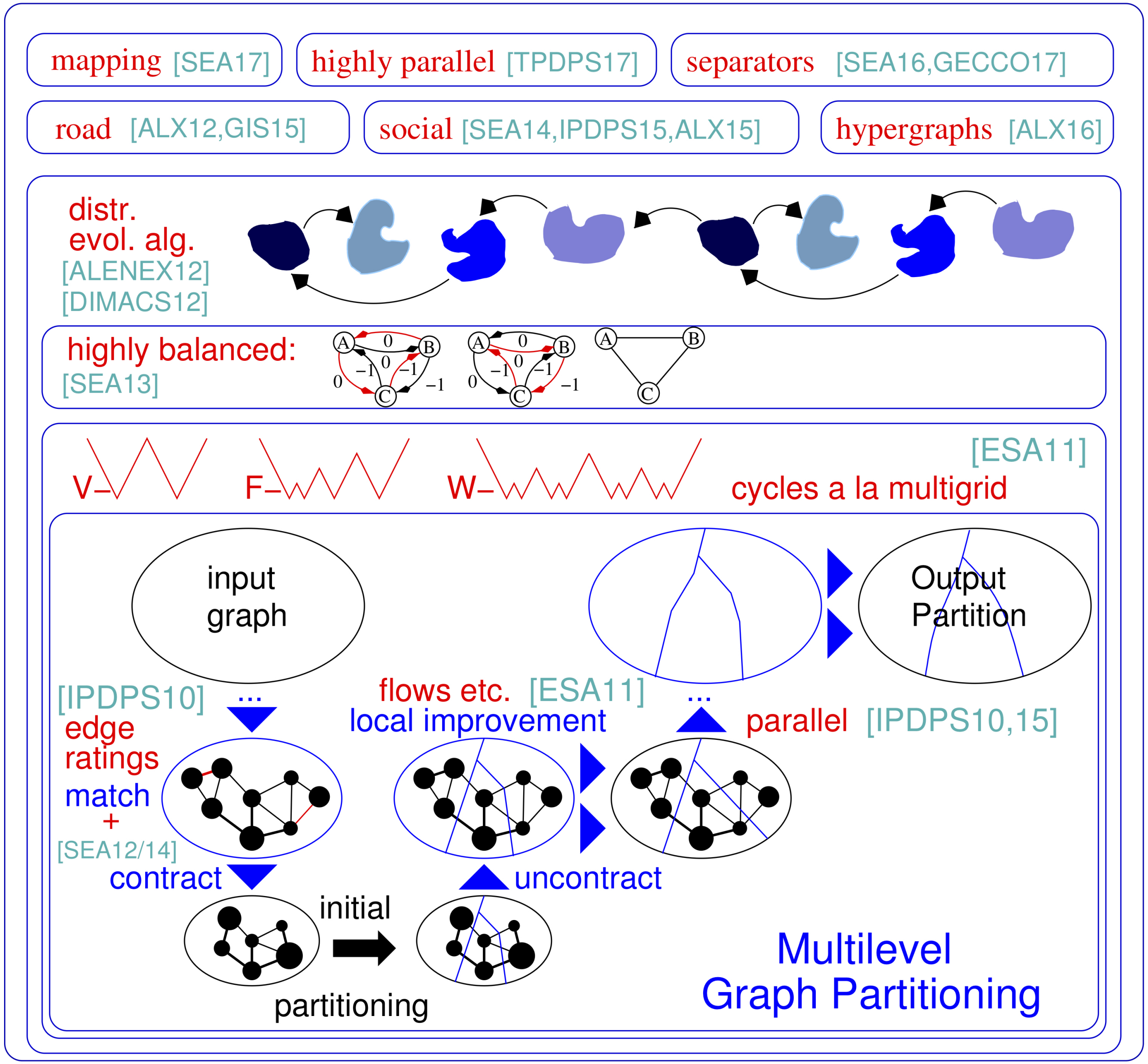

can worsen solution

- recalculate gain $g(v)$ of neighbors
- move each node at most once
- edge-cut: **7, 6,5,5,6**

rollback

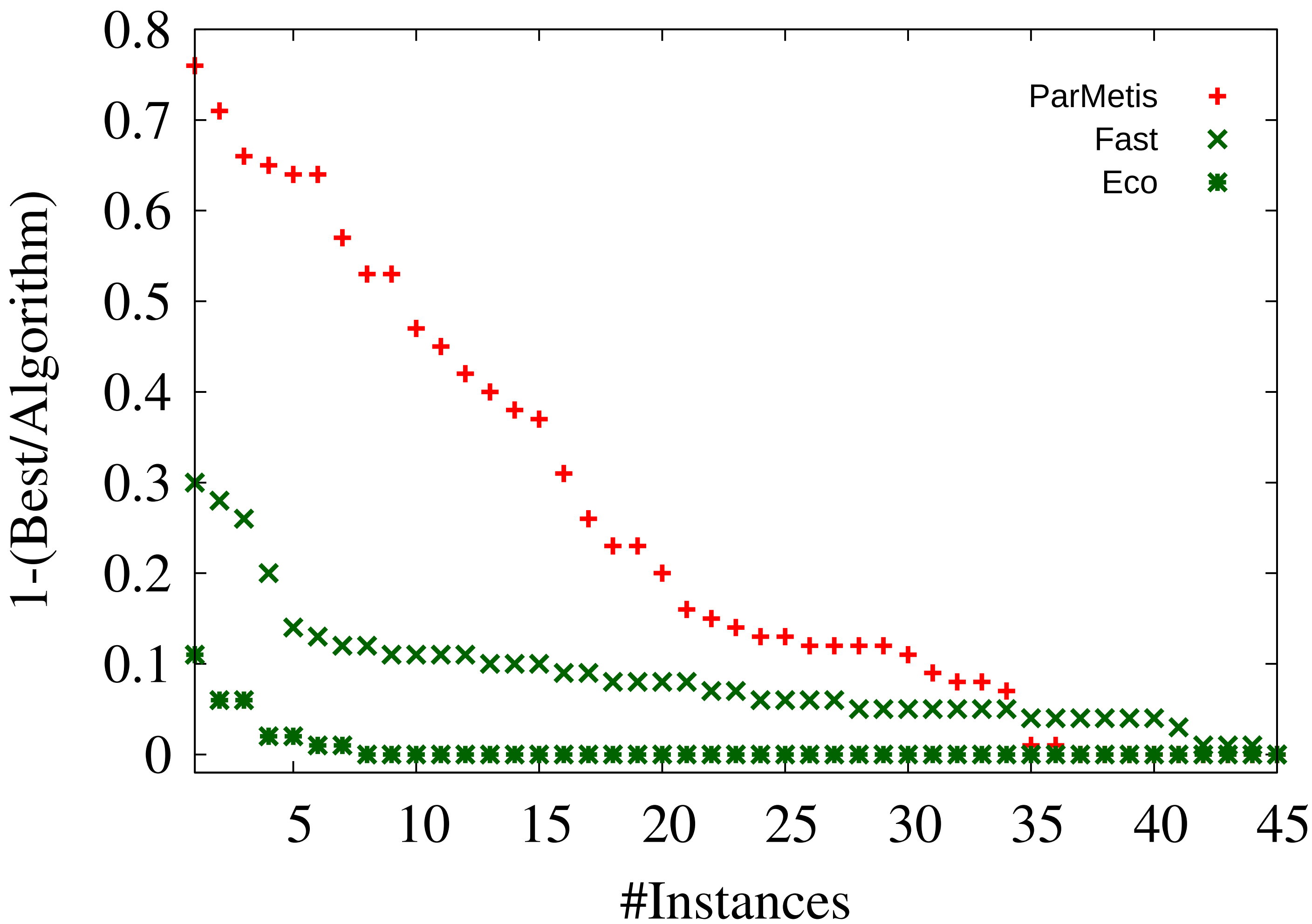


# KaHIP - Karlsruhe High Quality Partitioning




http://algo2.iti.kit.edu/kahip/

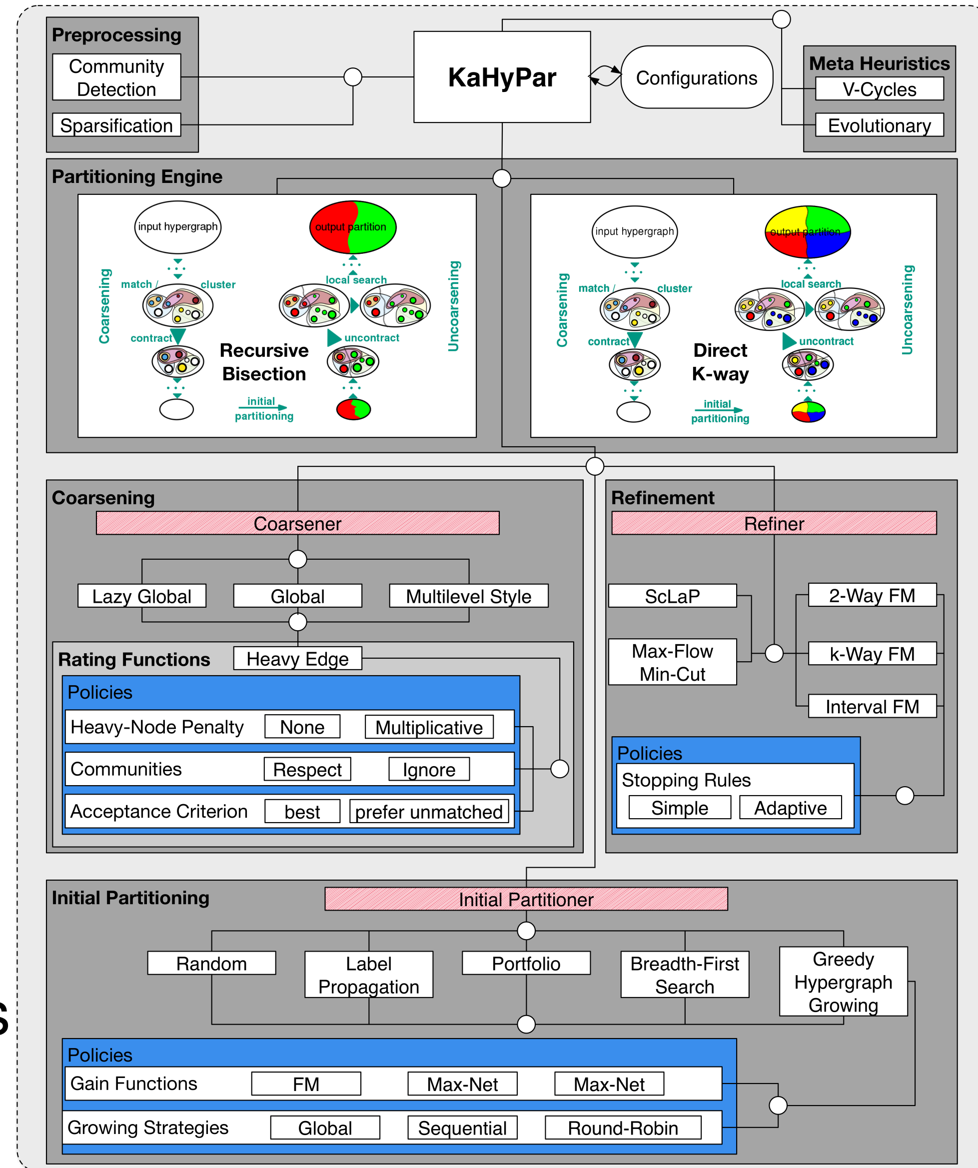
# Experimental Results – KaHIP (ParHIP)



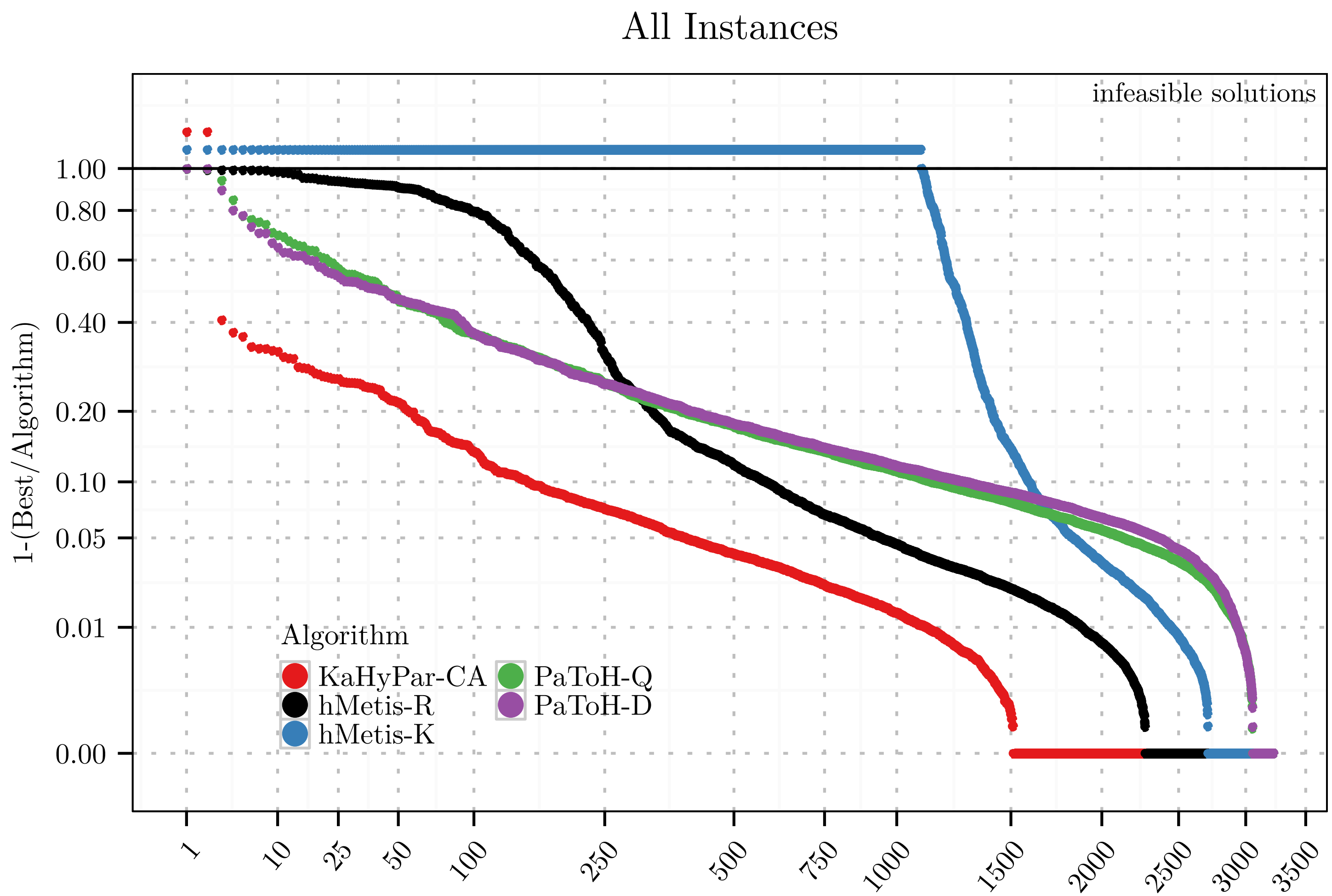
# KaHyPar - Karlsruhe Hypergraph Partitioning

Karlsruhe Institute of Technology

- *n*-Level Partitioning Framework
- Objectives:
  - hyperedge cut
  - connectivity $(\lambda - 1)$
- Partitioning Modes:
  - recursive bisection
  - direct $k$-way
- Upcoming Features:
  - evolutionary algorithm
  - flow-based refinement
  - advanced local search algorithms
- `http://www.kahypar.org`



# Experimental Results – KaHyPar



# Conclusion

## (Hyper)Graph Partitioning:

- fundamental graph problem with many application areas
- successful heuristic: multilevel approach + local search
- Graphs: KaHIP – `http://algo2.iti.kit.edu/kahip/`
- Hypergraphs: KaHyPar – `http://www.kahypar.org`