Subleading colour corrections in Herwig

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Section 1

Motivation

Why do subleading N_c showers?

- $1/N_c^2$ is not that small and $1/N_c$ suppression possible if there are two quark-lines.
- More energy
  - many more coloured partons.
  - many more colour suppressed terms.
- For a leading N_c shower, the number of colour connected pairs grow roughly as N_{partons}.
- The number of pairs of coloured partons grows as N_{partons}^2.
- Useful for exact NLO matching.

Section 2

Dipole showers

Dipole Factorization

Dipole factorization gives, whenever i and j become collinear or one of them soft:

$$|\mathcal{M}_{n+1}(..., p_i, ..., p_j, ..., p_k, ...)|^2 =$$

$$\sum_{k \neq i,j} \frac{1}{2p_i \cdot p_j} \langle \mathcal{M}_n(p_{\tilde{ij}}, p_{\tilde{k}}, ...) | \mathbf{V}_{ij,k}(p_i, p_j, p_k) | \mathcal{M}_n(p_{\tilde{ij}}, p_{\tilde{k}}, ...) \rangle$$

An emitter $\tilde{ij}$ splits into two partons i and j, with the spectator $\tilde{k}$ absorbing the momentum to keep all partons (before and after) on-shell.

Dipole Factorization

The spin averaged dipole insertion operator is

$$\mathbf{V}_{ij,k}(p_i, p_j, p_k) = -8\pi\alpha_s V_{ij,k}(p_i, p_j, p_k)\frac{\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_k}{\mathbf{T}^2_{\tilde{ij}}}$$

Where, for a final-final dipole configuration, we have for example

$$V_{q\to qg,k}(p_i, p_j, p_k) = C_F\left(\frac{2(1-z)}{(1-z)^2 + p_\perp^2/s_{ijk}} - (1+z)\right)$$

Emission probability

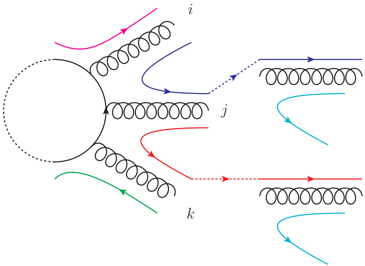
For a leading N_c shower, the emission probability would be

$$\mathrm{d}P_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{\mathrm{d}\phi_{n+1}(p_\perp^2, z)}{\mathrm{d}\phi_n} \times \frac{\delta(\tilde{ij}, \tilde{k} \text{ colour connected})}{1 + \delta_{\tilde{ij}\, g}}$$

Including subleading emissions, instead gives

$$\mathrm{d}P_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{\mathrm{d}\phi_{n+1}(p_\perp^2, z)}{\mathrm{d}\phi_n} \times \frac{-1}{\mathbf{T}_{\tilde{ij}}^2} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle}{|\mathcal{M}|^2}$$

Emission probability



- Leading N_c: i and j or j and k can radiate coherently.
- Subleading N_c: i and k can also radiate, but suppressed by colour factors.

Section 3

Colour Matrix Element Corrections

Overall picture

Using Herwigs dipole shower

- Instead of only colour connected emitter-spectator pairs radiating, all possible pairs can radiate.
- The emission probabilities are modified by a factor

$$\omega_{ik}^{n} = \frac{-1}{\mathbf{T}_{\tilde{ij}}^{2}} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle}{|\mathcal{M}|^2}$$

  which is included using the reweighting in Herwig.
- We evolve the colour structure to be able to evaluate the factor above for the next emission.
- Continue for a set number of emissions and then do the rest with the standard shower.

Density operator

Evaluating the first colour matrix element corrections, ω_{ik}^n, after the hard process is straightforward as the amplitude $|\mathcal{M}_n\rangle$ has been calculated. For the next emission we need $|\mathcal{M}_{n+1}\rangle$.
We can write the amplitude as a vector in some basis (trace, multiplet, etc.),

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle \leftrightarrow \mathcal{M}_n = (c_{n,1}, \ldots, c_{n,d_n})^T$$

Observe that

$$|\mathcal{M}_n|^2 = \mathcal{M}_n^\dagger S_n \mathcal{M}_n = \mathrm{Tr}\left(S_n \times \mathcal{M}_n \mathcal{M}_n^\dagger\right)$$

and

$$\langle\mathcal{M}_n|\mathbf{T}_{\tilde{i}\tilde{j}} \cdot \mathbf{T}_{\tilde{k}}|\mathcal{M}_n\rangle = \mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n}\mathcal{M}_n\mathcal{M}_n^\dagger T_{\tilde{i}\tilde{j},n}^\dagger\right)$$

Density operator

We construct an "amplitude matrix" $M_n = \mathcal{M}_n \mathcal{M}_n^\dagger$, that we evolve by

$$M_{n+1} = -\sum_{i \neq j} \sum_{k \neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{\tilde{ij}}^2} T_{\tilde{k},n} M_n T_{\tilde{ij},n}^\dagger$$

where

$$V_{ij,k} = \mathbf{T}_{\tilde{ij}}^2 \frac{p_i \cdot p_k}{p_j \cdot p_k}.$$

This allows us to calculate the "colour matrix element corrections".

Colour matrix element correction

With a way to evolve the density operator we can calculate the colour matrix element corrections for any number of emissions

$$\omega_{ik}^{n} = \frac{-1}{\mathbf{T}_{\tilde{ij}}^{2}} \frac{\mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n} M_n T_{\tilde{ij},n}^{\dagger}\right)}{\mathrm{Tr}\left(S_n \times M_n\right)}$$

- ω_{ik}^{n} can be negative, this is included through the weighted Sudakov algorithm (Bellm, J. et. al. arXiv:1605.08256).
- Initially this gave us large weights and large cancellations between positive and negative weights, when the number of subleading N_c corrected emissions was increased.

Convergence issue

- The weighted veto algorithm: uses a standard veto algorithm with a splitting kernel $\tilde{P}(q, z) > 0$ and a weight, at every veto/accept step the weight is updated. With this weight the algorithm gives the kernel $P(q, z)$ we want (that can be positive or negative).
- When there is more than one kernel the competition algorithm can be used.
- One can prove that one does not have to keep the weight for veto/accept steps that occur at a scale lower than the winning scale.
- This drastically reduces the weights we get (and could also be used for any other implementations of the weighted veto algorithm, e.g. scale variations).

Section 4

Preliminary results

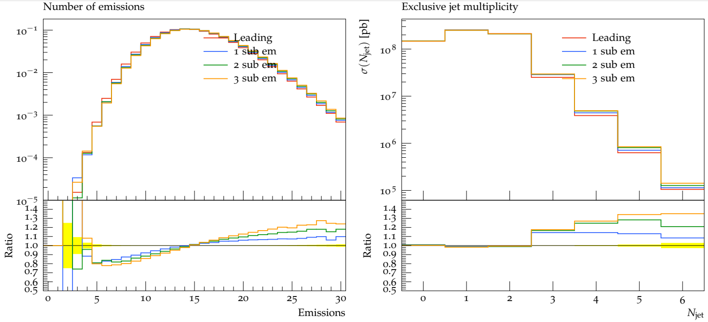
e^+e^- results

- Our results are in agreement with what S. Plätzer and M. Sjödahl found (Platzer, S., Sjodahl, M., arXiv:1206.0180).
- Differences are on the % level between leading and subleading shower ($\sim 10\%$ for tailored observables).

We have added

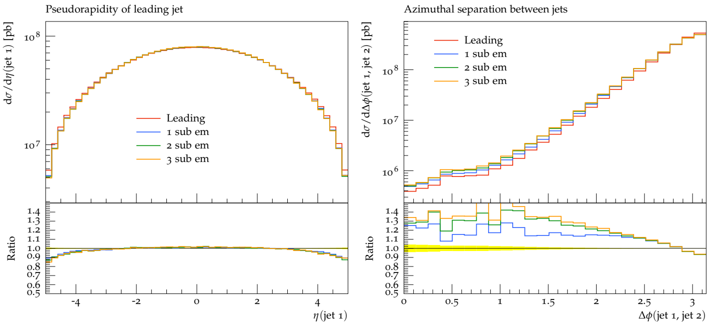
- Hadronic initial state, meaning initial state radiation (so we can do any process now, in particular LHC events).
- It is compatible with all of the additional functionality in Herwig 7.1.
- After the subleading N_c shower we continue with the standard Herwig dipole shower.
- $g \to q\bar{q}$ splittings.

Preliminary $pp \to jj$ results



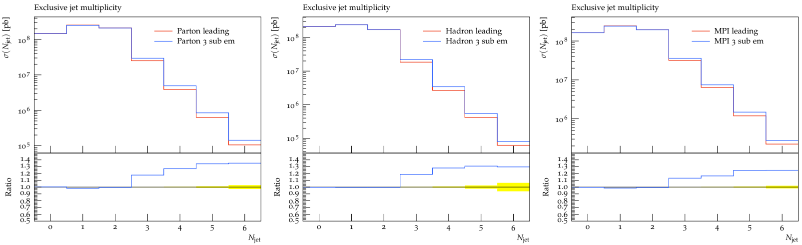
- Generation cut: $p_{\perp\,\text{cut}} = 20\,\text{GeV}$
- To describe the exclusive $2 + n$ jet multiplicity, we need n subleading emissions (as we get 2 from the hard process)

Pseudorapidity and $\Delta\phi_{12}$



- So far we have mainly been looking at standard observables for pp, it should not be hard to find observables with sizable corrections of order $1/N_c$.

How about hadronization and MPI?



- The effects of the subleading emissions are not washed out by either hadronization or MPI.

Section 5

Current status and future work

Current status

- Look at more processes (VBF, etc.)
- Look at the effect on analyses with data.
- Look for observables where subleading N_c has a large effect (Simon Plätzer and Malin Sjödahl looked at some interesting ones).

Future work

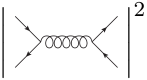
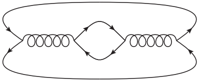
- Tuning with the subleading N_c shower.
- Virtual corrections, which rearrange the colour structure without any real emissions.
- Updated hadronization model.

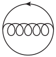
Section 6

Extra slides

Example of $1/N_c$ suppressed terms

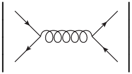
Leading colour structure:

 $=$ 

$= T_R$  $= T_R^2(N_c^2 - 1) \propto N_c^2.$

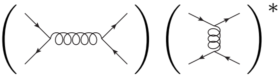
Example of $1/N_c$ suppressed terms

Leading colour structure:

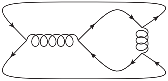


$$\left| \cdots \right|^2 \propto N_c^2.$$

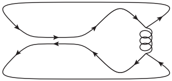
Interference term:



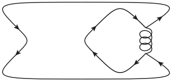
$$=$$



$$= T_R$$

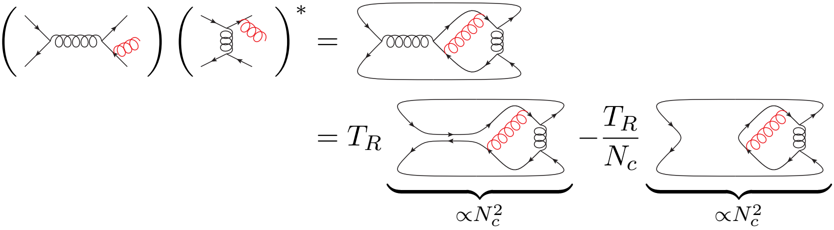


$$- \frac{T_R}{N_c}$$



$$= 0 - T_R^2 \frac{N_c^2 - 1}{N_c} \propto N_c.$$

Example of $1/N_c$ suppressed terms



Standard veto algorithm

Standard veto algorithm: we want to generate a scale q and additional splitting variables x (e.g. z and ϕ) according to a distribution dS_P.

$$\begin{aligned} \mathrm{d}S_P(\mu, x_\mu|q, x|Q) &= \mathrm{d}q\mathrm{d}^d x\, (\Delta_P(\mu|Q)\delta(q-\mu)\delta(x-x_\mu) \\ &\qquad + P(q,x)\theta(Q-q)\theta(q-\mu)\Delta_P(q|Q)) \end{aligned}$$

Where Δ_P is the Sudakov form factor,

$$\Delta_P(q|Q) = \exp\left(-\int_q^Q \mathrm{d}k \int \mathrm{d}^d z P(k,z)\right)$$

To do this we use an overestimate of the distribution (with nicer analytical properties) dS_R (change $P \to R$ in the above eqs.). Where we require $R(q,x) \geq P(q,x)$ for all q, x.

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Standard veto algorithm

$P(q, x) > 0$ and $R(q, x) \geq P(q, x)$. Set $k = Q$

1. Generate q and x according to $S_R(\mu, x_\mu | q, x | k)$.
2. If $q = \mu$, there is no emission above the cutoff scale.
3. Else, accept the emission with the probability

$$\frac{P(q, x)}{R(q, x)}.$$

4. If the emission was vetoed, set $k = q$ and go back to 1.

Weighted veto algorithm

Introduce an acceptance probability $0 \leq \epsilon(q, x|k, y) < 1$ and a weight ω. Set $k = Q$, $\omega = 1$.

1. Generate q and x according to $S_R(\mu, x_\mu|q, x|k)$.
2. If $q = \mu$, there is no emission above the cutoff scale.
3. Accept the emission with the probability $\epsilon(q, x|k, y)$, update the weight
$$\omega \to \omega \times \frac{1}{\epsilon} \times \frac{P}{R}$$
4. Otherwise update the weight to
$$\omega \to \omega \times \frac{1}{1-\epsilon} \times \left(1 - \frac{P}{R}\right)$$
and start over at 1 with $k = q$.