Theoretical Activities at SJTU

QCD research:

Xiangdong Ji, Jun Gao, Wei Wang

Heavy Quark Flavor Physics:

Wei Wang, Xiao-Gang He

Collider, Beyond SM, Neutrino, Grand Unification:

Pei-Hong Gu, Jun Gao, Hong-Jian He, Xiao-Gang He, Yue Zhao

Cosmology, Astrophysics, and Dark Matter Physics:

Hong-Jian He, Xiao-Gang He, Pei-Hong Gu, Yue Zhao, Jun Zhang

INPAC, Department of Astronomy, and T-D Lee Institute

A Theoretical Framework for Neutrino Mixing with $\delta = -\pi/2$ and $\theta_{23} = \pi/4$

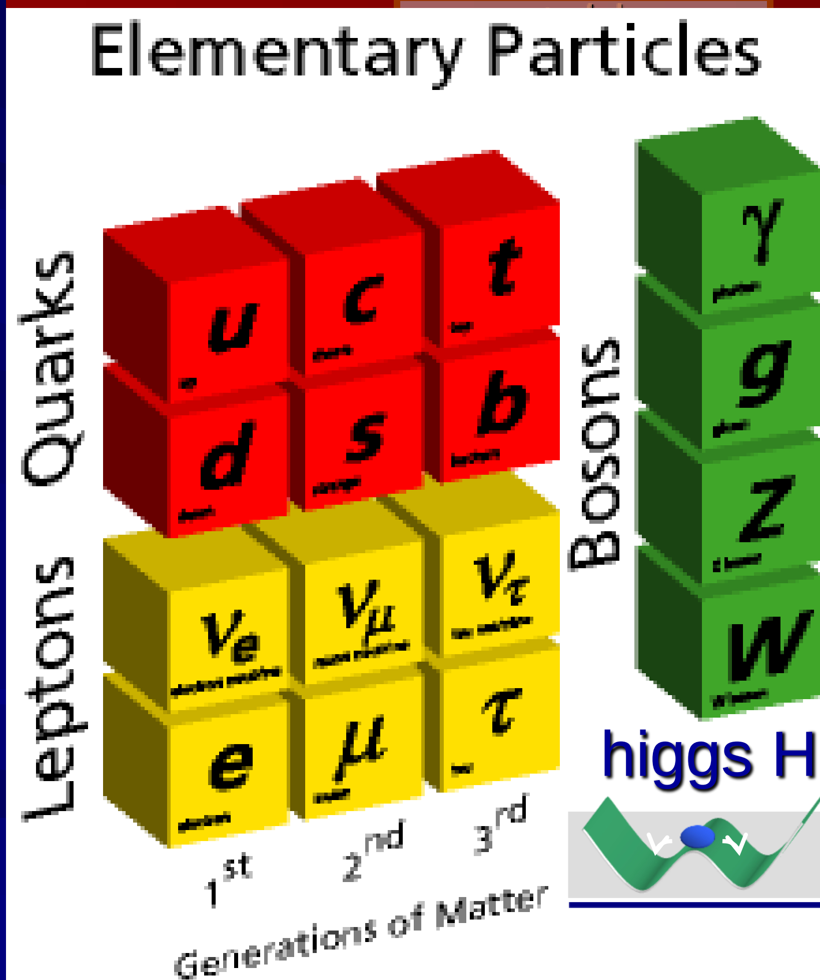
Xiao-Gang He

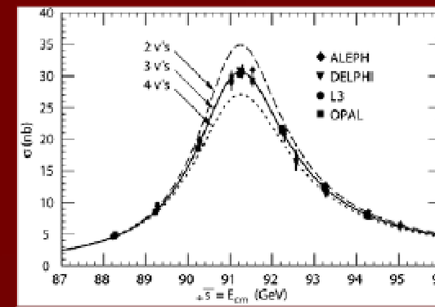
T-D Lee Institute, SJTU

2017 SJTU-KIT Collaborative Research Workshop
"Particles and the Universe"
KIT, Sept 6, 2017

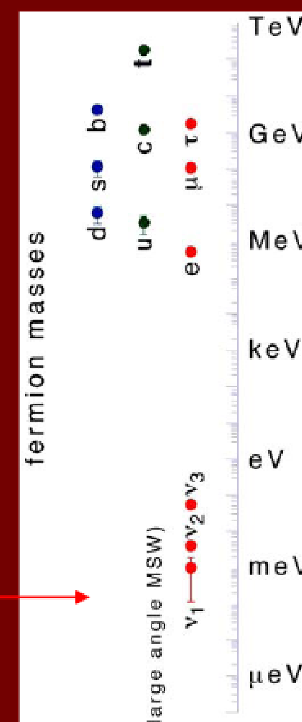
Three types of active light neutrinos

Nature's building blocks





Three flavors or generations, and no more, and we do not know why.



Some mass, but curiously little.

The Nobel Prize in Physics 2015

Ill. N. Elmehed. © Nobel Media AB 2015.

Takaaki Kajita

Prize share: 1/2

Ill. N. Elmehed. © Nobel Media AB 2015.

Arthur B. McDonald

Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

Four Nobel prizes so far directly related to neutrino physics

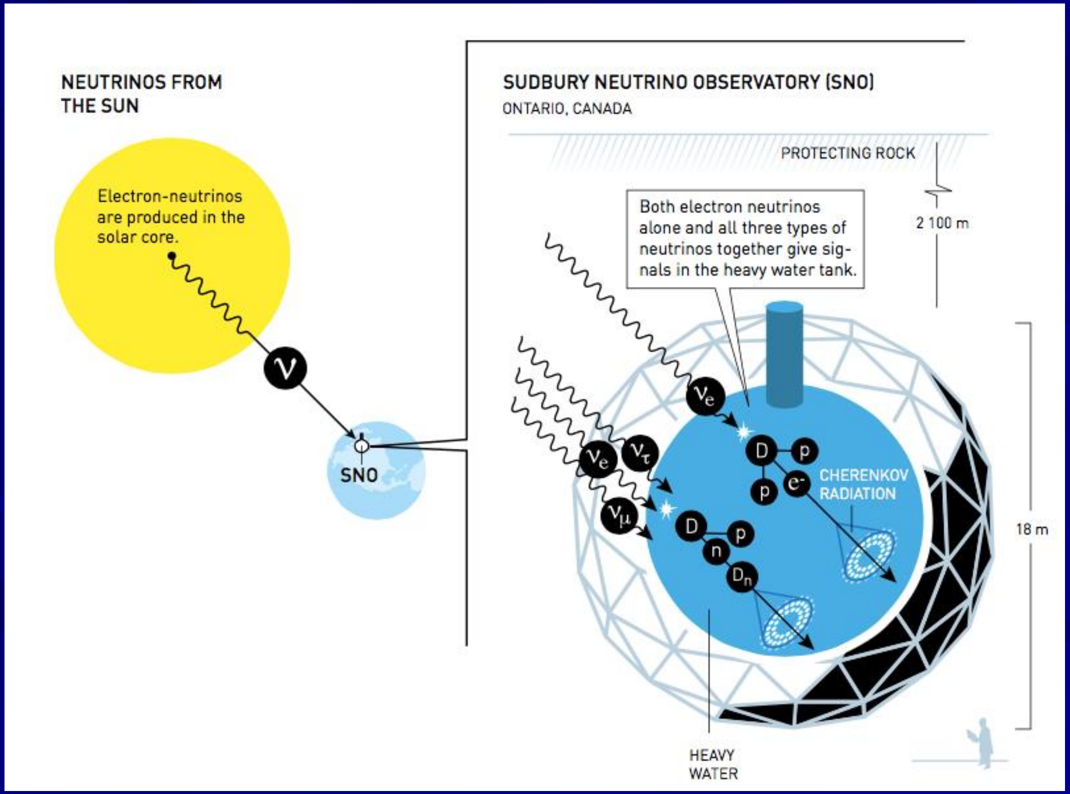
The SNO heavy water cherenkov detector

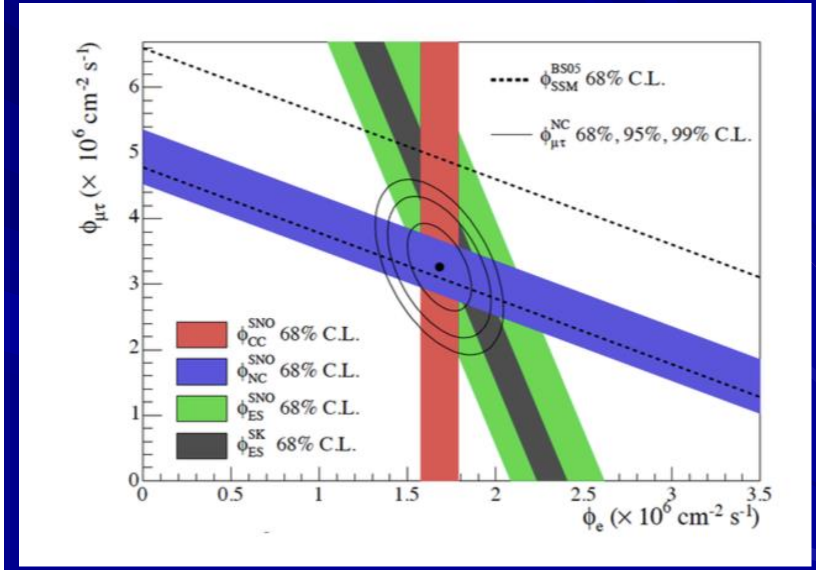
find out e-neutrios oscillated into other types of neutrinos

$$\nu_e + {}^2H \rightarrow e^- + p + p \quad \text{(CC)}$$
$$\nu_x + {}^2H \rightarrow \nu_x + p + n \quad \text{(NC)}$$
$$\nu_x + e^- \rightarrow \nu_x + e^- \quad \text{(ES)}$$

$$\phi(\nu_\mu) + \phi(\nu_\tau) = (3.26 \pm 0.25^{+0.40}_{-0.35}) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

$$\phi = \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) = 5.25 \pm 0.16(stat)^{+0.11}_{-0.13}(sys) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$





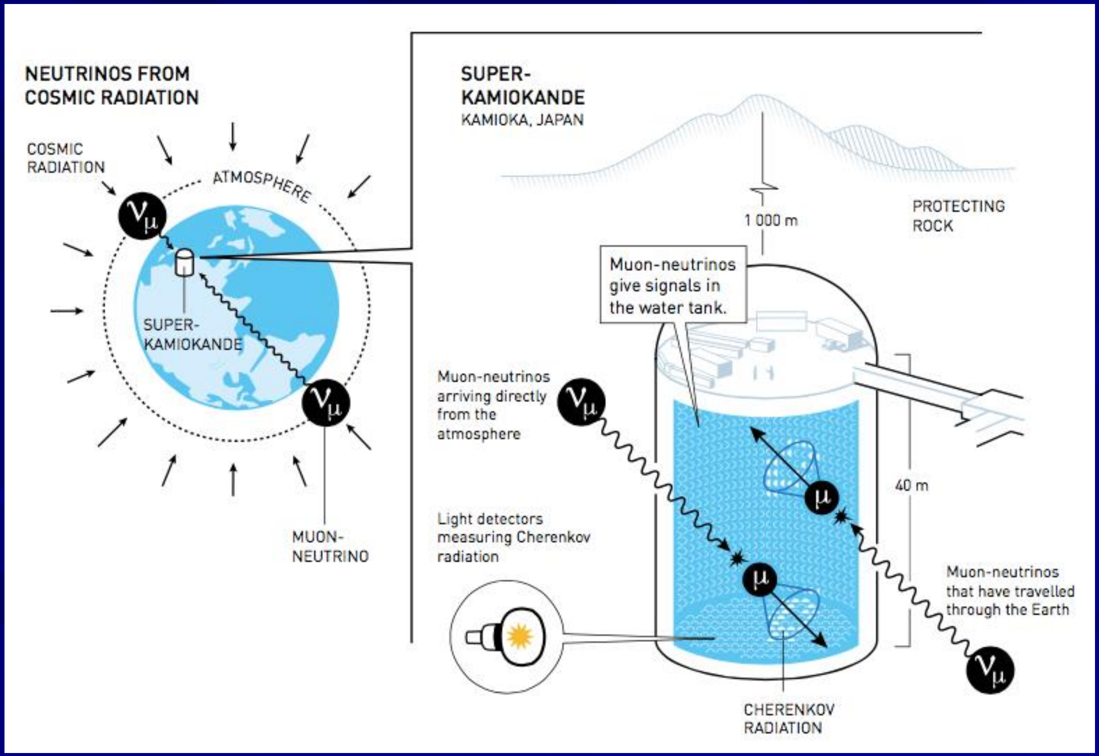
The super-Kamiokande water Chrenkov detector

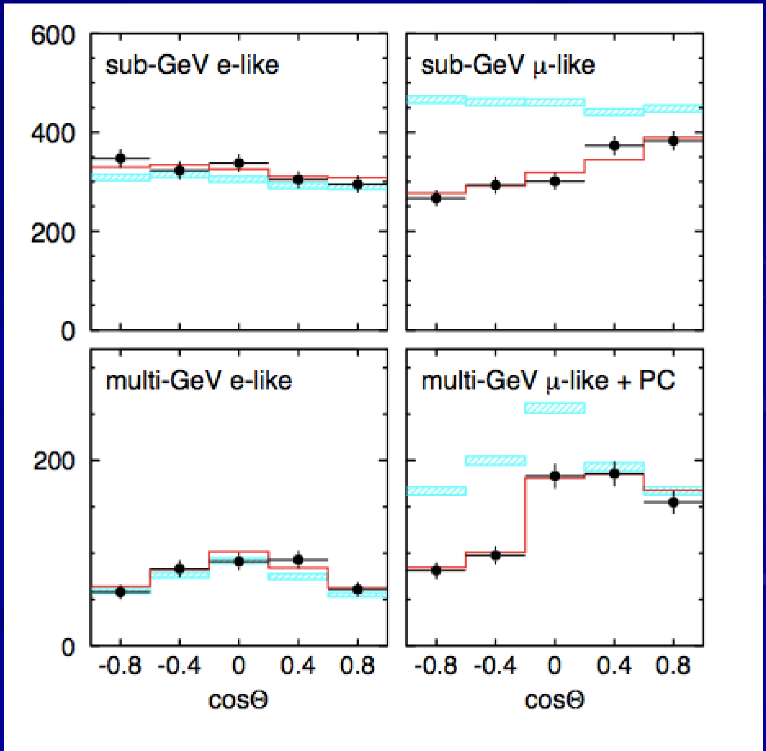
Cosmic protons hit atmosphere produce a lot of pions and then decay

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu}) \quad \mu^{\pm} \rightarrow e^{\pm} + \nu_{e}(\bar{\nu}_{e}) + \bar{\nu}_{\mu}(\nu_{\mu})$$

Expected Nμ/Ne ~2

$$R = (N_{\mu}/N_{e})_{obs} / (N_{\mu}/N_{e})_{theor}$$ ~ (0.5- 0.6)





Abundant data show that neutrinos have non-zero masses and mix with each other.

Solar neutrino oscillation: Homestake, Sage+Gallex/GNO, Super-K, SNO,Borexino …

Atmospherical neutrino oscillation: Super-Kamokande, …

Accelerator neutrino source: K2K, Minos , Nova …

Reactor neutrino source: Kamland, T2K, Chooz, Daya-Bay, Reno…

have observed neutrino oscillation phenomenon.

LSND and Miniboon…?

Observation of neutrino mixing implies:

Different neutrinos they mix with each other

Some of their masses must be non-zero!

$$P(\nu_1 \to \nu_2) = |<\nu_1(0)|\nu_2(t)>|^2 = \sin^2(2\theta)\sin^2(\Delta m^2_{21} L/4E),$$
$$\Delta m^2_{21} = m_2^2 - m_1^2.$$

Theoretical Models for Neutrinos

In the minimal SM: Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$G(8,1)\ (0), W(1,3)\ (0), B\ (1,1)(0)\ ,$$
$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}\ (3,2)(1/6)\ ,\ \ U_R\ (3,1)(2/3)\ ,\ \ D_R\ (3,1)(-1/3)\ ,$$
$$L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\ (1,2)(-1/2)\ ,\ \ E_R\ (1,1)(-1)\ ,$$
$$H = \begin{pmatrix} h^+ \\ (v+h^0)/\sqrt{2} \end{pmatrix}\ (1,2,1/2)\ ,\ \ \text{v - vev of Higgs}\ .$$

Quark and charged lepton masses are from the following Yukawa coupolings

$$\bar{Q}_L \tilde{H} U_R\ ,\ \ \bar{Q}_L H D_R\ ,\ \ \bar{L}_L H E_R\ .$$

Nothing to pair up with $L_L(\nu_L)$. In minimal SM, neutrinos are massless!

Extensions needed: Give neutrino masses and small ones!

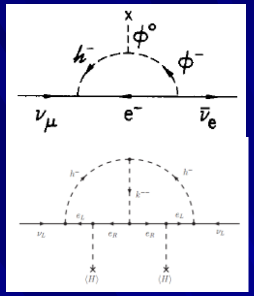
To have Dirac mass, need to introduce right handed neutrinos ν_R: (1,1)(0)

Dirac neutrino mass term

$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C\ , \rightarrow\ \ -\bar{\nu}_L m_\nu \nu_R\ , \rightarrow m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$

$m_{\nu_e} < 0.3$ eV, $\rightarrow\ \ Y_{\nu_e}/Y_e < 10^{-5}$, very much fine tuned!

Some theoretical models for neutrino masses



Loop generated neutrino masses:

The Zee Model(1980); Zee-Babu Model (zee 1980; Babu, 1988)

Other loop models: Babu&He; E. Ma; Mohapatra et al; Geng et al

Seesaw Models:

$$M_\nu = \begin{pmatrix} 0 & Y_\nu v/\sqrt{2} \\ Y_\nu^T v/\sqrt{2} & M_R \end{pmatrix}$$

$$m_\nu \approx Y_\nu^2 v^2/M_R\ , \quad M_N \approx M_R$$

Type I Introduce singlet neutrinos

The Seesaw Mechanism $\qquad L = \bar{\nu}_L (Y_\nu v/\sqrt{2})\nu_R + \bar{\nu}_R^c M_R \nu_R/2$

Once the neutral component in Δ gets the vev, $< \delta^0 >= v_\Delta/\sqrt{2}$

Type-III: Introduce triplet lepton representations Σ: (1,3,0)) (Foot, Lew, He and Joshi, 1989).

Neutrino Mixing

B. Pontecorvo (1957). "Mesonium and anti-mesonium". *Zh. Eksp. Teor. Fiz.* 33: 549–551.
B. Pontecorvo (1967). "Neutrino Experiments and the Problem of Conservation of Leptonic Charge". *Zh. Eksp. Teor. Fiz.* 53: 1717

Z. Maki, M. Nakagawa, and S. Sakata (1962). "Remarks on the Unified Model of Elementary Particles". *Progress of Theoretical Physics* 28 (5): 870.

S. Sakata Z. Maki M. Nakagawa

Three generation mixing in quarks and leptons

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix $V_{\rm CKM}$,

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix $U_{\rm PMNS}$

$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\rm CKM} D_L W_\mu^+ - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\rm PMNS} N_L W_\mu^- + H.C.\ ,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\rm CKM}$ or $U_{\rm PMNS}$ is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Where are we now?

parameter	best fit $\pm$ 1σ	2σ range	3σ range
Δm^2_{21} [10^{-5}eV^2]	7.56$\pm$0.19	7.20–7.95	7.05–8.14
$\|\Delta m^2_{31}\|$ [10^{-3}eV^2] (NO)	2.55$\pm$0.04	2.47–2.63	2.43–2.67
$\|\Delta m^2_{31}\|$ [10^{-3}eV^2] (IO)	2.49$\pm$0.04	2.41–2.57	2.37-2.61
$\sin^2\theta_{12}/10^{-1}$	$3.21^{+0.18}_{-0.16}$	2.89–3.59	2.73–3.79
$\theta_{12}/^\circ$	$34.5^{+1.1}_{-1.0}$	32.5–36.8	31.5–38.0
$\sin^2\theta_{23}/10^{-1}$ (NO)	$4.30^{+0.20}_{-0.18}$ [a]	3.98–4.78 & 5.60–6.17	3.84–6.35
$\theta_{23}/^\circ$	41.0$\pm$1.1	39.1–43.7 & 48.4–51.8	38.3–52.8
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.96^{+0.17}_{-0.18}$ [b]	4.04–4.56 & 5.56–6.25	3.88–6.38
$\theta_{23}/^\circ$	50.5$\pm$1.0	39.5–42.5 & 48.2–52.2	38.5–53.0
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.155^{+0.090}_{-0.075}$	1.98–2.31	1.89–2.39
$\theta_{13}/^\circ$	$8.44^{+0.18}_{-0.15}$	8.1–8.7	7.9–8.9
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.140^{+0.082}_{-0.085}$	1.97–2.30	1.89–2.39
$\theta_{13}/^\circ$	$8.41^{+0.16}_{-0.17}$	8.0–8.7	7.9–8.9
δ/π (NO)	$1.40^{+0.31}_{-0.20}$	0.85–1.95	0.00–2.00
$\delta/^\circ$	252^{+56}_{-36}	153–351	0–360
δ/π (IO)	$1.44^{+0.26}_{-0.23}$	1.01–1.93	0.00–0.17 & 0.79–2.00
$\delta/^\circ$	259^{+47}_{-41}	182–347	0–31 & 142–360

[a] There is a local minimum in the second octant, at $\sin^2\theta_{23}$=0.596 with $\Delta\chi^2 = 2.08$ with respect to the global minimum.
[b] There is a local minimum in the first octant, at $\sin^2\theta_{23}$=0.426 with $\Delta\chi^2 = 1.68$ with respect to the global minimum for IO.

Where have we come from and where will we go?

Theory for neutrino mixing

Early days, expecting neutrino mixing might be following a similar pattern as quarks, mixing angles are small.

For example, 1992 people are

trying to produce mixing on the right

10^{-3} to 0.3 eV^2 [4], and the allowed parameter

$\sin^2 2\theta$

FIG. 1. The figure shows (boldly outlined) the two reg
$\sin^2 2\theta_{12} - \delta m^2_{12}$ space allowed to solve the solar neutrin
lem. The solid contour lines are the ν_e flux from the Su

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neutrino problems via Fritzsch-type
ces

ang He†
ersity of Melbourne, Parkville 3052, Australia
92)

pheric neutrino problems by constructing a
mass matrices. Requiring the two neutri-
of the model and leads to a prediction of an
0 solar neutrino units in the ^{71}Ga detectors
e consistent with results from both experi-
parameter values recently reported from

(Davies and He, PRD46, 3208)

But both solar and atmospheric show large mixing angles after 1998!

Theory before and after Daya-Bay/Reno results

Before: popular mixing -The Tribimaximal Mixing
Harrison, Perkins, Scott (2002) , Z-Z. Xing (2002), He& Zee (2003)

The mixing pattern is consistent, within 2σ, with the tri-bimaximal mixing

$$V_{tri-bi} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

A4 a promising model (Ma&Ranjasekara, 2001) and realizations (Altarelli&Feruglio 2005, Babu&He 2005). Later many realizations: S4, D3, S3,D4, D7,A5,T',S4, Δ(27, 96), $PSL_2(7)$ … discrete groups Altarelli&Feruglio for review. (H. Lam; Mohapatra et al), T. Mahanthanpa&M-C. Chen; Frampton&Kephart; Y-L Wu, ….

After: Need to have a nonzero θ_{13}

Modification to tri-bimaximal mixing pattern need to be made. (Keum&He&Volkas; He&Zee, 2006).

In fact, more generically, A_4 symmetry leads to

$$V = \begin{pmatrix} \frac{2c}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \end{pmatrix} .$$

Tri-bimximal at higher scales and generate none zero θ_{13} at low energies?

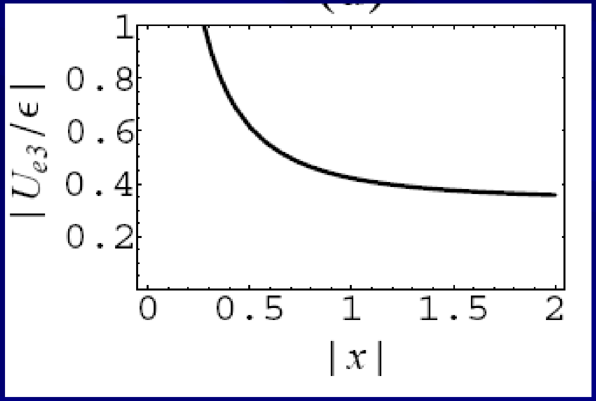
Baub and He, arxiv:0507217(hep-ph): A susy A4 model

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \,.$$

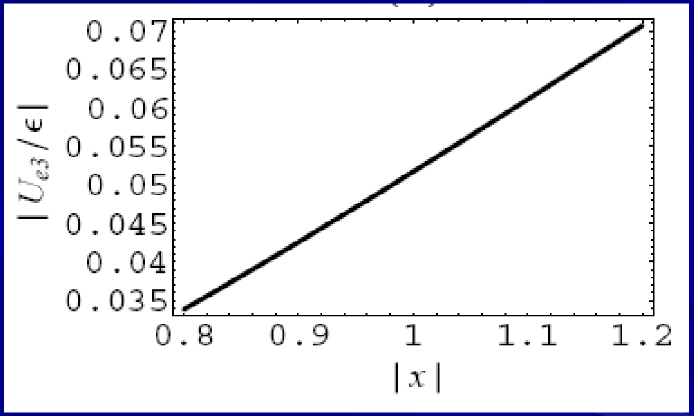
one–loop RGE $\quad \frac{dM_\nu^e}{d\ln t} = \frac{1}{32\pi^2}[M_\nu^e Y_e^\dagger Y_e + (Y_e^\dagger Y_e)^T M_\nu^e] + \ldots$

leading to the entries $M_{13,23}(1-\epsilon)$ and $M_{33}(1-2\epsilon)$

$\epsilon \simeq Y_\tau^2 \ln(M_{\rm GUT}/M_{EW})/32\pi^2.$



Inverted hierarchy



Normal hierarchy

Susy model, Y_t ~O(1), U_{e3} for inverted hierarchy, can be as large a 0.1, with RG effects!

SO(10)Grand Unification

SO(10) Yukawa couplings: $16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$

Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\, \mathbf{16}\, \mathbf{16}\, \mathbf{10}_H + Y_{126}\, \mathbf{16}\, \mathbf{16}\, \overline{\mathbf{126}}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

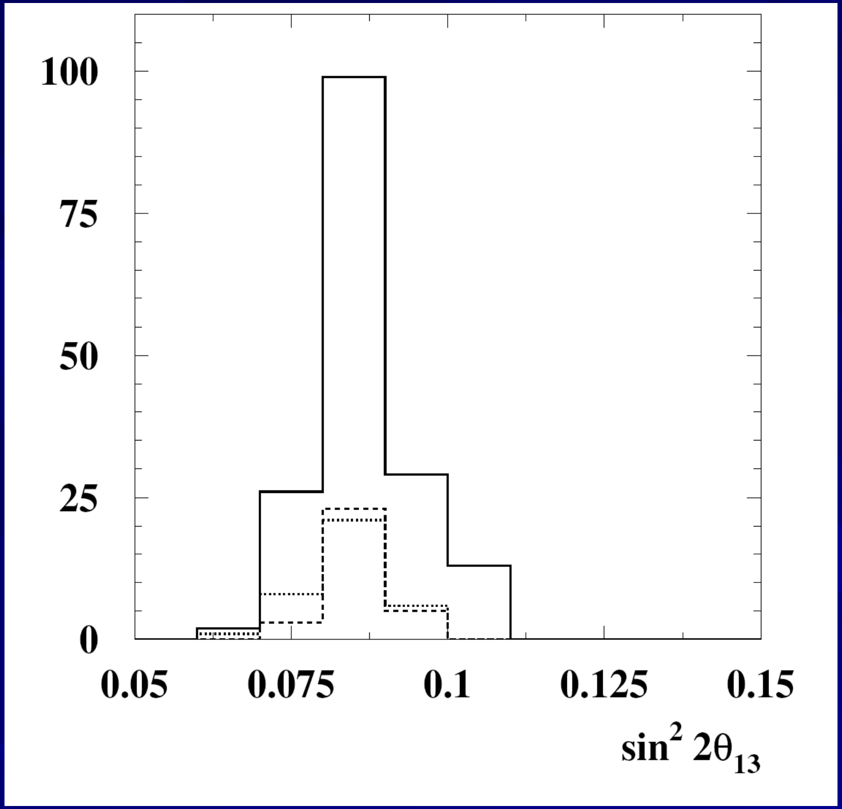
$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

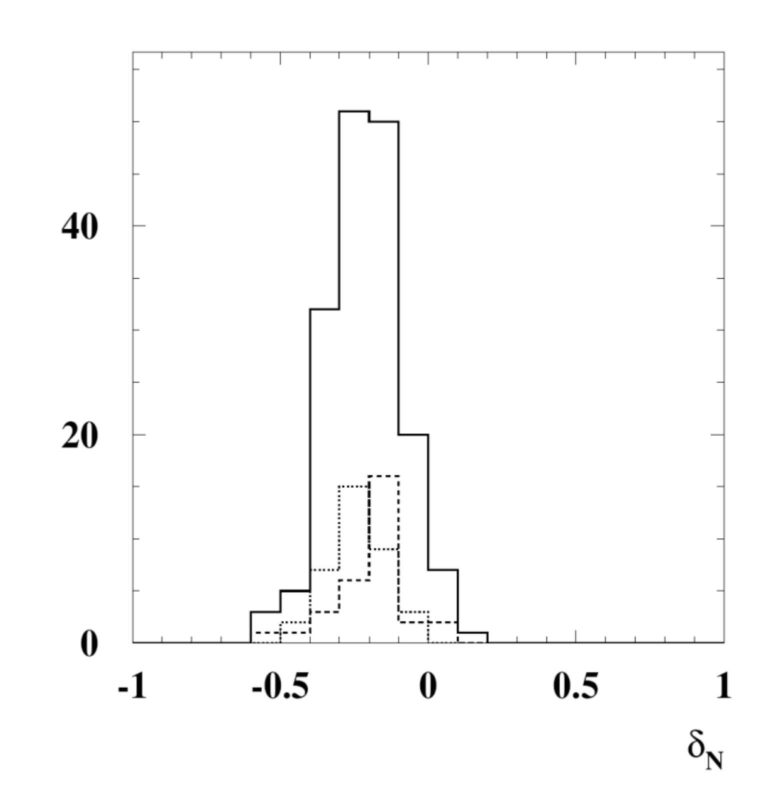
Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004)
Aulakh et al (2004)

Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006)
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek
Jushipura, Patel (2011)

θ_{13} in Minimal SO(10)

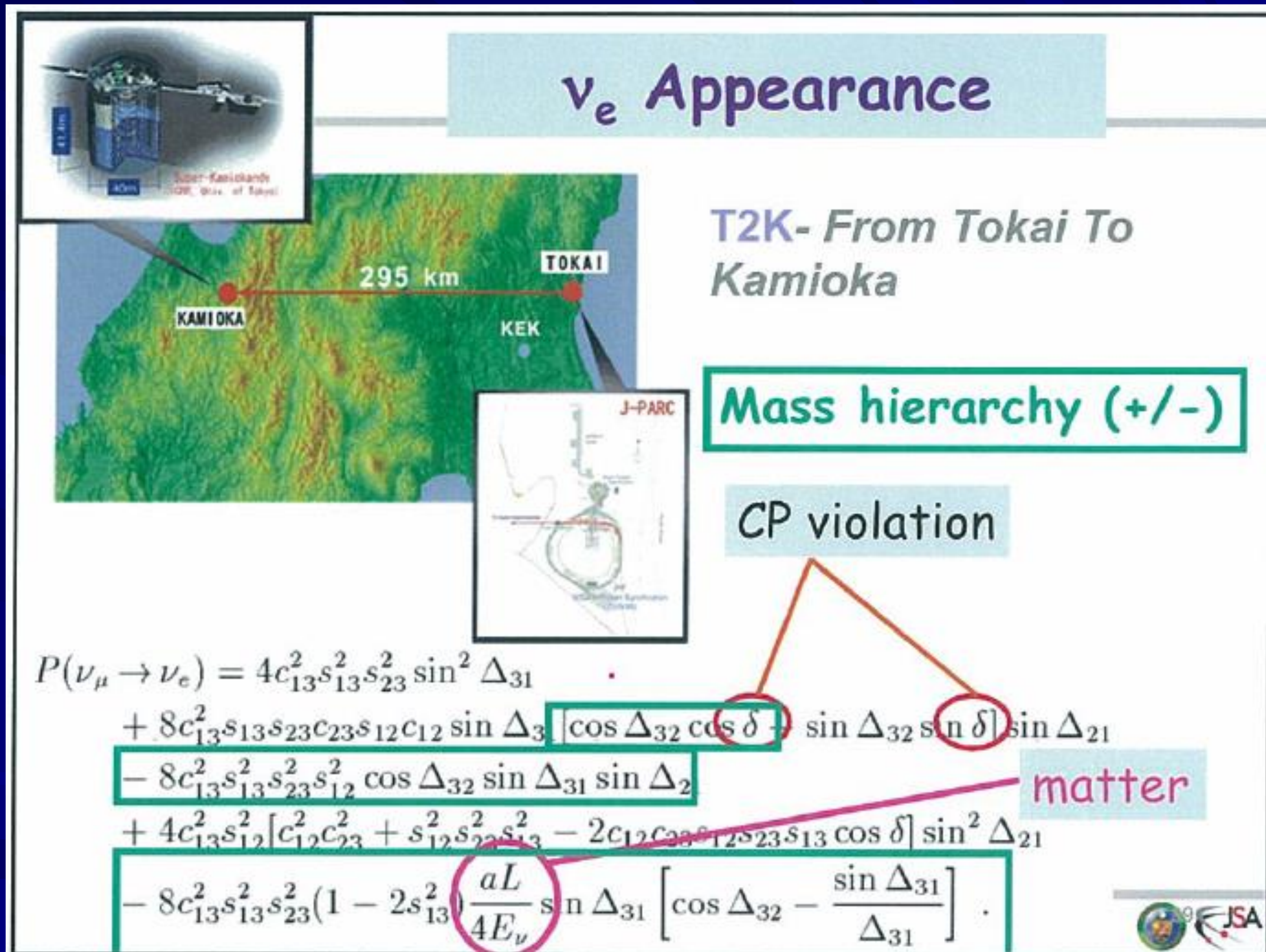




$\sin^2 2\theta_{13}$ and CP violating phase δ_N

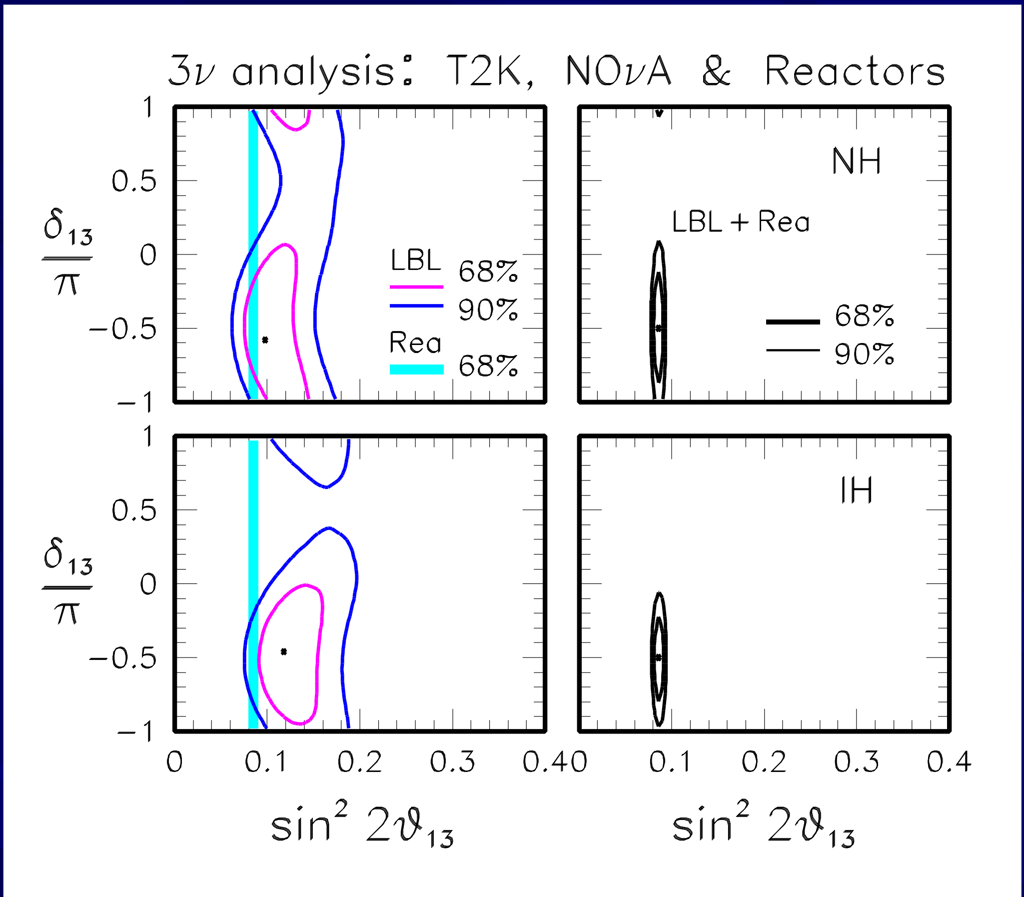
K.S. Babu and C. Macesanu (2005)

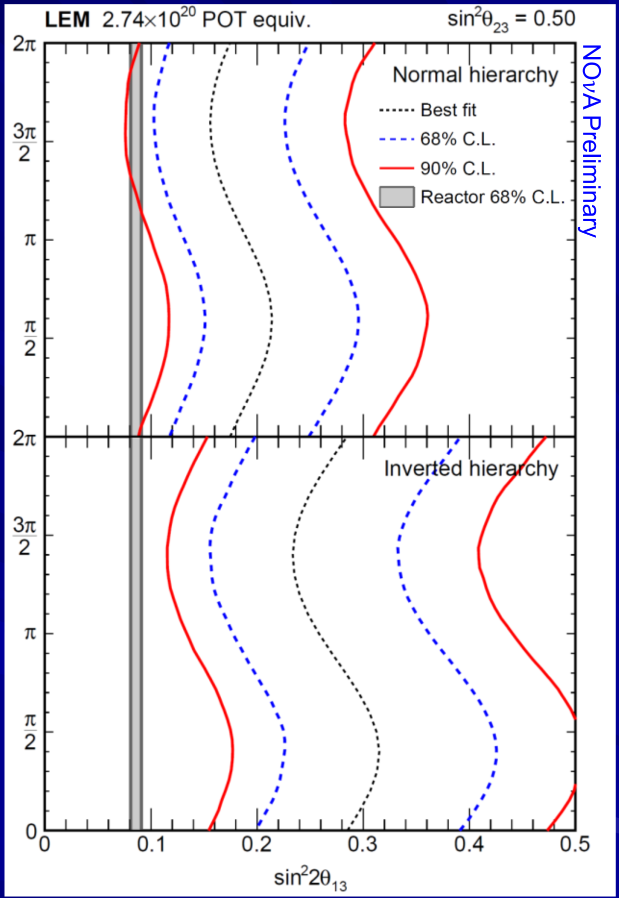
Evidence for CP Violation in Neutrino Mixing



Possible way of measuring mass hierarchy and CP phase

T2K, Nova old data fitting





arXiv:1509.03148
A. Palazzo

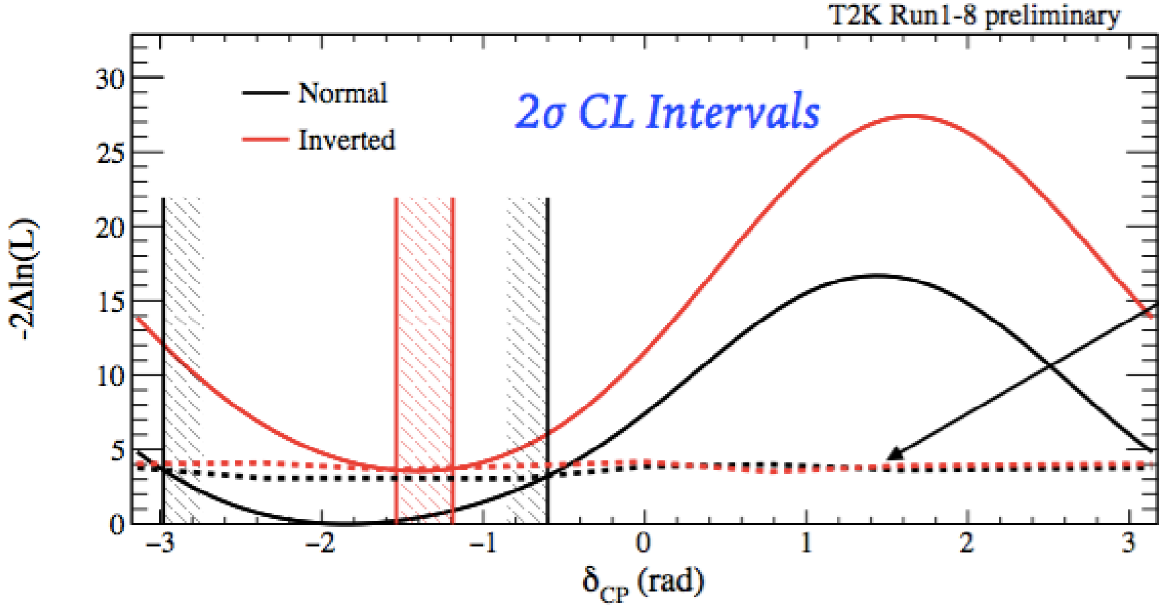
Fermilab seminar, August 6 2015
R. Patterson

New T2K data, August 4, 2017

PREDICTED AND OBSERVED EVENT RATES

	Predicted Rates				Observed
Sample	$\delta_{cp}=-\pi/2$	$\delta_{cp}=0$	$\delta_{cp}=\pi/2$	$\delta_{cp}=\pi$	Rates
CCQE 1-Ring e-like FHC	73.5	61.5	49.9	62.0	74
CC1π 1-Ring e-like FHC	$\boxed{6.92}$	6.01	4.87	5.78	$\boxed{15}$
CCQE 1-Ring e-like RHC	7.93	9.04	10.04	8.93	7
CCQE 1-Ring μ-like FHC	267.8	267.4	267.7	268.2	240
CCQE 1-Ring μ-like RHC	63.1	62.9	63.1	63.1	68

MEASUREMENT OF δ_{cp}



Best fit point: -1.83 radians in Normal Hierarchy

The 1σ CL confidence interval: Normal hierarchy: [-2.49, -1.23] radians

The 2σ CL confidence interval: Normal hierarchy: [-2.98, -0.60] radians
Inverted hierarchy: [-1.54, -1.19] radians

CP conserving values (0,π) fall outside of the 2σ CL intervals

Model Building with $\theta_{23}=\pi/4$ and $\delta_{CP} = 3\pi/2(-\pi/2)$

Structure of the mass matrix (charged lepton is already diagonal)

Assuming neutrinos are Majorana particles,

$$L = -\frac{1}{2}\bar{\nu}_L m_\nu \nu_L^C$$
$$m_\nu = V_{PMNS}\hat{m}_\nu V_{PMNS}^T \ , \qquad .$$

$\hat{m}_\nu = diag(m_1, m_2, m_3)$ with $m_i = |m_i| exp(i\alpha_i)$.

With $\delta = -\pi/2$ and $\theta_{23} = \pi/4$,
m_ν has the following form

$$m_\nu = \begin{pmatrix} a & c+i\beta & -(c-i\beta) \\ c+i\beta & d+i\gamma & \tilde{b} \\ -(c-i\beta) & \tilde{b} & d-i\gamma \end{pmatrix} ,$$

$$a = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 - m_3 s_{13}^2 \ ,$$
$$\tilde{b} = -\frac{1}{2}\left(m_1(s_{12}^2 + c_{12}^2 s_{13}^2) + m_2(c_{12}^2 + s_{12}^2 s_{13}^2) - m_3 c_{13}^2\right) \ ,$$
$$c = -\frac{1}{\sqrt{2}}(m_1 - m_2) s_{12} c_{12} c_{13} \ ,$$
$$d = \frac{1}{2}\left(m_1(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2(c_{12}^2 - s_{12}^2 s_{13}^2) + m_3 c_{13}^2\right) \ ,$$
$$\beta = \frac{1}{\sqrt{2}} s_{13} c_{13}\left(m_1 c_{12}^2 + m_2 s_{12}^2 + m_3\right) \ ,$$
$$\gamma = -(m_1 - m_2) s_{12} c_{12} s_{13} \ .$$

Note that in the most general case, because non-zero Majorana phases, the parameters a, $\tilde{b}$, c, d, β and γ are all complex.

Equavilent forms

$$m_\nu = \begin{pmatrix} e^{ip_1} & 0 & 0 \\ 0 & e^{ip_2} & 0 \\ 0 & 0 & e^{ip_3} \end{pmatrix} \begin{pmatrix} a & c+i\beta & -(c-i\beta) \\ c+i\beta & d+i\gamma & \tilde{b} \\ -(c-i\beta) & \tilde{b} & d-i\gamma \end{pmatrix} \begin{pmatrix} e^{ip_1} & 0 & 0 \\ 0 & e^{ip_2} & 0 \\ 0 & 0 & e^{ip_3} \end{pmatrix},$$

where the phases p_i are arbitrary.

For $p_1 = p_2 = 0$ and $p_3 = \pi$,

$$m_\nu = \begin{pmatrix} a & c+i\beta & (c-i\beta) \\ c+i\beta & d+i\gamma & b \\ (c-i\beta) & b & d-i\gamma \end{pmatrix},$$

eq A

where $b = -b$.

If neutrinos do not have non-trivial Majorana phases, α, β, γ are real

The mass matrix can be written asand

$$m_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}.$$

eq B

Mass matrix in forms eq A and eq B

always predict $\theta_{23}=\pi/4$ and $\delta_{CP}=3\pi/2$?

The answer is no!

For both is change $\delta_{CP}=\pi/2$, the mass matrix has the same form.

If α, β, γ are complex, θ_{23} and δ_{CP} may not be $\pi/4$ and $\pm\pi/2$.

This also points out a method to modify the solutions of $\theta_{23}=\pi/4$ and $\delta_{CP}=3\pi/2$

Need to be careful!

How to obtain m_ν discussed before?

μ-τ exchange + conjugation symmetry

(Generalized CP symmetry: Grimus&Lavoura (2004), Gui-Jun Ding et al., Xing …)

$$\nu_e \to \nu_e^C,\ \nu_\mu \to \nu_\tau^c,\ \nu_\tau \to \nu_\mu^C$$

Under this transformation

require invarance under the above transformatin

$$m_\nu = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \to m_\nu^\dagger = \begin{pmatrix} A_{11} & A_{13} & A_{12} \\ A_{13} & A_{33} & A_{23} \\ A_{12} & A_{23} & A_{22} \end{pmatrix} = \begin{pmatrix} A_{11}^* & A_{12}^* & A_{13}^* \\ A_{12}^* & A_{22}^* & A_{23}^* \\ A_{13}^* & A_{23}^* & A_{33}^* \end{pmatrix}$$

The above implies

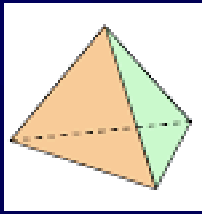
$$A_{11} = A_{11}^* \quad A_{23} = A_{23}^* \qquad m_\nu = \begin{pmatrix} A_{11} & A_{12} & A_{12}^* \\ A_{12} & A_{33}^* & A_{23} \\ A_{12}^* & A_{23} & A_{33} \end{pmatrix}$$

$$A_{22} = A_{33}^* \quad A_{12} = A_{13}^*$$

The above is the same as eq B.

What about charged leptons since (ν_L, e_L) are in one doublet, need to have all considered together! Not a trivial task!

Realization with A_4 symmetry



How to achieve in A_4 models?

A_4 group is defined as the set of all twelve even permutations of four object.

It has three one-dimensional representations 1, $1'$ and $1''$

and one three-dimensional irreduciblerepresentation 3.

Multiplication rules

$3 \times 3 = 3_s + 3_a + 1 + 1' + 1''$

$1 \times 1_i = 1_i,\ 1' \times 1' = 1'',\ 1'' \times 1'' = 1',\ 1' \times 1'' = 1$

$$(\underline{3} \otimes \underline{3})_{\underline{3}_s} = (x_2y_3 + x_3y_2\,,\ x_3y_1 + x_1y_3\,,\ x_1y_2 + x_2y_1)$$

$$(\underline{3} \otimes \underline{3})_{\underline{3}_a} = (x_2y_3 - x_3y_2\,,\ x_3y_1 - x_1y_3\,,\ x_1y_2 - x_2y_1)$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}} = x_1y_1 + x_2y_2 + x_3y_3,$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}'} = x_1y_1 + \omega x_2y_2 + \omega^2 x_3y_3,$$

$$(\underline{3} \otimes \underline{3})_{\underline{1}''} = x_1y_1 + \omega^2 x_2y_2 + \omega x_3y_3,$$

Some general properties

(X-G He, Chin. J. Phys 53, 100101(2015); X-G He and G-N Li, Phys. Lett. B750,620(2015); E Ma, Phys. Rev. D92, 051301(2015).

Assuming that the charged lepton mass matrix M_l is diagonalized from left by U_l ,

$$M_l = U_l \hat{m}_l U_r \ , \quad U_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \ ,$$

where $\omega = exp(i2\pi/3)$ and $\omega^2 = exp(i4\pi/3)$.

A_4 models usually have the above characteristic U_i.

U_r is a unitary matrix, but does not play a role in determining V_{PMNS}.

If neutrinos are Majorana particles, the most general mass matrix is

$$M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix} \ ,$$

In the basis where charged lepton is digonalized,
the neutrino mass matrix is of the form given by $U_l^\dagger M_\nu U_l^*$ with

$$A_{11} = \frac{1}{3}(w_1 + w_2 + w_3 + 2(x + y + z)) \ ,$$
$$A_{22} = \frac{1}{3}(w_1 + \omega w_2 + \omega^2 w_3 + 2(\omega^2 x + \omega y + z)) \ ,$$
$$A_{33} = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 + 2(\omega x + \omega^2 y + z)) \ ,$$
$$A_{12} = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 - \omega x - \omega^2 y - z) \ ,$$
$$A_{13} = \frac{1}{3}(w_1 + \omega w_2 + \omega^2 w_3 - \omega^2 x - \omega y - z) \ ,$$
$$A_{23} = \frac{1}{3}(w_1 + w_2 + w_3 - (x + y + z)) \ .$$

If the parameter set $P = (w_1, w_2, w_3, x, y, z)$ is real, then

$$A_{11} = A_{11}^* \ \ A_{23} = A_{23}^*$$

$$A_{22} = A_{33}^* \ \ A_{12} = A_{13}^*$$

The mass matrix is of the form given by eq B, $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$

To determine whether δ is $+\pi/2$ or $-\pi/2$

One has to check the sign of the Jarlskog parameter J = $\mathrm{Im}(V_{11}V^*_{12}V^*_{21}V_{22})$

$$\sin\delta = \frac{(1-|V_{13}|^2)\,Im(V_{11}V_{12}^*V_{21}^*V_{22})}{|V_{11}||V_{12}||V_{23}||V_{33}||V_{13}|}$$

A model with Type II seesaw

Particle contents and their transformation properties under standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge and A_4 family symmetry properties

$$
\begin{aligned}
& l_L : (1,2,-1)(3)\ , \quad l_R : (1,1,-2)(1+1''+1')\ , \\
& \phi : (1,2,-1)(1)\ , \quad \Phi : (1,2,-1)(3)\ , \\
& \Delta^{0,',''} : (1,3,-2)(1+1'+1'')\ , \quad \chi : (1,3,-2)(3)\ .
\end{aligned}
$$

The Lagrangian responsible for the lepton mass matrix is

$$
\begin{aligned}
L = {} & y_e \bar{l}_L \tilde{\Phi} l_R^1 + y_\mu \bar{l}_L \tilde{\Phi} l_R^2 + y_\tau \bar{l}_L \tilde{\Phi} l_R^3 \\
& + Y_\nu^0 \bar{l}_L \Delta^0 l_L^c + Y_\nu' \bar{l}_L \Delta' l_L^c + Y_\nu'' \bar{l}_L \Delta'' l_L^c + y_\nu \bar{l}_L \chi l_L^c + H.C.
\end{aligned}
$$

If the structure of the vacuum expectation value (vev) is of the form $< \Phi_{1,2,3} >= v^{\Phi}_{1,2,3} = v^{\Phi}$, $< \chi_i >= v^{\chi}_i$, $< \phi >= v_\phi$, and $< \Delta^{0,',''} >= v^{0,',''}_{\Delta}$,

$$M_l = U_l \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \ , \quad M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix} \ ,$$

In general w_i, x, y, z are complex!
where $m_{e,\mu,\tau} = \sqrt{3} y_{e,\mu,\tau} v^{\Phi}$ and

$$\begin{aligned} w_1 &= Y^0_\nu v^0_\Delta + Y^{'}_\nu v^{'}_\Delta + Y^{''}_\nu v^{''}_\Delta \ , \\ w_2 &= Y^0_\nu v^0_\Delta + \omega^2 Y^{'}_\nu v^{'}_\Delta + \omega Y^{''}_\nu v^{''}_\Delta \ , \\ w_3 &= Y^0_\nu v^0_\Delta + \omega Y^{'}_\nu v^{'}_\Delta + \omega^2 Y^{''}_\nu v^{''}_\Delta \ , \\ x &= y_\nu v^\chi_3 \ , \quad y = y_\nu v^\chi_2 \ , \quad z = y_\nu v^\chi_1 \ . \end{aligned}$$

In general w_i, x, y, z are complex!

Making parameters in the set P real

To obtain real w_i, x, y, and z,

one needs to make the Yukawa couplings and scalar vevs to be real

one can require the following generalized CP symmetry under

$$(l_{e,L}\ ,\ \ l_{\mu,L}\ ,\ l_{\tau,L}) \to (l_{e,L}^{CP}\ ,\ \ l_{\tau,L}^{CP}\ ,\ \ l_{\mu,L}^{CP})\ , \quad \Phi = (\Phi_1, \Phi_2, \Phi_3) \to (\Phi_1^\dagger, \Phi_3^\dagger, \Phi_2^\dagger)\ ,$$
$$(\Delta^0\ ,\ \Delta'\ ,\ \ \Delta'') \to (\Delta^{0\dagger}\ ,\ \ \Delta'^\dagger\ ,\ \ \Delta''^\dagger)\ , \quad (\chi_1\ ,\ \chi_2\ ,\ \ \chi_3) \to (\chi_1^\dagger\ ,\ \ \chi_3^\dagger\ ,\ \ \chi_2^\dagger)\ ,$$

and all other fields transform the same as those under the usual CP symmetry.

The above transformation properties will transform relevant terms

into their complex conjugate ones.

Requiring the Lagragian to be invariant under the above transformation

dictates the Yukawa couplings to be real.

The same requirement will dictates the scalar potential

to forbid spontaneous CP violation and vevs to be real.

One, however, notices that the parameters $w_{2,3}$ are in general complex even if the Yukawa couplings and the vevs of the scalar fields are made real because the appearance of ω^i.

To make them real, it is therefore required that

$$Im(\omega^2 Y_\nu^{'} v_\Delta^{'} + \omega Y_\nu^{''} v_\Delta^{''}) = Im(\omega Y_\nu^{'} v_\Delta^{'} + \omega^2 Y_\nu^{''} v_\Delta^{''}) = 0 \; .$$

The above can be achieved by the absent of the scalar fields $\Delta^{','' }$ or $Y_\nu^{'} v_{\Delta'} = Y_\nu^{''} v_{\Delta''}$.

An example with Z_2 residual group unbroken

If vev of χ_2 component of χ is non-zero,

but the vevs of $\chi_{1,3}$ are zero, the vev structure breaks A_4 down to a Z_2.

The charge lepton and neutrino mass matrice are given by

$$M_l = U_l \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} , \quad M_\nu = \begin{pmatrix} w_1 & 0 & y \\ 0 & w_2 & 0 \\ y & 0 & w_3 \end{pmatrix} .$$

The parameters in the set w_i, and y are in general complex.

X-G He, Chin. J Phys. 53, 100101(2015)

Diagonalizing the mass matrices, we have

$$V_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} c + se^{i\rho} & 1 & ce^{i\rho} - s \\ c + \omega se^{i\rho} & \omega^2 & \omega ce^{i\rho} - s \\ c + \omega^2 se^{i\rho} & \omega & \omega^2 ce^{i\rho} - s \end{pmatrix} ,$$

$\tan\rho = Im(yw_1^* + y^* w_3)/Re(yw_1^* + y^* w_3)$,
$s = \sin\theta$ and $c = \cos\theta$,

$$\tan 2\theta = \frac{2|yw_1^* + w_3 y^*|}{|w_1|^2 - |w_3|^2} .$$

Majorana phases α_i of m_i

$$\alpha_{1,3} = Arg(w_i(1 \pm \cos 2\theta) + w_2 e^{-i2\rho}(1 \mp \cos 2\theta) \pm 2\sin 2\theta y e^{-i\rho} , \quad \alpha_2 = Arg(w_2)$$

Translate into standard parameterization

$$s_{12} = \frac{1}{\sqrt{2}(1 + cs\cos\rho)^{1/2}} , \; s_{23} = \frac{(1 + cs\cos\rho + \sqrt{3}cs\sin\rho)^{1/2}}{\sqrt{2}(1 + cs\cos\rho)^{\frac{1}{2}}} ,$$
$$s_{13} = \frac{(1 - 2cs\cos\rho)^{1/2}}{\sqrt{3}} .$$

and

$$\sin\delta = (1 + \frac{4c^2 s^2 \sin^2\rho}{(c^2 - s^2)^2})^{-1/2}(1 - \frac{3c^2 s^2 \sin^2\rho}{(1 + cs\cos\rho)^2})^{-1/2} \times \begin{cases} -1 , & \text{if } c^2 > s^2 , \\ +1 , & \text{if } s^2 > c^2 . \end{cases}$$

It is clearly that if $\sin\rho$ is not zero,
$|\delta|$ and θ_{23} deviate from $\pi/2$ and $\pi/4$, respectively.
In the limit ρ goes to zero, that real parameter set P
$\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$.

Predictions: $|V_{i2}| = 1/\sqrt{3}$
$J = Im(V_{11}V^*_{12}V^*_{21}V_{22}) = (s^2-c^2)/6\sqrt{3}$
independent of ρ

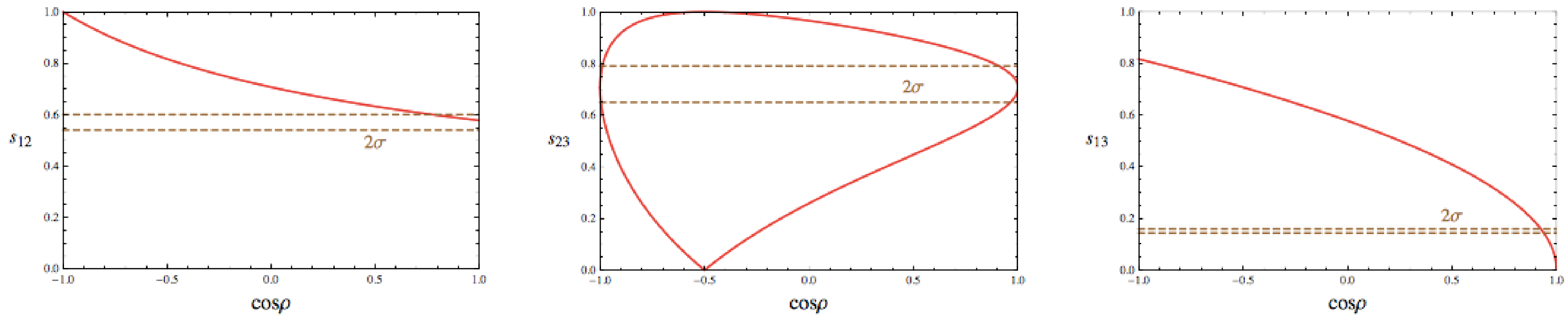
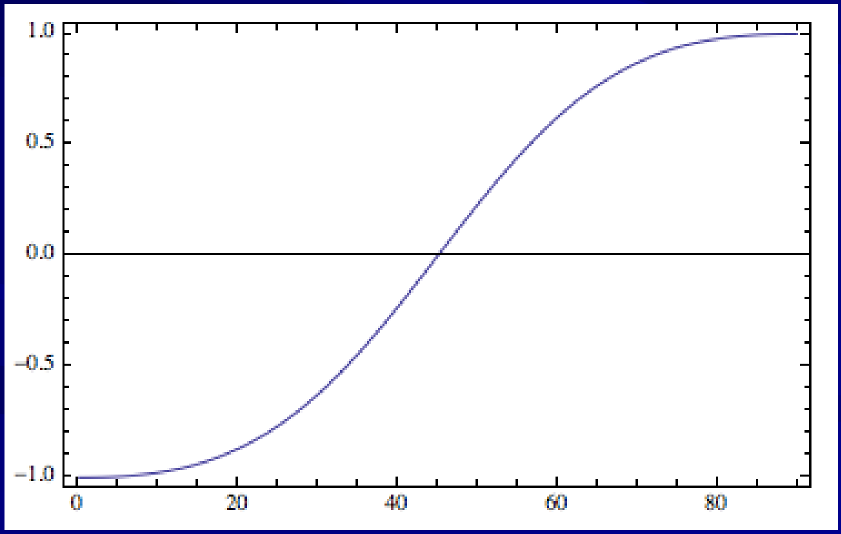


FIG. 1: s_{12}, s_{23}, s_{13} as functions of $cos\rho$ with $c = s = \sqrt{2}/2$ compared with allowed ranges for $(s_{13},\ s_{23},\ s_{12})$ at 1σ and 2σ, respectively, given by $(0.147 \sim 0.155,\ 0.731 \sim 0.773,\ 0.554 \sim 0.582)$ and $(0.143 \sim 0.159,\ 0.660 \sim 0.788,\ 0.540 \sim 0.597)$.



Sinδ vs θ (degree) (s=sinθ)

with cos ρ= 0.95

when c=s (θ=45º)

New model building guideline

First order: $\theta_{23} = \pi/4$ and $\delta_{CP} = 3\pi/2$, A non-zero θ_{13}.

A_4 family symmetry model provide an example to fully realize such mixing pattern. This class of A_4 models also provide direction for modifying the pattern, with complex Yukawa coefficients.

New experimental data will provide more clue about what the mixing pattern is and how theoretical model should be constructed.