PROJECT B1E:

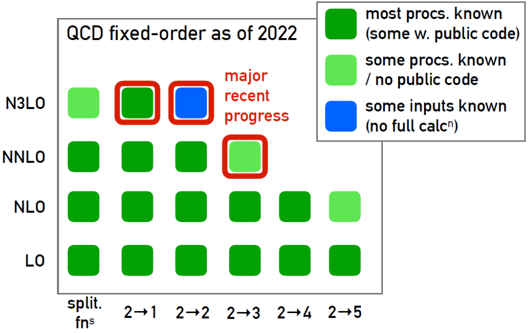
POWER CORRECTIONS IN COLLIDER PROCESSES

[GUIDO BELL, MICHAL CZAKON, KIRILL MELNIKOV]

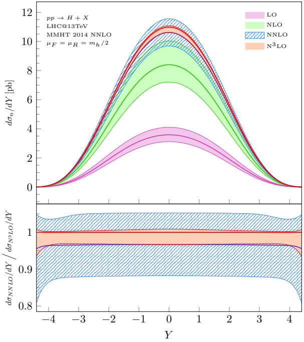
Precision calculations

Tremendous progress in perturbative QCD calculations in past decade

$$\mathrm{d}\sigma^{\text{part}} = \mathrm{d}\sigma^{\text{part}}_{\text{LO}} + \alpha_s\, \mathrm{d}\sigma^{\text{part}}_{\text{NLO}} + \alpha_s^2\, {\color{red}\mathrm{d}\sigma^{\text{part}}_{\text{NNLO}}} + \alpha_s^3\, {\color{red}\mathrm{d}\sigma^{\text{part}}_{\text{N}^3\text{LO}}} + \mathcal{O}(\alpha_s^4)$$



[Salam@ICHEP 2022]

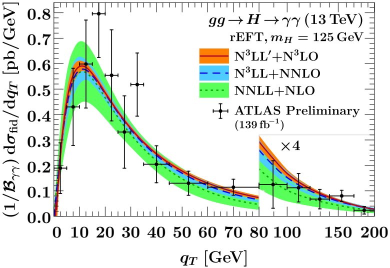


[Dulat, Mistlberger, Pelloni 18]

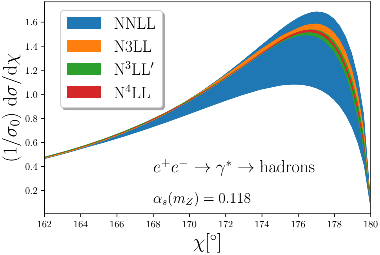
Precision calculations

Similar progress in understanding logarithmic corrections to all orders

$$d\sigma^{\text{part}} = \exp\left\{ \frac{1}{\alpha_s}\, g_{LL}(\alpha_s L) + g_{NLL}(\alpha_s L) + \alpha_s\, g_{NNLL}(\alpha_s L) + \alpha_s^2\, g_{N^3LL}(\alpha_s L) + \ldots \right\}$$



[Billis, Dehnadi, Ebert, Michel, Tackmann 21]



[Duhr, Mistlberger, Vita 22]

Precision calculations

For some observables the theory predictions are nowadays reaching %-level accuracy

$\Rightarrow$ are there other classes of corrections?

Precision calculations

For some observables the theory predictions are nowadays reaching %-level accuracy

$\Rightarrow$ are there other classes of corrections?

- electroweak corrections

  automated NLO calculations for multi-particle final states

  computation of mixed QCD-electroweak corrections

  implementation of electroweak Sudakov factors in event generators

Precision calculations

For some observables the theory predictions are nowadays reaching %-level accuracy

⇒ are there other classes of corrections?

- electroweak corrections
- parton distribution functions

  long-term goal to develop N^3LO PDFs with reliable uncertainties

  consistent implementation of QED effects

Precision calculations

For some observables the theory predictions are nowadays reaching %-level accuracy

$\Rightarrow$ are there other classes of corrections?

- electroweak corrections
- parton distribution functions
- power corrections

power corrections are highly process- and observable dependent

we currently only have a very limited understanding of power corrections

I will distinguish between perturbative and non-perturbative power corrections

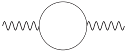
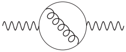
Non-perturbative power corrections

R-ratio

One of the simplest observables in QCD

$$R(Q) = \frac{\sigma_{e^+e^- \to \text{hadrons}}(Q)}{\sigma_{e^+e^- \to \mu^+\mu^-}(Q)}$$

Using dispersion relations it can be related to the hadronic vacuum polarisation

$\Pi(-Q^2) \sim$  $+$  $+ \; \dots$

- the R-ratio is currently known to N^4LO [Baikov, Chetyrkin, Kühn, Rittinger 12]

But how good is the partonic description?

Operator product expansion

In the Euclidean region with $Q^2 \gg \Lambda^2$

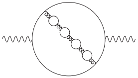
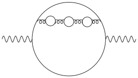
$$\Pi(-Q^2) \sim C_1(Q) + \frac{1}{Q^4} \left\{ C_{\bar{q}q}(Q) \, \langle \Omega | m_q \bar{q} q | \Omega \rangle + C_{GG}(Q) \, \langle \Omega | G^A_{\mu\nu} G^{A,\mu\nu} | \Omega \rangle \right\} + \ldots$$

- non-perturbative corrections scale as $(\Lambda/Q)^4$
- operator definition of non-perturbative matrix elements
- non-perturbative parameters accessible to lattice QCD and sum rule calculations

Unfortunately, the picture is far more complicated for other observables ...

Renormalon calculus

In DR the Wilson coefficients are sensitive to power corrections

$C_1 =$  $+$  $+ \dots$

$\Rightarrow$ IR sensitivity manifests as a factorial growth in the perturbative expansion

$$C_1 = \sum_{n=0}^{\infty} c_n \, \alpha_s^{n+1} \qquad \Rightarrow \qquad \mathcal{B}[C_1](t) = \sum_{n=0}^{\infty} c_n \, \frac{t^n}{n!}$$

$\Rightarrow$ shows up as a $t = -2/\beta_0 > 0$ renormalon in the Borel plane

$\Rightarrow$ ambiguity in the Borel integral

$$C_1 = \int_0^{\infty} dt \; e^{-t/\alpha_s} \, \mathcal{B}[C_1](t) \qquad \Rightarrow \qquad \delta C_1 \sim e^{\frac{2}{\beta_0 \alpha_s(Q)}} \sim \left(\frac{\Lambda}{Q}\right)^4$$

Without knowing anything about power corrections in the OPE . . .

Comparison

Factorisation

- + based on a rigorous expansion for $\Lambda \ll Q$ in QCD
- + operator definitions of non-perturbative parameters
- – highly process dependent
- – no OPE for processes with jet-like signatures

Renormalon calculus

- + universal approach for large class of observables
- + amounts to perturbative calculation with dressed gluon
- – model dependent (may blur universality aspects and even miss some effects)
- – can currently only be applied to processes without gluons at Born level

e^+e^- event shapes

Thrust $\quad T = \max_{\vec{n}} \frac{\sum_i |\vec{k}_i \cdot \vec{n}|}{\sum_i |\vec{k}_i|}$

Factorisation in the two-jet region ($\tau = 1 - T$)

$$\frac{d\sigma}{d\tau} \simeq H(Q) \int dp_L^2 \, dp_R^2 \, dk \; J(p_L^2) \, J(p_R^2) \, S(k) \, \delta\Big(\tau - \frac{p_L^2 + p_R^2}{Q^2} - \frac{k}{Q}\Big)$$

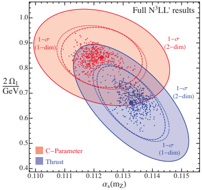
$\Rightarrow$ dominant non-perturbative correction results in a shift [Lee, Sterman 06]

$$\frac{d\sigma}{d\tau}(\tau) \xrightarrow{\text{NP}} \frac{d\sigma}{d\tau}\left(\tau - c_\tau \frac{\Omega_1}{Q}\right)$$

- linear power correction $\sim (\Lambda/Q)$
- operator definition $\Omega_1 = \langle \Omega | S_{\bar{n}}^\dagger S_n^\dagger \, \mathcal{E}_T(0) \, S_n S_{\bar{n}} | \Omega \rangle$
- calculable observable-dependent coefficient $c_\tau = 2 \,,\; c_C = 3\pi \,, \ldots$

α_s extractions

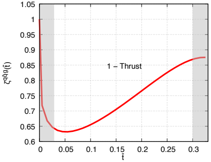
Global fit to thrust and C-parameter distributions [Hoang, Kolodrubetz, Mateu, Stewart 15]



- $\alpha_s(m_Z) = 0.1128 \pm 0.0012$ (thrust)
- $\sim 4\sigma$ below PDG world average

$\Rightarrow$ need better understanding of power corrections in 3-jet region [Luisoni, Monni, Salam 20]

First analysis based on renormalon calculus [Caola, Ferrario R., Limatola, Melnikov, Nason, Ozcelik 22]



- uses $q\bar{q}\gamma$ as a proxy for 3-jet configurations
- includes decay of dressed gluon into massless partons
- no operator definition of non-perturbative parameter

$\Rightarrow$ can this be combined with a factorisation-based approach?

LHC observables

Little is known about power corrections at hadron-hadron colliders

$\Rightarrow$ use renormalon calculus to identify observables with linear power corrections

First systematic study of linear power corrections* [Caola et al 21]

- for massless particles virtual corrections do not generate linear corrections
- collinear emissions (usually) do not generate linear corrections either
- linear corrections are associated with soft gluon emissions

* remember that renormalon calculus applies only to Born processes without gluons

Outlook part I

Main findings

- hadronically inclusive observables do not receive linear power corrections
- even true for single top production (provided one uses a short-distance top mass)
  [Makarov, Melnikov, Nason, Ozcelik 23]
- no linear power corrections to Z-boson p_T distribution

Project B1e:

- study additional processes with massive quarks in renormalon model
- explore ways to incorporate non-abelian interactions into renormalon calculus
- investigate connections to SCET (e.g. for event-shape studies in 3-jet region)

Perturbative power corrections I

Back to the perturbative world

Power corrections associated with ratio of two perturbative scales

- thrust in the two-jet region

$$\left.\begin{aligned} \mu_H &\sim Q \\ \mu_J &\sim \sqrt{\tau} Q \\ \mu_S &\sim \tau Q \end{aligned}\right\} \quad \frac{\mu_S^2}{\mu_J^2} \sim \frac{\mu_J^2}{\mu_H^2} \sim \tau$$

$\Rightarrow$ leading power resums $\left[\frac{\ln^n \tau}{\tau}\right]_+$, but what about $\ln^n \tau$ terms at subleading power?

SCET provides the ingredients to study these corrections

- effective Lagrangian known to $\mathcal{O}(\lambda^2)$
- subleading-power operator bases
- anomalous dimensions of subleading hard / jet / soft functions

Key problem

SCET factorisation theorems involve matrix elements of non-local operators

$\Rightarrow$ convolutions of hard coefficient functions with jet and soft functions

At subleading power it happens that these convolutions diverge at the endpoints

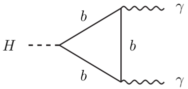
$$\int_0^1 \mathrm{d}z \;\; h(z)\, j(z) \; = \; \int_0^1 \mathrm{d}z \;\; z^{-\varepsilon} \; z^{-1-\varepsilon} \; \neq \; \int_0^1 \mathrm{d}z \left[1 - \varepsilon \ln z + \ldots\right] \left[-\frac{1}{\varepsilon}\delta(z) + \frac{1}{z_+} + \ldots\right]$$

$\Rightarrow$ convolution and renormalisation of EFT operators do not commute

$\Rightarrow$ prevents one from using RG techniques at subleading power

A specific example

Bottom-quark contribution to $H \to \gamma\gamma$



- scale hierachy $m_b \ll M_H$
- subleading power due to helicity suppression

Bare factorisation theorem [Liu, Neubert 19]

$$\mathcal{M}_b(H \to \gamma\gamma) \sim H_1 \, \langle O_1 \rangle + 2 \int_0^1 \frac{dz}{z} \, \bar{H}_2(z) \, \langle O_2(z) \rangle + H_3 \int_0^\infty \frac{d\ell_-}{\ell_-} \int_0^\infty \frac{d\ell_+}{\ell_+} \, J(M_H \ell_+) \, J(M_H \ell_-) \, S(\ell_+ \ell_-)$$

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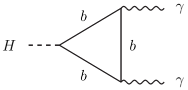

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Bare factorisation theorem is spoilt by endpoint divergences [Liu, Neubert 19]

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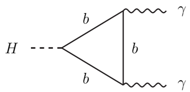
- in the endpoint regions the two terms describe the same physics

$\Rightarrow$ is there a way to combine these contributions?

A specific example

Bottom-quark contribution to $H \to \gamma\gamma$



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Bare factorisation theorem is free from endpoint divergences [Liu, Neubert 19]

$$\mathcal{M}_b(H \to \gamma\gamma) \sim (H_1 + \Delta H_1)\,\langle O_1 \rangle$$

$$+\,2\int_0^1 \frac{dz}{z}\left\{ \bar{H}_2(z)\,\langle O_2(z)\rangle - \Big[\!\Big[\bar{H}_2(z)\,\langle O_2(z)\rangle \Big]\!\Big]_0 - \Big[\!\Big[\bar{H}_2(z)\,\langle O_2(z)\rangle \Big]\!\Big]_1 \right\}$$

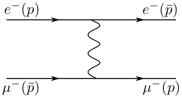
$$+\,H_3 \int_0^{M_H} \frac{d\ell_-}{\ell_-} \int_0^{M_H} \frac{d\ell_+}{\ell_+}\; J(M_H \ell_+)\, J(M_H \ell_-)\, S(\ell_+ \ell_-)$$

- rearrangement based on refactorisation conditions [Böer 18; Liu, Neubert 19]

$$\Big[\!\Big[\bar{H}_2(z)\,\langle O_2(z)\rangle \Big]\!\Big]_0 = \lim_{z\to 0} \bar{H}_2(z)\,\langle O_2(z)\rangle = \frac{H_3}{2}\int_0^\infty \frac{d\ell_+}{\ell_+}\; J(M_H \ell_+)\, J(z\, M_H^2)\, S(\ell_+ z\, M_H)$$

A counterexample

Muon-electron scattering in backward direction



- scale hierachy $m_e \sim m_\mu \ll \sqrt{s} \quad (t \approx -s,\; u \approx 0)$
- leading power QED process

Bare factorisation theorem [GB, Böer, Feldmann 22]

$$F_1(\lambda) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y}\; f_c(x)\; H(xy)\; f_{\bar{c}}(y)$$

$$+ \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \int_0^\infty \frac{d\rho}{\rho} \int_0^\infty \frac{d\omega}{\omega}\; f_c(x)\; J(x\rho)\; S(\rho\omega)\; J(\omega y)\; f_{\bar{c}}(y)$$

A counterexample

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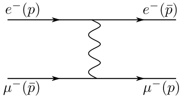

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A counterexample

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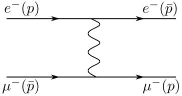

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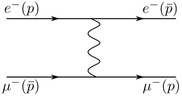
$$+ \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \int_0^{\sqrt{s}} \frac{d\rho}{\rho} \int_0^{\infty} \frac{d\omega}{\omega}\, f_c(x)\, J(x\rho)\, S(\rho\omega)\, J(\omega y)\, f_{\bar{c}}(y)$$

A counterexample

Muon-electron scattering in backward direction



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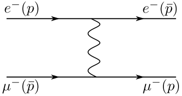
$$+ f_c \otimes J \otimes S \otimes J \otimes S \otimes J \otimes f_{\bar{c}} + \ldots$$

$\Rightarrow$ generates iterated pattern of endpoint-divergent convolution integrals

A counterexample

Muon-electron scattering in backward direction



- scale hierachy $m_e \sim m_\mu \ll \sqrt{s} \quad (t \approx -s,\ u \approx 0)$
- leading power QED process

Double logarithms descend from consistency relation [GB, Böer, Feldmann 22]

$$F_1(\lambda) = \mathcal{F}_1(z) = 1 + z \int_0^1 d\xi \int_0^1 d\eta\, \mathcal{F}_1(\xi^2 z)\, \theta(1-\xi-\eta)\, \mathcal{F}_1(\eta^2 z)$$

in terms of the logarithmic variables $z = \frac{\alpha}{2\pi} \ln^2 \lambda^2 , \qquad \xi = \frac{\ln x}{\ln \lambda^2} , \qquad \eta = \frac{\ln y}{\ln \lambda^2}$

$\Rightarrow$ generates a modified Bessel function [Gorshkov, Gribov, Lipatov, Frolov 66]

$$F_1(\lambda) = \frac{I_1(2\sqrt{z})}{\sqrt{z}} = 1 + \frac{z}{2} + \frac{z^2}{12} + \frac{z^3}{144} + \mathcal{O}(z^4)$$

Outlook part II

Why is this relevant?

- generic feature of $2 \to 2$ processes and beyond
- closely related to factorisation of hadronic matrix elements in exclusive B decays

$\Rightarrow$ μe scattering provides a simple setup to study generic structure of endpoint singularities

Project B1e:

- resummation beyond double-logarithmic approximation
- cross-check resummed results against fixed-order calculation
- apply the technology to related QCD processes

Perturbative power corrections II

Slicing methods

Power corrections are also relevant for fixed-order calculations

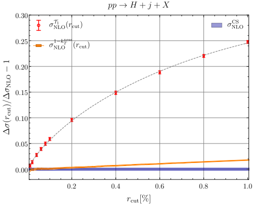
$$\underbrace{\sigma(X)}_{\text{N}^k\text{LO}} = \int_0 d\tau \, \frac{d\sigma(X)}{d\tau} = \underbrace{\int_0^{\tau_{\text{cut}}} d\tau \, \frac{d\sigma(X)}{d\tau}}_{\substack{\text{soft + collinear emissions} \\ \text{to Born process at N}^k\text{LO}}} + \underbrace{\int_{\tau_{\text{cut}}} d\tau \, \frac{d\sigma(X)}{d\tau}}_{\substack{\text{Born + 1 resolved jet} \\ \text{at N}^{k-1}\text{LO}}}$$

Compute unresolved contribution with methods from factorisation

$$\int_0^{\tau_{\text{cut}}} d\tau \, \frac{d\sigma(X)}{d\tau} = \underbrace{\int_0^{\tau_{\text{cut}}} d\tau \, \frac{d\sigma(X)}{d\tau}\Big|_{\text{LP}}}_{\substack{\text{requires N}^k\text{LO hard, jet,} \\ \text{beam and soft functions}}} + \underbrace{\mathcal{O}(\tau_{\text{cut}})}_{\substack{\text{crucial to improve the} \\ \text{numerical efficiency}}}$$

Two strategies

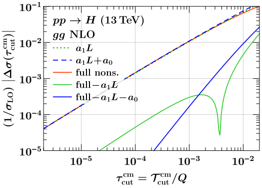
Design a resolution variable with small power corrections [Buonocore et al 22]



- at NLO k_T^{ness}-slicing seems to be less sensitive to power corrections than jettiness-slicing

$\Rightarrow$ Why? And is this also true at NNLO?

Control the dominant power corrections analytically [Ebert et al 18]



- at NLO the cutoff can be significantly relaxed if power corrections are included analytically
- at NNLO so far only the $\tau \ln^3 \tau$ terms are known

Outlook part III

Project B1e:

- compute NNLO power corrections for q_T and 0-jettiness slicing
- extend the method to processes with jets
- understand structure of power corrections for other slicing variables

Conclusions

Project B1e addresses power corrections to collider processes from various angles

- non-perturbative power corrections

  $\Rightarrow$ renormalon calculus, factorisation

- resummation at subleading power

  $\Rightarrow$ endpoint singularities, consistency relations

- power corrections to slicing techniques

  $\Rightarrow$ analytic NNLO calculations