New projects: C1c: Non-perturbative matrix elements for B-mixing and lifetimes II

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Calculation of $\Gamma(B_q)$

$$\Gamma(B_q) = \sum_i \Gamma_i \langle \mathcal{O}_i \rangle$$

Perturbative Γ_i:

- Dimensional regularization with $D = 4 - 2\epsilon$
- Operators mix through renormalization, also with evanescent operators (vanish in $D = 4$):

$$\mathcal{O}^{\mathrm{R}} = Z_{\mathcal{O}\mathcal{O}}\mathcal{O} + Z_{\mathcal{O}\mathrm{E}}E$$

- Γ_i scheme dependent:
  1. Explicit dependence on μ
  2. Scheme for γ_5
  3. Choice of evanescent operators

Lattice $\langle \mathcal{O}_i \rangle$:

- Lattice spacing a as UV regulator
- Have to take continuum limit $a \to 0$ in the end
- Operators mix through renormalization:

$$\mathcal{O}^{\mathrm{R}} = Z_{11}\mathcal{O}_1 + Z_{12}\mathcal{O}_2$$

- $\langle \mathcal{O}_i \rangle$ scheme dependent

$\Rightarrow$ Scheme matching between lattice and perturbative results highly nontrivial

Gradient flow

- Introduce parameter *flow time* $t \geq 0$ [Narayanan, Neuberger 2006; Lüscher 2009; Lüscher 2010]
- *Flowed fields* in $D+1$ dimensions obey differential *flow equations*:

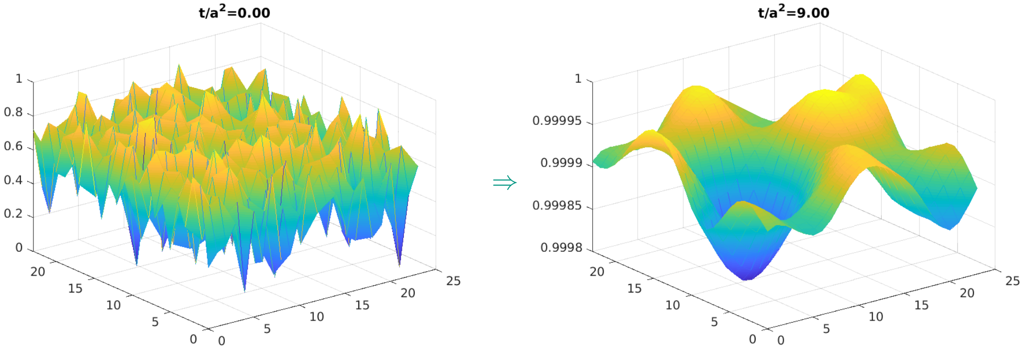
Flow equations [Narayanan, Neuberger 2006; Lüscher 2010; Lüscher 2013]

$$\partial_t B^a_\mu = \mathcal{D}^{ab}_\nu G^b_{\nu\mu} \quad \text{with} \quad B^a_\mu(t,x)\big|_{t=0} = A^a_\mu(x),$$
$$\partial_t \chi = \Delta\chi \quad \text{with} \quad \chi(t,x)\big|_{t=0} = \psi(x)$$

$$\mathcal{D}^{ab}_\mu = \delta^{ab}\partial_\mu - f^{abc}B^c_\mu, \qquad G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + f^{abc}B^b_\mu B^c_\nu, \qquad \Delta = (\partial_\mu + B^a_\mu T^a)(\partial_\mu + B^b_\mu T^b)$$

$\Rightarrow$ CRC Colloquium by Robert Harlander and Oliver Witzel on February 01, 2023

Smearing



[Courtesy of Oliver Witzel]

Applications of the gradient flow

- Inherent smearing to remove small-distance fluctuations [Narayanan, Neuberger 2006; Lüscher 2010; ...]
- Scale setting with the gradient flow extremely precise and cheap [Lüscher 2010; Borsányi et al. 2012; ...]
- Composite operators do not require renormalization [Lüscher, Weisz 2011]

$\Rightarrow$ Define gradient-flow scheme which is valid both on the lattice and perturbatively:

$$\mathcal{O}_j(x)\ \text{divergent} \quad \rightarrow \quad \tilde{\mathcal{O}}_j(t,x)\ \text{finite}$$

- Define gradient-flow coupling [Lüscher 2010; ...]
- Define the energy-momentum tensor of QCD on the lattice [Suzuki 2013; Makino, Suzuki 2014; Harlander, Kluth, **FL** 2018]
  $\Rightarrow$ Studies of thermodynamics on the lattice [FlowQCD since 2014]
- Apply to electroweak Hamiltonian [Suzuki, Taniguchi, Suzuki, Kanaya 2020; Harlander, **FL** 2022]
- ...

Flowed operator product expansion

- Small flow-time expansion [Lüscher, Weisz 2011]:

$$\tilde{\mathcal{O}}_i(t,x) = \sum_j \zeta_{ij}(t)\mathcal{O}_j(x) + O(t)$$

- Invert to express operators through flowed operators [Suzuki 2013; Makino, Suzuki 2014; Monahan, Orginos 2015]:

Flowed OPE

$$T = \sum_i \Gamma_i \mathcal{O}_i = \sum_{i,j} \Gamma_i \zeta_{ij}^{-1}(t)\tilde{\mathcal{O}}_j(t) \equiv \sum_j \tilde{\Gamma}_j(t)\tilde{\mathcal{O}}_j(t)$$

- T defined in regular QCD expressed through finite flowed operators $\tilde{\mathcal{O}}_j(t)$
- Gradient-flow definition of T valid both on the lattice and perturbatively

Calculation of $\Gamma(B_q)$ in the gradient-flow scheme

$$\Gamma(B_q) = \sum_i \Gamma_i \langle \mathcal{O}_i \rangle = \sum_{i,j} \Gamma_i \zeta_{ij}^{-1} \langle \tilde{\mathcal{O}}_j \rangle \equiv \sum_j \tilde{\Gamma}_j \langle \tilde{\mathcal{O}}_j \rangle$$

Perturbative $\tilde{\Gamma}_j$:

- Dimensional regularization with $D = 4 - 2\epsilon$
- Finite and scheme indepedent:
  1. No explicit dependence on μ
  2. No dependence on scheme for γ_5
  3. Independent of evanescent operators

Lattice $\langle \tilde{\mathcal{O}}_j \rangle$:

- Lattice spacing a as UV regulator
- Finite for $a \to 0$
- No operator mixing

$\Rightarrow$ Gradient-flow scheme convenient for matching

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- Γ_i: project C1b
- ζ_{ij}^{-1}: this project
- $\langle \tilde{\mathcal{O}}_j \rangle$: this project

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Gradient-flow Lagrangian

- Write Lagrangian for the gradient flow as [Lüscher, Weisz 2011; Lüscher 2013]

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_B + \mathcal{L}_\chi,$$

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=1}^{n_f} \bar{\psi}_f (\not{D}^{\mathrm{F}} + m_f) \psi_f + \dots$$

- Construct flowed Lagrangian using Lagrange multiplier fields $L^a_\mu(t,x)$ and $\lambda_f(t,x)$:

$$\mathcal{L}_B = -2 \int_0^\infty \mathrm{d}t \, \mathrm{Tr}\left[L^a_\mu T^a \left(\partial_t B^b_\mu T^b - \mathcal{D}^{bc}_\nu G^c_{\nu\mu} T^b \right) \right], \qquad \partial_t B^a_\mu = \mathcal{D}^{ab}_\nu G^b_{\nu\mu}$$

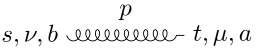
$$\mathcal{L}_\chi = \sum_{f=1}^{n_f} \int_0^\infty \mathrm{d}t \left(\bar{\lambda}_f \left(\partial_t - \Delta \right) \chi_f + \bar{\chi}_f \left(\overleftarrow{\partial_t} - \overleftarrow{\Delta} \right) \lambda_f \right), \qquad \partial_t \chi = \Delta \chi, \;\; \partial_t \bar{\chi} = \bar{\chi} \overleftarrow{\Delta}$$

$\Rightarrow$ Flow equations automatically satisfied

$\Rightarrow$ QCD Feynman rules + gradient-flow Feynman rules (complete list in [Artz, Harlander, **FL**, Neumann, Prausa 2019])

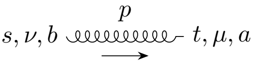
Gradient-flow Feynman rules

- Flowed propagators



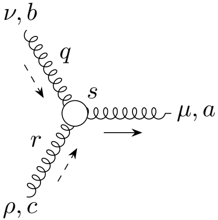
$$= \delta^{ab} \frac{1}{p^2} \delta_{\mu\nu} \, e^{-(t+s)p^2}$$

- Flow lines



$$= \delta^{ab} \, \theta(t-s) \delta_{\mu\nu} \, e^{-(t-s)p^2}$$

- Flow vertices



$$= -igf^{abc} \int_0^\infty \mathrm{d}s \left(\delta_{\nu\rho}(r-q)_\mu + 2\delta_{\mu\nu} q_\rho - 2\delta_{\mu\rho} r_\nu \right)$$

Operator basis

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$$\mathcal{H}_{\text{eff}} = \sum_i \Gamma_i \mathcal{O}_i = \sum_{i,j} \Gamma_i \zeta_{ij}^{-1} \tilde{\mathcal{O}}_j \equiv \sum_j \tilde{\Gamma}_j \tilde{\mathcal{O}}_j$$

- So far we focused on the current-current operators and work in CMM basis [Chetyrkin, Misiak, Münz 1997]
- Define finite flowed operators:

$$\mathcal{O}_1 = -\left(\bar{\psi}_{1,\mathrm{L}}\gamma_\mu T^a \psi_{2,\mathrm{L}}\right)\left(\bar{\psi}_{3,\mathrm{L}}\gamma_\mu T^a \psi_{4,\mathrm{L}}\right) \quad \Rightarrow \quad \tilde{\mathcal{O}}_1 = -\left(\bar{\chi}_{1,\mathrm{L}}\gamma_\mu T^a \chi_{2,\mathrm{L}}\right)\left(\bar{\chi}_{3,\mathrm{L}}\gamma_\mu T^a \chi_{4,\mathrm{L}}\right)$$
$$\mathcal{O}_2 = \left(\bar{\psi}_{1,\mathrm{L}}\gamma_\mu \psi_{2,\mathrm{L}}\right)\left(\bar{\psi}_{3,\mathrm{L}}\gamma_\mu \psi_{4,\mathrm{L}}\right) \quad \Rightarrow \quad \tilde{\mathcal{O}}_2 = \left(\bar{\chi}_{1,\mathrm{L}}\gamma_\mu \chi_{2,\mathrm{L}}\right)\left(\bar{\chi}_{3,\mathrm{L}}\gamma_\mu \chi_{4,\mathrm{L}}\right)$$

- Reminder: $\tilde{\mathcal{O}}_i$ do not require renormalization
- Compute matching matrix $\zeta_{ij}(t)$:

$$\tilde{\mathcal{O}}_i(t, x) = \sum_j \zeta_{ij}(t)\mathcal{O}_j(x) + O(t)$$

Method of projectors

- Define projectors [Gorishny, Larin, Tkachov 1983; Gorishny, Larin 1987]

$$P_k[\mathcal{O}_i] \equiv D_k \langle 0|\mathcal{O}_i|k\rangle \stackrel{!}{=} \delta_{ik} + O(\alpha_s)$$

- Apply to small flow-time expansion:

$$P_k[\tilde{\mathcal{O}}_i(t)] = \sum_j \zeta_{ij}(t) P_k[\mathcal{O}_j]$$

- $\zeta_{ij}(t)$ only depend on t

⇒ Set all other scales to zero

⇒ No perturbative corrections to $P_k[\mathcal{O}_j]$, because all loop integrals are scaleless

“Master formula”

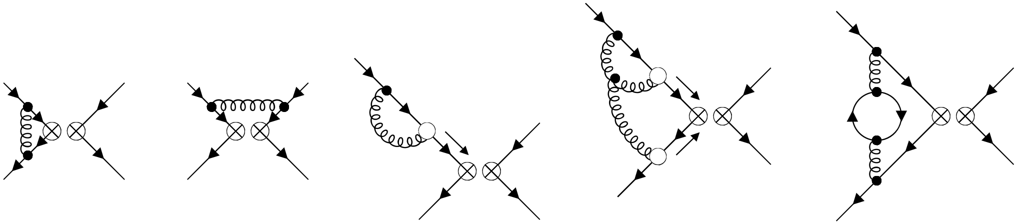
$$\zeta_{ij}(t) = P_j[\tilde{\mathcal{O}}_i(t)]\Big|_{p=m=0}$$

Calculation of ζ_{ij}^{-1}

- Projector for $\mathcal{O}_2 = \left(\bar{\psi}_{1,\mathrm{L}}\gamma_\mu\psi_{2,\mathrm{L}}\right)\left(\bar{\psi}_{3,\mathrm{L}}\gamma_\mu\psi_{4,\mathrm{L}}\right)$:

$$P_2[\mathcal{O}] = \frac{1}{16N_c^2}\,\mathrm{Tr}_{\text{line 1}}\mathrm{Tr}_{\text{line 2}}\,\langle 0|\left(\psi_{4,\mathrm{L}}\gamma_\nu\bar{\psi}_{3,\mathrm{L}}\right)\left(\psi_{2,\mathrm{L}}\gamma_\nu\bar{\psi}_{1,\mathrm{L}}\right)\mathcal{O}|0\rangle\Big|_{p=m=0}$$

- Sample diagrams:



Results in CMM basis

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- Matching matrix ζ^{-1}:

$$(\zeta^{-1})_{11}(t) = 1 + a_s\left(4.212 + \frac{1}{2}L_{\mu t}\right) + a_s^2\left[22.72 - 0.7218\, n_f + L_{\mu t}\,(16.45 - 0.7576\, n_f) + L_{\mu t}^2\left(\frac{17}{16} - \frac{1}{24}\, n_f\right)\right],$$

$$(\zeta^{-1})_{12}(t) = a_s\left(-\frac{5}{6} - \frac{1}{3}L_{\mu t}\right) + a_s^2\left[-4.531 + 0.1576\, n_f + L_{\mu t}\left(-3.133 + \frac{5}{54}\, n_f\right) + L_{\mu t}^2\left(-\frac{13}{24} + \frac{1}{36} n_f\right)\right],$$

$$(\zeta^{-1})_{21}(t) = a_s\left(-\frac{15}{4} - \frac{3}{2}L_{\mu t}\right) + a_s^2\left[-23.20 + 0.7091\, n_f + L_{\mu t}\left(-15.22 + \frac{5}{12}\, n_f\right) + L_{\mu t}^2\left(-\frac{39}{16} + \frac{1}{8}\, n_f\right)\right],$$

$$(\zeta^{-1})_{22}(t) = 1 + a_s\, 3.712 + a_s^2\left[19.47 - 0.4334\, n_f + L_{\mu t}\,(11.75 - 0.6187\, n_f) + \frac{1}{4}\, L_{\mu t}^2\right]$$

- $a_s = \alpha_s(\mu)/\pi$ renormalized in $\overline{\text{MS}}$ scheme and $L_{\mu t} = \ln 2\mu^2 t + \gamma_E$
- Set $N_c = 3$, $T_R = \frac{1}{2}$, and transcendental coefficients replaced by floating-point numbers

Lattice simulation with gradient flow

Central idea of lattice QCD: numerically solve path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{O} \mathrm{e}^{-S}$$

Without gradient flow:

1. Produce gauge configurations e^{-S}, i.e. simulate dynamics of gluons and sea quarks
2. Measurement: Calculate correlation functions
3. Extract bare matrix elements $\langle \mathcal{O} \rangle$
4. Renormalize
5. Continuum limit
6. Match lattice scheme to continuum scheme like $\overline{\text{MS}}$

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6. Match lattice scheme to continuum scheme like $\overline{\text{MS}}$

With gradient flow:

1. Produce gauge configurations e^{-S}, i.e. simulate dynamics of gluons and sea quarks
2. Evolve to flow time t by numerically solving flow equations for each configuration
3. Measurement: Calculate correlation functions for each t
4. Extract gradient-flow matrix elements for each t
5. Continuum limit $\Rightarrow$ better behaved?
6. Match gradient-flow scheme to continuum scheme like $\overline{\text{MS}}$ $\Rightarrow$ ζ

Summary

- Goal of this project: compute nonperturbative matrix elements for B-meson mixing and lifetimes
- Two approaches:
  - Sum rules in HQE
  - Lattice with gradient-flow scheme

⇒ Crosscheck within the project

- Final goal: phenomenological studies together with Wilson coefficients from C1b