

Robert Harlander
Alexander Lenz
Ulrich Nierste

**C1c | Non-perturbative matrix elements for
B-mixing and lifetimes**

TRR 257 P3H: Particle Physics Phenomenology after the Higgs discovery

Project C1c - Overview:

Summary: the main focus is **lifetimes and inclusive decays of B and D mesons**. Based on the **Heavy Quark Expansion** a systematic expansion of the physical quantities is obtained as an expansion in the inverse heavy quark mass.

$$\Gamma(H_Q) = \Gamma_3 + \sum_{i=2}^{\infty} \Gamma_{3+i} \frac{\langle \mathcal{O}_{3+i} \rangle}{m_Q^i} + 16\pi^2 \sum_{i=3}^{\infty} \tilde{\Gamma}_{3+i} \frac{\langle \tilde{\mathcal{O}}_{3+i} \rangle}{m_Q^i} \longrightarrow \text{In case of only mixing}$$

Non-perturbative matrix elements of four-quark operators of dimension six $\langle \tilde{\mathcal{O}}_6 \rangle$ and dimension seven $\langle \tilde{\mathcal{O}}_7 \rangle$ are computed with sum rules and lattice QCD and their phenomenological implications are studied.

Participating Scientists

Principal investigators

- R. Harlander Aachen
- A. Lenz Siegen
- U. Nierste Karlsruhe

Further Participating Scientists

- O. Witzel Siegen
- A. Pivovarov Siegen
- F. Lange Karlsruhe

Funded Positions

- Martin Lang, postdoctoral fellow Siegen
- Janosch Borgulat, PhD Aachen

Project C1c - Experimental Status:

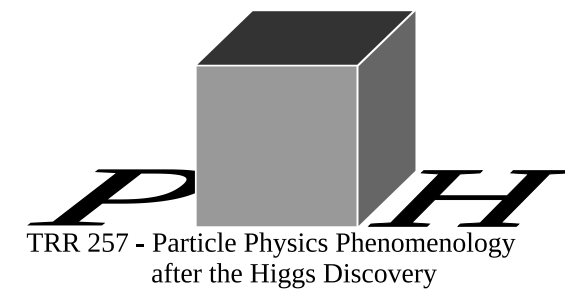
<i>b</i> -hadron species	average lifetime	lifetime ratio
B^0	1.519 ± 0.004 ps	
B^+	1.638 ± 0.004 ps	$B^+/B^0 = 1.076 \pm 0.004$
B_s^0	1.520 ± 0.005 ps	$B_s^0/B^0 = 1.001 \pm 0.004$
B_{sL}	1.429 ± 0.007 ps	
B_{sH}	1.624 ± 0.009 ps	
B_c^+	0.510 ± 0.009 ps	
Λ_b	1.471 ± 0.009 ps	$\Lambda_b/B^0 = 0.969 \pm 0.006$
Ξ_b^-	1.572 ± 0.040 ps	
Ξ_b^0	1.480 ± 0.030 ps	$\Xi_b^0/\Xi_b^- = 0.929 \pm 0.028$
Ω_b^-	$1.64 +0.18 -0.17$ ps	
<i>b</i> -hadron average (weighted by fractions in Z decays)	1.5672 ± 0.0029 ps	

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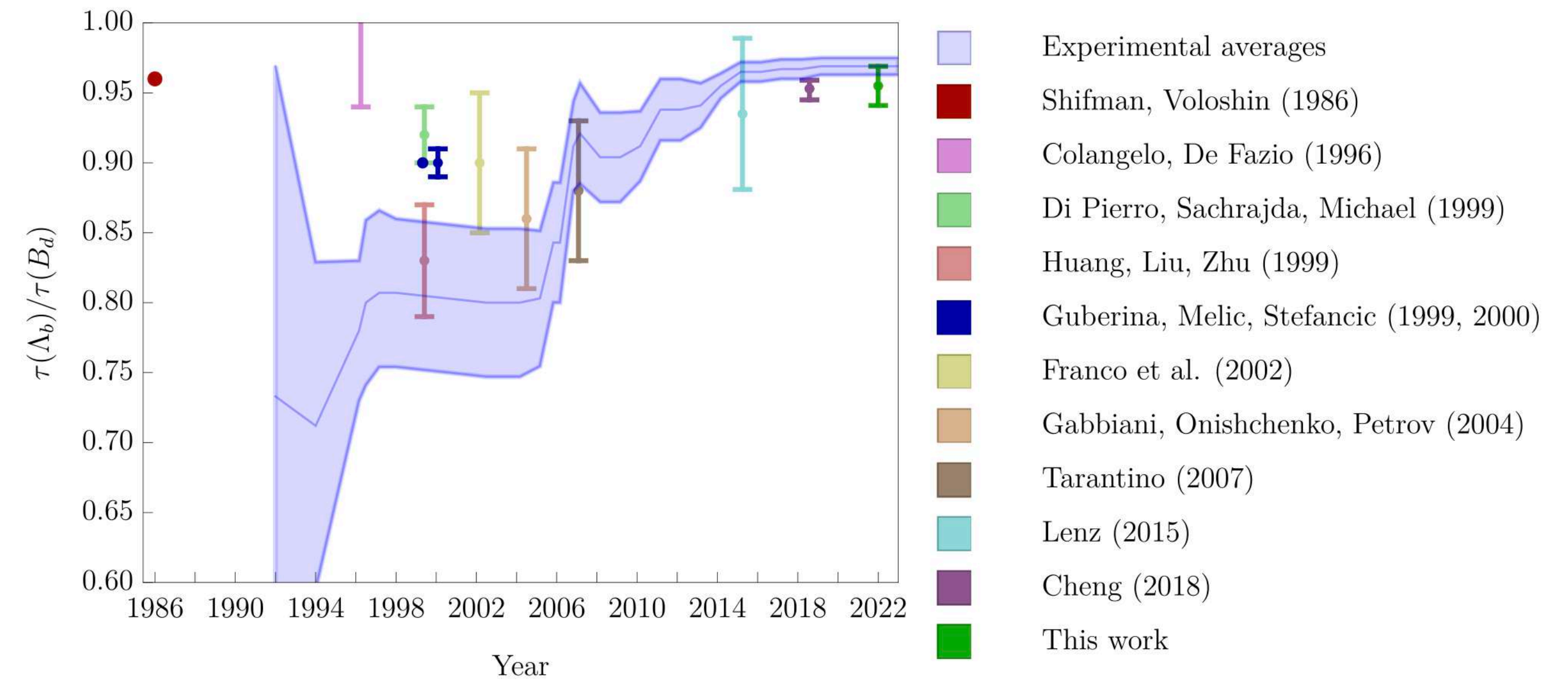
Project C1c - Experimental Status:

Quark-hadron duality at work: lifetimes of bottom baryons

James Gratx (Boskovic Inst., Zagreb), Alexander Lenz (Siegen U.), Blaženka Melić (Boskovic Inst., Zagreb), Ivan Nišandžić (Boskovic Inst., Zagreb), Maria Laura Piscopo (Siegen U.) et al. (Jan 18, 2023)
e-Print: 2301.07698 [hep-ph]



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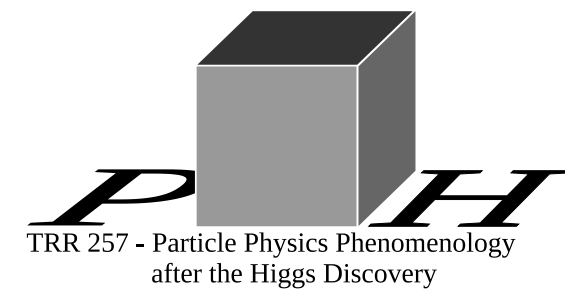


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Project C1c - Experimental Status:

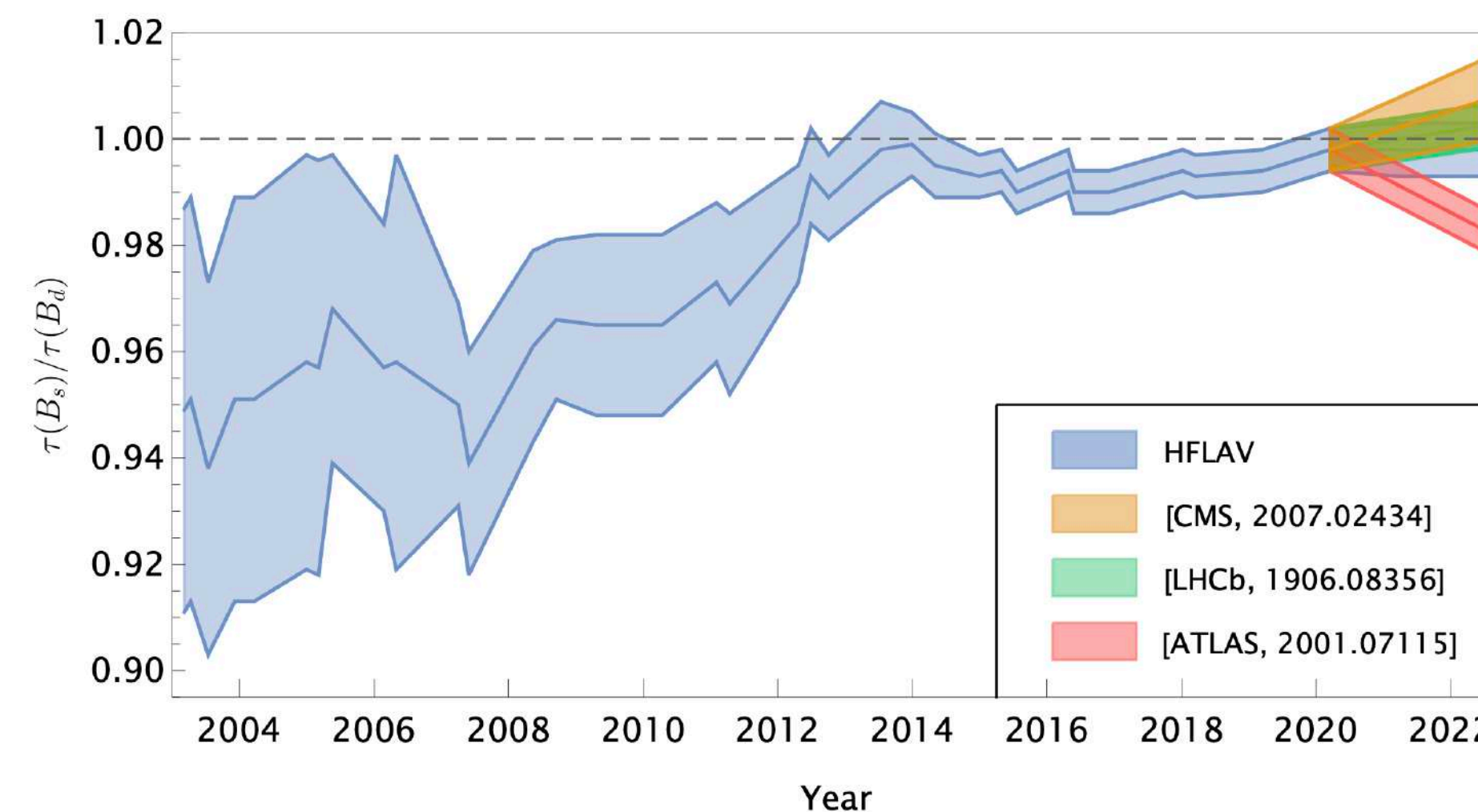
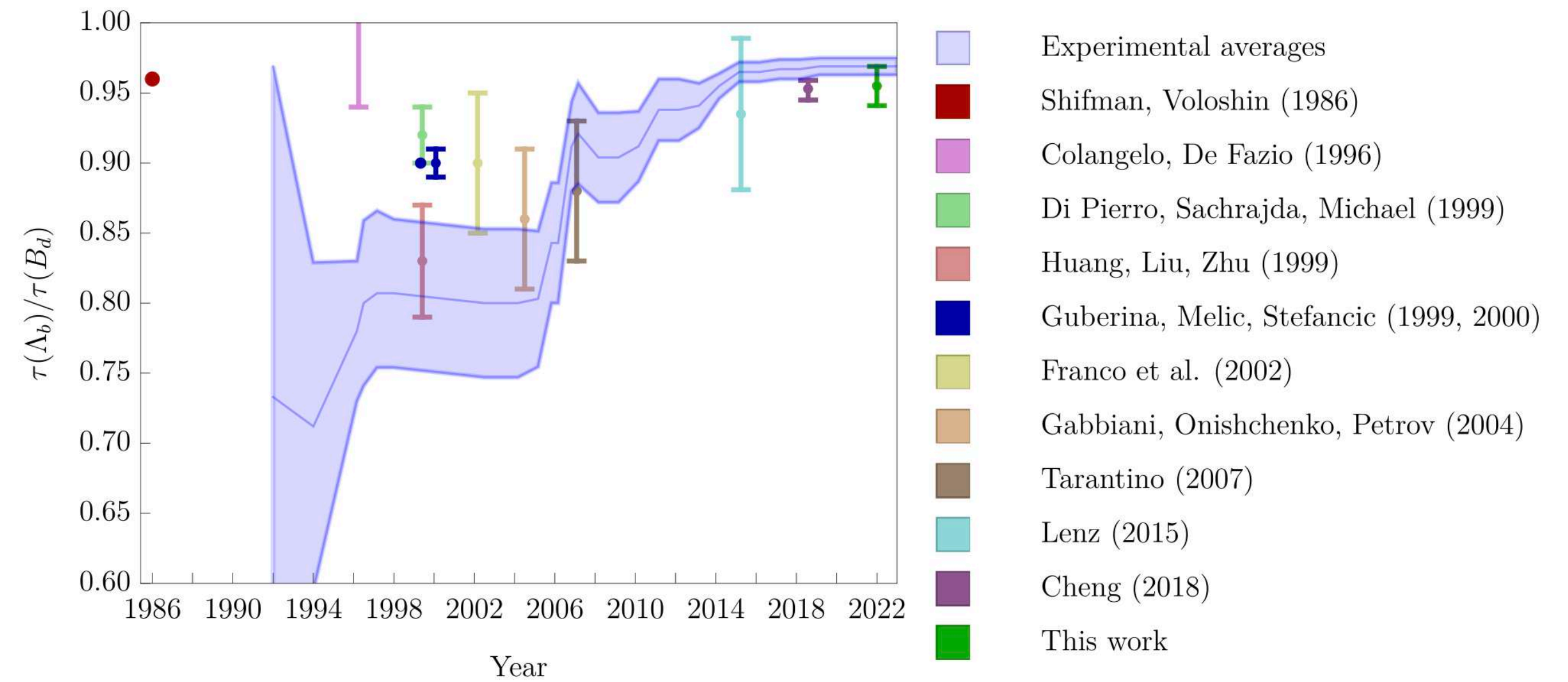
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HFLAV 2023



Disintegration of beauty: a precision study

Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Aleksey V. Rusov (Siegen U.) (Aug 4, 2022)

Published in: JHEP 01 (2023) 004 • e-Print: 2208.02643 [hep-ph]

Project C1c - Experimental Status (Charm):

	D^0	D^+	D_s^+
τ [ps]	0.4101(15)	1.040(7)	0.504(4)
Γ [ps ⁻¹]	2.44(1)	0.96(1)	1.98(2)
$\tau(D_q)/\tau(D^0)$	1	2.54(2)	1.20(1)
$\text{Br}(D_q \rightarrow X e^+ \nu_e)$ [%]	6.49(11)	16.07(30)	6.30(16)
$\frac{\Gamma(D_q \rightarrow X e^+ \nu_e)}{\Gamma(D^0 \rightarrow X e^+ \nu_e)}$	1	0.977(26)	0.790(26)

PDG 2023

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CLEO [5, 7]	179.6 ± 8.2	503 ± 50	N/A	N/A
FOCUS [6, 8, 9, 11]	203.5 ± 4.2	439 ± 24	118_{-13}^{+15}	72 ± 16
SELEX [4, 12]	198.1 ± 9.0	N/A	N/A	65 ± 16^2
LHCb [1, 2]	203.5 ± 2.2	457 ± 6	154.5 ± 2.6	268 ± 26
LHCb 2021 [3]	N/A	N/A	148.0 ± 3.2	276.5 ± 14.1
PDG 2018 [10]	200 ± 6	442 ± 26	112_{-10}^{+13}	69 ± 12
PDG 2020 [21]	202.4 ± 3.1	456 ± 5	153 ± 6	$268 \pm 24 \pm 10$
Reference values	202.4 ± 3.1 [21]	456 ± 5 [21]	152.0 ± 2.0 [3]	274.5 ± 12.4 [3]

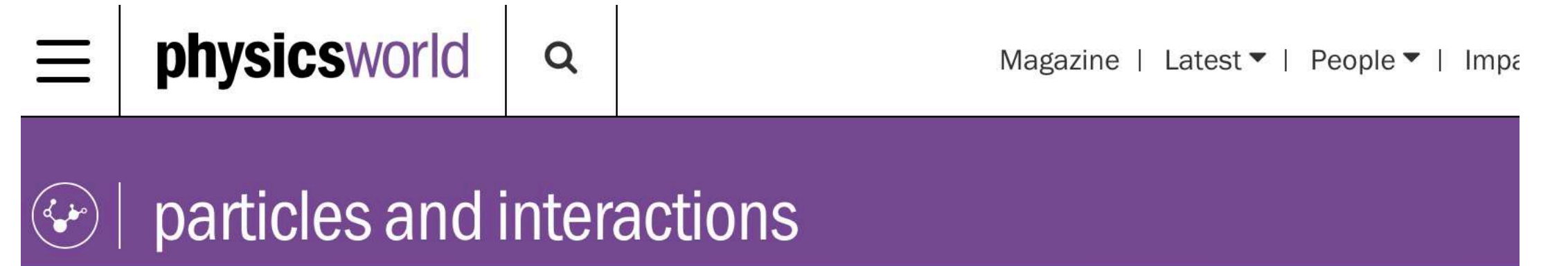
PDG 2023

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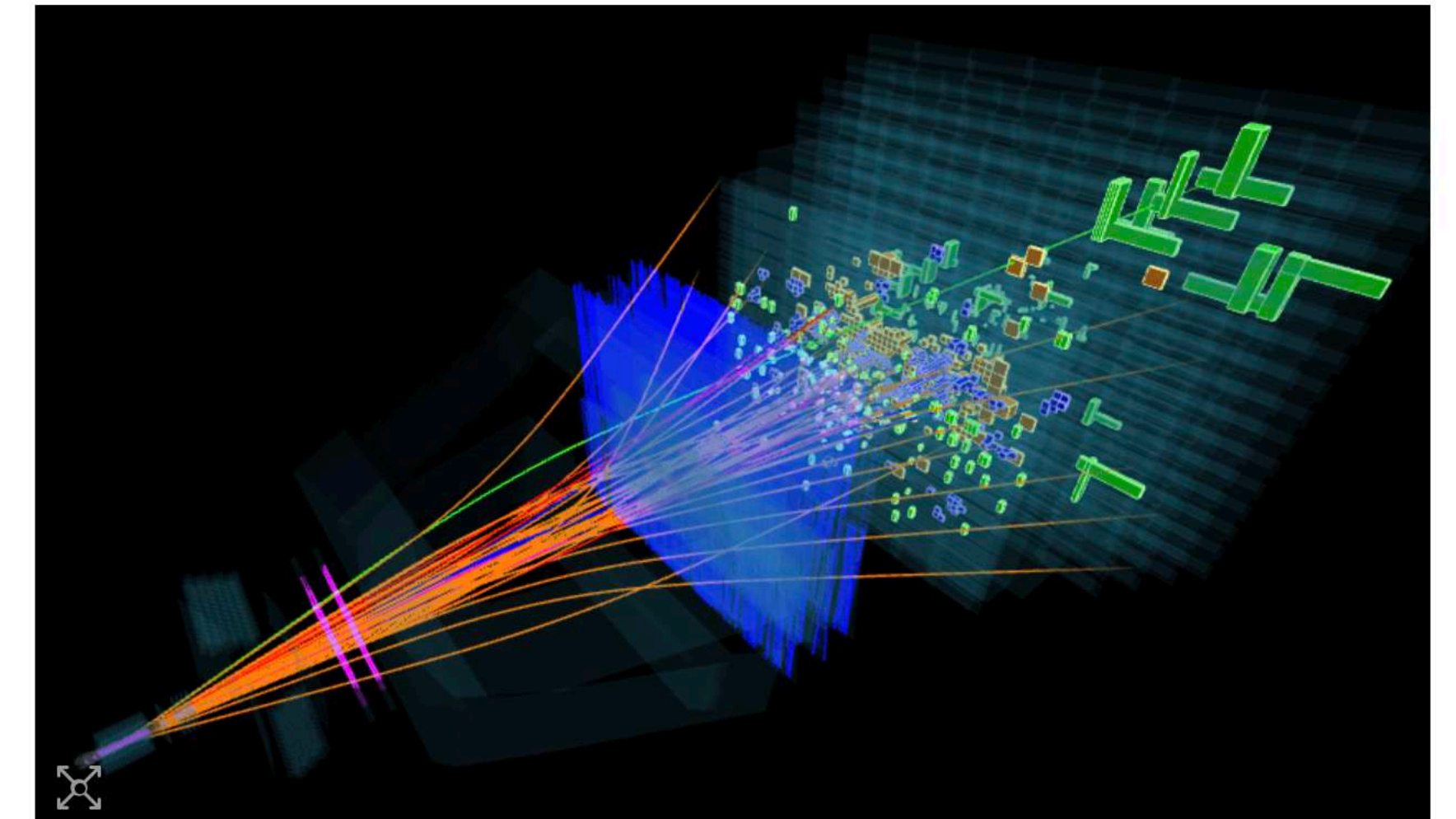
particles and interactions



PARTICLES AND INTERACTIONS | RESEARCH UPDATE

Charmed baryon puzzles particle physicists by living longer

14 Aug 2018



Colliding protons: a collision captured by LHCb. (Courtesy: LHCb/CERN)

The most precise measurement of the lifetime of the Ω_c^0 particle has been made by physicists working on the LHCb experiment at CERN. The charmed baryon decays within femtoseconds after being produced in proton-proton collisions at the Large Hadron Collider

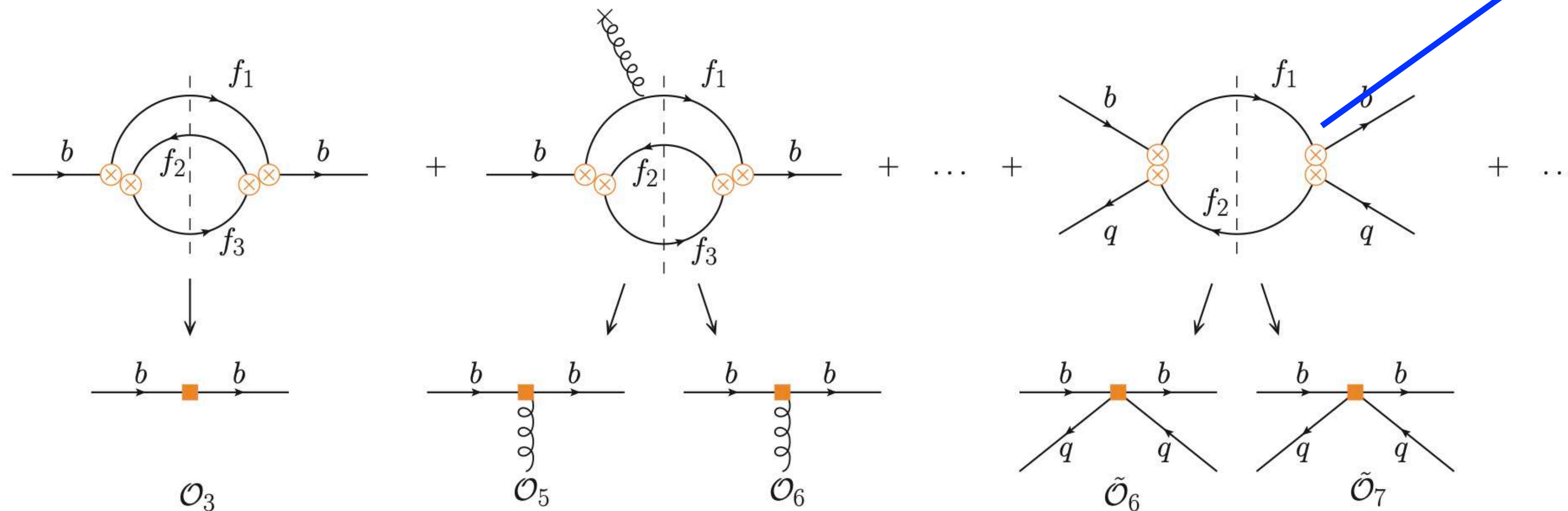
Project C1c - Theoretical Status:

The total decay rate of a B_q meson can be expressed as

$$\Gamma(B_q) = \frac{1}{2m_{B_q}} \sum_X \int_{\text{PS}} (2\pi)^4 \delta^{(4)}(p_B - p_X) |\langle X(p_X) | \mathcal{H}_{\text{eff}} | B_q(p_B) \rangle|^2,$$

$\Delta B = 1$ effective Hamiltonian

Heavy Quark Expansion:
Shifman, Voloshin 1983,...
For history see: [AL 1405.3601](#)



Using the optical theorem this can be rewritten as

$$\Gamma(B_q) = \frac{1}{2m_{B_q}} \text{Im} \langle B_q | \mathcal{T} | B_q \rangle,$$

$$\mathcal{T} = i \int d^4x T \{ \mathcal{H}_{\text{eff}}(x), \mathcal{H}_{\text{eff}}(0) \}.$$

QCD AND HEAVY QUARKS
In Memoriam Nikolai Uraltsev
Ikaros I Bigi • Paolo Gambino • Thomas Mannel editors



Project C1c - Theoretical Status:

The total decay rate of a B_q meson can be expressed as

$$\Gamma(B_q) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \dots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \dots \right),$$

- **Perturbative Wilson coefficients** $\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_i^{(1)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \Gamma_i^{(2)} + \dots,$

Project C1b
Lenz, Nierste, Steinhauser

- **Non-perturbative matrix elements of 2 quark operators** $\langle \mathcal{O}_i \rangle$

- **Non-perturbative matrix elements of 4 quark operators** $\langle \tilde{\mathcal{O}}_i \rangle$

Project C1c
Harlander, Lenz, Nierste

Project C1c - Theoretical Status:

The total decay rate of a B_q meson can be expressed as

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Project C1b
Lenz, Nierste, Steinhauser

Γ_i independent of B meson

- Non-perturbative matrix elements of 2 quark operators $\langle \mathcal{O}_i \rangle$

- Non-perturbative matrix elements of 4 quark operators $\langle \tilde{\mathcal{O}}_i \rangle$

Project C1c
Harlander, Lenz, Nierste

Project C1c - Theoretical Status:

The total decay rate of a B_q meson can be expressed as

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Project C1b
Lenz, Nierste, Steinhauser

Γ_i independent of B meson - $\tilde{\Gamma}_i$ dependent of B meson

- Non-perturbative matrix elements of 2 quark operators $\langle \mathcal{O}_i \rangle$: dependent of B meson

- Non-perturbative matrix elements of 4 quark operators $\langle \tilde{\mathcal{O}}_i \rangle$: dependent of B meson

Project C1c
Harlander, Lenz, Nierste

Project C1c - Theoretical Status Quo

Perturbative semi-leptonic

$\Gamma_3^{(1)}$	1983	HoKim, Pham
$\Gamma_3^{(2)}$	1997 – 2013	Czarnecki, Melnikov, vanRitbergen, Pak, Dowling, Bonciani, Ferroglia, Biswas, Brucherseifer, Caola
$\Gamma_3^{(3)}$	2020 2021	Fael, Schoenwald, Steinhauser Czakon, Czarnecki, Dowling
$\Gamma_5^{(0)}$	1992	Bigi, Uraltsev, Vainshtein, Blok, Shifman
$\Gamma_5^{(1)}$	2013 – 2015	Alberti, Gambino, Nandi, Mannel, Pivovarov, Rosenthal
$\Gamma_6^{(0)}$	1996	Gremm, Kapustin
$\Gamma_6^{(1)}$	2019	Mannel, Pivovarov
$\Gamma_7^{(0)}$	2006	Dassinger, Mannel, Turczyk
$\Gamma_8^{(0)}$	2010	Mannel, Turczyk, Uraltsev

Perturbative non-leptonic

$\Gamma_3^{(1)}$	1983 – 2013	HoKim, Pham, Altarelli, Petrarca, Voloshin, Bagan, Ball, Braun, Goszinsky, Fiol, Lenz, Nierste, Ostermaier, Krinner, Rauh, Greub, Liniger
$\Gamma_3^{(2)}$	2005	partly by : Czarnecki, Slusarczyk
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$\tilde{\Gamma}_6^{(1)}$	2002	Beneke, Buchalla, Greub, Lenz, Nierste Franco, Lubicz, Mescia, Tarantino, Rauh
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Non-perturbative matrix elements

Piscopo 2302.14590

	B_d, B^+	B_s	Λ_b	$\Xi_b^-, \Xi_b^0, \Omega_b^-$
$\langle \mathcal{O}_5 \rangle$	<i>Fits to SL data [26–29]</i> <i>HQET sum rules [30, 31]</i> <i>Lattice QCD [32, 33]</i>	<i>Spectroscopy [35]</i>	<i>Spectroscopy [37]</i>	<i>Spectroscopy [41]</i>
$\langle \mathcal{O}_6 \rangle$	<i>Fits to SL data [26–29]</i> <i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>	<i>Sum rules estimates [35]</i> <i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>	<i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>	<i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>
$\langle \tilde{\mathcal{O}}_6 \rangle$	<i>HQET sum rules [34]</i>	<i>HQET sum rules [36]</i>	<i>HQET SR [38]; NRCQM + spectroscopy [39, 40]</i>	<i>NRCQM + spectroscopy [39, 40]</i>
$\langle \tilde{\mathcal{O}}_7 \rangle$	<i>Vacuum insertion approximation</i>		—	—

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People in
Aachen,
Karlsruhe,
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$\langle \tilde{\mathcal{O}}_7 \rangle$	Vacuum insertion approximation	—	—	—

People in
Aachen,
Karlsruhe,
Siegen

No lattice values

Project C1c - Theoretical Status: Ratios

Lifetime ratios

$$\begin{aligned} \frac{\tau(B^+)}{\tau(B_d)} &= 1 + [\Gamma(B_d) - \Gamma(B^+)] \cdot \tau(B^+) \\ &= 1 + \left[\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle_{B_d} - \langle \mathcal{O}_5 \rangle_{B^+}}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle_{B_d} - \langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \dots + 16\pi^2 \left\{ \tilde{\Gamma}_{6B_d} \frac{\langle \mathcal{O}_6 \rangle_{B_d}}{m_b^3} - \tilde{\Gamma}_{6B^+} \frac{\langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \tilde{\Gamma}_{7B_d} \frac{\langle \mathcal{O}_7 \rangle_{B_d}}{m_b^4} - \tilde{\Gamma}_{7B^+} \frac{\langle \mathcal{O}_7 \rangle_{B^+}}{m_b^4} + \dots \right\} \right] \cdot \tau(B^+) \end{aligned}$$

Project C1c - Theoretical Status: Ratios

Lifetime ratios

$$\frac{\tau(B^+)}{\tau(B_d)} = 1 + [\Gamma(B_d) - \Gamma(B^+)] \cdot \tau(B^+)$$

$$= 1 + \left[\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle_{B_d} - \langle \mathcal{O}_5 \rangle_{B^+}}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle_{B_d} - \langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \dots + 16\pi^2 \left\{ \tilde{\Gamma}_{6B_d} \frac{\langle \mathcal{O}_6 \rangle_{B_d}}{m_b^3} - \tilde{\Gamma}_{6B^+} \frac{\langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \tilde{\Gamma}_{7B_d} \frac{\langle \mathcal{O}_7 \rangle_{B_d}}{m_b^4} - \tilde{\Gamma}_{7B^+} \frac{\langle \mathcal{O}_7 \rangle_{B^+}}{m_b^4} + \dots \right\} \right] \cdot \tau(B^+)$$

- Free quark decay Γ_3 cancels exactly

Project C1c - Theoretical Status: Ratios

Lifetime ratios

$$\frac{\tau(B^+)}{\tau(B_d)} = 1 + [\Gamma(B_d) - \Gamma(B^+)] \cdot \tau(B^+)$$

$$= 1 + \left[\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle_{B_d} - \langle \mathcal{O}_5 \rangle_{B^+}}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle_{B_d} - \langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \dots + 16\pi^2 \left\{ \tilde{\Gamma}_{6B_d} \frac{\langle \mathcal{O}_6 \rangle_{B_d}}{m_b^3} - \tilde{\Gamma}_{6B^+} \frac{\langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \tilde{\Gamma}_{7B_d} \frac{\langle \mathcal{O}_7 \rangle_{B_d}}{m_b^4} - \tilde{\Gamma}_{7B^+} \frac{\langle \mathcal{O}_7 \rangle_{B^+}}{m_b^4} + \dots \right\} \right] \cdot \tau(B^+)$$

- Free quark decay Γ_3 cancels exactly
- Kinetic, chromo-magnetic, Darwin term proportional to isospin violating effects

Project C1c - Theoretical Status: Ratios

Lifetime ratios

$$\frac{\tau(B^+)}{\tau(B_d)} = 1 + [\Gamma(B_d) - \Gamma(B^+)] \cdot \tau(B^+)$$

$$= 1 + \left[\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle_{B_d} - \langle \mathcal{O}_5 \rangle_{B^+}}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle_{B_d} - \langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \dots + 16\pi^2 \left\{ \tilde{\Gamma}_{6B_d} \frac{\langle \mathcal{O}_6 \rangle_{B_d}}{m_b^3} - \tilde{\Gamma}_{6B^+} \frac{\langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \tilde{\Gamma}_{7B_d} \frac{\langle \mathcal{O}_7 \rangle_{B_d}}{m_b^4} - \tilde{\Gamma}_{7B^+} \frac{\langle \mathcal{O}_7 \rangle_{B^+}}{m_b^4} + \dots \right\} \right] \cdot \tau(B^+)$$

- Free quark decay Γ_3 cancels exactly
- Kinetic, chromo-magnetic, Darwin term proportional to isospin violating effects
- Sizable effects expected due to spectator effects: Pauli interference

Project C1c - Theoretical Status: Ratios

Lifetime ratios

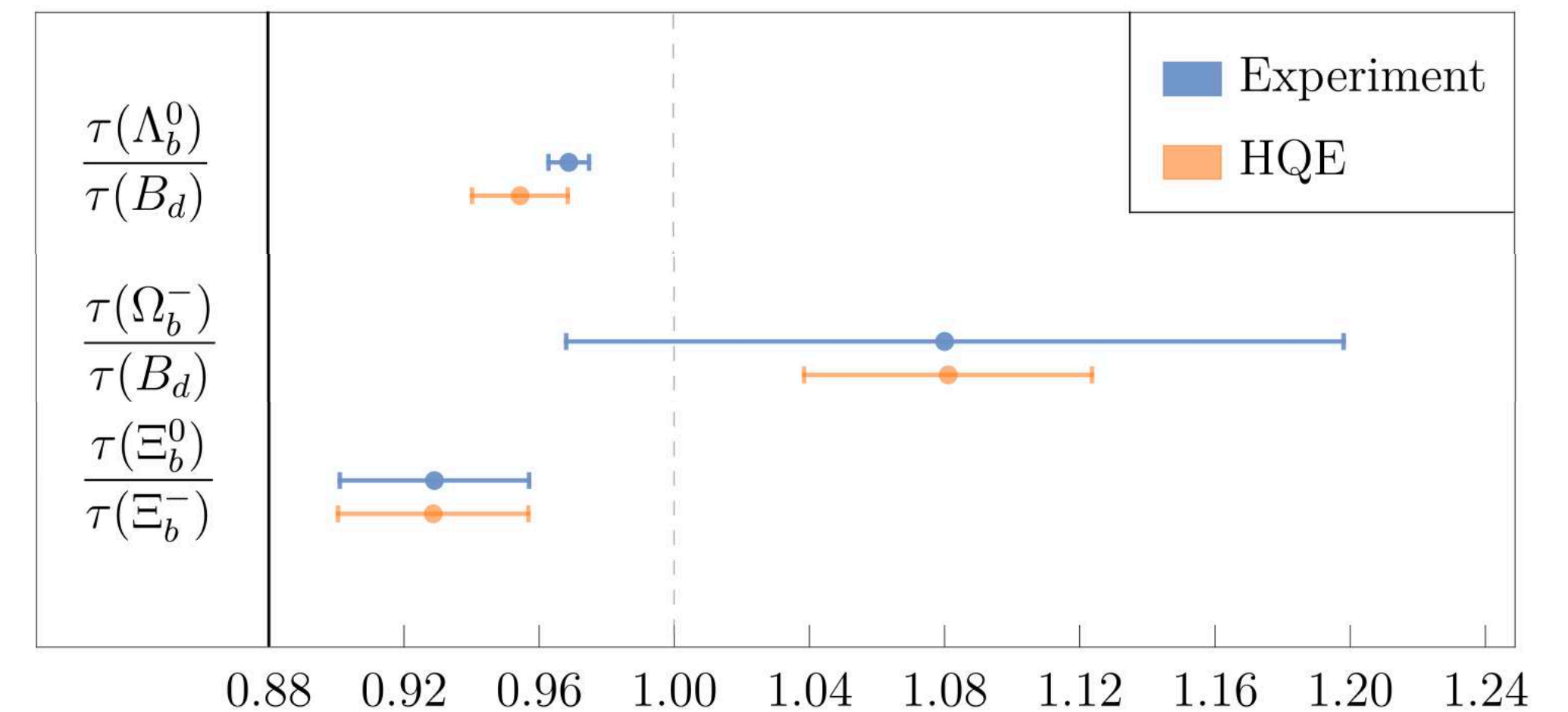
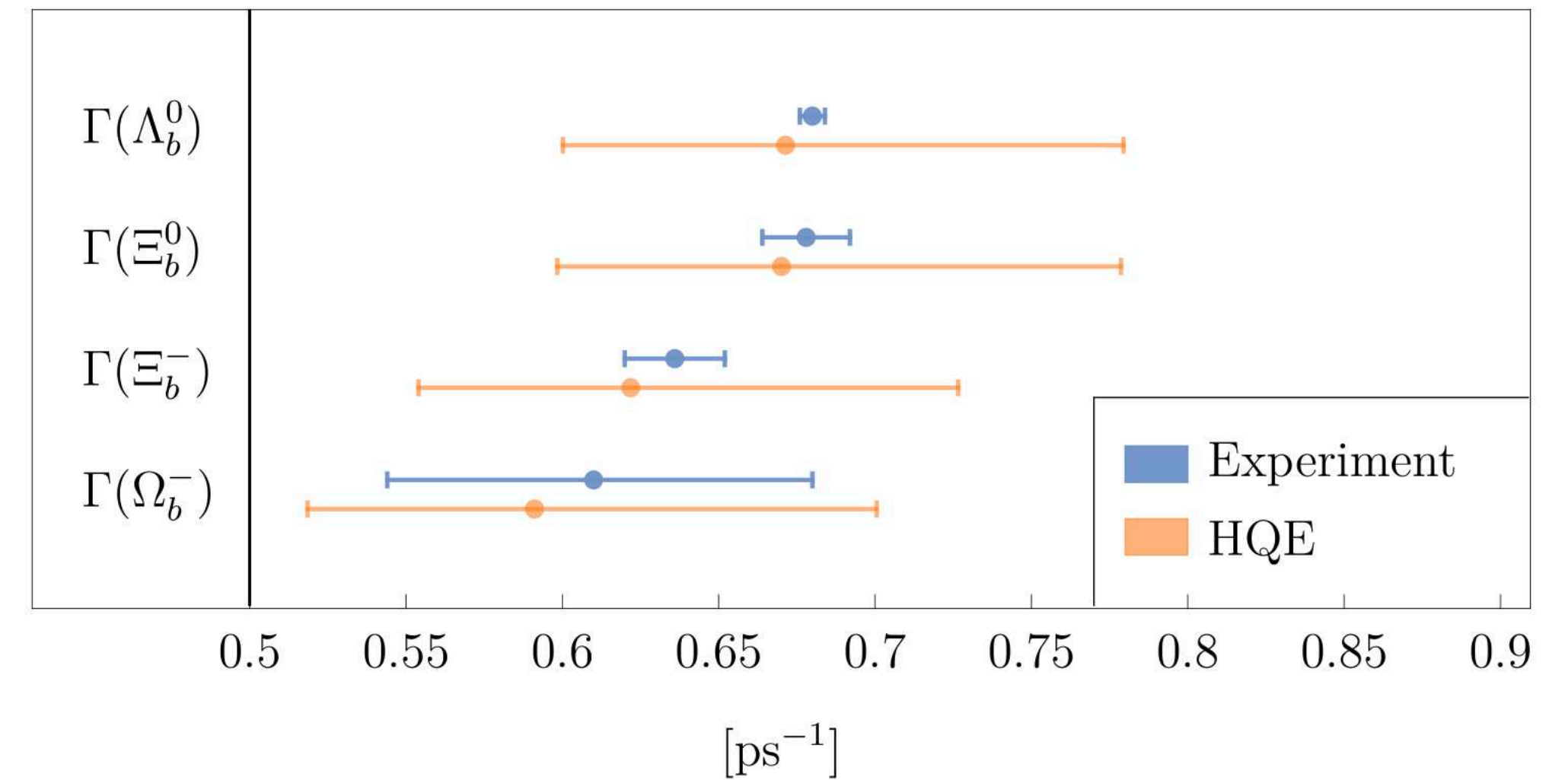
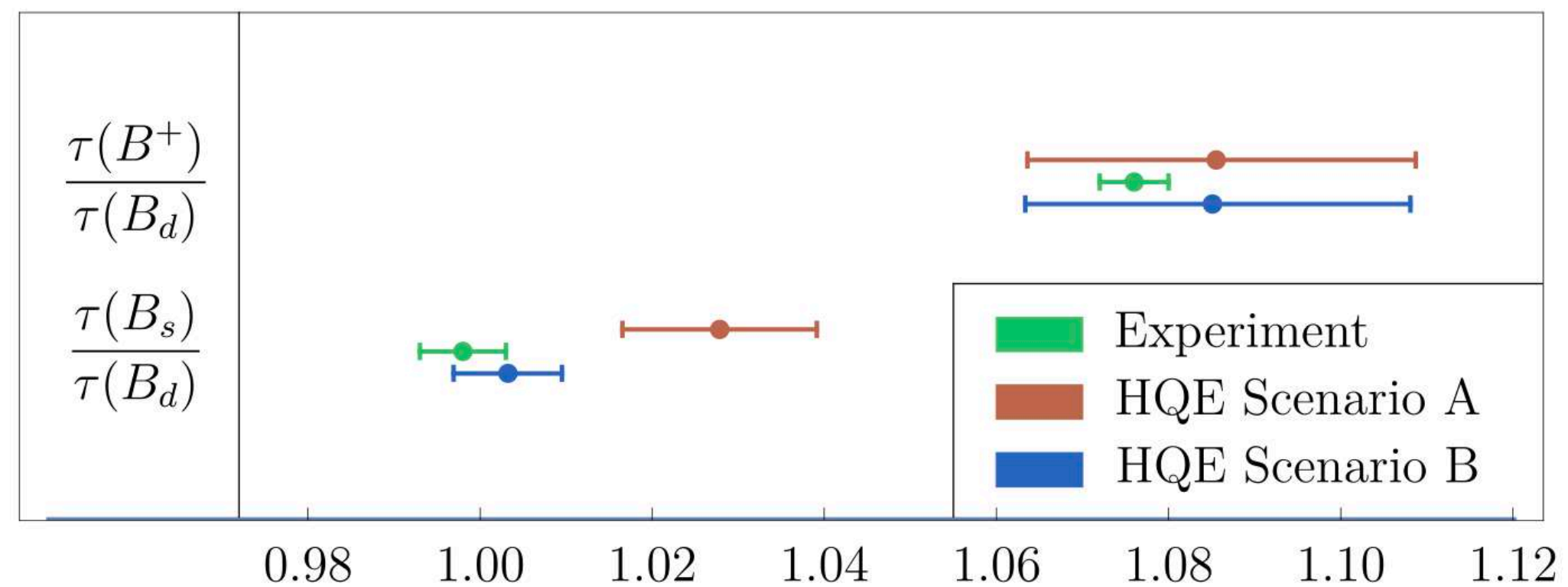
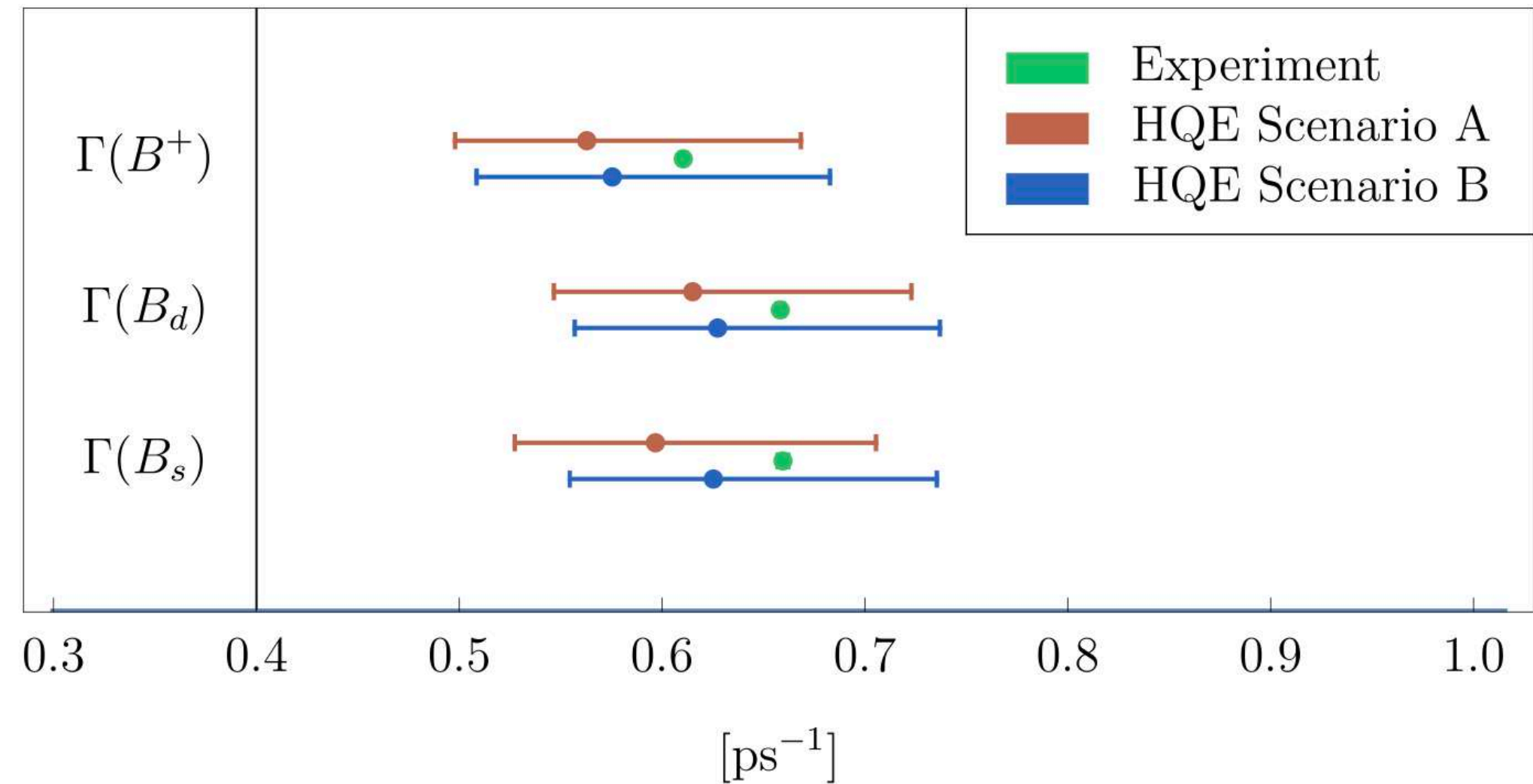
$$\frac{\tau(B^+)}{\tau(B_d)} = 1 + [\Gamma(B_d) - \Gamma(B^+)] \cdot \tau(B^+)$$

$$= 1 + \left[\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle_{B_d} - \langle \mathcal{O}_5 \rangle_{B^+}}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle_{B_d} - \langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \dots + 16\pi^2 \left\{ \tilde{\Gamma}_{6B_d} \frac{\langle \mathcal{O}_6 \rangle_{B_d}}{m_b^3} - \tilde{\Gamma}_{6B^+} \frac{\langle \mathcal{O}_6 \rangle_{B^+}}{m_b^3} + \tilde{\Gamma}_{7B_d} \frac{\langle \mathcal{O}_7 \rangle_{B_d}}{m_b^4} - \tilde{\Gamma}_{7B^+} \frac{\langle \mathcal{O}_7 \rangle_{B^+}}{m_b^4} + \dots \right\} \right] \cdot \tau(B^+)$$

Could be dominant for $\frac{\tau(B_s)}{\tau(B_d)}$

- Free quark decay Γ_3 cancels exactly
- Kinetic, chromo-magnetic, Darwin term proportional to isospin violating effects
- Sizable effects expected due to spectator effects: Pauli interference
- For B_s : large contribution due to Darwin term, $\langle \mathcal{O}_6 \rangle_{B_s}$ unknown! ,
 $\langle \mathcal{O}_6 \rangle_{B_d}$ extracted from inclusive V_{cb} fits - 2 different results! Gambino et al. vs Vos et al.

Project C1c - Theoretical Status: Results for bottom Hadrons



Disintegration of beauty: a precision study

Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Aleksey V. Rusov (Siegen U.) (Aug 4, 2022)

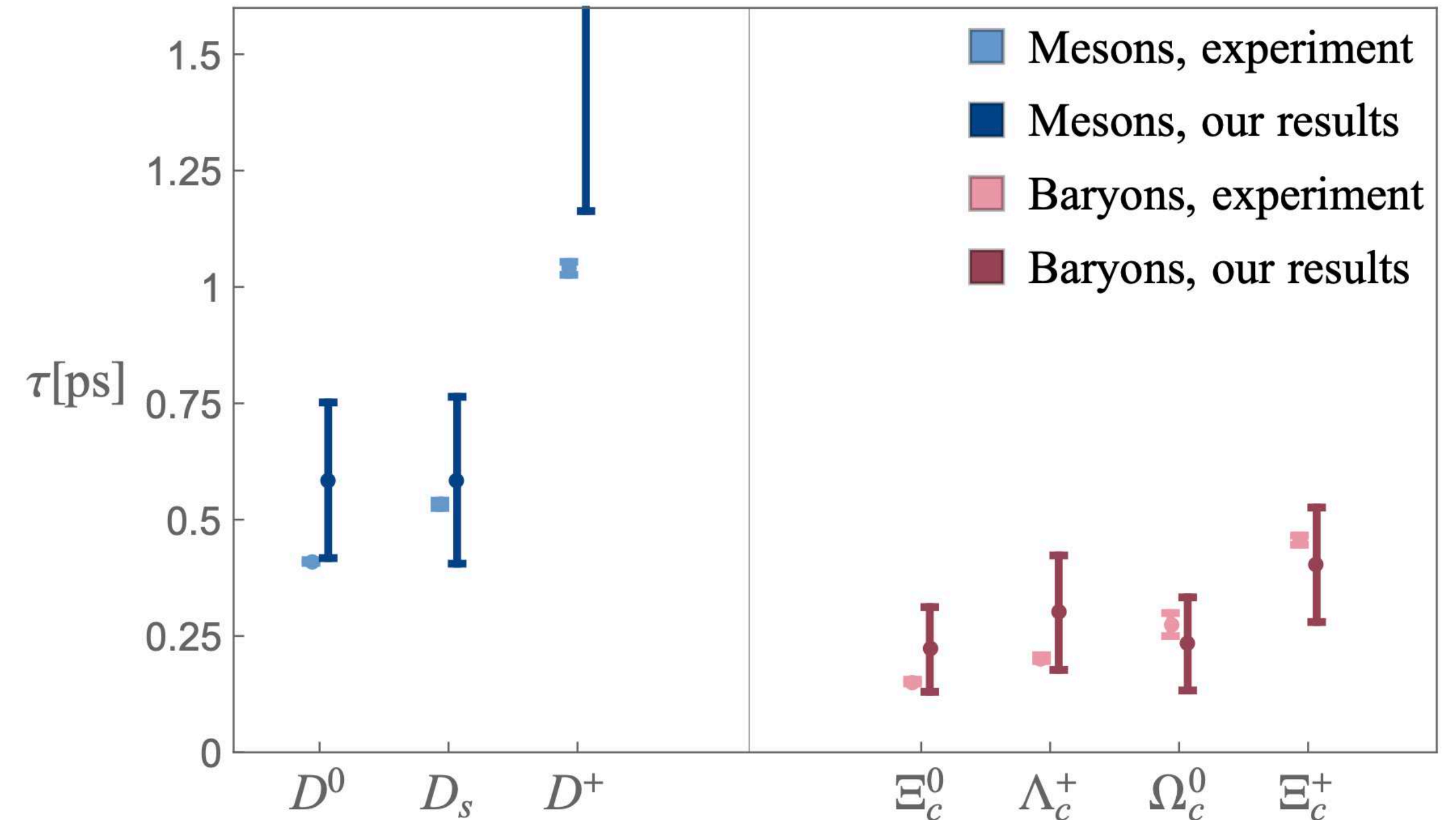
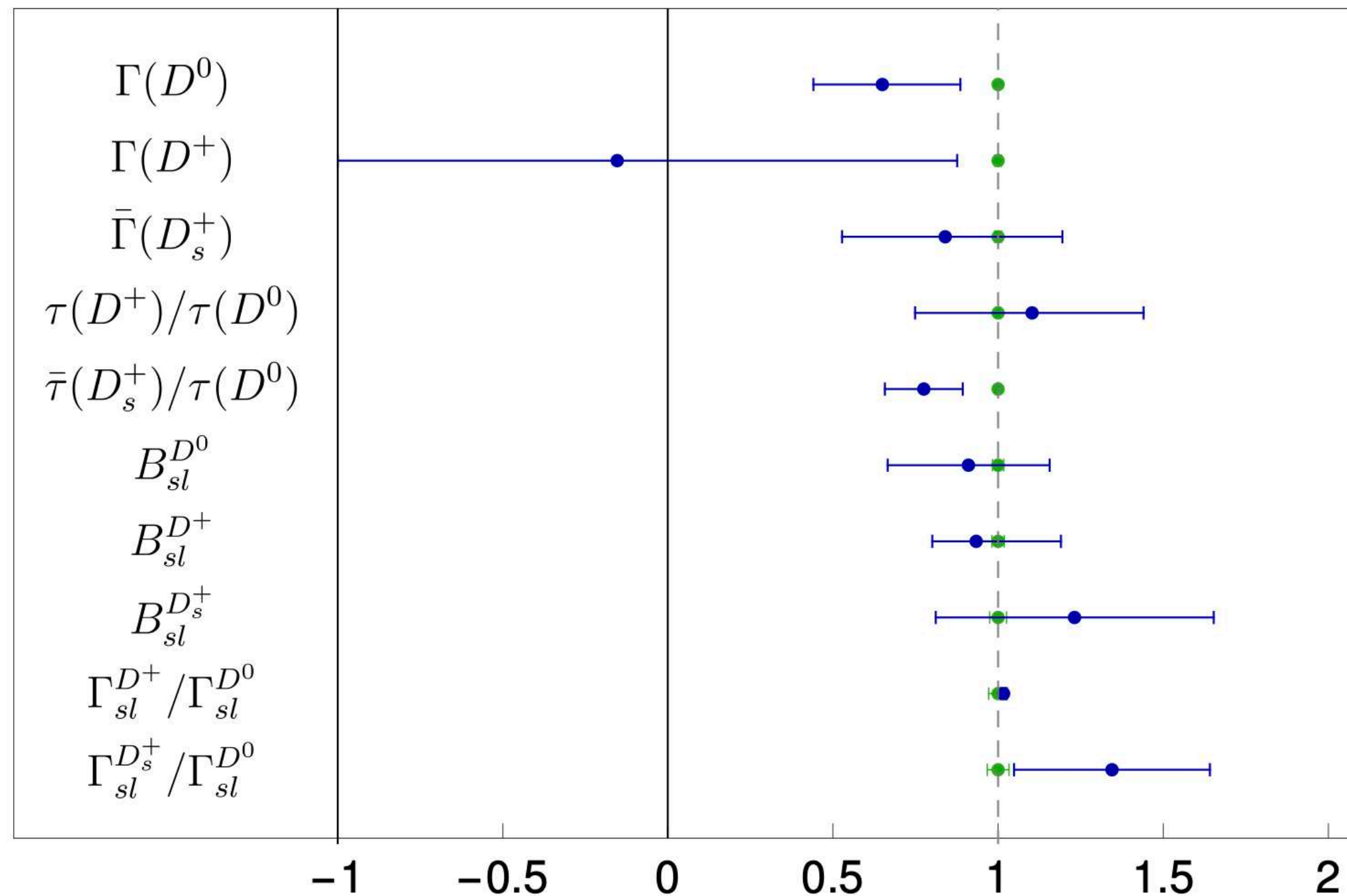
Published in: *JHEP* 01 (2023) 004 • e-Print: [2208.02643](https://arxiv.org/abs/2208.02643) [hep-ph]

Quark-hadron duality at work: lifetimes of bottom baryons

James Gratex (Boskovic Inst., Zagreb), Alexander Lenz (Siegen U.), Blaženka Melić (Boskovic Inst., Zagreb), Ivan Nišandžić (Boskovic Inst., Zagreb), Maria Laura Piscopo (Siegen U.) et al. (Jan 18, 2023)

e-Print: [2301.07698](https://arxiv.org/abs/2301.07698) [hep-ph]

Project C1c - Theoretical Status: Results for charm hadrons



Revisiting inclusive decay widths of charmed mesons

Daniel King (Durham U., IPPP and Durham U.), Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Thomas Rauh (U. Bern, AEC), Aleksey V. Rusov (Siegen U.) et al. (Sep 27, 2021)

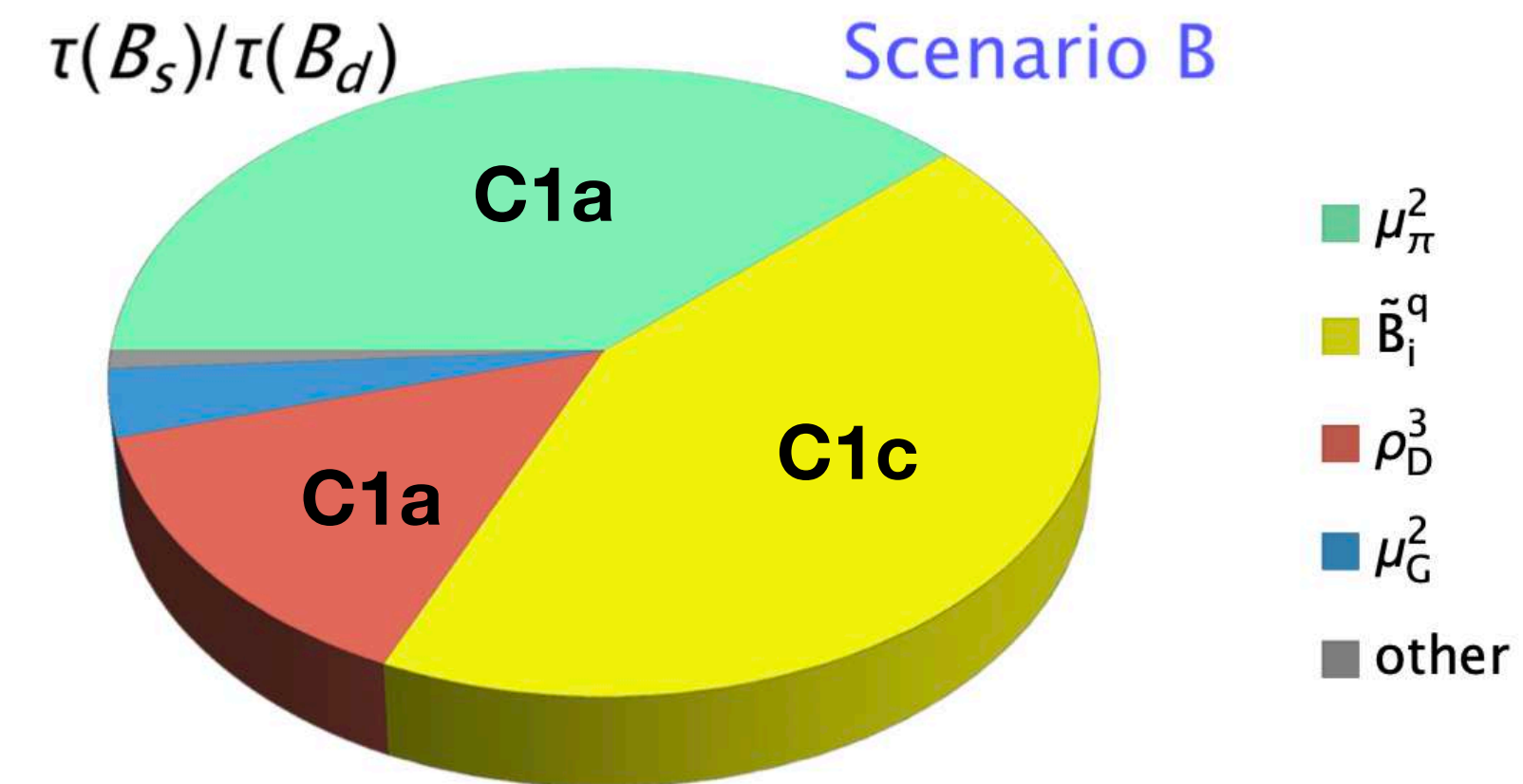
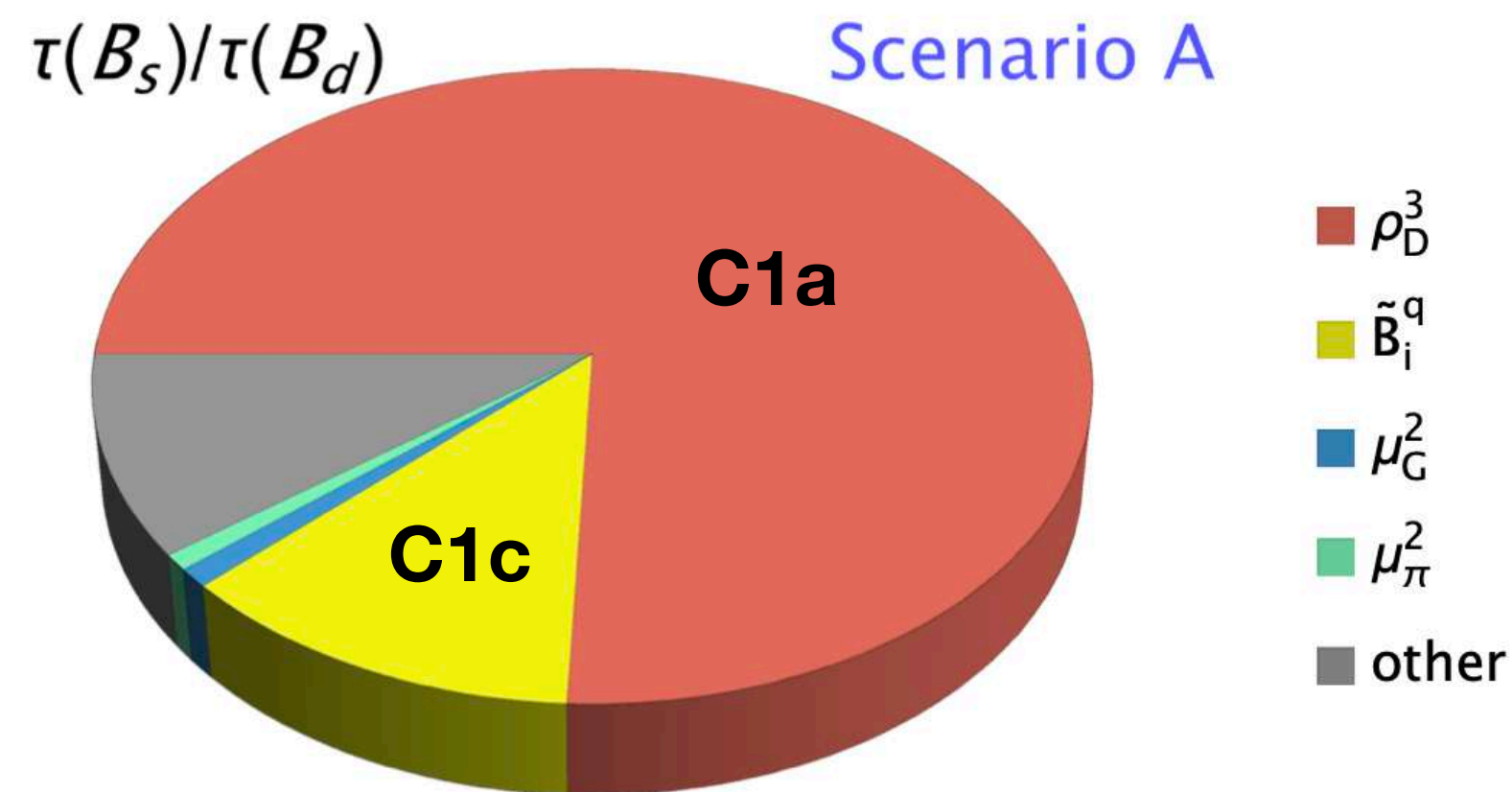
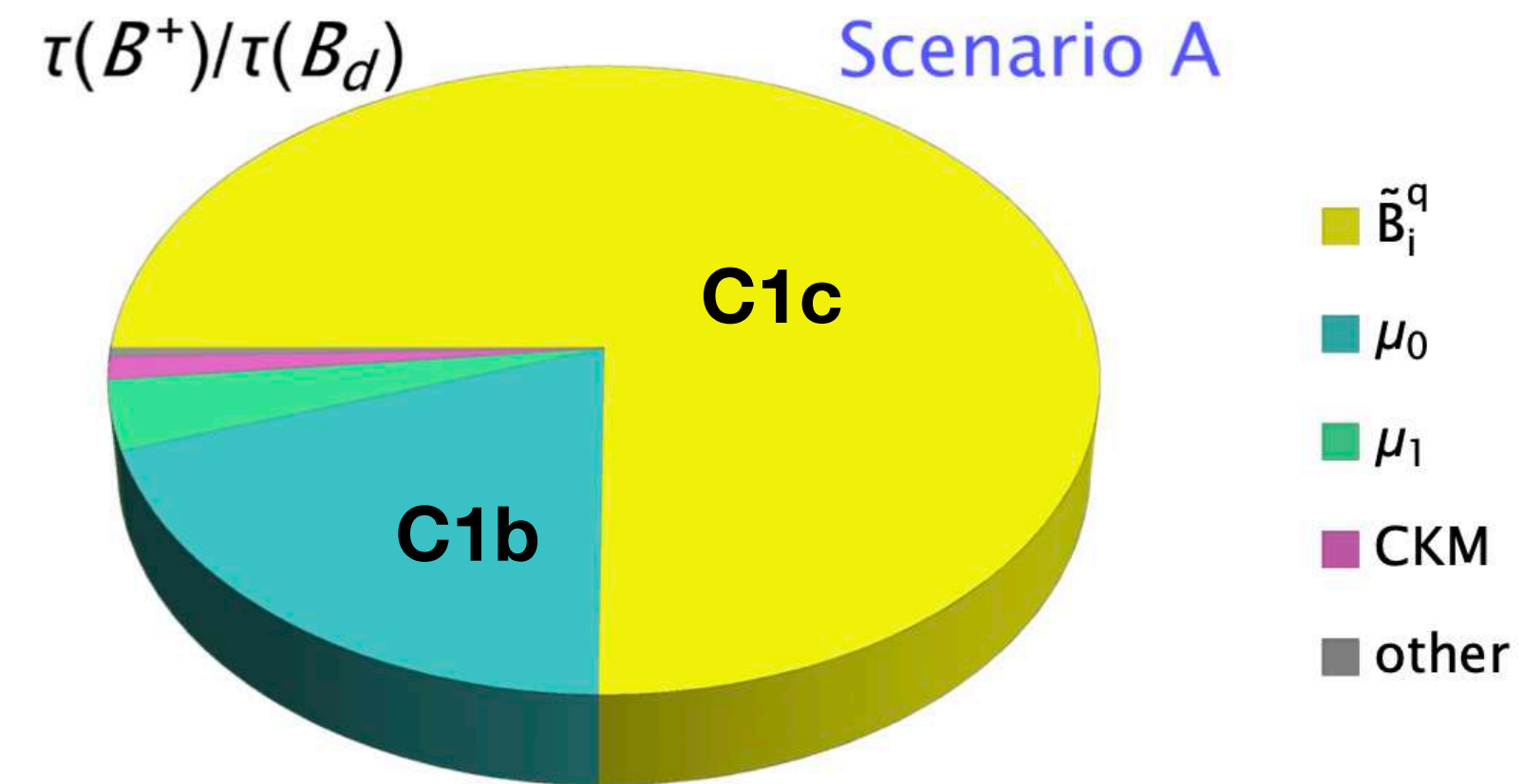
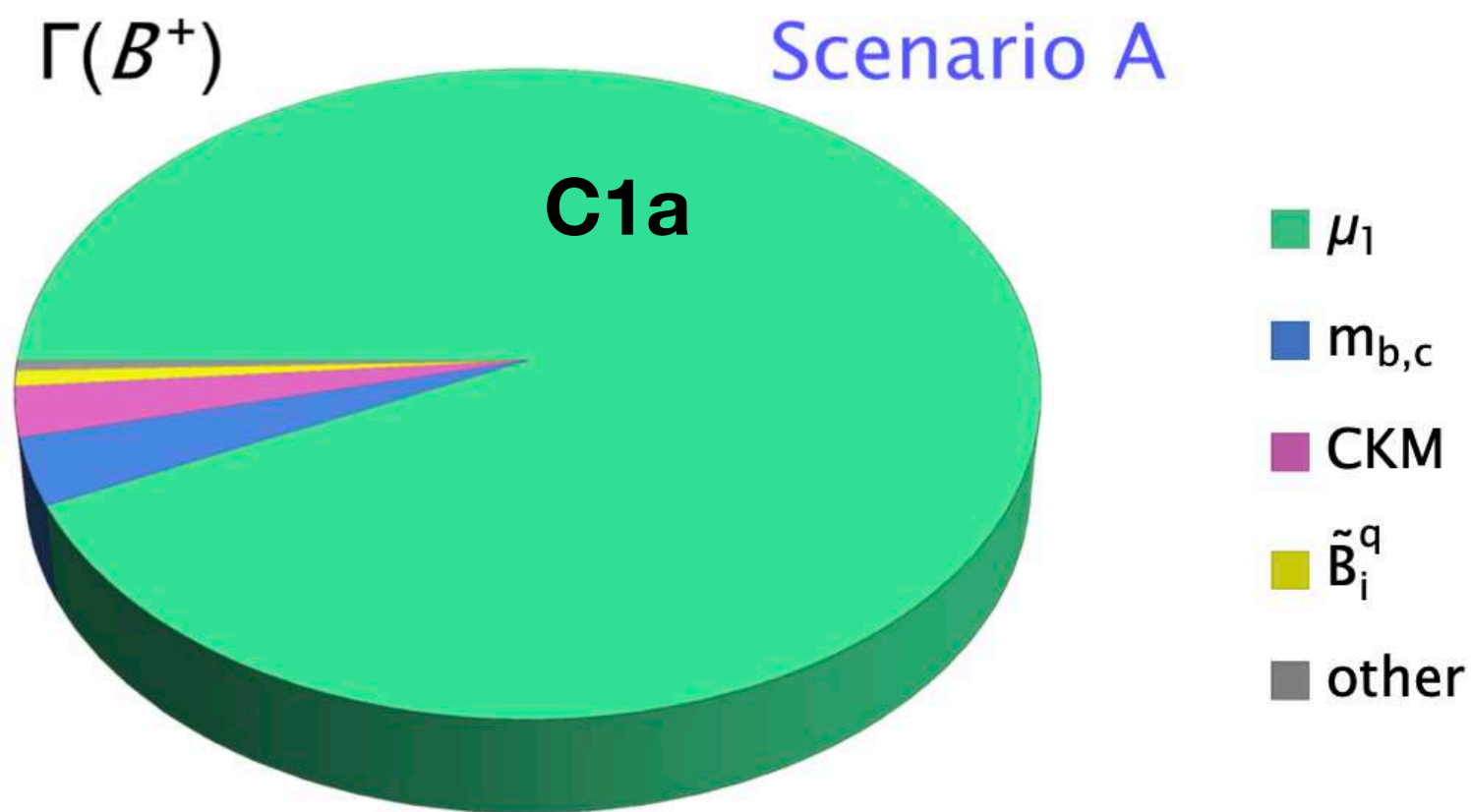
Published in: *JHEP* 08 (2022) 241 • e-Print: [2109.13219](https://arxiv.org/abs/2109.13219) [hep-ph]

Lifetimes of singly charmed hadrons

James Gratx (Boskovic Inst., Zagreb), Blaženka Melić (Boskovic Inst., Zagreb), Ivan Nišandžić (Boskovic Inst., Zagreb) (Apr 25, 2022)

Published in: *JHEP* 07 (2022) 058, *JHEP* 07 (2022) 058 • e-Print: [2204.11935](https://arxiv.org/abs/2204.11935) [hep-ph]

Project C1c: Current theory uncertainties



Lenz, Piscopo, Rusov; 2208.02643

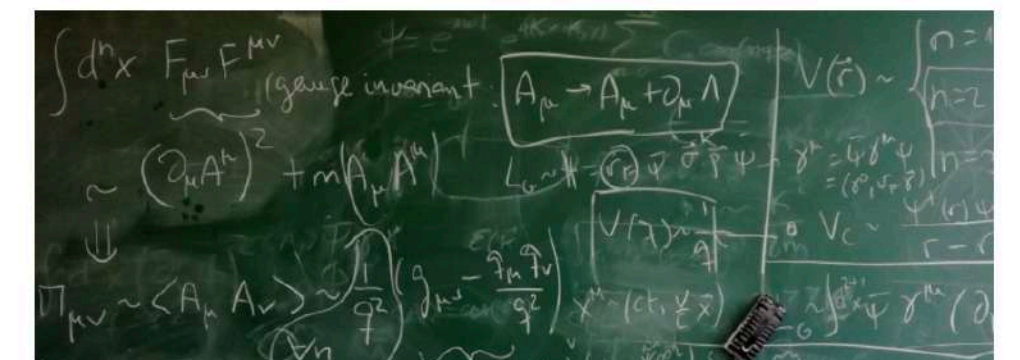
Project C1c - Project plans:

1. Calculation of sum rules for $\langle \tilde{\mathcal{O}}_7 \rangle$ for mixing
2. Calculation of sum rules for $\langle \tilde{\mathcal{O}}_7 \rangle$ for lifetimes
3. Perturbative calculation of flowed matching matrix $\zeta_{nm}(t)$ through NNLO
4. Lattice calculation of flowed matrix elements $\langle \tilde{\mathcal{O}}_n(t) \rangle$
5. Extrapolation to zero flow time
6. Phenomenological studies

Currently
Matthew Black,
Soon Martin Lang

Matthew Black,
Robert Harlander,
Fabian Lange,
Oliver Witzel

THE GRADIENT FLOW IN QCD AND
OTHER STRONGLY COUPLED FIELD
THEORIES



20 March 2023 — 24 March 2023

ECT* - Villa Tambosi

Strada delle Tabarelle, 286
Trento - Italy

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The gradient flow field transformation is a continuous smoothing transformation that removes ultraviolet fluctuations. It can serve as a tool to renormalize quantum field theories, allowing numerical studies of strongly coupled systems. The flow has been used extensively in lattice gauge theory calculations, both in QCD and in beyond the standard model settings for applications including scale setting, the determination of the running coupling constant and the corresponding renormalization group beta function, and the topological structure of the vacuum. Many of the newly emerging applications of the gradient flow depend on the perturbative connection of the gradient flow and continuum renormalization schemes, requiring difficult perturbative calculations that match the non-perturbative lattice methods. This workshop will bring together experts in lattice and perturbative QCD to discuss recent progress in the application of the gradient flow, develop common ideas, identify needs and possibilities for the gradient flow and to spark collaborative efforts.

Organizers

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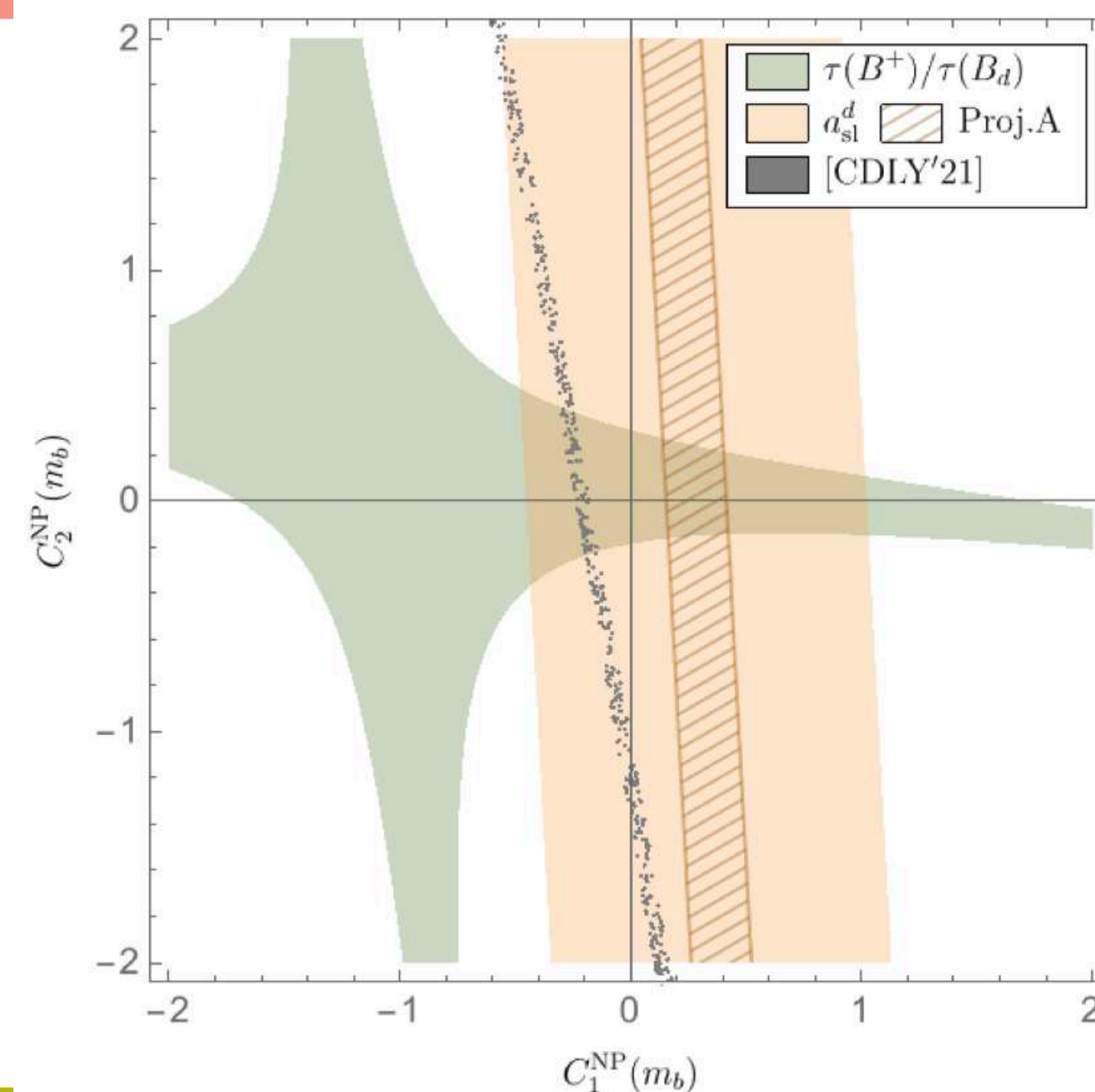
oliver.witzel@uni-siegen.de

Project C1c - Work package 6:

Lifetimes and mixing parameter to test the SM and extensions of it

Improved knowledge of the values of the matrix elements of dimension-6 and -7 four-quark operators will considerably increase the precision of the SM predictions for B meson lifetimes and B mixing parameter, providing crucial tests of the SM. Within the SM such an increased precision can further be used to directly determine CKM parameters involving the top quark, or V_{cb} indirectly. Extensions of the SM can significantly modify the theory predictions for lifetimes and mixing, and we will investigate the impact of our results on different BSM approaches, which are studied within the CRC.

$$\frac{\tau(B^+)^{\text{HQE}}}{\tau(B_d)} = 1 + \left[\Gamma^{\text{SM}}(B_d) - \Gamma^{\text{SM}}(B^+) \right] \tau^{\text{Exp.}}(B^+) + \left[\Gamma^{\text{BSM}}(B_d) - \Gamma^{\text{BSM}}(B^+) \right] \tau^{\text{Exp.}}(B^+)$$



Taming New Physics in $b \rightarrow c\bar{u}d(s)$ with $\tau(B^+)/\tau(B_d)$ and a_{sl}^d #
 Alexander Lenz (Siegen U.), Jakob Müller (Siegen U.), Maria Laura Piscopo (Siegen U.), Aleksey V. Rusov (Siegen U.) (Nov 4, 2022)
 e-Print: 2211.02724 [hep-ph]

Spare slides

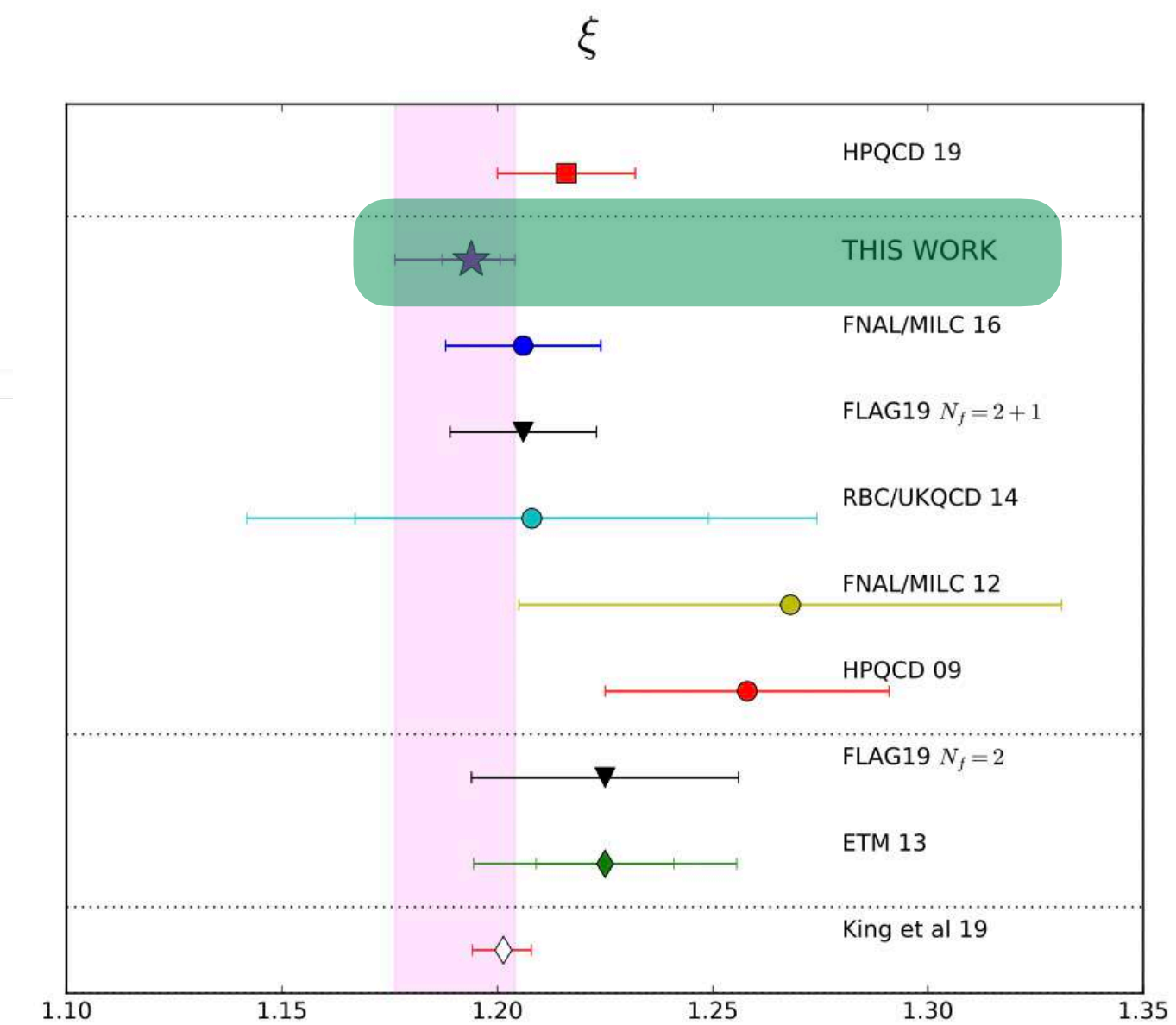
Project C1c - Overview 2:

Previous work of the participating scientists:

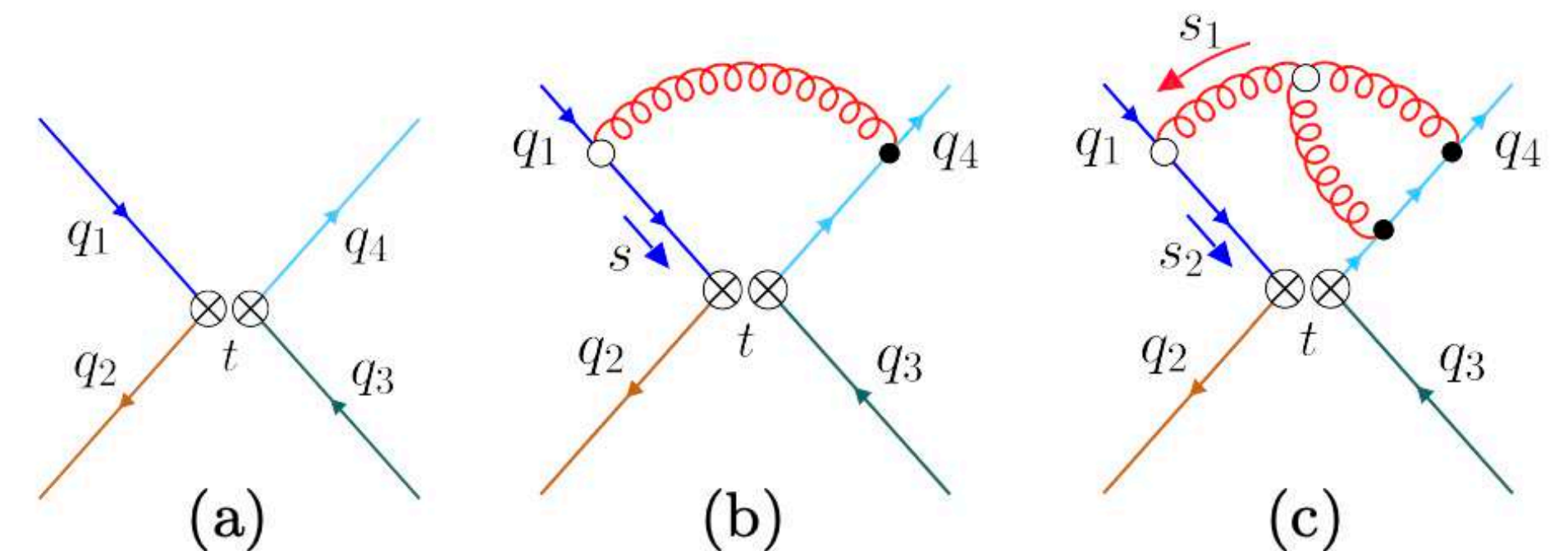
- HQET sum rule: calculation of dimension six matrix elements $\langle \tilde{\mathcal{O}}_6 \rangle$ for B_d and D mixing and for lifetimes of B_d , B^+ , D^0 and D^+ mesons and of m_s corrections to describe B_s mixing and lifetimes and D_s^+ lifetimes



- Lattice: calculation of dimension six matrix elements $\langle \tilde{\mathcal{O}}_6 \rangle$ for B_s mixing within **RBC-UKQCD**



- Gradient flow: software framework for perturbative calculations in the gradient flow formalism / various applications / flowed coefficients of dimension-6 current-current operators

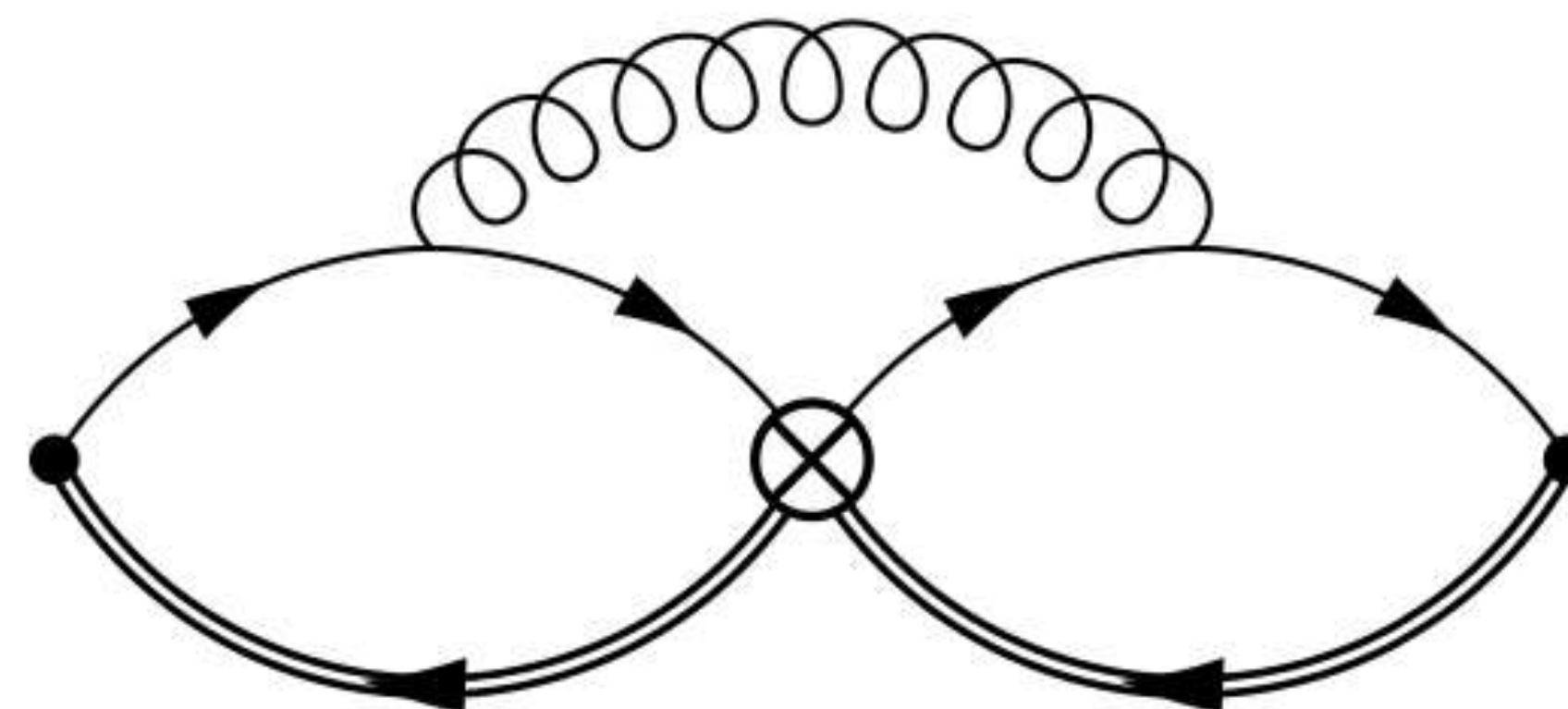


Project C1c - Work package 1:

Sum rule determination of matrix elements of dimension-7 operators for mixing

We will calculate the perturbative three-loop correction to the three-point sum rules with an insertion of the dimension-7 four-quark $\Delta B = 2$ operators. This work proceeds analogously to the calculations performed in (Kirk, Lenz, Rauh, 2017) for dimension-6 operators. First the case of massless spectator quarks will be considered and then extended to massive spectator quarks by expanding in light quark masses, as done in (King, Lenz, Rauh, 2019).

$$\begin{aligned}
 R_1 &= \frac{m_s}{m_b} \bar{s}_\alpha (1 + \gamma_5) b_\alpha \bar{s}_\beta (1 - \gamma_5) b_\beta \\
 R_2 &= \frac{1}{m_b^2} \bar{s}_\alpha \overleftarrow{D}_\rho \gamma^\mu (1 - \gamma_5) D^\rho b_\alpha \bar{s}_\beta \gamma_\mu (1 - \gamma_5) b_\beta \\
 R_3 &= \frac{1}{m_b^2} \bar{s}_\alpha \overleftarrow{D}_\rho (1 + \gamma_5) D^\rho b_\alpha \bar{s}_\beta (1 + \gamma_5) b_\beta
 \end{aligned}$$



Project C1c - Work package 2:

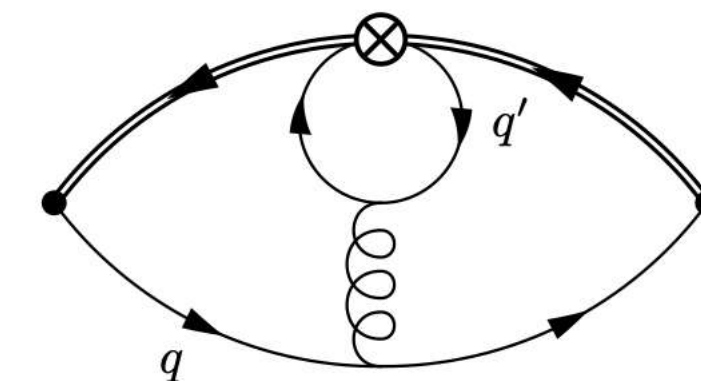
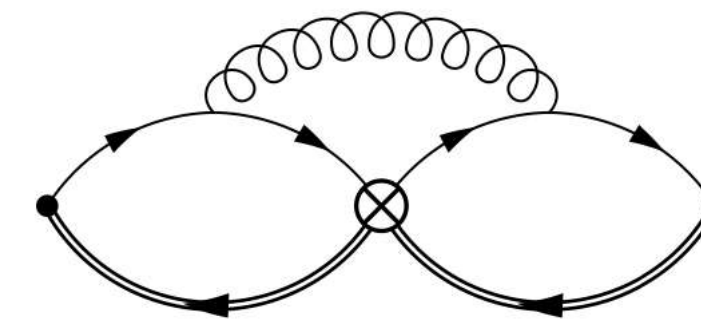
Sum rule determination of matrix elements of dimension-7 operators for lifetimes

For the lifetimes we plan to calculate non-perturbative and perturbative three-loop corrections to the three-point sum rules with an insertion of the dimension-7 four-quark operators. Again, we will start with the massless spectator quark case and later also expand in light spectator quark masses, following the calculations performed for dimension-6 operators (Kirk, Lenz, Rauh, 2017) (King, Lenz, Rauh, 2022). In addition we will study the size of eye-contractions at dimension seven.

$$P_1^q = m_q (\bar{c}(1 - \gamma_5)q)(\bar{q}(1 - \gamma_5)c),$$

$$P_2^q = \frac{1}{m_c} (\bar{c} \overleftarrow{D}_\nu \gamma_\mu (1 - \gamma_5) D^\nu q)(\bar{q} \gamma^\mu (1 - \gamma_5) c),$$

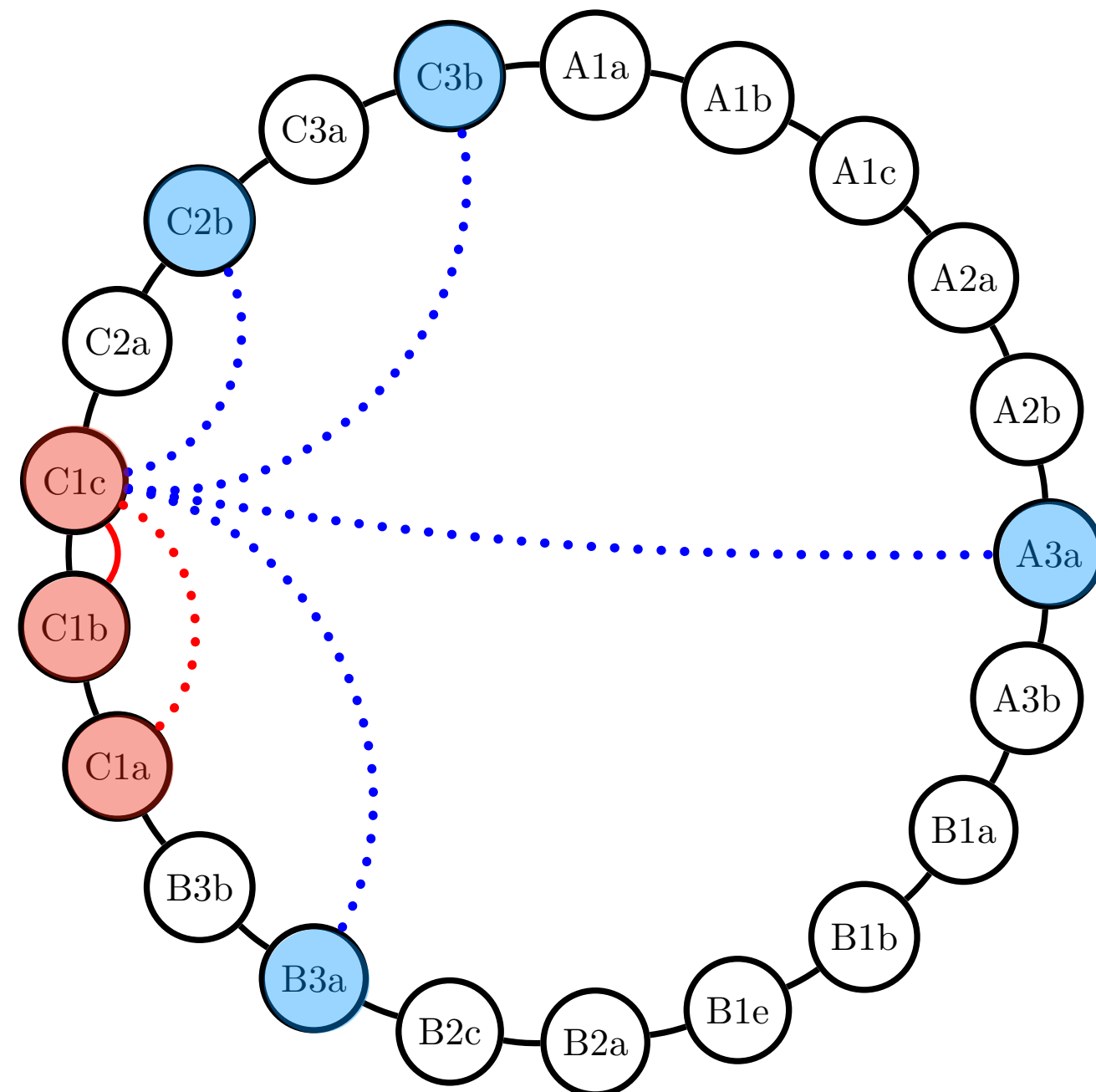
$$P_3^q = \frac{1}{m_c} (\bar{c} \overleftarrow{D}_\nu (1 - \gamma_5) D^\nu q)(\bar{q}(1 + \gamma_5)c),$$



Project C1c - Overview 3:

References

Role within the CRC



- C1b: perturbative Wilson coefficients
- C1a: Parameter of HQE
- A3a, B3a, C3b: BSM models
- C2b: constraints on BSM in non-leptonic decays

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