Solving Beautiful Puzzles

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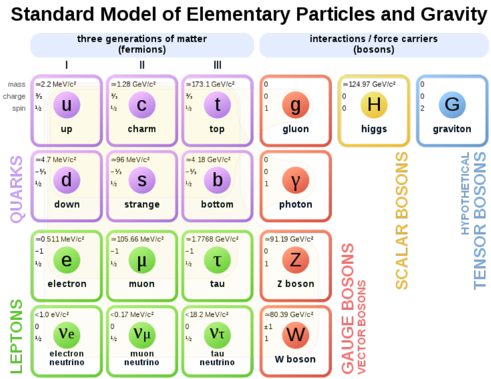

Passed all tests up to 100 GeV

The Flavour Puzzle

- Flavour symmetry broken by Yukawa couplings to the Higgs field
- Origin of mixing between families described by unitary CKM matrix
- Visualized by unitary triangles
- Dominant source of CP violation (antiparticle-particle asymmetry)

$$\begin{pmatrix} \boldsymbol{V_{ud}} & V_{us} & V_{ub} \\ V_{cd} & \boldsymbol{V_{cs}} & V_{cb} \\ V_{td} & V_{ts} & \boldsymbol{V_{tb}} \end{pmatrix}$$

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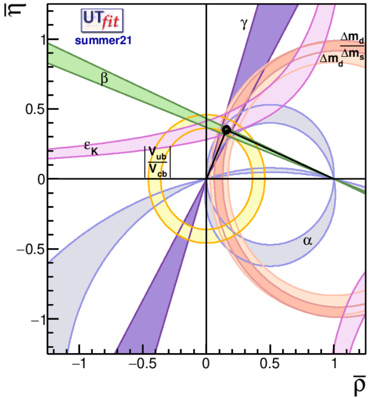
Our understanding of Flavour is unsatisfactory

The Flavour Puzzle

Thanks to Marcella Bona for providing the 2021 plots

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



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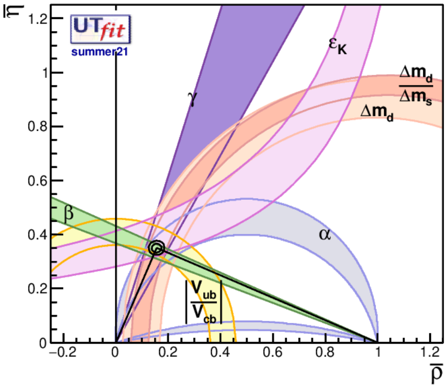

Huge amounts of data + theory advances = Precision frontier

Tiny deviations from SM predictions constrain effects of New Physics

SM or beyond?

Challenge:

Disentangle SM long-distances effects from the effects of new interactions

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- Some puzzles related to B decays

SM or beyond?

Challenge:

Disentangle SM long-distances effects from the effects of new interactions

- Some puzzles related to B decays
- Revise previous assumptions: reliable theory uncertainties
- Look for the cleanest observables/methods

Challenge:

Disentangle SM long-dist… effects fr… e effects … ctions

Puzzles in semileptonic decays

- Inclusive versus Exclusive
- V_{cb} and V_{ub}
- LFUV in R_D and R_{D*}

Puzzles in nonleptonic decays

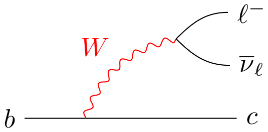
- Missing CP violation
- $B \to \pi K$ puzzle
- $B \to D\pi$ puzzle

Puzzles in rare decays

- Anomalies in $b \to s\ell\ell$

Puzzles in semileptonic decays: V_{ub} and V_{cb}

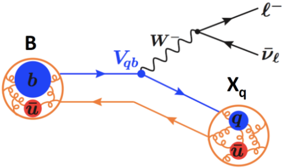
Exclusive versus Inclusive Theory



- Theory (Weak interaction): Transitions between quarks/partons

Exclusive versus Inclusive Theory

Figure from Marzia Bordone



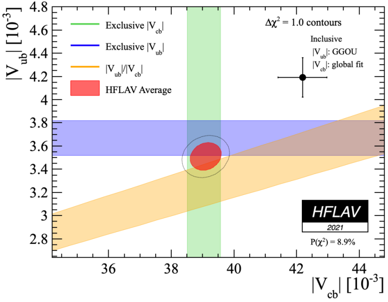
- Theory (Weak interaction): Transitions between quarks/partons
- Observation: Transitions between hadrons

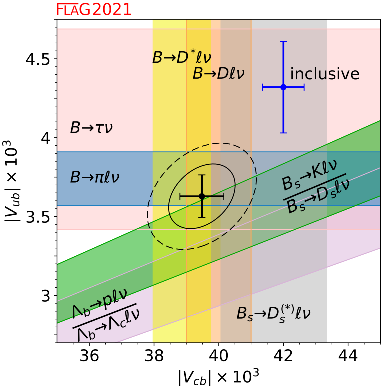
Challenge:

- Dealing with QCD at large distances/small scales
- Parametrize fundamental mismatch in non-perturbative objects
  - Calculable: Lattice or Light-cone sumrules
  - Measurable: from data

Longstanding Puzzle

2021 compilations





Inclusive $B \to X_c$ decays or the power of the heavy quark

Inclusive Decays

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to $\Lambda_{\rm QCD}$
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem → (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B|O_i^{(k)}|B \right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B| \ldots |B\rangle$ non-perturbative matrix elements → string of iD
- operators contain chains of covariant derivatives
  $\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v (iD_\mu) \ldots (iD_{\mu_n}) b_v|B\rangle$

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  - $C_i^{(k)}$ perturbative Wilson coefficients
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  - operators contain chains of covariant derivatives $\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v (iD_\mu) \ldots (iD_{\mu_n}) b_v|B\rangle$

- HQE parameters extracted from lepton energy and hadronic mass moments

Decay rate

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD,iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD,iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\left\langle (M_X^2)^n \right\rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

$$R^*(E_{\rm cut}) = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use lepton energy cut

State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

- Include terms up to $1/m_b^3$* see also Gambino, Healey, Turczyk [2016]
- Recent progress: α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- Recent progress: $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Includes all known α_s, α_s^2 and α_s^3 corrections!

Recent update:

$$|V_{cb}|_{\rm incl} = (42.16 \pm 0.51) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679

Towards the ultimate precision in inclusive V_{cb}

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)}\right) \right.$$
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Challenge:

- Include higher-order $1/m_b$ and α_s corrections
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

Alternative V_{cb} determination

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\, v_\mu = \delta v_\mu \;\text{ and }\; \delta_{RP}\, iD_\mu = -m_b \delta v_\mu$$

  - links different orders in $1/m_b \to$ reduction of parameters
  - up to $1/m_b^4$: 8 parameters (previous 13)

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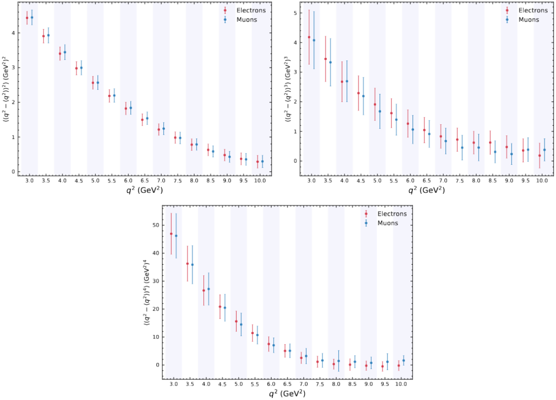
- Caveat: standard lepton energy and hadronic mass moments are not RPI quantities
- Alternative determination using only RPI q^2 moments including $1/m_b^4$
- First measurements of q^2 moments available Belle [2109.01685], Belle II [2205.06372]

q^2 moments

Belle Collaboration [2109.01685, 2105.08001]



Centralized moments as function of $q^2_{\rm cut}$

New V_{cb} Determination

$$R^*(q^2_{\text{cut}}) \qquad \langle (q^2)^n \rangle_{\text{cut}}$$

$$\downarrow$$

$$\mu_3, \mu_G^2, \tilde{\rho}_D^3, r_E^4, r_G^4, s_E^4, s_B^4, s_{qB}^4, m_b, m_c$$

$$\downarrow$$

$$\text{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3}\mu_3 + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D}\frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E}\frac{r_E^4}{m_b^4} + \Gamma_{r_G}\frac{r_G^4}{m_b^4} + \Gamma_{s_B}\frac{s_B^4}{m_b^4} + \Gamma_{s_E}\frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}}\frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb} = (41.69 \pm 0.63) \cdot 10^{-3}$$

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

New V_{cb} determination

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Agreement at the $1-2\sigma$ level
- First pure data extraction of $1/m_b^4$ terms
- Important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\mathrm{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\mathrm{GeV}^4$$

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- Inputs for $B \to X_u \ell \nu$ Next, B lifetimes Alex's project and $B \to X_s \ell\ell$ KKV, Huber, et al.
- Extraction of $\rho_D^3 = 0.03 \pm 0.02$ much smaller than previous!
- In progress: New analysis including all the available data
- In progress: $1/m_c^2 1/m_b^3$ contributions

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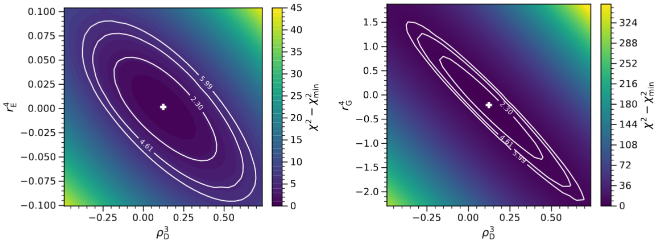

Exclusive V_{cb}

$B \to D$ and $B \to D^*$

- Form factors extracted from lattice, LC sumrules (+data)
- Knowledge on the q^2 dependence crucial
- BGL Boyd, Grinstein,Lebed or CLN/HQE Caprini, Lellouch, Neubert parametrization
  - Start of many discussions Gambino, Jung, Schacht, Bordone, van Dyck, Gubernari, ...
  - BGL: model independent parametrization using analyticity
  - CLN*: uses HQE at $1/m_b$ + assumptions *justified at time of introduction
- Improved HQE treatment including $1/m_c^2$ corrections Bordone, van Dyk, Jung [1908.09398]

$$|V_{cb}|_{\text{excl}} = (40.3 \pm 0.8) \times 10^{-3}$$

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  - BGL: model independent parametrization using analyticity
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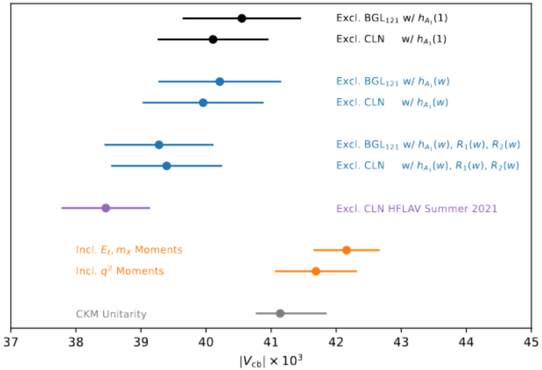
$$|V_{cb}|_{\rm excl} = (40.3 \pm 0.8) \times 10^{-3}$$

- $B \to D^*$ form factors at nonzero recoil Fermilab/MILC [2105.14019]
  - tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D^{(*)}}$
- New experimental and lattice data needed!

Latest Belle analysis [2301.07529]



- Using form factors without shape information:

$$|V_{cb}|_{\text{excl}} = (40.6 \pm 0.9) \times 10^{-3}$$

- Also quotes CLN values $\rightarrow$ shows the 2023 V_{cb} puzzle

The challenge of inclusive $B \to X_u$ decays

The challenge of V_{ub}

Exclusive $B \to \pi \ell \nu$

- Only one form factor
- Combining Lattice QCD [FNAL/MILC, RBC/UKQCD] and QCD sum rules

Recent update:

Leljak, Melic, van Dyk [2102.07233]

$$|V_{ub}|_{\text{excl}} = (3.77 \pm 0.15) \cdot 10^{-3}$$

The challenge of V_{ub}

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- Only one form factor
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Inclusive $B \to X_u \ell \nu$

- Experimental cuts necessary to remove charm background
- Local OPE as in $b \to c$ cannot work
- Switch to different set-up using light-cone OPE
- Introduce non-perturbative shape functions ($\sim$ parton DAs in DIS)
- Different frameworks: BLNP, GGOU, DGE, ADFR

Recent update:

Belle [2102.00020]

$$|V_{ub}|_{\text{incl}} = (4.10 \pm 0.28) \cdot 10^{-3}$$

BLNP update - in progress -

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

$\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
$\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
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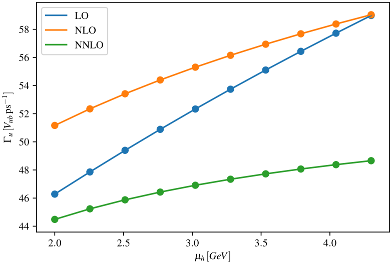
- In progress: include known α_s^2 corrections

Shape function parametrization

Preliminary! Olschewsky, Lange, Mannel, KKV [2304.xxxx]



- α_s^2 corrections give large corrections [see also Pezcjak 2019]
- Required to make precision predictions

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- In progress: include known α_s^2 corrections
- Moments of shape functions can be linked to HQE parameters in $b \to c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \to c$

BLNP update - in progress -

Bosch, Lange, Neubert, Paz [2005]
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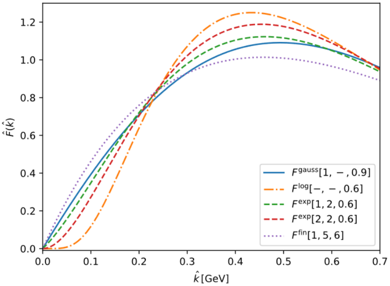
  $\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
  $\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- In progress: include known α_s^2 corrections
- Moments of shape functions can be linked to HQE parameters in $b \rightarrow c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \rightarrow c$
- Shape function is non-perturbative and cannot be computed
  - In progress: new flexible parametrization

Shape function parametrization

Olschewsky, Lange, Mannel, KKV [2304.xxxx]



- All moments of shape functions are linked to HQE parameters
- Allows for a range of different shapes $\rightarrow$ systematic uncertainty

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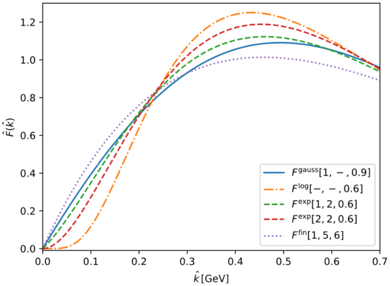

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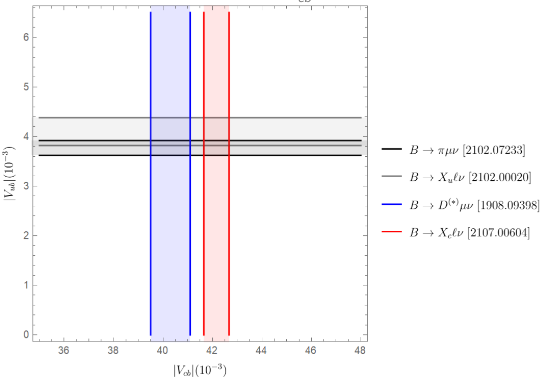
In progress:

Gunawardana, Lange, Mannel, Paz, Olschewsky, KKV [in progress]

$$|V_{ub}|_{\text{incl}} = \quad \text{Stay Tuned!}$$

Inclusive versus Exclusive semileptonic decays

*Illustration of V_{cb} exclusive



- Recently a lot of attention for the V_{cb} puzzle! [Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari]

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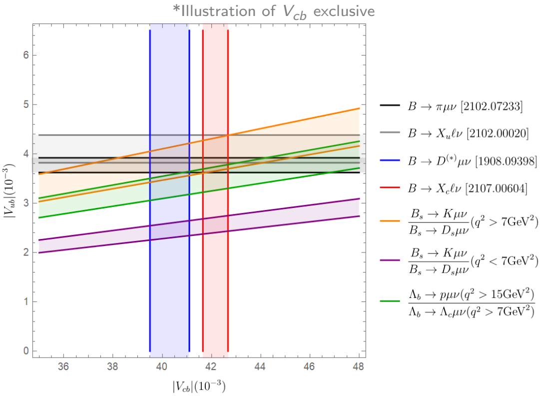
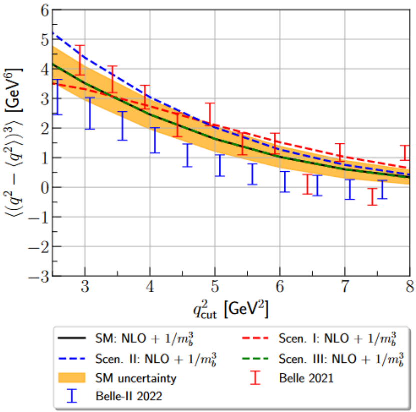


- Recently a lot of attention for the V_{cb} puzzle! [Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari]
- Recent progress: $B_s \to K\mu\nu$ [LHCb [2012.05143], Khodjamirian, Rusov [2017]]
- Unlikely to be due to NP Jung, Straub [2018]
- New data necessary: stay tuned!

New Physics?

Rahimi, Fael, Vos [2208.04282]



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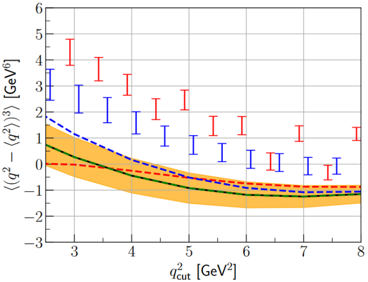

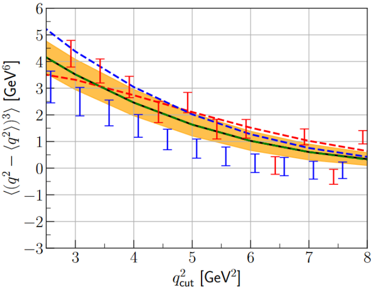

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Heavy quark expansion for charm?

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\mathrm{QCD}}/m_c$ less than unity, but not so small ...

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- Exploit the full physics potential of BES III, LHCb ...
- Lifetimes? $\rightarrow$ Job of Alex and co.
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \rightarrow s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?

Challenges:

- Valence and non-valence WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

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In short: how to handle the charm mass?

The HQE for charm

I: $\boldsymbol{m_Q \sim m_q \gg \Lambda_{\rm QCD}}$ OPE for $b \to c\ell\bar{\nu}$

- q is treated as a heavy degree of freedom
- two-quarks operators: $\bar{Q}_v(iD^\alpha \cdots iD^\beta)Q_v$
- IR sensitivity to mass m_q

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 8\log\rho + \ldots\right]\frac{\rho_D^3}{m_Q^3}, \quad \text{with } \rho = (m_q/m_Q)^2$$

II: $\boldsymbol{m_Q \gg m_q \gg \Lambda_{\rm QCD}}$ start with q dynamical

- four-quark operators $(\bar{Q}_v\Gamma q)(q\bar{\Gamma}Q_v)$

$\rightarrow$ removed when matching onto two-quark operators

- RGE running gives $\log(m_q/m_Q)$

III: $\boldsymbol{m_Q \gg m_q \sim \Lambda_{\rm QCD}}$ OPE for $c \to s\ell\bar{\nu}$

- q dynamical degree of freedom
- four-quark operators remain in OPE
- no explicit $\log(m_q/m_Q)$: hidden inside new non-perturbative HQE parameters

IV: $\boldsymbol{m_Q \gg \Lambda_{\rm QCD} \gg m_q}$ for $b \to u$ and $c \to d$ transitions

HQE for charm revisited

$\rho = m_s^2/m_c^2$

Fael, Mannel, KKV [1910.05234]

$$\frac{\Gamma(D \to X_s \ell \nu)}{\Gamma_0} = \left(1 - 8\rho - 10\rho^2\right)\mu_3 + (-2 - 8\rho)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3} + \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

- RPI quantities (q^2 moments) depend on reduced set
- Data required to test description
- Comparison of extracted HQE parameters with B decays

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Key question: HQE indeed applicable to inclusive charm decays?

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How to handle the charm mass?

Short-Distances Masses

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- $\overline{\mathrm{MS}}$ for scales μ above heavy quark mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_Q(\mu)^{\mathrm{kin}} = m_Q^{\mathrm{Pole}} - \left[\overline{\Lambda}\right]_{\mathrm{pert}} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\mathrm{pert}} + \ldots$$

$$[\overline{\Lambda}]_{\mathrm{pert}} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi}\mu \qquad [\mu_\pi^2]_{\mathrm{pert}} = C_F \frac{\alpha_s(m_c)}{\pi}\mu^2$$

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- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.
- $\Lambda_{\rm QCD} < \mu < m_Q$: expansion parameters μ/m_Q
  - Well established for m_B: $\mu/m_B \simeq 0.2$
  - Charm??
    - $\rightarrow$ $\mu = 1$ GeV $\rightarrow \mu/m_c \simeq 1$
    - $\rightarrow$ $\mu = 0.5$ GeV $\rightarrow \mu/m_c \simeq 0.4$

Kinetic Mass

Putting all power corrections to zero!

- $m_c^{\rm kin}(1 \text{ GeV}) = 1.16 \text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\rm kin} = \Gamma_0 \left[1 + 7.7\frac{\alpha_s(m_c)}{\pi} + 69\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

- $m_c^{\rm kin}(0.5 \text{ GeV}) = 1.4 \text{ GeV}$ ($m_s \to 0$ limit)

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$\mu = 0.5$ GeV touches upon the non-perturbative regime?

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

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- Start from vacuum correlator

$$\int d^4x\, e^{-iqx} \langle 0| T[j_\mu(x) j_\nu(0)] |0\rangle = (g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi(q^2)$$

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- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int\frac{ds}{s^{n+1}}R(s)$$

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- Replace m_c:

$$m_c = \frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/(2n)}$$

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Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344 Boushmelev, Mannel, KKV [2301.05607]

$$\Gamma(b \to u\ell\nu) = \frac{G_F^2|V_{ub}|^2}{192\pi^3}\left(\frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/2}\right)^5\left(1+\frac{\alpha_s(\mu)}{\pi}a_1+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 a_2+\cdots\right)$$

$$= \frac{G_F^2|V_{cs}|^2}{6144\pi^3}\left(\frac{\bar{C}_n^{(0)}}{M_n}\right)^{5/2}\left(1+\frac{\alpha_s(\mu)}{\pi}\left[a_1+\frac{5}{2n}\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}}\right]\right.$$

$$\left.+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\left[a_2+\frac{5}{2n}a_1\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}}+\frac{5}{2n}\frac{\bar{C}_n^{(2)}}{\bar{C}_n^{(0)}}+\frac{5}{4n}\left(\frac{5}{4n}-1\right)\left(\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}}\right)^2\right]+\cdots\right)$$

- Conclusion: pert. series improves a bit
- Scale at which α_s^2 vanishes rather low: 0.7 m_b
- In progress: Similar approach for the charm + power corrections

Solving Beautiful Puzzles

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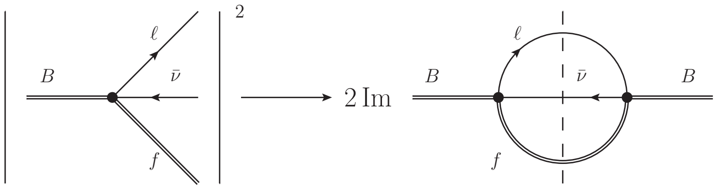
Close collaboration between theory and experiment necessary!

Backup

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$



Optical Theorem

$$\begin{aligned}
\Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2 \\
&= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) | B(v) \rangle \\
&= 2 \, \text{Im} \int d^4x \, e^{-iq\cdot x} \langle B(v) | T \left\{ \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) \right\} | B(v) \rangle
\end{aligned}$$

where $\mathcal{H}_{eff} = J^\mu_c L_\mu, \quad J^\mu_c = \bar{b}\gamma^\mu P_L c$

Inclusive Decays: the OPE

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$

Heavy Quark Expansion

- B meson: $p_B = m_B v$
- Split the momentum b quark: $p_b = m_b v + k$, expand in $k \sim iD\, Q_v$
- Field-redefinition of the heavy field $Q(x) = exp(-im(v \cdot x))Q_v(x)$

$$\begin{aligned}\Gamma \quad &= 2\ \text{Im} \int d^4x\, e^{-iq\cdot x} \langle B(v)| T\left\{\mathcal{H}_{eff}(x)\mathcal{H}^{\dagger}_{eff}(0)\right\}|B(v)\rangle \\ &= 2\ \text{Im} \int d^4x\, e^{i(m_b v - q)\cdot x} \langle B(v)| T\left\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^{\dagger}_{eff}(0)\right\}|B(v)\rangle\end{aligned}$$

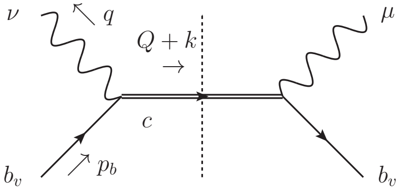
where $\widetilde{\mathcal{H}}_{eff} = \tilde{J}^{\mu}_{c} L_{\mu}, \quad \tilde{J}^{\mu}_{c} = \bar{b}_v \gamma^{\mu} P_L c, \quad \Gamma \propto 2\text{Im}\, T^{\mu\nu} L_{\mu\nu}$

Inclusive Decays: the OPE

$\Gamma(B \to X_c \ell \nu_\ell) \propto 2\text{Im}\, T^{\mu\nu} L_{\mu\nu}$

$$T^{\mu\nu} = i \int dx^4 e^{i(m_b v - q)\cdot x} T\left\{\bar{b}_v(x)\gamma^\mu P_L c(x), \bar{c}(0)\gamma^\nu P_L b_v(0)\right\}$$

$$Q = m_b v - q$$



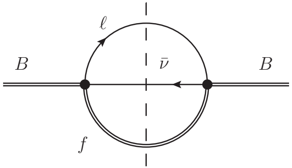
$$= \bar{b}_v \gamma_\mu P_L \left[\frac{i}{\not{Q} + i\not{D} - m_c}\right] \gamma_\nu P_L b_v$$

$$\frac{i}{\not{Q}+i\not{D}-m_c} = \frac{i}{\not{Q}-m_c} + \frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c} + \frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c} + \dots$$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, · · ·

Operator Product Expansion (OPE)

$2\,\mathrm{Im}$



$$= \sum_{n,i} \frac{C_i^{(n)}(\mu,\alpha_s)}{m_b^i} \langle B|\mathcal{O}_i^{(n)}|B\rangle_\mu$$

- $\mathcal{C}_i(\mu)$: short distance, perturbative coeficients
- $\langle B|\mathcal{O}_i|B\rangle_\mu$: non-perturbative forward matrix elements of local operators
- operators contain chains of covariant derivatives

$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\ldots(iD_{\mu_n})b_v|B\rangle$$

HQE for Charm revisited

- The general structure of the expansion for $D \to X_s \ell \bar{\nu}$:

$$d\Gamma = d\Gamma_0 + d\Gamma_{(2,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 + d\Gamma_{(2,2)} \left(\frac{m_s}{m_c}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^3 + d\Gamma_{(4,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 + d\Gamma_{(4,2)} \left(\frac{m_s}{m_c}\right)^4 + \cdots$$

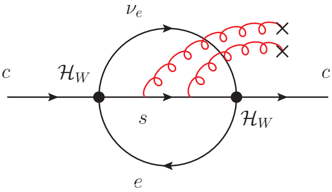
- Expansion parameters:
  - $1/m_c$
  - α_s
  - m_s/m_c

Fael, Mannel, KKV, hep-ph/1910.05234

- Systematic treatment of four-quark operators order by order in $1/m_Q$
- Set up OPE directly for $\Gamma_{\rm tot}$ and $\langle M^{(n)} \rangle$

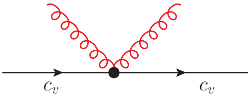
  following the idea in Bauer, Falk, Luke hep-ph/9604290

$2\,\mathrm{Im}$

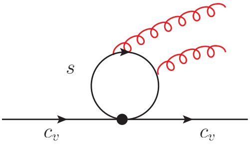


$\longrightarrow$

$C^{2\mathrm{q}}(\mu) \times$



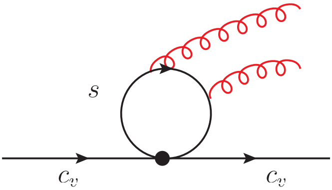
$+ \, C^{4\mathrm{q}}(\mu) \times$



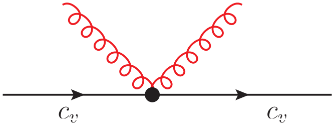
HQE for Charm revisited

- $\log(m_c/m_b)$ in $B \to X\ell\nu$ corresponds to $\log(\mu/m_c)$ in $D \to X\ell\nu$
- caused by mixing of four-quark operators into two-quark operators:

$$C_i^{2q}(\mu) = C_i^{2q}(m_c) + \log\left(\frac{\mu}{m_c}\right) \sum_j \hat{\gamma}_{ij}^T C_j^{4q}(m_c)$$



$\to$



Fael, Mannel, KKV

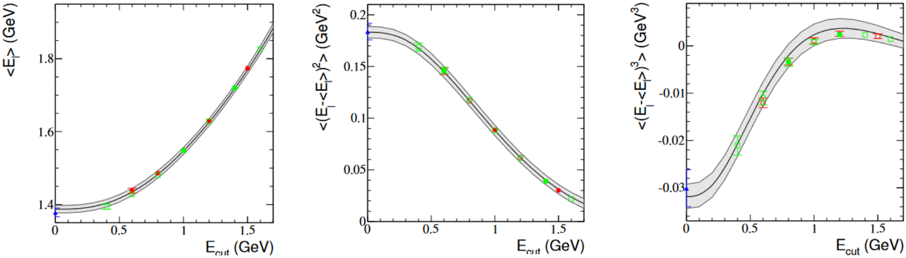
- Additional HQE parameters for $c \to q$: $T_i \equiv \frac{1}{2m_D}\langle D|O_i^{4\mathrm{q}}|D\rangle$
- Up to $1/m_c^3$ only <u>one</u> extra HQE param:

$$\tau_0 = 128\pi^2\left(T_1(\mu) - T_2(\mu) - 2\frac{T_3(\mu)}{m_c} + \frac{T_4(\mu)}{m_c}\right)$$
$$+ \log\left(\frac{\mu^2}{m_c^2}\right)\left[8\tilde{\rho}_D^3 + \frac{1}{m_c}\left(\frac{16}{3}r_G^4 - \frac{16}{3}r_E^4 + \frac{8}{3}s_E^4 - \frac{1}{3}s_{qB}^4 - 12m_s^4\right)\right]$$

- Up to $1/m_c^4$ only <u>two</u> extra HQE params: $\boldsymbol{\tau_m}$ and $\boldsymbol{\tau_\epsilon}$.

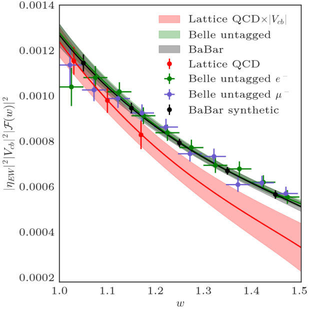
Moments of the spectrum

Gambino, Schwanda Phys. Rev. D 89, 014022 (2014)



$B \to D^*$ form factors

Fermilab-MILC [2105.14019]



- Tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D^{(*)}}$
- New experimental and lattice data needed!

Ratios and isospin sumrules

Beneke, Boer, Toelstede, KKV, JHEP 11 (2020) 081 [2008.10615]

- QED gives sub-percent corrections to Branching ratios

Ratios and isospin sumrules

Beneke, Boer, Toelstede, KKV, JHEP 11 (2020) 081 [2008.10615]

- Beneficial to consider ratios in which QCD is suppressed

$$R_L = \frac{2\mathrm{Br}(\pi^0 K^0) + 2\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- K^0) + \mathrm{Br}(\pi^+ K^-)} = R_L^{\mathrm{QCD}} + \cos\gamma \mathrm{Re}\,\delta_{\mathrm{E}} + \delta_U$$

- new structure dependent QED corrections enter linearly, QCD only quadratically

$$\delta_E = (-1.12 + 0.16i) \cdot 10^{-3}$$

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- Combined QED effect larger than QCD uncertainty!

The V_{cb} puzzle: Inclusive versus Exclusive decays

Exclusive $B \to D^{(*)}\ell\bar{\nu}$

- Form factor required (only for $B \to D$ available at different kinematic points)
- Different parametrizations for form factors: CLN Caprini, Lellouch, Neubert [1997] and BGL Boyd, Grinstein, Lebed [1995]
  - BGL: model independent based on unitarity and analyticity
  - CLN: Simple parametrization using HQE relations
- Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone, Gubernari [2104.02094]

Inclusive $B \to X_c\ell\nu$

- Determined fully data driven including $1/m_b$ power corrections

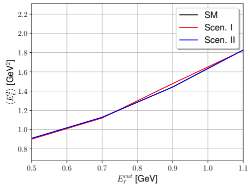
Recently a lot of attention for the V_{cb} puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari

Stay tuned!

Mannel, Rahimi, KKV [in progress]

NP in the τ sector

- Affects also inclusive $B \to X_c \tau \nu$ Rusov, Mannel, Shahriaran [2017]
- Lepton and hadronic moments challenging to measure
- Recently moments of the five-body decay $B \to X_c \tau (\to \mu \nu \nu) \nu$ investigated Mannel, Rahimi, KKV [2105.02163]
- Would also be influenced by NP [in progress]
- Specific NP scenarios from global fit Mandal, Murgui, Penuela, Pich [2004.06726]



Preliminary!

Five-body τ contribution

Rahimi, Mannel, KKV JHEP 09 (2021) 051 [arXiv: 2105.02163];

Contribution from five-body charm decay to $b \to c\ell\nu$ via

$$B(p_B) \to X_c(p_{X_c})(\tau(q_{[\tau]}) \to \mu(q_{[\mu]})\nu_\mu(q_{[\bar{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]}))\bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

:

- Phase space suppressed:

$$\frac{\Gamma_{\rm tot}(b \to c\tau(\to \ell\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau)}{\Gamma_{\rm tot}(b \to c\ell\bar{\nu})} \sim 4.0\%$$

- Experimentally effects diminished by cutting on the invariant mass of the B
- Can be calculated exactly in the HQE

$$\frac{d^8\Gamma}{dq^2\, dq^2_{\nu\bar{\nu}}\, dp^2_{X_c}\, d^2\Omega\, d\Omega^*\, d^2\Omega^{**}} = -\frac{3G_F^2|V_{cb}|^2\sqrt{\lambda}(q^2-m_\tau^2)(m_\tau^2-q^2_{\nu\bar{\nu}})\mathcal{B}(\tau\to\mu\nu\nu)}{2^{17}\pi^5 m_\tau^8 m_b^3 q^2} W_{\mu\nu}L^{\mu\nu}$$

  - $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for τ)
  - $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Shape functions

Bigi, Shifman, Uraltsev, Luke, Neubert, Mannel, $\cdots$

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D)) b_v |B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega \theta(m_b(1-y) - \omega) f(\omega)$$

- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{m_b^3}\delta'''(\omega) + \cdots$$

- Shape function is non-perturbative and cannot be computed

Shape functions

Lange, Neubert, Bosch, Paz

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

$\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
$\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
$\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- Framework to include radiative corrections (+ NNLL resummation)
- Introduces 3 subleading shape functions

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- Other approach: OPE with hard-cutoff μ Gambino, Giordano, Ossola, Uraltsev
  - Use pert. theory above cutoff and parametrize the infrared
  - Different definition of the shape functions
- Shape functions have to be parametrized and obtained from data

New Physics explanation?

- Too many to count: exclusive $B \to D^{(*)}$ in combination with

$$R_{D^{(*)}} = \frac{B \to D^{(*)}\tau\nu}{B \to D^{(*)}\mu\nu}$$

- For inclusive $b \to c$ less analyses
  - RH-current, scalar and tensor NP contributions to rate Jung, Straub [2018]
  - RH-current to moments Feger, Mannel, et. al. [2010]
  - NP for moments KKV, Fael, Rahimi [in progress]