Standard Model precision calculations for the LHC

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Based on: [arXiv:2211.05722], [arXiv:1910.10059], [arXiv:2103.02671], [arXiv:2009.10386]
in collaboration with:
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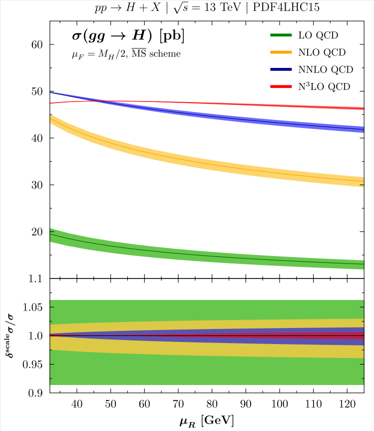
Precision matters

Colour-singlet production at N^{3}LO



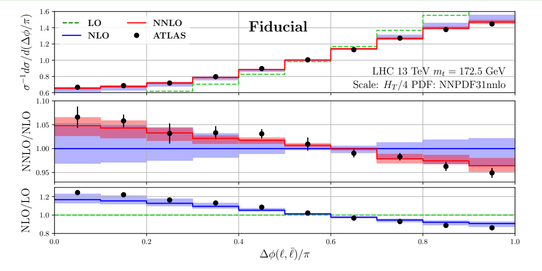
[Baglio, Duhr, Mistlberger, Szafron '22]

- Encompasses many interesting processes at the LHC: Higgs production, Drell-Yan, *VH* production, vector boson pair production, …
- Some results for inclusive cross sections available at N^{3}LO in QCD
- First differential distributions were published
- Single Higgs production is famous for large perturbative corrections

Precision matters

Colour-singlet production at N^{3}LO

$t\bar{t}$ spin correlations



[AB, Czakon, Mitov, Papanastasiou, Poncelet '19]

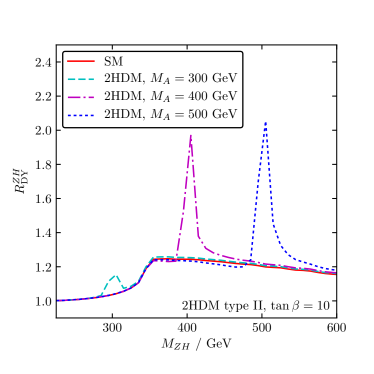
- $t\bar{t}$ pairs have spin correlations
- Effects can be measured via decay products
- In 2018 ATLAS had found slight deviations from SM
- NNLO corrections improved agreement with the data

Precision matters

Colour-singlet production at N^{3}LO

$t\bar{t}$ spin correlations

BSM searches with WH/ZH



[Harlander, Klappert, Pandini, Papaefstathiou '18]

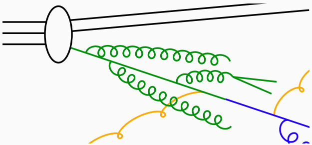
- WH and ZH production very similar in the Standard Model
- Main difference: loop-induced $gg \to ZH$ channel only exists for ZH
- Very good theoretical control over the ratio ZH/WH allows to search for new physics

- NNLO corrections improved agreement with the data

Focus of this talk

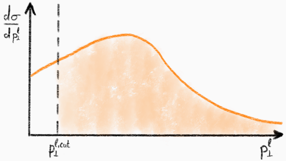
Beam functions for N-jettiness at N^{3}LO

- Important building block for differential distributions at N^{3}LO in QCD
- CRC 257: Their calculation was an objective of project B1a



Mixed QCD-EW corrections and the W boson mass

- m_W is measured to very high precision at hadron colliders
- Previously negligible effects can become important
- Assess the impact of $O(\alpha\alpha_s)$ corrections on m_W measurements.



Beam functions for N-jettiness

What is N-jettiness?

Observable to describe: “How much does an event look like an N jet event?”

$\tau_N = 0$

Yes, this looks like an N jet event.

$\tau_N > 0$

There are more than N jets in this event.

What is N-jettiness?

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Yes, this looks like an N jet event.

$\tau_N > 0$

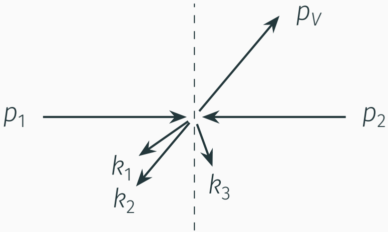
There are more than N jets in this event.

Example: Zero-jettiness (N=0)

Definition: $\tau_0 = \sum_j \min_{i \in \{1,2\}} \frac{p_i \cdot k_j}{Q_i}$

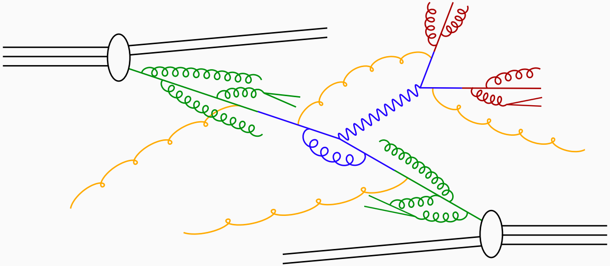
Q_i: Normalisation scales

$p_i \cdot k_j = p_i^0 k_j^0 (1 - \cos\theta_{ij})$

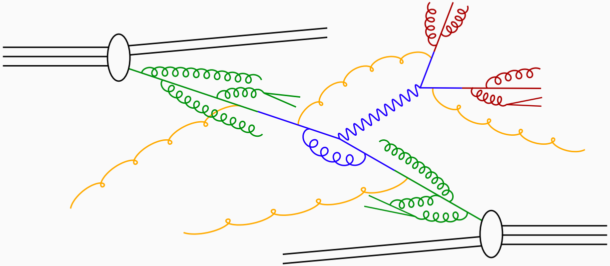


Zero-jettiness τ_0 vanishes if *all* real emissions become soft or collinear with initial state momenta

What are beam functions?



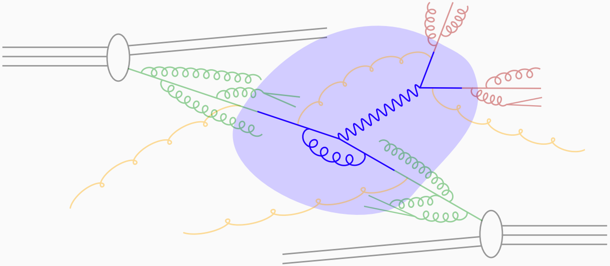
What are beam functions?



Factorisation theorem: $\mathrm{d}\sigma \sim B \otimes B \otimes S \otimes H \otimes \mathrm{d}\sigma_{\mathrm{LO}} \otimes J \otimes J$

- Factorisation often occurs if kinematics become Born-like
- Enforced by limits of suitable observables
    - q_T of color-singlet system
    - N-jettiness
    - …

What are beam functions?

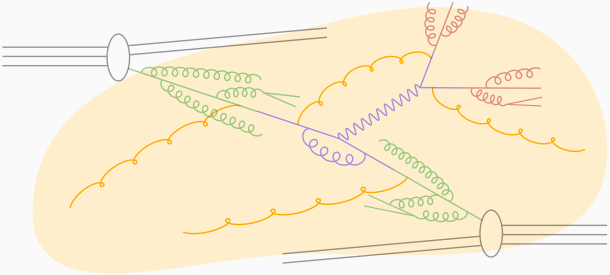


Factorisation theorem: $d\sigma \sim B \otimes B \otimes S \otimes H \otimes d\sigma_{LO} \otimes J \otimes J$

Hard function H

- Describes hard scattering process

What are beam functions?

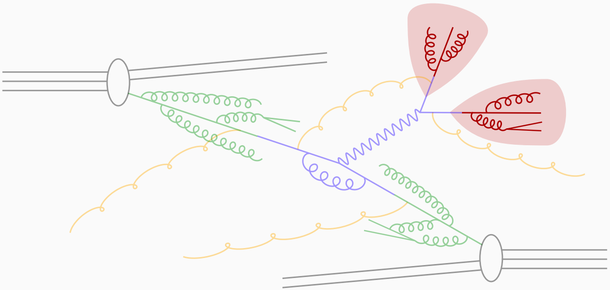


Factorisation theorem: $\mathrm{d}\sigma \sim B \otimes B \otimes S \otimes H \otimes \mathrm{d}\sigma_{\mathrm{LO}} \otimes J \otimes J$

Soft function S

- Describes soft radiation

What are beam functions?

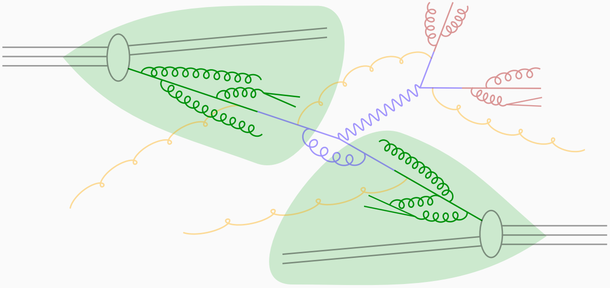


Factorisation theorem: $\mathrm{d}\sigma \sim B \otimes B \otimes S \otimes H \otimes \mathrm{d}\sigma_{\mathrm{LO}} \otimes J \otimes J$

Jet functions J

- Describe collinear radiation in the final state (jets)

What are beam functions?

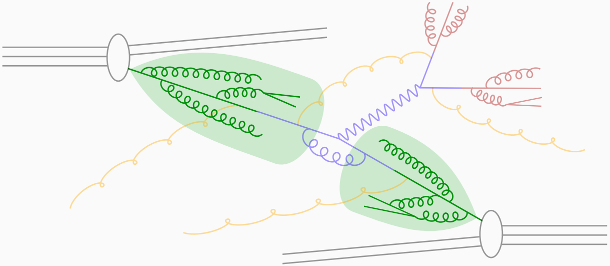


Factorisation theorem: $\mathrm{d}\sigma \sim B \otimes B \otimes S \otimes H \otimes \mathrm{d}\sigma_{\mathrm{LO}} \otimes J \otimes J$

Beam functions B

- Describe collinear radiation off the initial state
- Similar to PDFs, but more differential: $f(z, \mu)$ vs. $B(z, t, \mu)$
- Non-perturbative objects, but ...

What are beam functions?



Factorisation theorem: $\mathrm{d}\sigma \sim B \otimes B \otimes S \otimes H \otimes \mathrm{d}\sigma_{\mathrm{LO}} \otimes J \otimes J$

Beam functions B

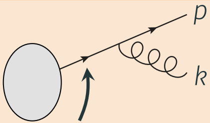
- …can be related to PDFs via matching relation:

$$B_i(z,t,\mu) = \sum_j \mathcal{I}_{ij}(z,t,\mu) \otimes f_j(z,\mu)$$

- Matching coefficients $\mathcal{I}_{ij}(z,t,\mu)$ can be calculated perturbatively

What can you do with N-jettiness beam functions?

IR singularities in fixed-order calculations



$$\frac{1}{(p-k)^2} = \frac{1}{E_p E_k (1-\cos\theta)}$$

- Appear in soft or collinear limits of massless particles
- Cancel between real and virtual contributions
- Numerical phase space integration requires scheme to treat those singularities

Subtraction schemes

$$\int_0^1 \mathrm{d}x \, \frac{f(x)}{x^{1+\varepsilon}} = \int_0^1 \mathrm{d}x \, \frac{f(x)-f(0)}{x^{1+\varepsilon}} + \int_0^1 \mathrm{d}x \, \frac{f(0)}{x^{1+\varepsilon}}$$

- Subtract (and add back) behaviour in singular limit
- Available up to NNLO

Slicing schemes

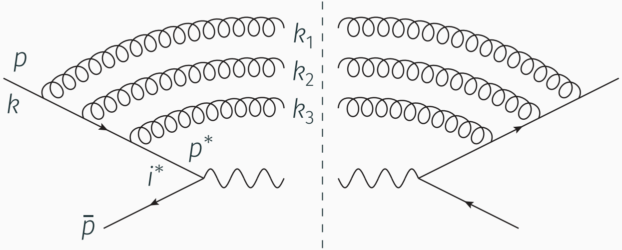
$$\int_0^1 \mathrm{d}x \, \frac{f(x)}{x^{1+\varepsilon}} = \int_\Lambda^1 \mathrm{d}x \, \frac{f(x)}{x^{1+\varepsilon}} + \int_0^\Lambda \mathrm{d}x \, \frac{f(0)}{x^{1+\varepsilon}}$$

- Use observable to isolate most singular configuration
- Approximate $f(x)$ below threshold
- Conceptually simpler
  $\rightarrow$ first candidate for going to N^3LO

How to calculate beam functions

Beam functions have operator definitions in SCET, but [Ritzmann, Waalewijn ’14] observed: Matching coefficients can be calculated from collinear limits of QCD amplitudes

$$\int \prod_{i=1}^{n_R} \frac{\mathrm{d}^d k_i}{(2\pi)^{d-1}} \delta_+(k_i^2)\, \delta\left(2p \cdot k_{1\ldots n_R} - \frac{t}{z}\right) \delta\left(\frac{2\bar{p} \cdot k_{1\ldots n_R}}{s} - (1-z)\right) \frac{\hat{C}_p |M(p, \bar{p}, \{k_i\})|^2}{|M_0(zp, \bar{p})|^2}$$



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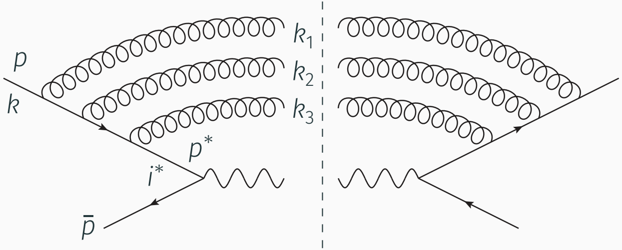

- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]

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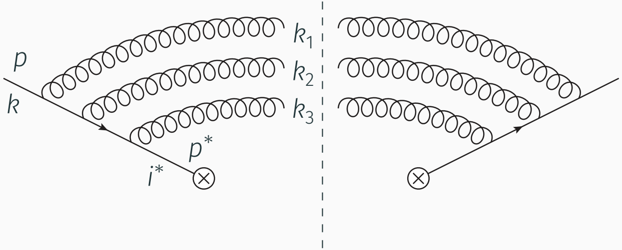

- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]
  - Consider only one leg

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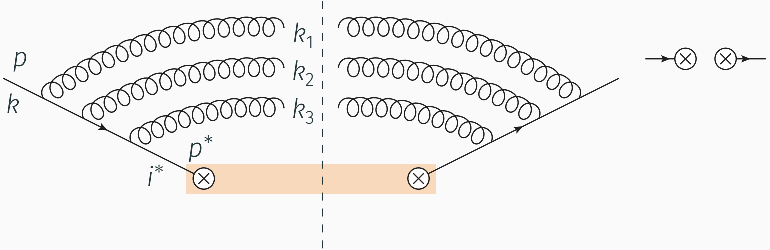


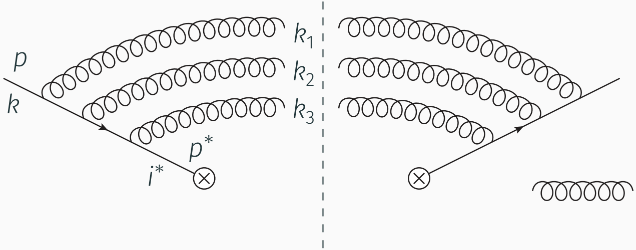
$$\rightarrow \frac{\bar{\not{p}}}{4N_c p^* \cdot \bar{p}}$$

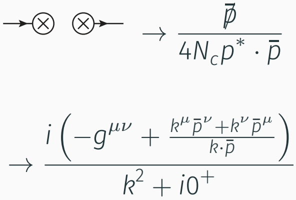
- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]
  - Consider only one leg
  - Replace hard process by suitable projector

How to calculate beam functions

Beam functions have operator definitions in SCET, but [Ritzmann, Waalewijn '14] observed:
Matching coefficients can be calculated from collinear limits of QCD amplitudes

$$\int \prod_{i=1}^{n_R} \frac{\mathrm{d}^d k_i}{(2\pi)^{d-1}} \delta_+(k_i^2)\,\delta\left(2p\cdot k_{1\ldots n_R} - \frac{t}{z}\right)\delta\left(\frac{2\bar{p}\cdot k_{1\ldots n_R}}{s} - (1-z)\right)\frac{\hat{C}_p |M(p,\bar{p},\{k_i\})|^2}{|M_0(zp,\bar{p})|^2}$$





- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]
  - Consider only one leg
  - Replace hard process by suitable projector
  - Work in axial gauge

How to calculate beam functions

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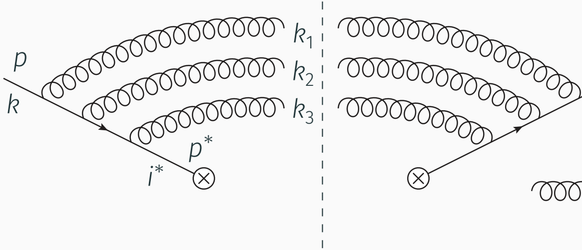
$$\int \prod_{i=1}^{n_R} \frac{\mathrm{d}^d k_i}{(2\pi)^{d-1}} \delta_+(k_i^2)\, \delta\left(2p\cdot k_{1\ldots n_R} - \frac{t}{z}\right) \delta\left(\frac{2\bar{p}\cdot k_{1\ldots n_R}}{s} - (1-z)\right) \frac{\hat{C}_p |M(p,\bar{p},\{k_i\})|^2}{|M_0(zp,\bar{p})|^2}$$



 $\rightarrow \dfrac{\not{\bar{p}}}{4N_c p^*\cdot\bar{p}}$

 $\rightarrow \dfrac{i\left(-g^{\mu\nu} + \frac{k^\mu\bar{p}^\nu + k^\nu\bar{p}^\mu}{k\cdot\bar{p}}\right)}{k^2 + i0^+}$

- Integrate over constrained phase space ($k_{1\ldots n_R} = k_1 + \cdots + k_{n_R}$)
  - Implement δ distributions via reverse unitarity

$$\delta(x) = \frac{1}{2\pi i}\left(\frac{1}{x-i0} - \frac{1}{x+i0}\right)$$

Calculation details

Standard toolchain:

Diagrams: `QGRAF`

↓

Diagram processing: `FORM` & `Mathematica`

↓

IBPs: `Reduze` & `Kira`

↓

Differential equations

↓

Canonical basis

↓

Boundary constants

↓

Itereated integrals

Calculation details

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Itereated integrals

Peculiarities of the calculation

Diagram processing

- Axial gauge propagators and phase space constraints require partial fraction decomposition.
- Due to $\delta(\frac{2}{s}(k_1 + k_2) \cdot \bar{p} - (1 - z))$ we have

$$\frac{1}{(k_1 \cdot \bar{p})(k_2 \cdot \bar{p})} = \frac{2}{s(1-z)} \left[\frac{1}{k_1 \cdot \bar{p}} + \frac{1}{k_2 \cdot \bar{p}} \right]$$

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Itereated integrals

Peculiarities of the calculation

Integration-by-parts reduction

- Number of master integrals: 851
- Apply partial fraction relations again to find identities between MI
- Afterwards: Just 473 master integrals

Calculation details

Standard toolchain:

Diagrams: QGRAF

↓

Diagram processing: FORM & Mathematica

↓

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Differential equations

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Itereated integrals

Peculiarities of the calculation

Differential equations

- 13 letters appear in the differential equation
- Three different square roots appear:

$$\sqrt{z}\sqrt{4-z},\ \sqrt{z}\sqrt{4+z},\ \sqrt{4+z^2}$$

Canonical basis

- Due to simultaneous square roots we had to construct parts of the canonical basis by hand

Calculation details

Standard toolchain:

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↓

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Differential equations

↓

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Itereated integrals

Peculiarities of the calculation

Boundary constants

- Fix boundary constants in limit $z \to 1$
- These integrals can be related to master integrals for $gg \to H$ in threshold limit
- We independently recalculated large fraction of boundary constants for N^3LO Higgs production

Calculation details

Standard toolchain:

Diagrams: QGRAF

↓

Diagram processing: FORM & Mathematica

↓

IBPs: Reduze & Kira

↓

Differential equations

↓

Canonical basis

↓

Boundary constants

↓

Itereated integrals

Peculiarities of the calculation

Results in terms of iterated integrals

- Final results have only 5 letters
- Only one square root survives: $\sqrt{z}\sqrt{4-z}$

Results

- There are five independent matching coefficients: $\mathcal{I}_{q_iq_j}, \mathcal{I}_{q_i\bar{q}_j}, \mathcal{I}_{qg}, \mathcal{I}_{gq}, \mathcal{I}_{gg}$ (Recall: $B_i = \sum_j \mathcal{I}_{ij} \otimes f_j$)
- We have computed all of them at N^{3}LO in QCD [AB, Melnikov, Rietkerk, Tancredi, Wever '19], [Baranowski, AB, Melnikov, Tancredi, Wever '22]
- Results agree with [Ebert, Mistlberger, Vita '20]

$$
\begin{aligned}
&+ \frac{21}{8}L_{1,1} - \frac{28}{3}L_{0,0,1} - \frac{82}{9}L_{0,1,0} + \frac{21}{2}L_{0,1,1} - \frac{1}{3}L_{1,0,-1} - \frac{59}{6}L_{1,0,0} \\
&+ \frac{64}{9}L_{1,0,1} + \frac{73}{9}L_{1,1,0} - \frac{14}{3}L_{1,1,1} - \frac{22}{9}L_{1,2,1}\bigg)\pi^2 + \bigg(13L_1 + 34L_{0,1} \\
&+ 42L_{1,0} - \frac{76}{3}L_{1,1}\bigg)\zeta_3 + \frac{371}{1080}L_1\pi^4\Bigg) + \frac{1}{1080}(187\bar{z} + 92)\pi^4 \\
&- \frac{3\sqrt{1-\bar{z}}(5\bar{z}+14)}{\sqrt{3+\bar{z}}}\left[L_{r,0,1} + \frac{2}{3}L_{r,1,1} - \frac{\pi^2}{6}L_r\right] - 15\bar{z}\bigg[L_{r,r,0,1} \\
&+ \frac{2}{3}L_{r,r,1,1} - \frac{\pi^2}{6}L_{r,r}\bigg] + 6(\bar{z}-2)\left[L_{1,r,r,0,1} + \frac{2}{3}L_{1,r,r,1,1} - \frac{\pi^2}{6}L_{1,r,r}\right] \\
&+ \frac{\bar{z}^2 - 2\bar{z} + 2}{\bar{z}}\Bigg(18\left[L_{0,r,r,0,1} + \frac{2}{3}L_{0,r,r,1,1} - \frac{\pi^2}{6}L_{0,r,r}\right] \\
&- 6\left[L_{1,r,r,0,1} + \frac{2}{3}L_{1,r,r,1,1} - \frac{\pi^2}{6}L_{1,r,r}\right]\Bigg) + \bigg(\frac{1}{18}(-\bar{z}^2 - 22\bar{z} + 26) \\
&+ \frac{1}{36}(-108\bar{z} + 11)L_0 + \frac{1}{18}(55\bar{z} - 47)L_1 - \frac{16}{9}\bar{z}L_{0,0} + \frac{1}{9}(18\bar{z} - 1)L_{0,1} \\
&+ \frac{1}{12}(55\bar{z} - 52)L_{1,0} + \frac{1}{72}(-553\bar{z} + 586)L_{1,1}\bigg)\pi^2 + \bigg(\frac{1}{6}(235\bar{z} - 18)
\end{aligned}
$$

Results

- There are five independent matching coefficients: $\mathcal{I}_{q_iq_j}, \mathcal{I}_{q_i\bar{q}_j}, \mathcal{I}_{qg}, \mathcal{I}_{gq}, \mathcal{I}_{gg}$ (Recall: $B_i = \sum_j \mathcal{I}_{ij} \otimes f_j$)
- We have computed all of them at N^{3}LO in QCD [AB, Melnikov, Rietkerk, Tancredi, Wever '19], [Baranowski, AB, Melnikov, Tancredi, Wever '22]
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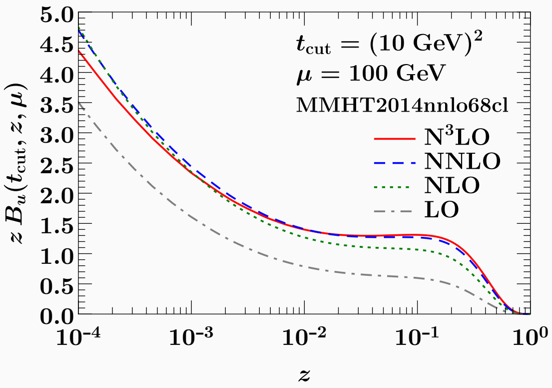


Plot: [Ebert, Mistlberger, Vita '20]

$$B_i(t_{\text{cut}}, z, \mu) = \int_0^{t_{\text{cut}}} \mathrm{d}t\, B_i(t, z, \mu)$$

Outlook

Almost all building blocks for N^{3}LO zero-jettiness slicing are available:

$$\mathrm{d}\sigma \sim B \otimes B \otimes S \otimes H \otimes \mathrm{d}\sigma_{\mathrm{LO}}$$
$$\checkmark \quad \checkmark \quad (\checkmark) \quad \checkmark \quad \checkmark$$

Soft function S for zero-jettiness:

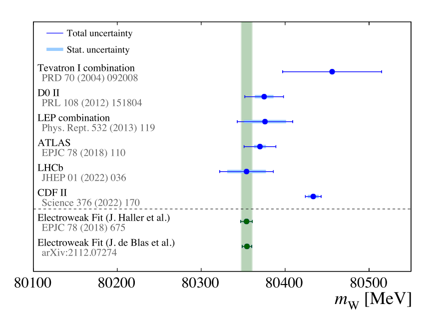
- Partial N^{3}LO results available:
  [Baranowski, Delto, Melnikov, Wang ’22] [Chen, Feng, Jia, Liu ’22]
- Completion is one of the objectives of project B1a of this CRC

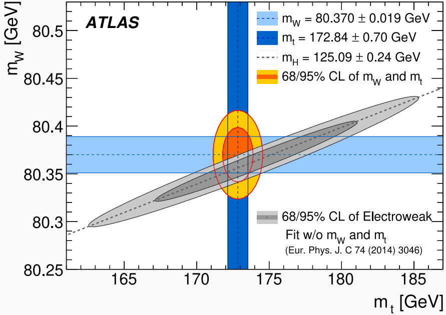
Beyond zero-jettiness?

- Beam functions are identical for zero- and N-jettiness.
- Soft function depends on N $\rightarrow$ more challenging for $N > 0$

Mixed QCD-EW corrections and the W boson mass

Why is m_W interesting?





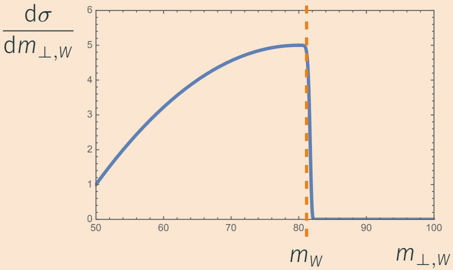
[ATLAS '17]

- High precision W mass measurements allow to cross-check the Standard Model
- ATLAS has measured $m_W = (80\,370 \pm 19)$ MeV [ATLAS '17]
- ATLAS and CMS collaborations aim to reduce uncertainty to $\mathcal{O}(10\,\text{MeV})$
  $\to$ would rival precision from global electroweak fits
  $\to$ would mean $\mathcal{O}(0.01\%)$ uncertainty
- CDF has measured $m_W = (80\,433.5 \pm 9.4)$ MeV [CDF '22]
  $\to$ 7σ discrepancy from global EW fits

How to measure m_W at the LHC

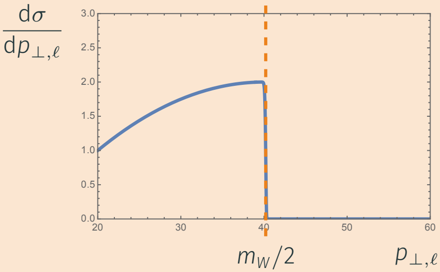
Need observables that are sensitive to m_W:

Transverse mass of W



$$m_{\perp,W} = \sqrt{2p_{\perp,\ell}p_{\perp,\nu}(1 - \cos\phi_{\ell\nu})}$$

Transverse momentum of ℓ

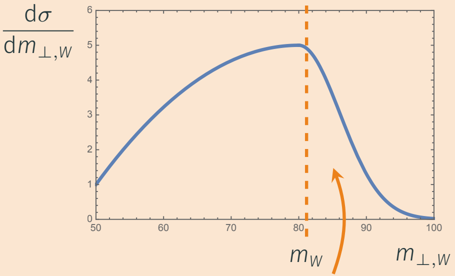


At LO and with idealized detectors both observables have sharp kinematic edges.
$\rightarrow$ Very sensitive observables

How to measure m_W at the LHC

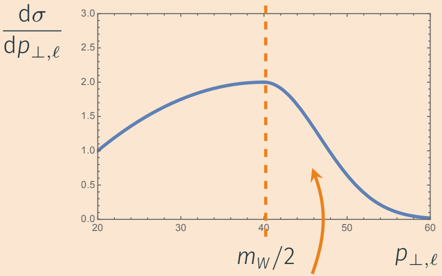
Need observables that are sensitive to m_W:

Transverse mass of W



Beyond the edge: Mostly detector effects

Transverse momentum of ℓ



Mostly QCD & QED initial state radiation

Starting from NLO and with realistic detectors the edges are washed out

Theory predictions for for m_W measurements at hadron colliders

Problem

Typical theory uncertainties at N^3LO: $\mathcal{O}(1\%)$ vs. Precision goal for m_W at LHC: $\mathcal{O}(10\,\text{MeV}) \sim \mathcal{O}(0.01\%)$

$\rightarrow$ We cannot hope to predict distributions to this precision from first principles directly.

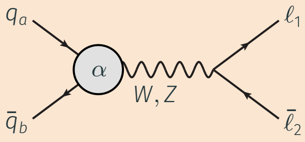
Strategy

- Measure Z distributions
- Parametrise them in QCD-motivated way
- Transfer information from Z to W distributions

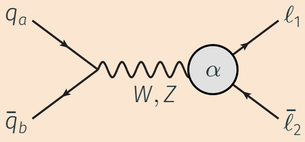
$\rightarrow$ Requires precise control over effects that distinguish between W and Z bosons.
$\rightarrow$ Electroweak corrections certainly fall into this category.

Electroweak corrections for on-shell W and Z production

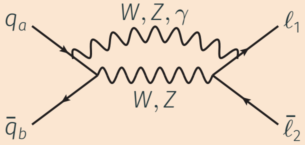
NLO EW: Initial state



NLO EW: Final state



NLO EW: non-factorisable

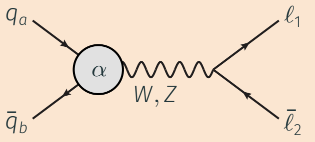


[Dittmaier, Huss, Schwinn '14]:

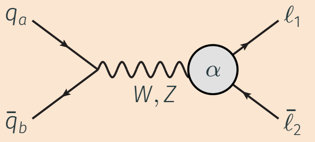
$$\sim \mathcal{O}\left(\alpha \frac{\Gamma}{m_V}\right) \sim \mathcal{O}\left(\alpha^2\right)$$

Electroweak corrections for on-shell W and Z production

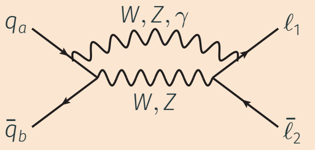
NLO EW: Initial state



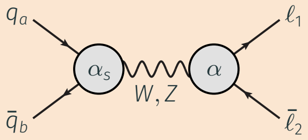
NLO EW: Final state



NLO EW: non-factorisable

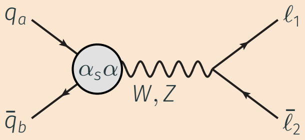


Mixed QCD-EW: Initial-Final



- Previously investigated
  [Dittmaier, Huss, Schwinn '15] [Carloni Calame et al. '16]
- Estimated impact on m_W:
  $\delta m_W \sim O(15\,\text{MeV})$

Mixed QCD-EW: Initial-Initial

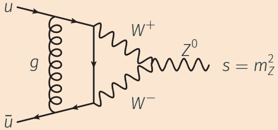


- Focus of this talk

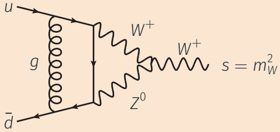
Calculation of mixed QCD-EW corrections

Amplitudes

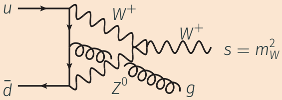
- Z form factor: [Kotikov, Kühn, Veretin '07]



- W form factor: Calculated by us



- RV amplitudes: OpenLoops 2



Subtraction scheme

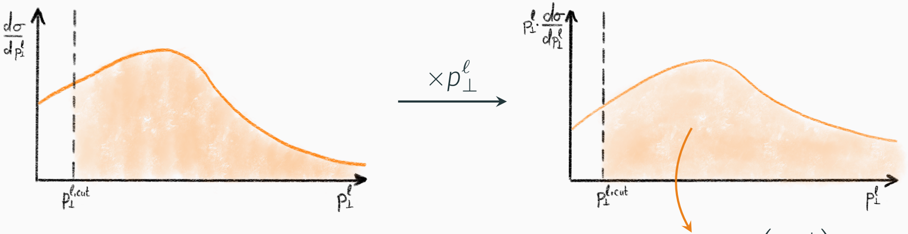
- Build on progress with NNLO QCD subtraction schemes
- We use the Nested Soft Collinear Subtraction Scheme
  [Caola, Melnikov, Röntsch '17] [Caola, Melnikov, Luisoni, Röntsch '17] [Caola, Melnikov, Röntsch '19a] [Caola, Melnikov, Röntsch '19b] [Asteriadis, Caola, Melnikov, Röntsch '19]
  (closely related to Sector-improved Residue Subtraction Scheme)
  [Czakon '10] [Czakon '11] [Czakon, Heymes '14] [Czakon, van Hameren, Mitov, Poncelet '19]
- Z production: Abelianisation of NNLO QCD subtraction is sufficient
- W production: New contributions from radiating W bosons

Our goal

Goal: Estimate impact of new corrections on W boson mass measurements

We use the average transverse momentum of the charged lepton ($V = W, Z$):

$$\langle p_\perp^{\ell,V} \rangle = \frac{\int \mathrm{d}\sigma_V \times p_\perp^\ell}{\int \mathrm{d}\sigma_V}$$



$$\langle p_\perp^{\ell,V} \rangle = m_V f \left(\frac{p_\perp^{\text{cut}}}{m_V} \right)$$

with $f^{\text{LO}}(0) = \frac{15\pi}{128}$

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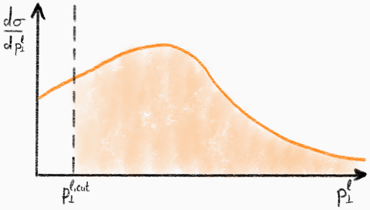
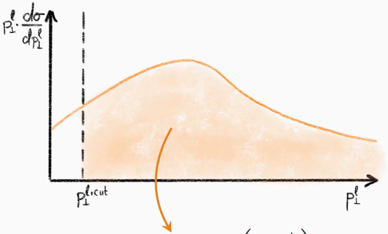


$\times p_\perp^\ell$



$$\langle p_\perp^{\ell,V} \rangle = m_V f\left(\frac{p_\perp^{\text{cut}}}{m_V}\right)$$

with $f^{\text{LO}}(0) = \frac{15\pi}{128}$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \, m_Z \, C_{\text{th}}$$

Construction of our observable (cont.)

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Measurement from LHC

$\propto m_W$

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$\propto m_Z$

Measurement from LHC

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

Measurement from LEP

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} m_Z C_{\text{th}}$$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \, m_Z \, C_{\text{th}}$$

Theoretical correction factor

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Theoretical correction factor

$\rightarrow$ Calculate via $C_{\text{th}} = \frac{m_W}{m_Z} \frac{\langle p_\perp^{\ell,Z} \rangle^{\text{th}}}{\langle p_\perp^{\ell,W} \rangle^{\text{th}}}$

Construction of our observable (cont.)

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Theoretical correction factor

$\rightarrow$ Calculate via $C_{\text{th}} = \frac{m_W}{m_Z} \frac{\langle p_\perp^{\ell,Z} \rangle^{\text{th}}}{\langle p_\perp^{\ell,W} \rangle^{\text{th}}}$

Adding a new correction to the theory
$\rightarrow$ changes C_{th}
$\rightarrow$ changes extracted mass m_W^{meas}

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle}$$

Estimating the shift of m_W

Estimates for shifts on W mass measurement via $\delta m_W = \left(\frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle} \right) m_W$

	Inclusive	Fiducial cuts	Tuned cuts
NLO EW	1 MeV	3 MeV	−3 MeV
Mixed QCD-EW	−7 MeV	−17 MeV	−1 MeV

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Mixed QCD-EW corrections appear to have larger impact than NLO EW corrections

- G_μ input parameter scheme reduces size of NLO EW corrections
- Strong cancellation between changes in Z and W

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NLO EW: $\delta m_W \approx +32\,\text{MeV} - 31\,\text{MeV}$

Mixed QCD-EW: $\delta m_W \approx -61\,\text{MeV} + 54\,\text{MeV}$

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- Cuts inspired by [ATLAS '17] analysis
- Larger shifts than for inclusive setup

W production:

- $p_\perp^{e^+} > 30$ GeV
- $p_\perp^{\text{miss}} > 30$ GeV
- $|\eta_{e^+}| < 2.4$
- $m_T^W > 60$ GeV

Z production:

- $p_\perp^{e^\pm} > 25$ GeV
- $|\eta_{e^\pm}| < 2.4$

Estimating the shift of m_W

Estimates for shifts on W mass measurement via $\delta m_W = \left(\frac{\delta\langle p_\perp^{\ell,Z}\rangle}{\langle p_\perp^{\ell,Z}\rangle} - \frac{\delta\langle p_\perp^{\ell,W}\rangle}{\langle p_\perp^{\ell,W}\rangle} \right) m_W$

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- Cuts inspired by [ATLAS '17] analysis
- Larger shifts than for inclusive setup
  - Relevant momenta: $p_\perp^{e^+}/M_V$
  - ATLAS applies larger $p_\perp^{e^+}$ cuts to W bosons than to Z bosons
  - Leads to small decorrelation between W and Z bosons

W production:

- $p_\perp^{e^+} > 30$ GeV
- $p_\perp^{\text{miss}} > 30$ GeV
- $|\eta_{e^+}| < 2.4$
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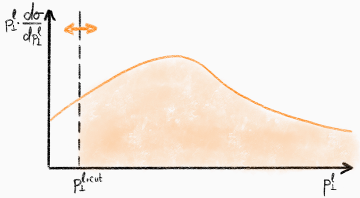
- Size of shifts strongly depends on fiducial cuts
- Tune cuts ($p_\perp^{e^+}$ from W^+) such that $C_{\text{th}} = 1$ at LO
- Impact of mixed QCD-EW gets reduced a lot



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Summary

- Fiducial cuts are an important factor for impact of mixed QCD-EW corrections.
- Mixed QCD-EW corrections might play a role for m_W measurements at the LHC.
- Close collaboration with experimental colleagues is necessary to properly assess the impact at the LHC.

Conclusions

Conclusions

- Precision calculations allow us to make the most of the LHC.

- We calculated the matching coefficients for the N-jettiness beam functions at N^3LO in QCD.
- The beam functions are important building blocks for pushing N-jettiness slicing to N^3LO (and a great success for the CRC).

- We calculated mixed QCD-EW corrections to the on-shell W and Z production.
- We used these calculations to estimate the impact on m_W measurements at the LHC.
- The relevance depends strongly on the fiducial cuts.
- Further investigation together with experimental collaborations is necessary.