Karlsruhe Institute of Technology

Multi-Scale 2-Loop Amplitudes

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Aachen – 2. March 2023

Annual Meeting of the CRC TRR 257

Introduction

Timeline of NNLO calculations for hadronic collisions

“NNLO revolution” starting in 2014

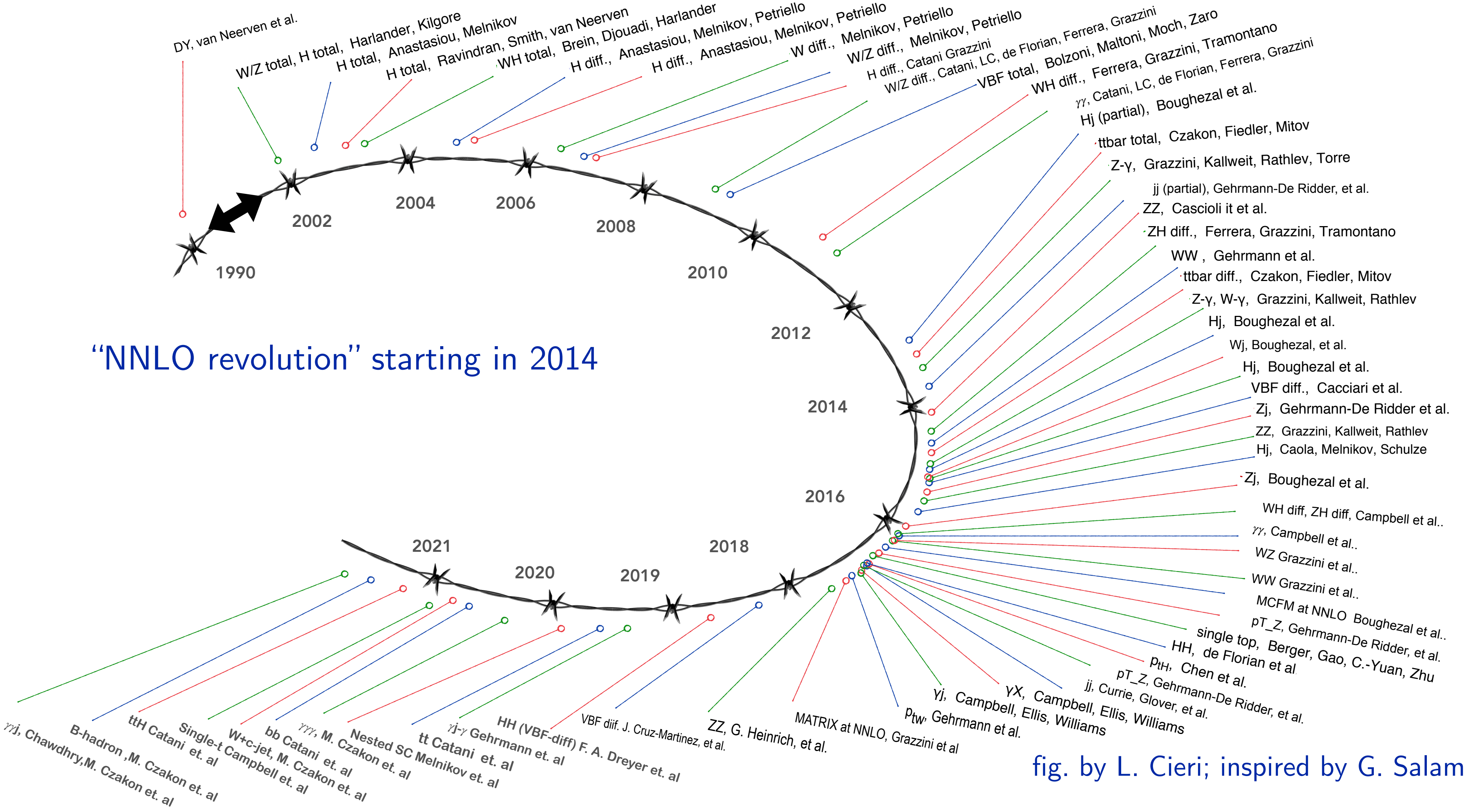
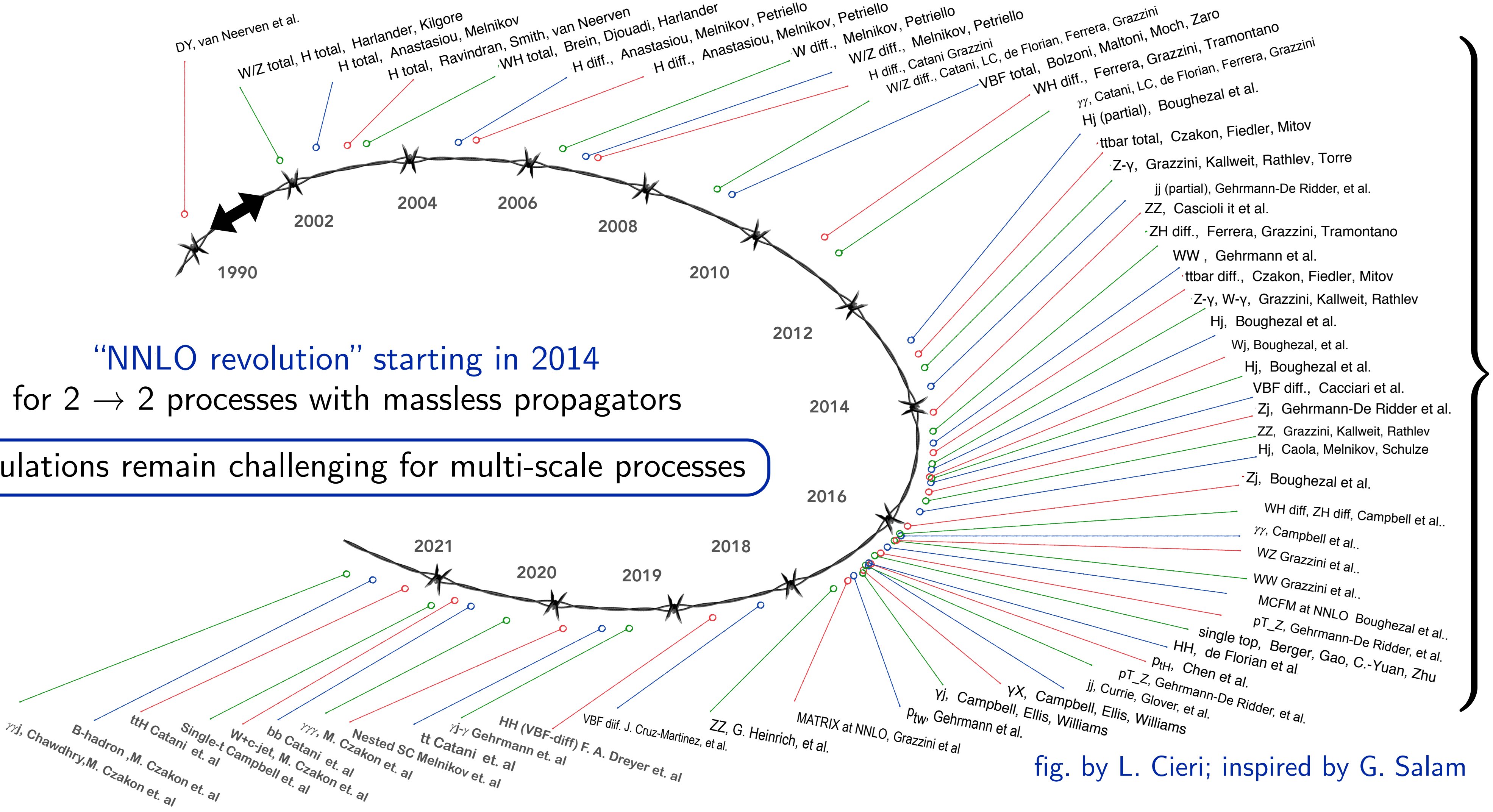


fig. by L. Cieri; inspired by G. Salam

Introduction

Timeline of NNLO calculations for hadronic collisions



“NNLO revolution” starting in 2014
for 2 $\rightarrow$ 2 processes with massless propagators

Calculations remain challenging for multi-scale processes

Fast progress for 2 $\rightarrow$ 2 processes with massless legs

some 2 $\rightarrow$ 3 calculations

fig. by L. Cieri; inspired by G. Salam

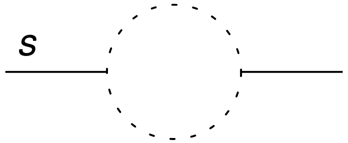
Introduction

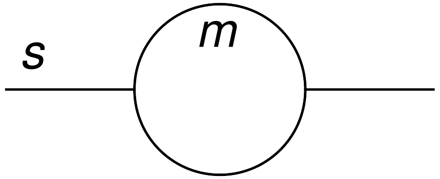
The NNLO revolution was partly caused by the observation, that Differential Equations can usually be brought into canonical form [Henn 13]

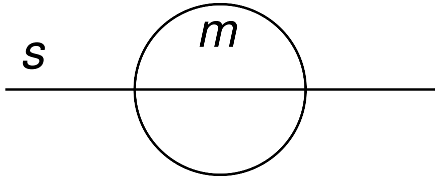
$$\partial_{x_i} \vec{I}(\vec{x}) = \epsilon A_i(\vec{x}) \vec{I}(\vec{x})$$

→ DEQ can be solved order-by-order in ε
→ Solution in terms of iterated integrals
for massless integrals: mostly HPLs/GPLs resp. PolyLogs

However, with massive propagators, the calculation can quickly become complicated







Most complicated terms:

$$\log(s/\mu^2)$$

$$\sqrt{s(s-4m)}\,\log\left(\frac{\sqrt{s(s-4m)}+2m-s}{2m}\right)$$

$$-\frac{4\,\mathrm{K}(\lambda)}{(p^2+m^2)\sqrt{a_{13}a_{24}}}\left[2\mathcal{E}_4\left(\begin{smallmatrix}0 & -1\\ 0 & \infty\end{smallmatrix};1,\vec{a}\right)+\mathcal{E}_4\left(\begin{smallmatrix}0 & -1\\ 0 & 0\end{smallmatrix};1,\vec{a}\right)+\mathcal{E}_4\left(\begin{smallmatrix}0 & -1\\ 0 & 1\end{smallmatrix};1,\vec{a}\right)\right]$$

$$\mathrm{K}(\lambda)=\int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-\lambda x^2)}} \qquad \mathcal{E}_4\left(\begin{smallmatrix}n_1 & \dots & n_k\\ c_1 & \dots & c_k\end{smallmatrix};x,\vec{a}\right)=\int_0^x dt\,\Psi_{n_1}(c_1,t,\vec{a})\,\mathcal{E}_4\left(\begin{smallmatrix}n_2 & \dots & n_k\\ c_2 & \dots & c_k\end{smallmatrix};t,\vec{a}\right)$$

see Duhr 19

→ square roots

→ elliptic integrals

Outline of the Talk

Multi-Scale 2-Loop Amplitudes

$$\# \text{ scales} = \# \text{ indep. Mandelstam invariants} + \# \text{ indep. masses}$$

- Introduction
- The standard (analytic) approach
  - application to massless $2 \to 3$ processes
- $2 \to 2$ processes with massive internal legs
  - heavy top limit
  - series expansions
  - numerical methods
  - new ideas
- Common issues
  - expression swell
  - mass-scheme uncertainties
- Conclusion

Disclaimer:
The focus of this talk is on an overview of the methods used, not on the presentation of all calculations

The Standard Approach to Multi-Loop Calculations

1. Identify possible tensor structures using (gauge/permutation) symmetries, e.g. for HH production

New ideas: direct calculation of polarized amplitudes
L. Chen 19; Peraro, Tancredi 19,20

$$\mathcal{M}^{\mu\nu} = A_1(s,t,m_H^2,m_t^2,D)\, T_1^{\mu\nu} + A_2(s,t,m_H^2,m_t^2,D)\, T_2^{\mu\nu}$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu\, p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2\,(p_1 \cdot p_2)} \left\{ m_H^2 p_1^\nu\, p_2^\mu - 2\,(p_1 \cdot p_3)\, p_3^\nu\, p_2^\mu - 2\,(p_2 \cdot p_3)\, p_3^\nu\, p_1^\mu + 2\,(p_1 \cdot p_2)\, p_3^\nu\, p_3^\mu \right\}$$

2. Calculate the form factors using projectors

$$P_1^{\mu\nu}\mathcal{M}_{\mu\nu} = A_1(s,t,m_H^2,m_t^2,D) \qquad P_2^{\mu\nu}\mathcal{M}_{\mu\nu} = A_2(s,t,m_H^2,m_t^2,D)$$

$\rightarrow$ all Lorentz indices are contracted
L-loop N-propagator integrals written as

$$I(\{\nu_i\}) = \int \prod_{l \le L} d^d l_l \frac{\prod_{i>N} D_i(q_i^2 - m_i^2)^{-\nu_i}}{\prod_{i \le N} D_i(q_i^2 - m_i^2)^{\nu_i}} \qquad \nu_i \in \mathbb{Z}$$

3. IBP reduction [Chetyrkin, Tkachov; Laporta]

$$\int \mathrm{d}^d p_i \frac{\partial}{\partial p_i^\mu} \left[q^\mu\, \mathbf{I}'(p_1,\ldots,p_l;k_1,\ldots,k_m) \right] = 0$$

q: loop or external momentum

$\rightarrow$ employ linear relations to express to reduce all loop integrals to minimal set of independent master integrals

The Standard Approach to Multi-Loop Calculations

4. Differentiate master integrals wrt. kin. invariants x_j
   $\rightarrow$ system of differential equations (DEQs)

$$\frac{\partial \vec{I}(\vec{x},\epsilon)}{\partial x_j} = A_{x_j}(\vec{x},\epsilon)\vec{I}(\vec{x},\epsilon)$$

5. Find transformation U to canonical basis, such that

$$I'(\vec{x},\epsilon) = U(\vec{x},\epsilon)I(\vec{x},\epsilon) \qquad \frac{\partial \vec{I}'(\vec{x},\epsilon)}{\partial x_j} = \epsilon A'_x(\vec{x})\vec{I}'(\vec{x},\epsilon)$$

For multi-scale problems, this often introduces square roots or elliptic functions into the system of DEQs

can sometimes be removed by variable change, e.g. $\sqrt{s(s-4m)} \xrightarrow[s\to -m\frac{(1-x)^2}{x}]{} m\frac{1-x^2}{x}$

6. Solve DEQs order by order in ε,
   integration constants can be fixed, e.g. known integral result in specific limits (easier to calculate)

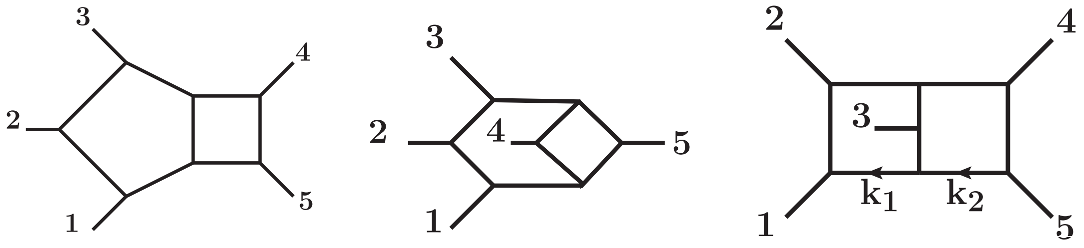
The integration is straightforward in terms of GPLs, if A'_x contains only simple poles

$$G(z_1,\ldots,z_n;y) = \int_0^y \frac{dx}{x-z_1} G(z_2,\ldots,z_n;x) \qquad G(;x) = 1$$

The z_i are called letters

Massless 2 → 3 scattering

Gehrmann, Henn, Presti 18
Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia 18
Chicherin, Sotnikov 20

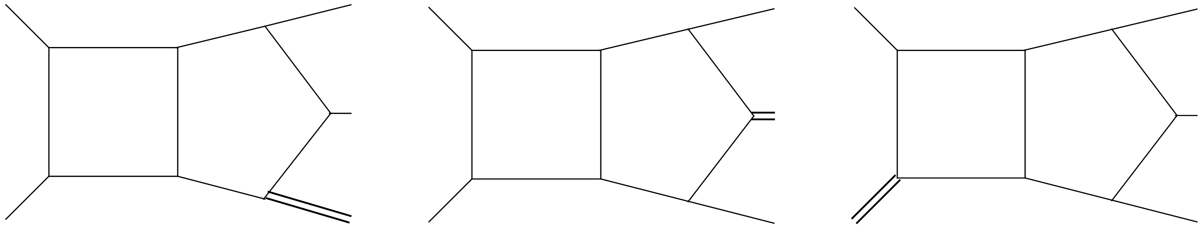


scales: 5 independent Mandelstam invariants
+ parity-odd invariant $\epsilon_5 = \mathrm{tr}\left[\gamma_5 \not{p}_1 \not{p}_2 \not{p}_3 \not{p}_4\right]$

Letter	v notation	momentum notation	cylic
W_1	v_1	$2p_1 \cdot p_2$	+ cyclic (4)
W_6	$v_3 + v_4$	$2p_4 \cdot (p_3 + p_5)$	+ cyclic (4)
W_{11}	$v_1 - v_4$	$2p_3 \cdot (p_4 + p_5)$	+ cyclic (4)
W_{16}	$v_4 - v_1 - v_2$	$2p_1 \cdot p_3$	+ cyclic (4)
W_{21}	$v_3 + v_4 - v_1 - v_2$	$2p_3 \cdot (p_1 + p_4)$	+ cyclic (4)
W_{26}	$\frac{v_1v_2 - v_2v_3 + v_3v_4 - v_1v_5 - v_4v_5 - \sqrt{\Delta}}{v_1v_2 - v_2v_3 + v_3v_4 - v_1v_5 - v_4v_5 + \sqrt{\Delta}}$	$\frac{\mathrm{tr}[(1-\gamma_5)\not{p}_4\not{p}_5\not{p}_1\not{p}_2]}{\mathrm{tr}[(1+\gamma_5)\not{p}_4\not{p}_5\not{p}_1\not{p}_2]}$	+ cyclic (4)
W_{31}	$\sqrt{\Delta}$	$\mathrm{tr}[\gamma_5\not{p}_1\not{p}_2\not{p}_3\not{p}_4]$	

many different letters, but integration in terms of polylogarithms possible

Planar integrals with one off-shell leg are also known [Canko, Papadopoulos, Syrrakos 20]



Applications:

γγγ. [Chawdhry, Czakon, Mitov, Poncelet]
[Kallweit, Sotnikov, Wieseman]
[Abreu, Page, Pascual, Sotnikov]

γγj [Agarwal, Buccioni, von Manteuffel, Tancredi]
[Chawdhry, Czakon, Mitov, Poncelet]
[Badger, Gehrmann, Marcoli, Mood]

jjj [Abreu, Cordero, Ita, Page, Sotnikov]

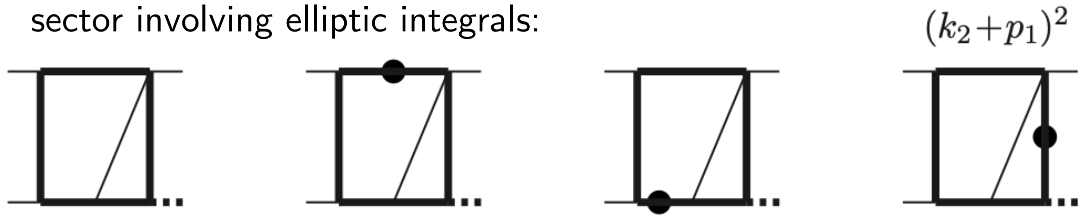
Wγj [Badger, Hartanto, Kryś, Zoia]

Wbb [Badger, Hartanto, Zoia]
[Hartanto, Poncelet, Popescu, Zoia]

$2 \to 2$ scattering with massive propagators

Representative example: NLO QCD corrections to Higgs + Jet production

sector involving elliptic integrals:



- Involves elliptic integrals
- Many square roots, cannot be rationalized simultaneously

(semi-)analytic result for planar integrals: [Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16]

- weight 2 contributions written in terms of Li_2 for polylogarithmic sectors
- weight 3&4 and elliptic contributions via integration (over elliptic kernel)

$\to$ analytic continuation & physics application not straightforward

First full calculations of NLO HJ production:

- Using HE expansion [Lindert, Kudashkin, Melnikov, Wever 18]
- Using sector decomposition [Jones, Luisoni, MK 18]
- Using DiffExp [Frellesvig, Hidding, Maestri, Moriello, Salvatori 19; Bonciani, Del Duca, Frellesvig, Hidding, Hirschi, Moriello, Salvatori, Somogyi, Tramontano 22]

This process also contributes to H production at NNLO [Czakon, Harlander, Klappert, Niggetiedt 21]

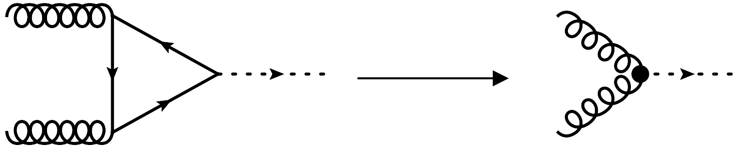
Analytic integration often not first choice for massive integrals

Nevertheless, lots of progress on elliptic integrals:

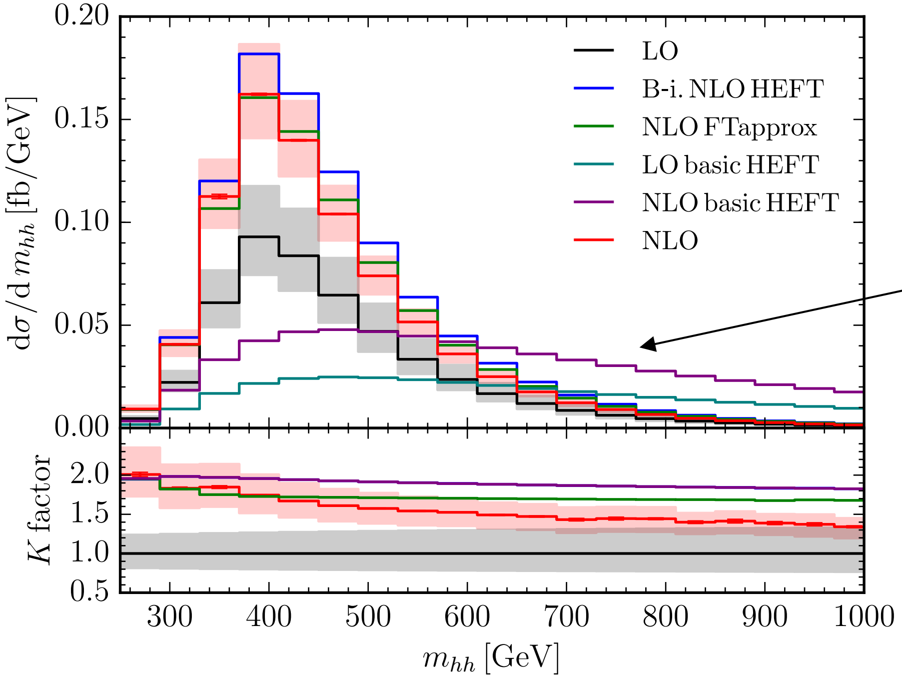
Abreu, Adams, Beccetti, Bezuglov, Bogner, Bourjaily, Broedel, Chaubey, Duhr, Dulat, Frellesvig, Laporta, Müller, Onishchenko, Ozcelik, Primo, Remiddi, Schweitzer, Tancredi, Veretin, Walden, Weinzierl

Heavy Top Limit

Typical simplification in Higgs physics:
Heavy Top Limit (HTL/HEFT)



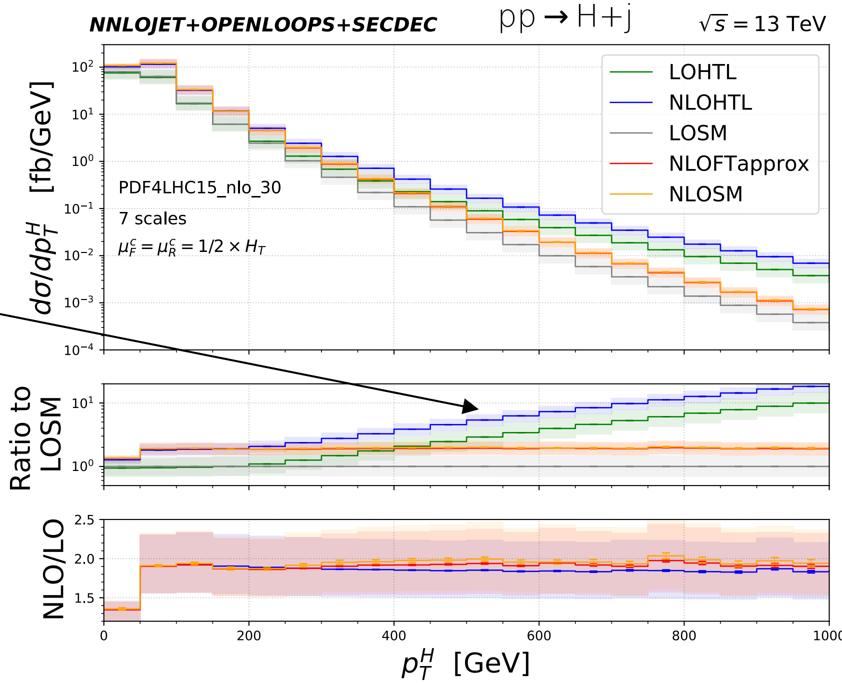
HH production



Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16

HTL predicts wrong shape of distribution

HJ production



Chen, Huss, Jones, MK, Lang, Lindert, Zhang 21

Important improvement: Born-improved HTL:

$$d\sigma_{NLO} \approx d\sigma_{NLO}^{HEFT} = \frac{d\sigma_{NLO}(m_t \to \infty)}{d\sigma_{LO}(m_t \to \infty)}\, d\sigma_{LO}(m_t)$$

Keeping (at least some) m_t dependence is important

Further improvement: m_t-dependence in reals: $\mathrm{FT}_{\mathrm{approx}}$

Series Expansions

Many integrals have been calculated as Series Expansions in various limits → simpler integrals

The expansion can be performed using expansion by regions [Beneke, Smirnov 98]
using the tool asy [Pak, Smirnov; Jantzen, Smirnov, Smirnov]

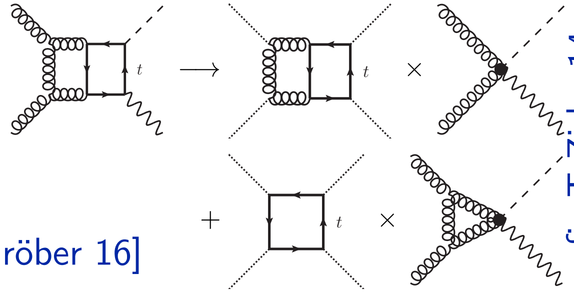
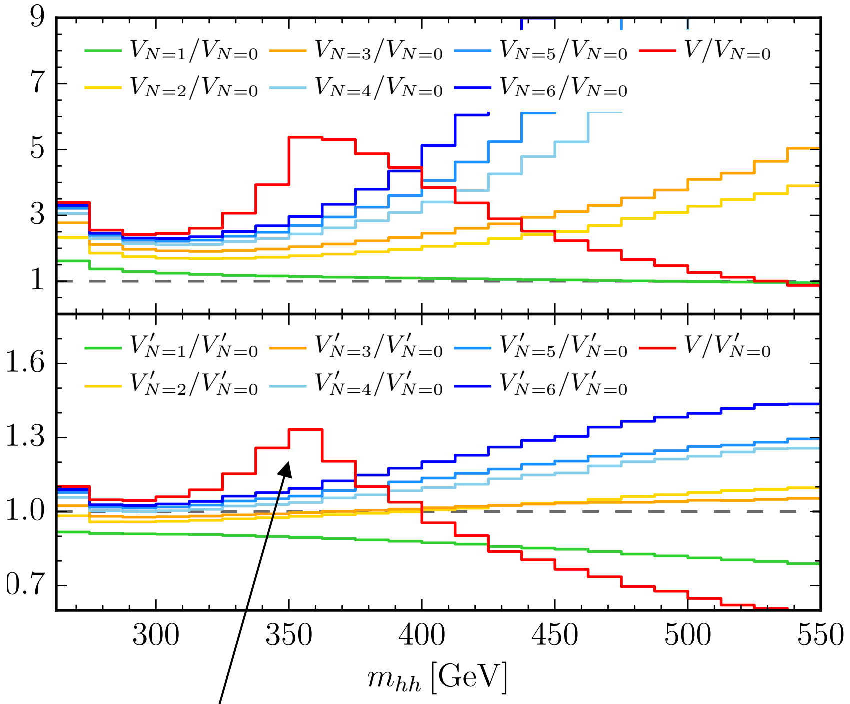


fig:T Zirke 14

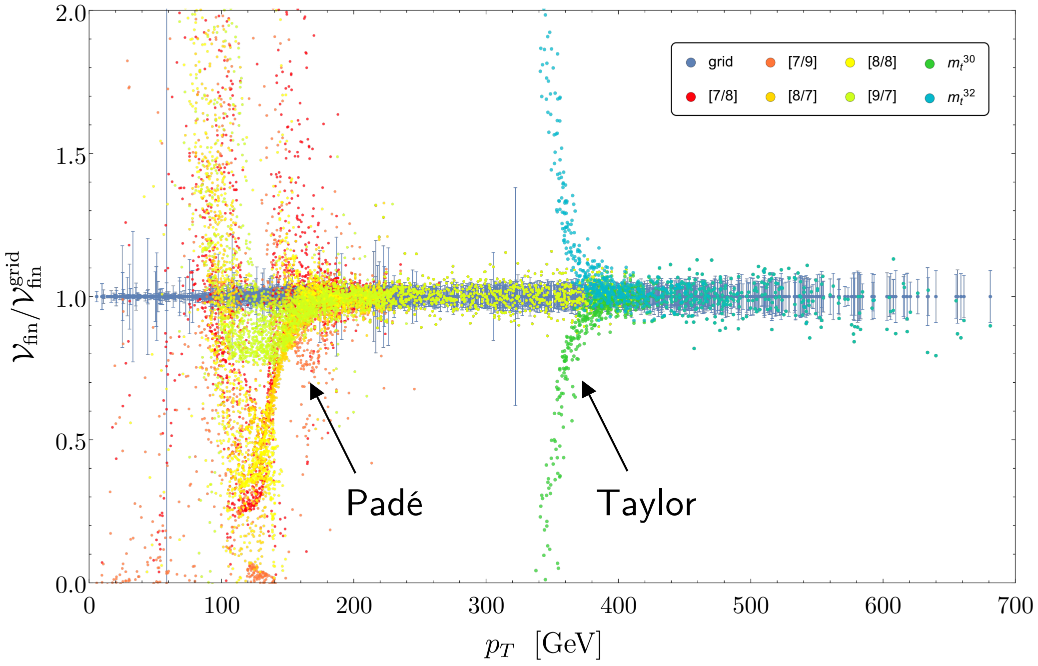
Example HH production:

- HTL ($m_t \to \infty$) [Grigo, Hoff, Melnikov, Steinhauser 13, 15; Degrassi, Giardino, Gröber 16]
- High Energy ($m_t \to 0$) [Davies, Mishima, Steinhauser, Wellmann 18]

Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16



Expansion not valid for $m_{hh} \geq 2m_t$



Davies, Heinrich, Jones, MK, Mishima, Steinhauser, Wellmann 19

The expansions can be improved using Padé ansatz:

$$\mathcal{V}_{\text{fin}}^{N} = \frac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots + b_m x^m} \equiv [n/m](x)$$

Series Expansions

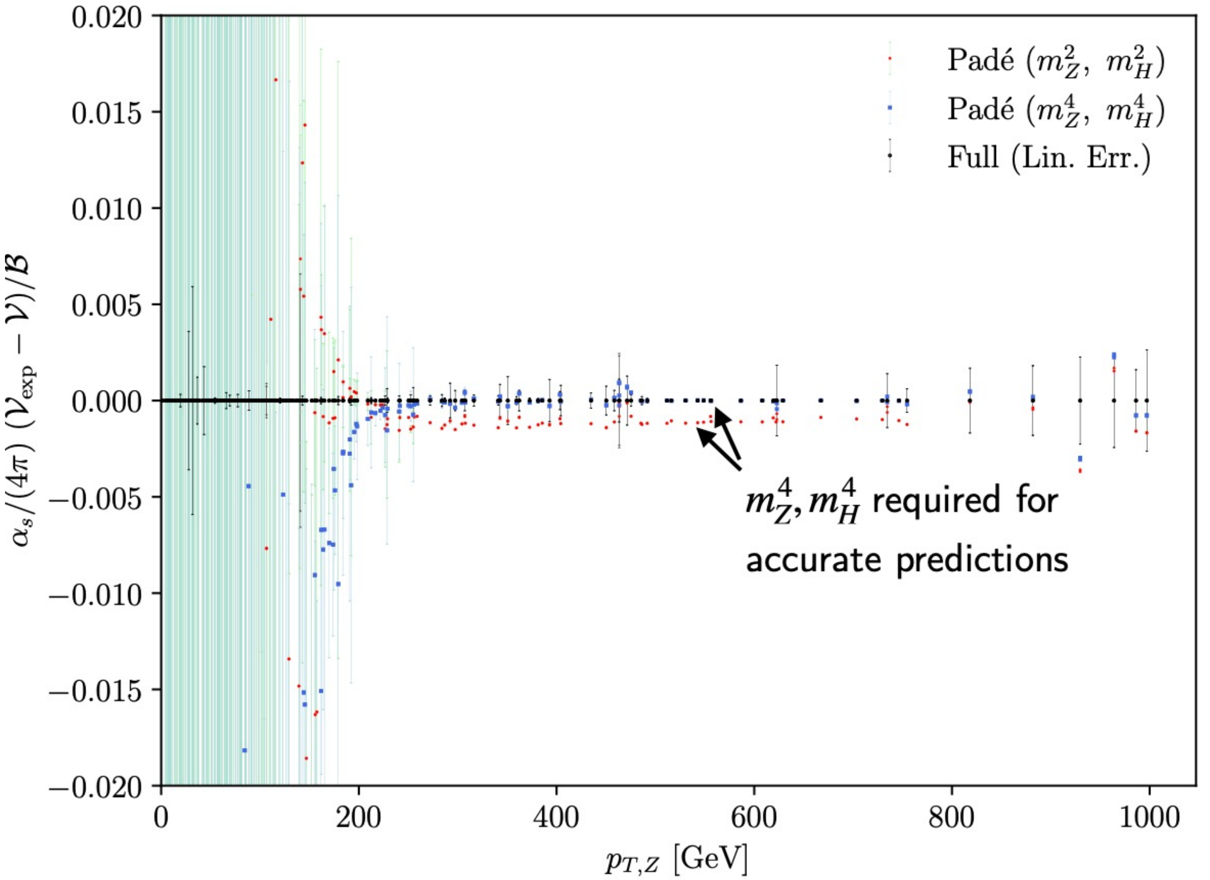
Davies, Mishima, Steinhauser 20;
Chen, Davies, Heinrich, Jones, MK, Mishima, Schlenk, Steinhauser 22

Integrals typically depend on multiple scales

e.g. gg → ZH in HE region:

$$m_Z, m_H < m_t \ll s, t$$

expansion around small masses up to m_Z^4, m_H^4, m_t^{32}



Many more results using m_t-expansions:

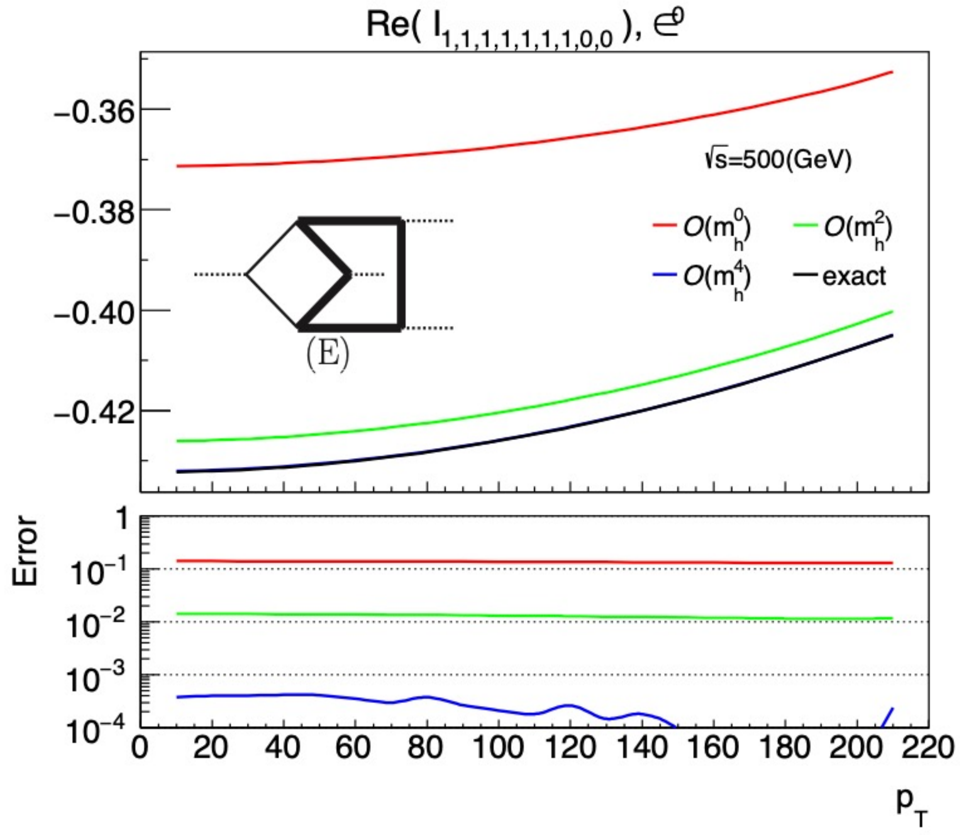
HH: Grigo, Hoff, Steinhauser; Davies, Herren, Mishima, Steinhauser; Degrassi, Giardino, Gröber
HJ: Harlander, Neumann, Ozeren, Wiesemann; Neumann, Wiesemann; Kudashkin, Melnikov, Wever
ZZ: Davies, Mishima, Steinhauser, Wellmann; Gröber, Rauh
ZH: Hasselhuhn, Luthe, Steinhauser; Davies, Mishima, Steinhauser; Degrassi, Gröber, Vitti, Zhao

Series Expansions

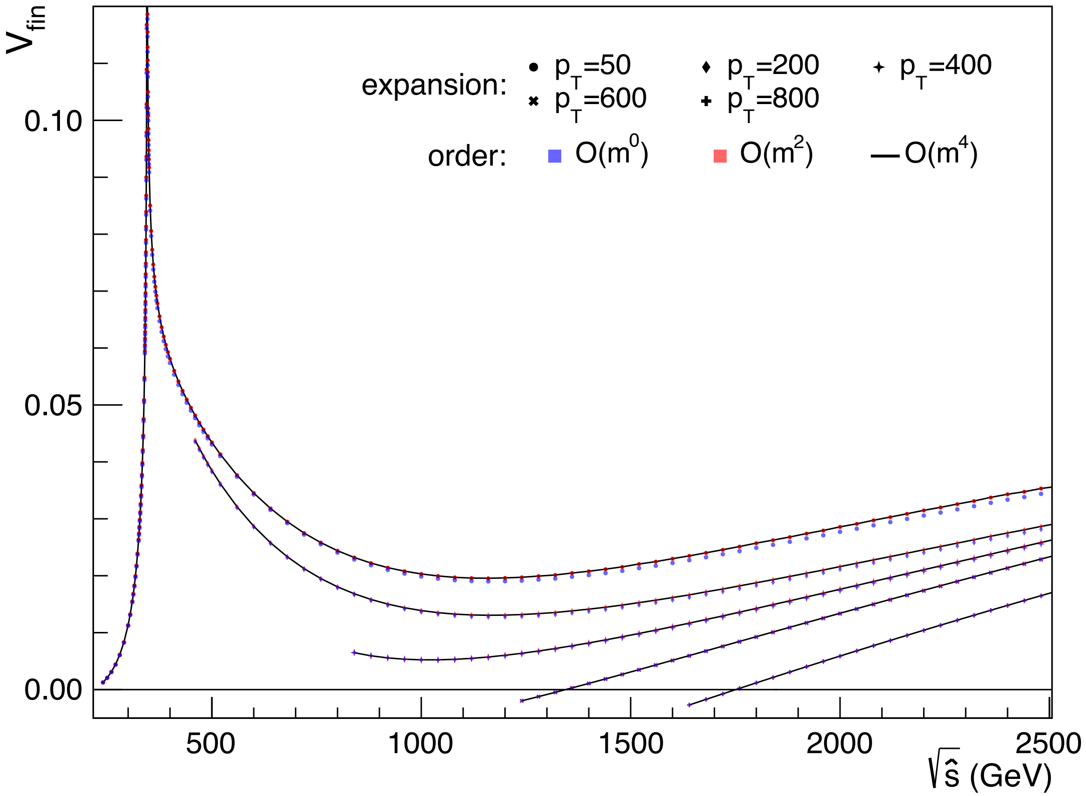
Different method using expansions:

- Expand in masses of external particles
- Solve remaining integrals with full m_t dependence

HH production [Xu, Yang 18]



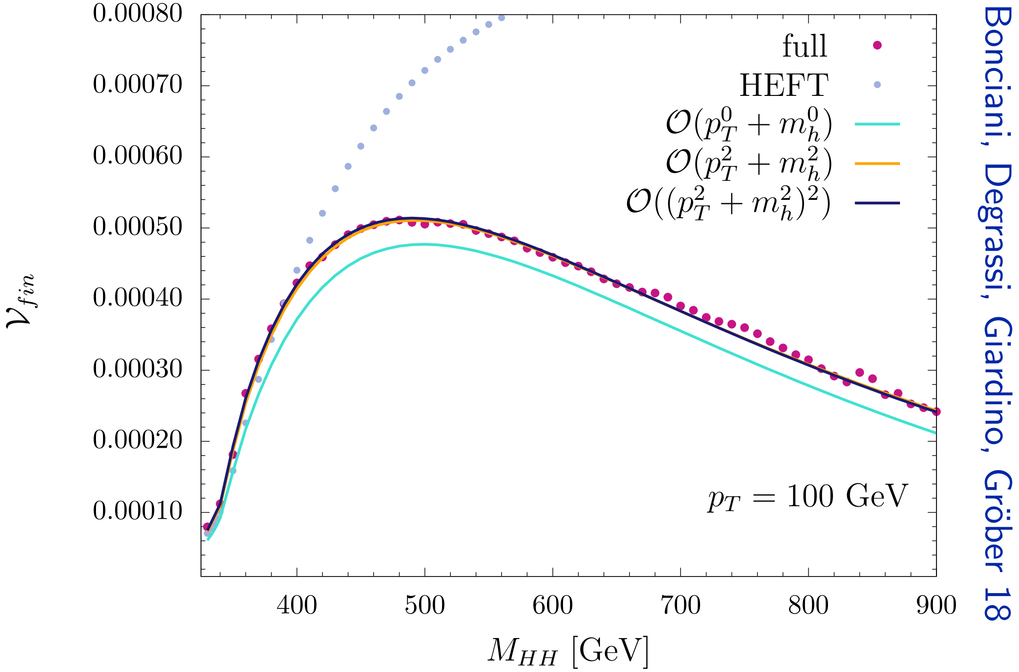
ZH production [Wang, Xu, Xu, Yang 21]



Series Expansions

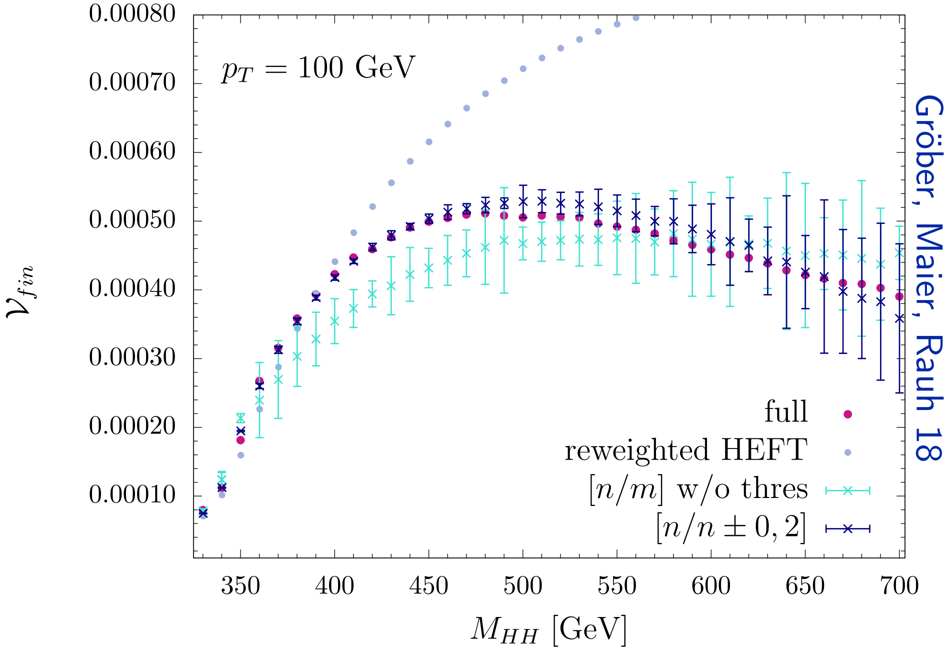
Further results based on expansions for HH production

Expand in $p_T^2 + m_h^2$



Works well for $p_T \lesssim 300$ GeV

Combine large mass expansion
with Padé ansatz and threshold logarithms



Works well below and at top-quark pair threshold

ZH production: combination of HE and p_T expansion [Bellafronte, Degrassi, Giardino, Gröber, Vitti]

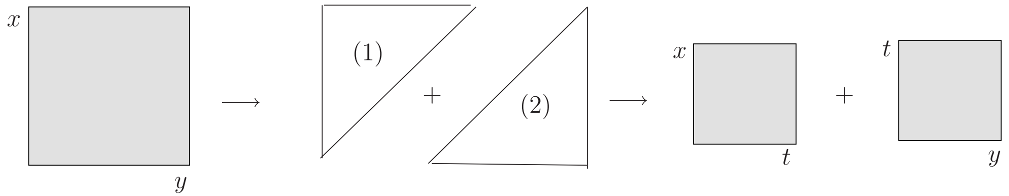
Sector Decomposition

Numerical evaluation of loop integrals with pySecDec

[Borowka, Heinrich, Jahn, Jones, MK, Langer, Magerya, Olsson, Põldaru, Schlenk, Villa]

Available at
github.com/gudrunhe/secdec

- Sector decomposition [Binoth, Heinrich 00]
  factorizes overlapping singularities



- Subtraction of poles & expansion in ε
- Contour deformation [Soper 00; Binoth et.al. 05, Nagy, Soper 06; Borowka et al. 12]

→ Finite integrals at each order in ε

→ Numerical integration possible

New in version 1.5:

- expansion by regions
- evaluation of linear combinations of integrals, with automated optimization of sampling points per sector, minimizing

$$T = \sum_{\text{integral}\, i} t_i + \lambda\left(\sigma^2 - \sum_i \sigma_i^2\right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

$\sigma_i =$ error estimate (including coefficients in amplitude)
$\lambda =$ Lagrange multiplier $\qquad \sigma =$ precision goal

- automated reduction of contour-def. parameter
- automatically adjusts FORM settings

pySecDec integral libraries can be directly linked to amplitude code

pySecDec – Quasi-Monte Carlo

Review: Dick, Kuo, Sloan 13
First application to loop integrals:
Li, Wang, Yan, Zhao 15

Our preferred integration algorithm is a
Quasi-Monte Carlo using rank-1 shifted lattice rule

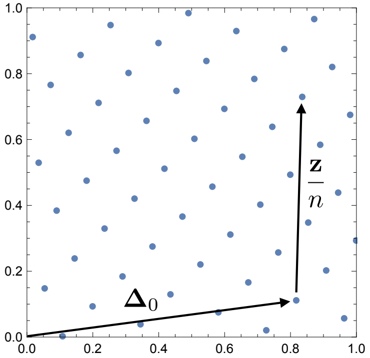
$$I[f] \approx I_k = \frac{1}{N} \cdot \sum_{i=1}^{N} f(\mathbf{x}_{i,k}), \quad \mathbf{x}_{i,k} = \left\{ \frac{i \cdot \mathbf{z}}{N} + \mathbf{\Delta}_k \right\}$$

$\{\ldots\} =$ fractional part ($\to x \in [0;1[$)

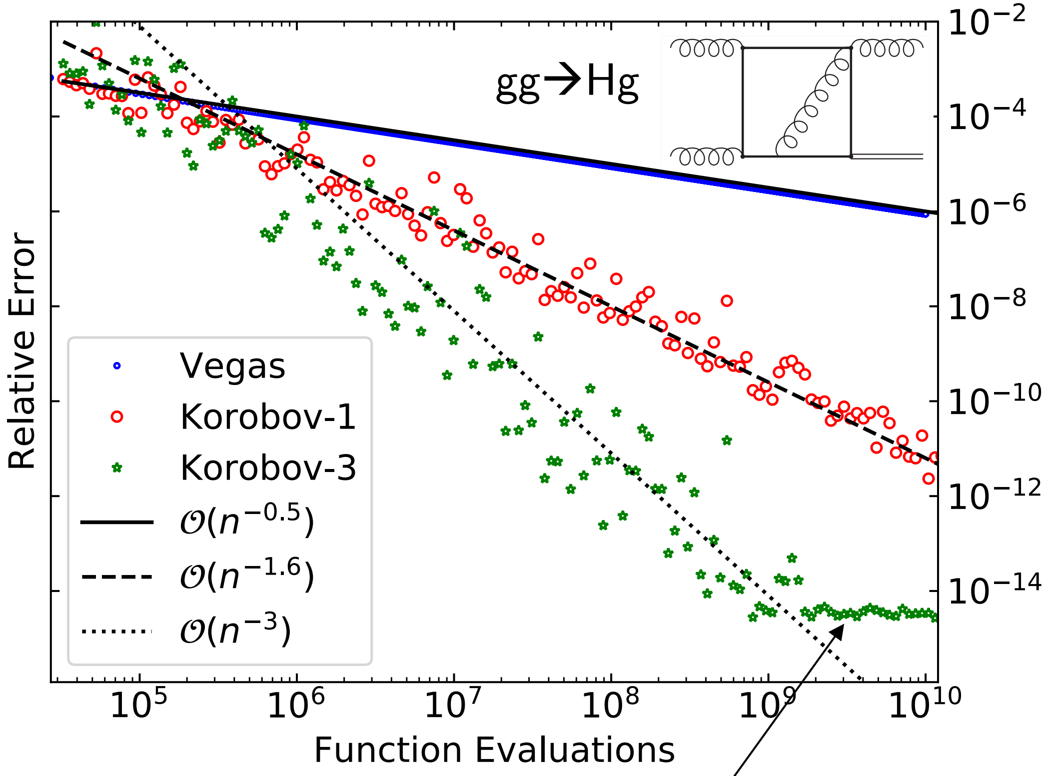
$\mathbf{\Delta}_k =$ randomized shifts
$\to$ m different estimates of Integral
$\to$ error estimate of result

$\mathbf{z} =$ generating vector
constructed component-by-component [Nuyens 07]
minimizing worst-case error

$\to$ integration error scales as $\mathcal{O}(n^{-1})$ or better



Integrator available at github.com/mppmu/qmc
[Borowka, Heinrich, Jahn, Jones, MK, Schlenk]



Limited by double precision arithmetic

Processes calculated using (py-)SecDec with QMC:

HH [Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16]
HJ [Jones, MK, Luisoni 18]
AA [Chen, Heinrich, Jahn, Jones, MK, Schlenk, Yokoya 19]
ZH [Chen, Heinrich, Jones, MK, Klappert, Schlenk 20]
ZZ [Agarwal, Jones, Manteuffel 20]

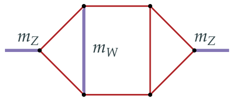
pySecDec – Work in Progress

Vitaly Magerya, L&L `22

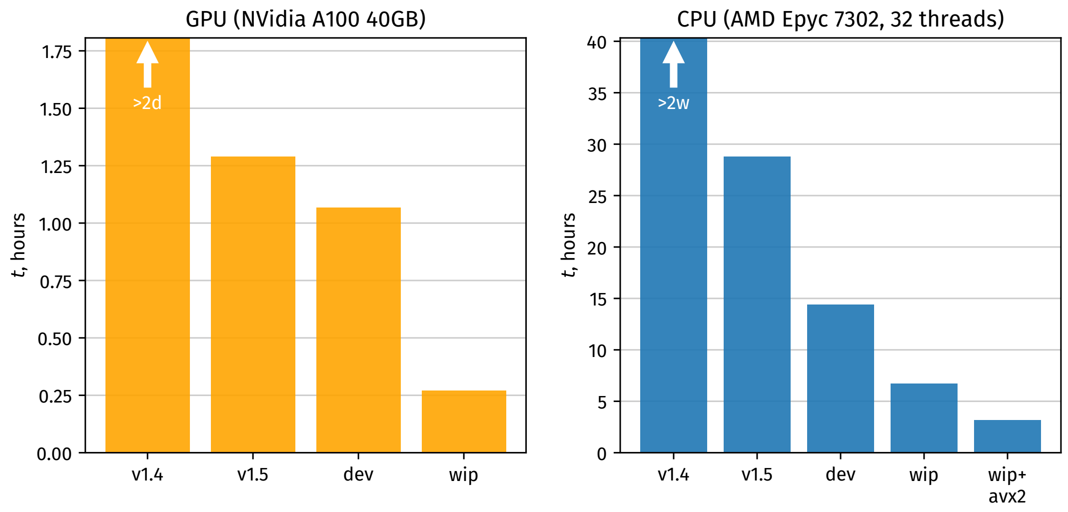
Coming soon:

pySecDec v1.6
→ significant speed improvements

Performance improvements by pySecDec version

Time to integrate 

to 7 digits of precision with pySecDec + QMC:



Speedup sources:

- *v1.5:* adaptive sampling, automatic contour deformation adjustment;
- *dev:* separation of real and complex variables in the integrand code;
- *wip:* simlification of the integrand code, vectorization on CPU (AVX2).

The latest release is fast; *the next release will be faster.*

Auxiliary Mass Flow

Liu, Ma, Wang 17; Liu, Ma, Tao et.al. 20; Liu, Ma 22
Brønnum-Hansen, Wang 21

$$I \propto \lim_{\eta\to 0^+} \int \prod_l^L d^d k_i \prod_i^N \frac{1}{[q_i^2 - (m_i^2 - i\eta)]^{\nu_i}}$$

Idea:

- Solve DEQ in $x \propto -i\eta$

$$\partial_x \boldsymbol{I} = \boldsymbol{M}\boldsymbol{I}$$

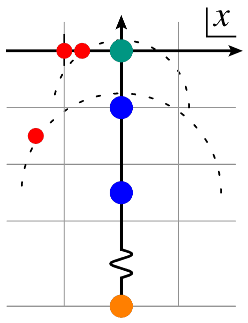
- Start from boundary point $x = -i\infty$
  $\rightarrow$ (massive vacuum graphs) x (massless graphs)
- Transfer to x=0 using power-log expansions

Expand $\boldsymbol{I}$ around **boundary** in variable $y = x^{-1} = 0$:

$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_k^N \sum_l \boldsymbol{c}_{jkl} y^k \ln^l y + \dots$$

Evaluate and expand around **regular points**:

$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_{k=0}^N \boldsymbol{c}_{jk} x'^k + \dots$$



$m_t^2 \to m_t^2(1+x)$

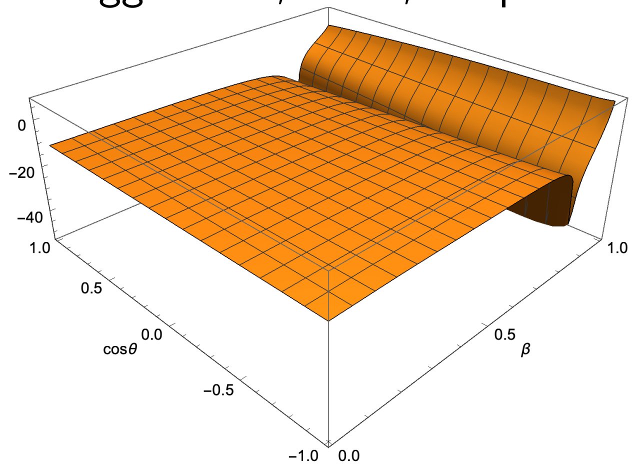
Brønnum-Hansen, L&L 22

Public Implementation: AMFlow [Liu, Ma 22]

Full amplitudes evaluated using this method:
gg $\rightarrow$ WW, gg $\rightarrow$ ZZ [Brønnum-Hansen, Wang 21]

can achieve 15-20 digits precision in ~1h CPU time

gg $\rightarrow$ ZZ, LLLL, CF part:



Series Expansion Along Path – DiffExp & SeaSyde

General Idea: Solve DEQs along path $\gamma(t): \; t \mapsto \{x_1(t), \ldots, x_m(t)\}, \qquad \vec{x}(t_a) = \vec{a}, \;\; \vec{x}(t_b) = \vec{b}$

Moriello 19
see also Lee, Smirnov, Smirnov 17

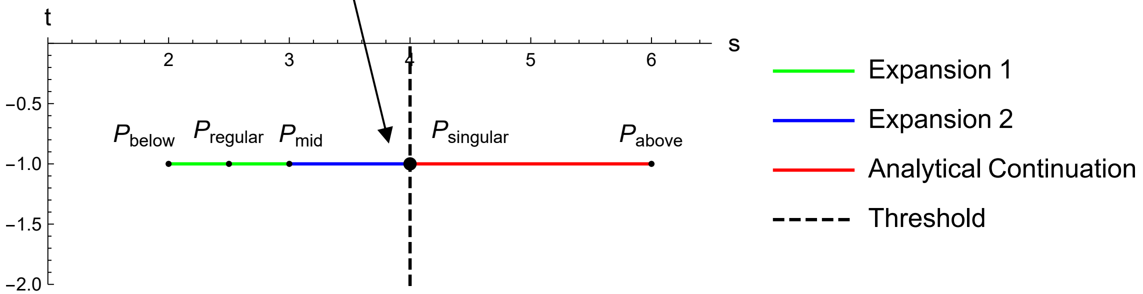
$$\frac{\partial}{\partial t}\vec{f}(t,\epsilon) = \mathbf{A}_t(t,\epsilon)\vec{f}(t,\epsilon) \qquad \mathbf{A}_t(t,\epsilon) = \sum_{i=1}^{m} \mathbf{A}_{x_i}(t,\epsilon)\frac{\partial x_i(t)}{\partial t} \qquad \vec{f}(\vec{a}) \text{ known}$$

Series expansion near singular/regular points: $\vec{f}^{(i)}_{\text{sing}}(t) = \sum_{j_1 \in S_i}\sum_{j_2=0}^{\infty}\sum_{j_3=0}^{N_i} \vec{c}^{\,(i,j_1,j_2,j_3)}(t-\tau)^{w_{j_1}+j_2}\log(t-\tau)^{j_3} \qquad \vec{f}^{(i)}_{\text{reg}}(t) = \sum_{j=0}^{\infty}\vec{c}^{\,(i,j)}(t-\tau)^j$

2 public implementations:

DiffExp [Hidding 20]

- works for real parameters
- arbitrary path
- specify $i\delta$-prescription for each physical singularity, cross threshold by expanding in singular points
  e. g. $\sqrt{4m^2 - s - i\delta}$

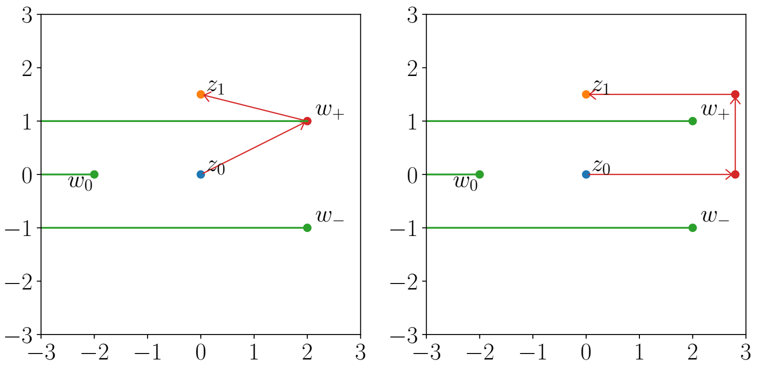


Physics application: NLO QCD (2L) corrections to HJ
Bonciani, Del Duca, Frellesvig, Henn, Hidding, Maestri, Moriello, Salvatori,Smirnov 19;
Frellesvig, Hidding, Maestri, Moriello, Salvatori 19

SeaSyde

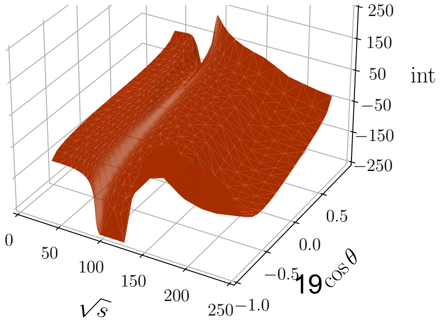
[Armadillo, Bonciani, Devoto, Rana, Vicini 22]

- transform 1 variable at a time
- choose path avoiding branch cuts
- also works with complex masses



Physics application:

2L mixed QCD-EW corrections to NC DY



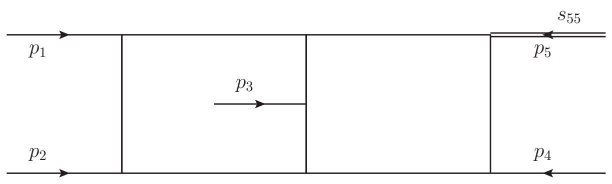
Iterative Application of Feynman's trick

Hidding, Usovitsch 22

Feynman's trick $\quad D_1 = -(k+p)^2 + m_1^2\,, \qquad D_2 = -(k+q)^2 + m_2^2 \longrightarrow xD_1 + (1-x)D_2 = -(k+P)^2 + M^2$

can be used to combine pairs of propagators into one propagator with generalized kinematics.
This procedure can be applied iteratively:

κ	input	output	Number of master integrals
0	-	uncombined	142
1	$\{D_1, D_2\}$	$D_{12} = D_1 x_1 + (1-x_1) D_2$	69
2	$\{D_4, D_5\}$	$D_{45} = D_4 x_2 + (1-x_2) D_5$	32
3	$\{D_7, D_8\}$	$D_{78} = D_7 x_3 + (1-x_3) D_8$	16
4	$\{D_{12}, D_3\}$	$D_{123} = D_{12} x_4 + (1-x_4) D_3$	8
5	$\{D_{45}, D_6\}$	$D_{456} = D_{45} x_5 + (1-x_5) D_6$	4
6	$\{D_{123}, D_{456}\}$	$D_{123456} = D_{123} x_6 + (1-x_6) D_{456}$	2
7	$\{D_{123456}, D_{78}\}$	$D_{12345678} = D_{123456} x_7 + (1-x_7) D_{78}$	1



generalized tadpole:
- x_7-dependence can be obtained using DiffExp
- using const s_{ij}, x_j as boundary

Integrate 1 Feynman parameter at a time

Integration split into multiple, simpler 1-dimensional problems
$\rightarrow$ computationally efficient

Computational Challenge: Expression Swell

The size of the expressions can become huge, with reduction tables of size O(1TB)
$\rightarrow$ it can be beneficial to
- fix mass ratios during reduction
- or perform numerical reduction for each phase-space point

Strategies to avoid Expression Swell:

- Finite-field methods
  - FiniteFlow [Peraro]
  - Firefly [J. Klappert, Klein, Lange]

  New tool for faster evaluation of numerical samples:
  Ratracer [V. Magerya]
- Syzygy Equations
  Gluza, Kajda, Kosower 10; Schabinger 11; Lee 14;
  Ita 15; Larsen, Zhang 15; Bitoun, Bogner, Klausen, Panzer 17;
  Manteuffel, Panzer, Schabinger 20; Agarwal, Jones, Manteuffel 20;
- good basis of master integrals,
  with d-dependence factorizing from kinematic dependence in denominators of reduction and possibly additional properties
  Usovitsch 20; Smirnov, Smirnov 20; see also MK Radcor `19 proc.
- direct projection to physical amplitudes
  typically simpler; avoid evanescent tensor structures
  L. Chen 19; Peraro, Tancredi 19,20

Interesting new development:

Direct construction of reduction coefficients
using Intersection Numbers
In contrast to Laporta's algorithm, no need for full reduction tables

Mizera 17; Mastrolia Mizera 18; Frellesvig, Gasparotto, Laporta, Mandal, Mastrolia, Mattiazzi, Mizera 19,20; Abreu, Britto, Duhr, Gardi, Matthew 19; Weinzierl 20; Caron-Huot, Pokraka 21; Chen, Jiang, Ma, Xu, Yang 22; Chestnov, Gasparotto, Mandal, Mastrolia, Matsubara-Heo, Munch, Takayama 22

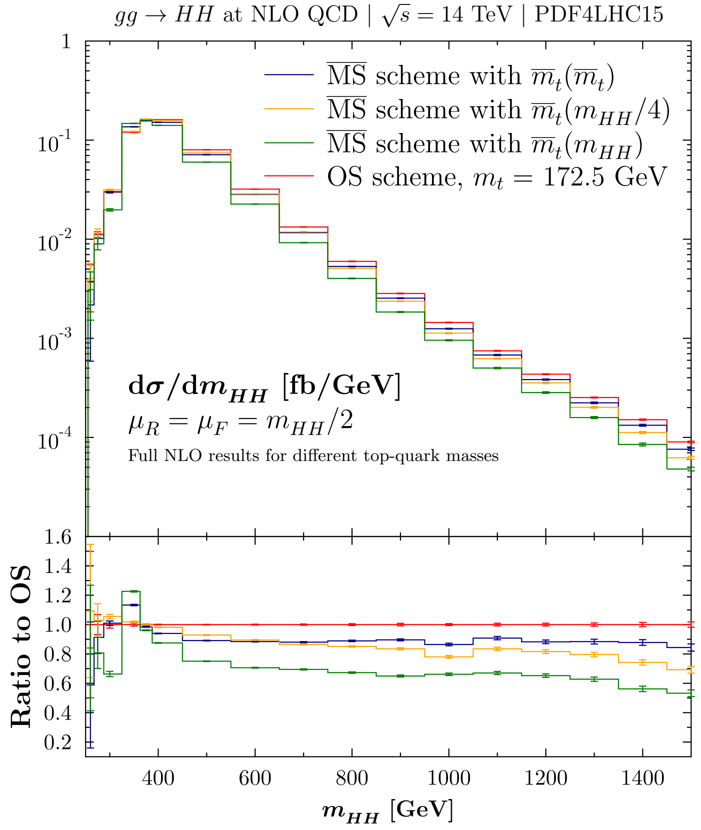
Renormalization Scheme Uncertainties

Amplitude with massive internal particles depends on mass-renormalization scheme
$\rightarrow$ additional uncertainty can be estimated by comparing OS and $\overline{MS}$ results

HH production [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 19,20]

$$\left.\frac{d\sigma(gg \to HH)}{dQ}\right|_{Q=300\text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%}\text{ fb/GeV}$$

$$\left.\frac{d\sigma(gg \to HH)}{dQ}\right|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%}\text{ fb/GeV}$$



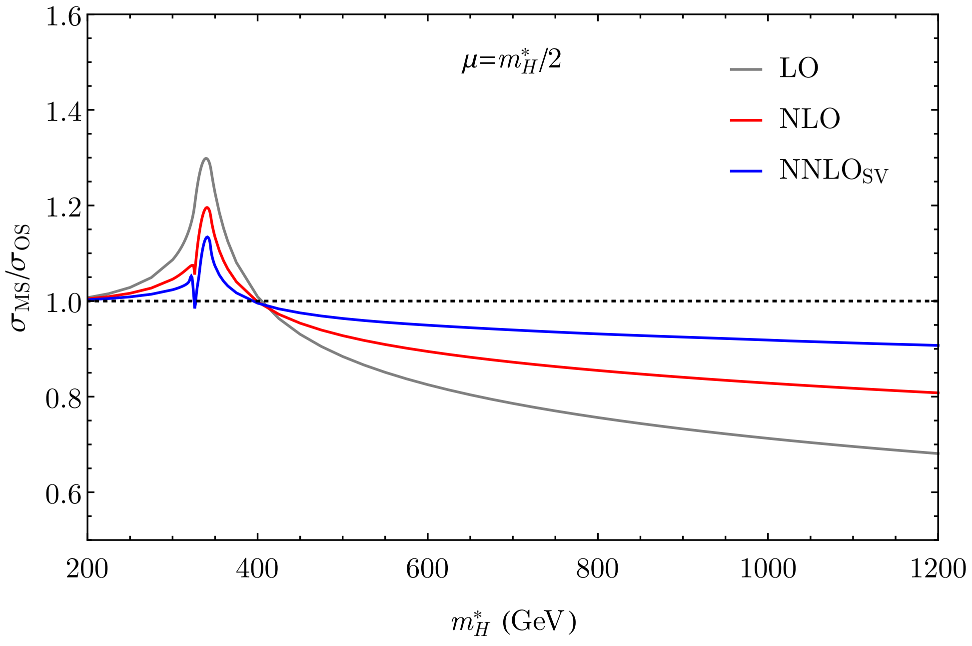
Dependence on mass-renormalization scheme can be large for large $\sqrt{s}$

off-shell H production [see Jones, Spira in Les Houches `19 Mazzitelli 22]

$$\left.\sigma(gg \to H^*)\right|_{Q=125\text{ GeV}} = 42.17^{+0.4\%}_{-0.5\%}\text{ pb}$$

$$\left.\sigma(gg \to H^*)\right|_{Q=600\text{ GeV}} = 1.97^{+0.0\%}_{-15.9\%}\text{ pb}$$

Only small dependence for physical m_H

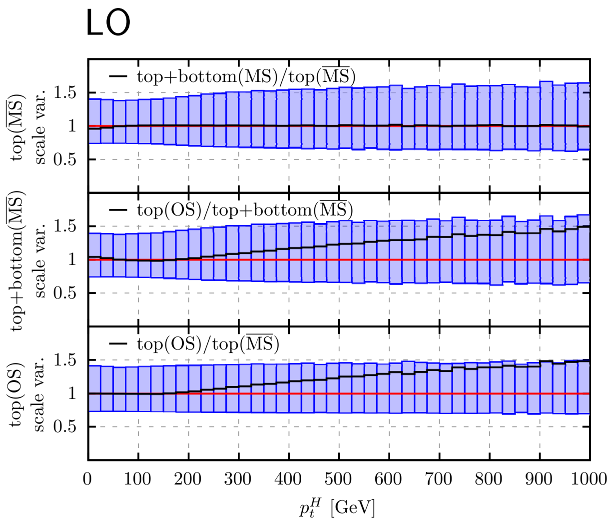


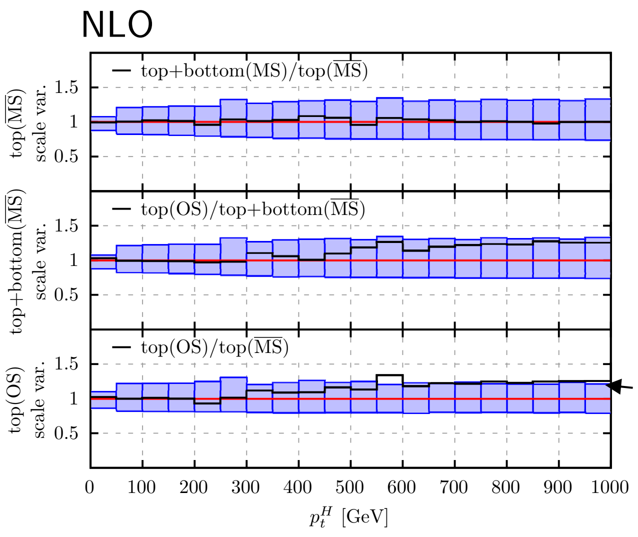
Renormalization Scheme Uncertainties

Scheme uncertainties of similar size also for other processes:

HJ production (using DiffExp approach)

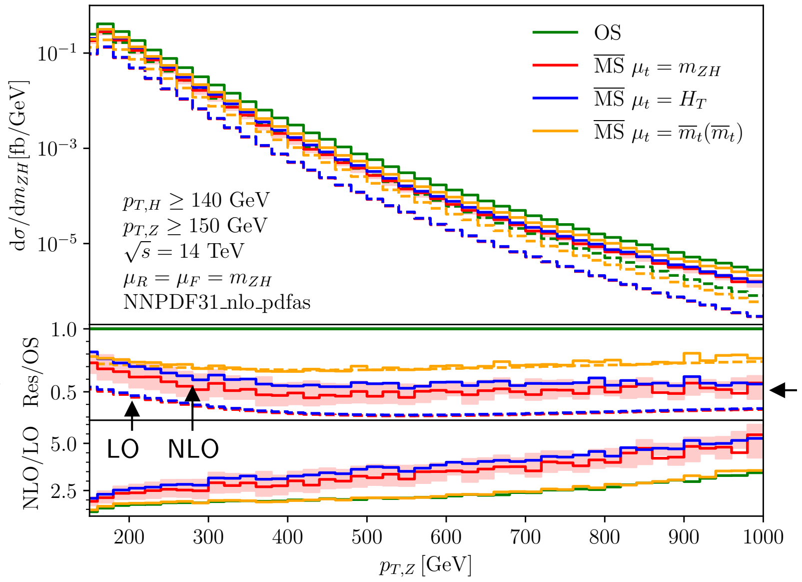
Bonciani, Del Duca, Frellesvig, Hidding, Hirschi, Moriello, Salvatori, Somogyi, Tramontano 22





ZH production (using SecDec & HE expansion)

Chen, Davies, Heinrich, Jones, MK, Mishima, Schlenk, Steinhauser 22



Scheme uncertainty typically reduced by factor of ~2 going from LO to NLO, but still O(20-50%) at large $\sqrt{s}$, p_T

Renormalization Scheme Uncertainties

Leading HE contributions in gg $\to$ HH and gg $\to$ ZH production

$$A_i^{\text{fin}} = a_s A_i^{(0),\text{fin}} + a_s^2 A_i^{(1),\text{fin}} + \mathcal{O}(a_s^3)$$

HH

$$A_i^{(0)} \sim m_t^2 f_i(s,t)$$

$$A_i^{(1)} \sim 6 C_F A_i^{(0)} \log\left[\frac{m_t^2}{s}\right]$$

LO: m_t^2 from y_t^2
NLO: leading $\log(m_t^2)$ from mass c.t.
converting to $\overline{MS}$ gives $\log(\mu_t^2/s)$
motivating scale choice of $\mu_t^2 = s$

ZH

$$A_i^{(0)} \sim m_t^2 f_i(s,t) \log^2\left[\frac{m_t^2}{s}\right],$$

$$A_i^{(1)} \sim \frac{(C_A - C_F)}{6} A_i^{(0)} \log^2\left[\frac{m_t^2}{s}\right]$$

LO: one m_t from y_t
NLO: leading $\log(m_t^2)$ not coming from mass c.t.

$\to$ The leading contributions seem to have different origins for the 2 processes

Open questions:

- Can the origin of these logarithms be understood and predicted for higher orders?
- Does this result in a preferred mass-renormalization scheme?
- Can these uncertainties be reduced w/o calculating the next order?

For H production, these logarithms were also studied in [Liu, Modi, Penin 22]

Conclusions

- Fully analytical methods remain challenging for multi-scale processes
- Powerful alternatives to analytic calculations:
  - Series Expansions in Kinematics
  - Sector Decomposition
  - Solve DEQs via series expansions
- Mass renormalization important source of uncertainty

Conclusions

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Thank you for your attention!