

# Decoding the EOS of neutron star-like matter via flow patterns of nuclear cluster emitted in HI collisions

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for the  
HADES Collaboration

MU Days 2023

15<sup>th</sup> September 2023

GEFÖRDERT VOM



Bundesministerium  
für Bildung  
und Forschung

**HFHF**

GOETHE  
UNIVERSITÄT  
FRANKFURT AM MAIN

**HADES**





# Outline

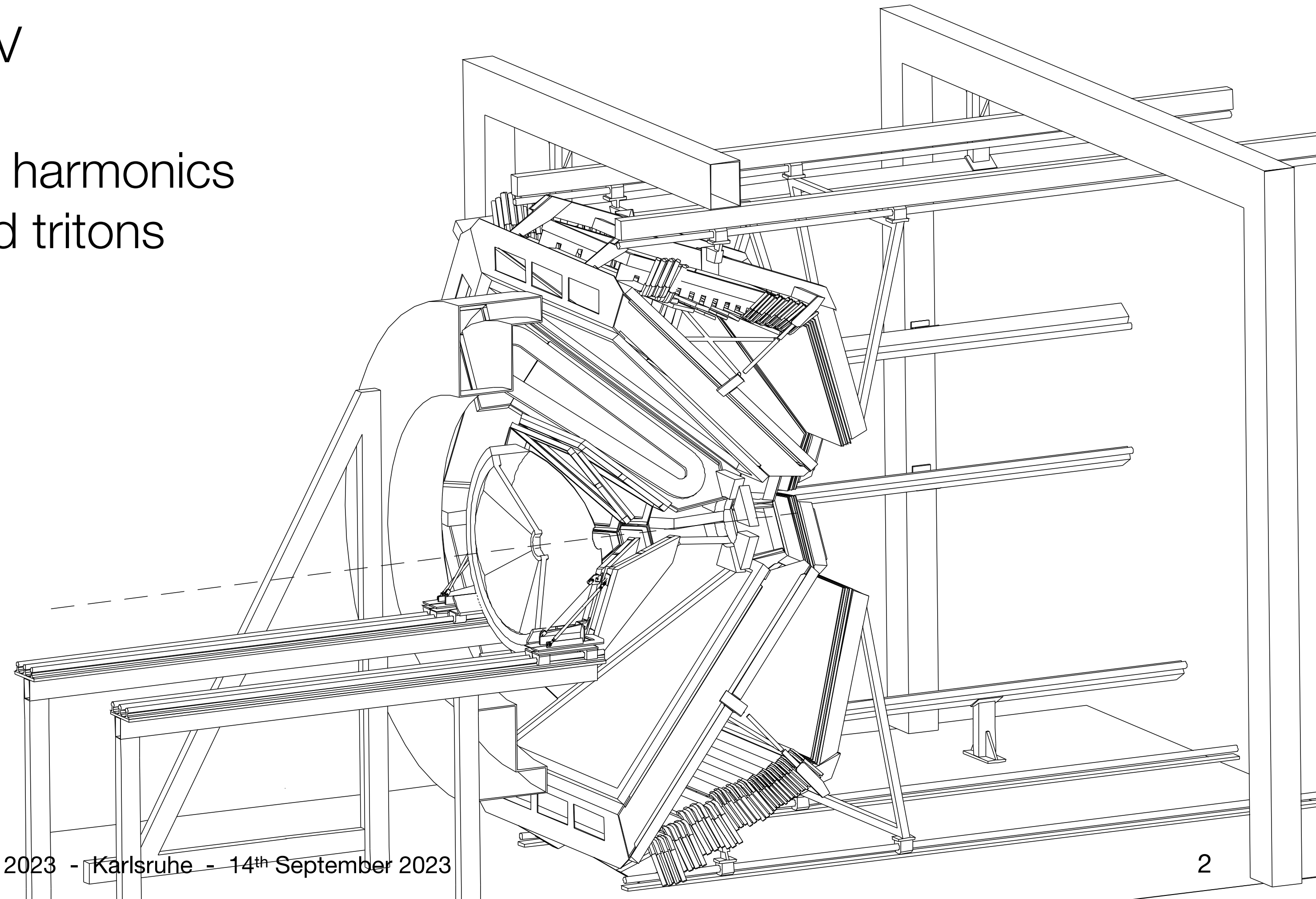
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- Dense nuclear matter and collective phenomena
- HADES and Au+Au data at 1.23 AGeV
- Directed  $v_1$ , elliptic  $v_2$ , and higher flow harmonics ( $v_3, v_4, v_5, v_6$ ) of protons, deuterons and tritons
- Model comparisons
- Event-wise flow correlations

**Talk based on following publication:**

**HADES, PRL 125 (2020) 262301 [arXiv:2005.12217](#) [hepdata]**

**HADES, EPJ A 59 (2023) 80 [arXiv:2208.02740](#) [hepdata soon]**





# Nuclear Matter under Extreme Conditions

What is the nature of matter?

And what are the properties of nuclear matter under the most extreme conditions?

Equation-of-state of dense matter in the *universe* and in the *laboratory*

## Neutron Star Merger

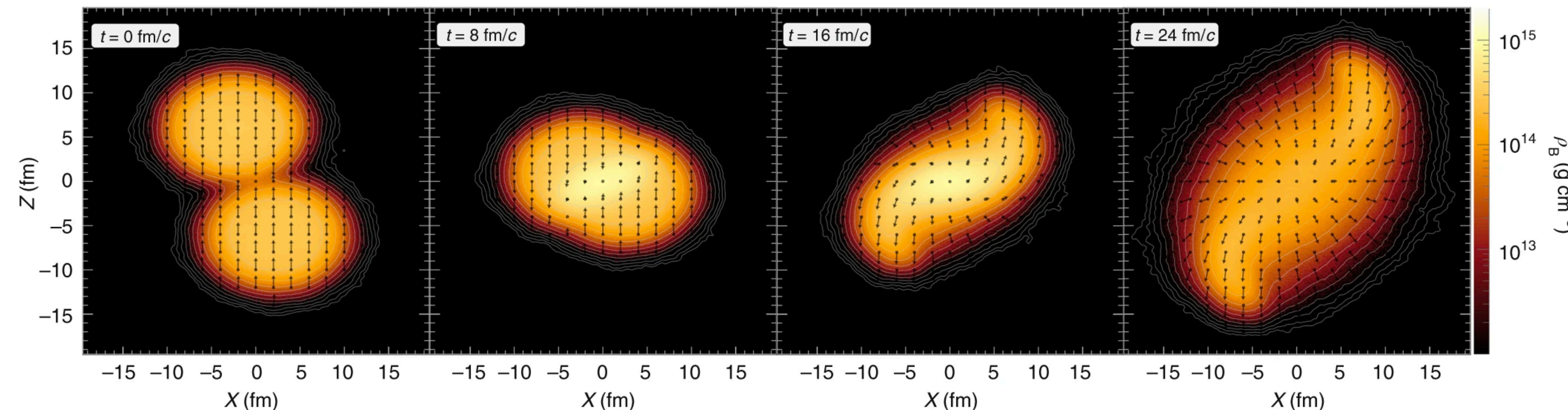
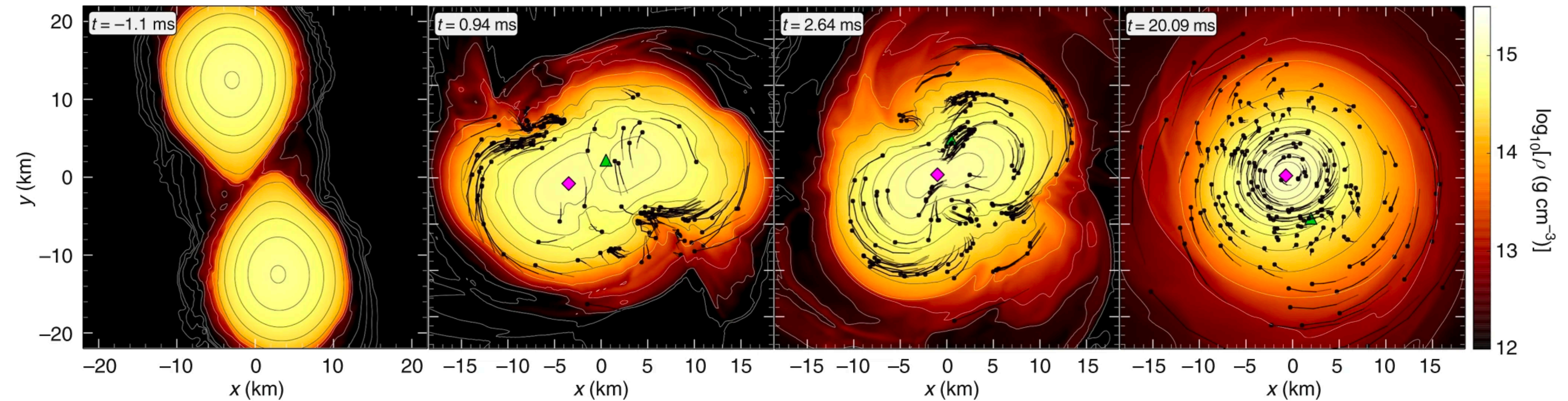
Observation via gravitational waves

**GW170817**: B.P. Abbott et al. (LIGO + VIRGO)  
PRL **119** (2017) 1611001

## Heavy-ion Collision

Equation-of-state of dense matter

HADES, Nature Phys. **15** (2019) 1040





# Collective Effects

## Flow Phenomenology

### Emission relative to event plane

Interactions in medium, nuclear stopping  
 $\Rightarrow$  buildup of non-uniform pressure gradients  
 provides accelerating forces in different directions

Access to medium properties, e.g. viscosity,  
 equation-of-state

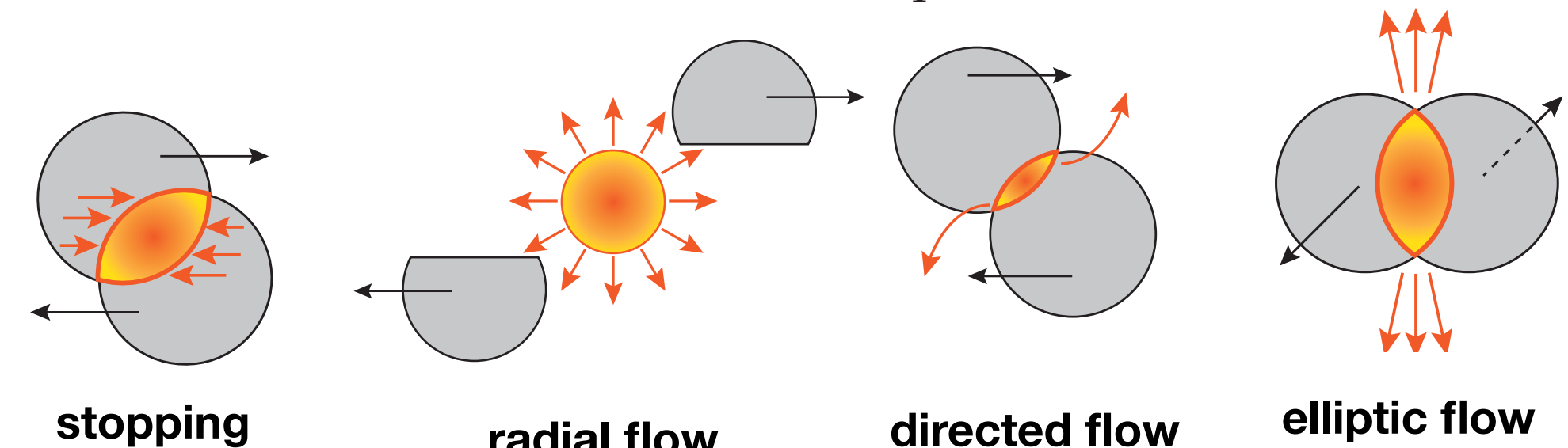
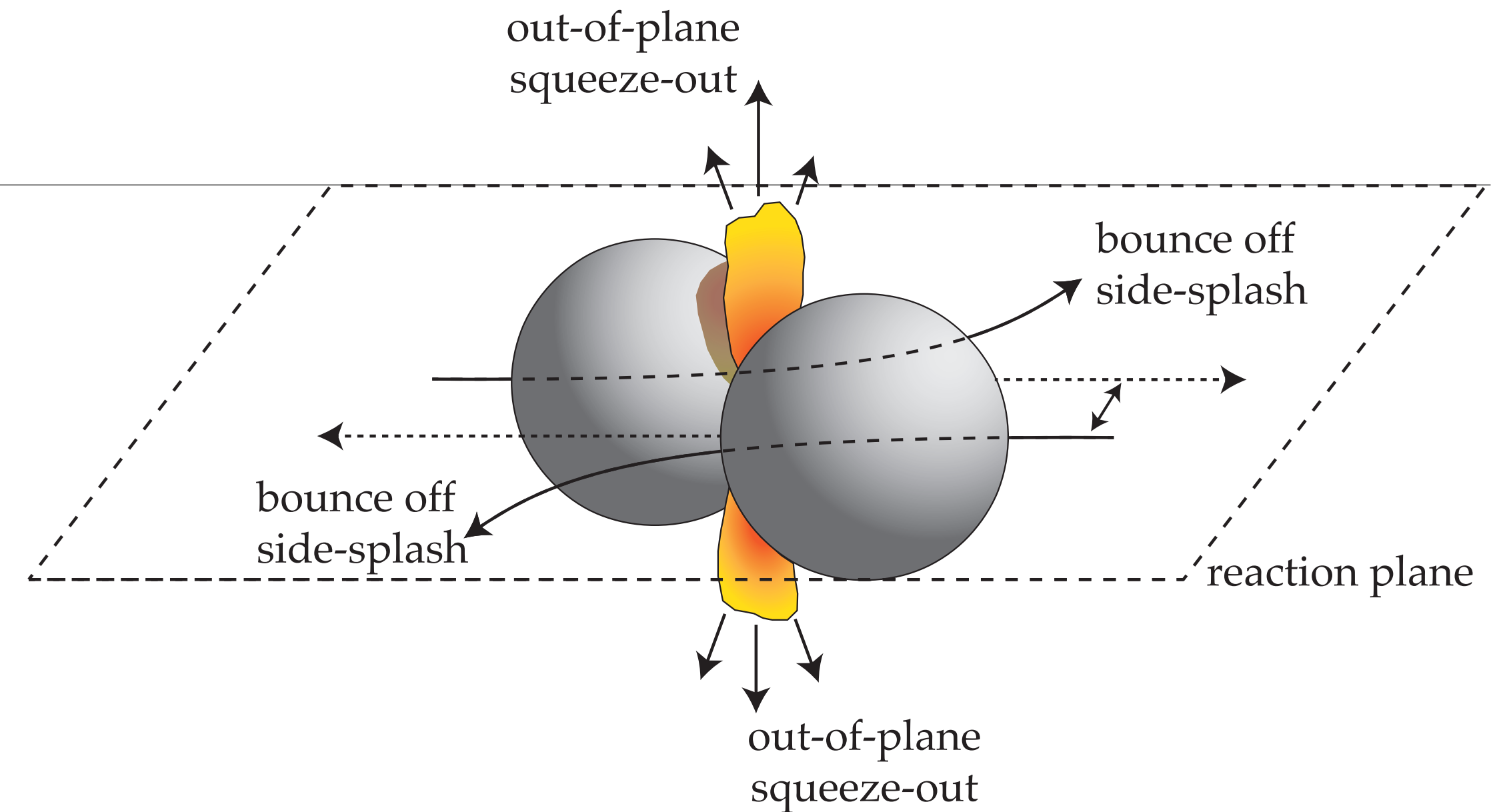
### Fourier-decomposition

of the triple differential invariant cross section

$$E \frac{d^3 N}{dp^3} = \frac{1}{2\pi} \frac{d^2 N}{p_t dp_t dy} \left( 1 + 2 \sum_{n=1}^{\infty} v_n(p_t, y) \cos(n\phi) \right)$$

Extraction of azimuthal moments  $v_n$   $\phi = (\varphi - \Psi_{RP})$

$$v_n(p_t, y) = \langle \cos(n\phi) \rangle$$

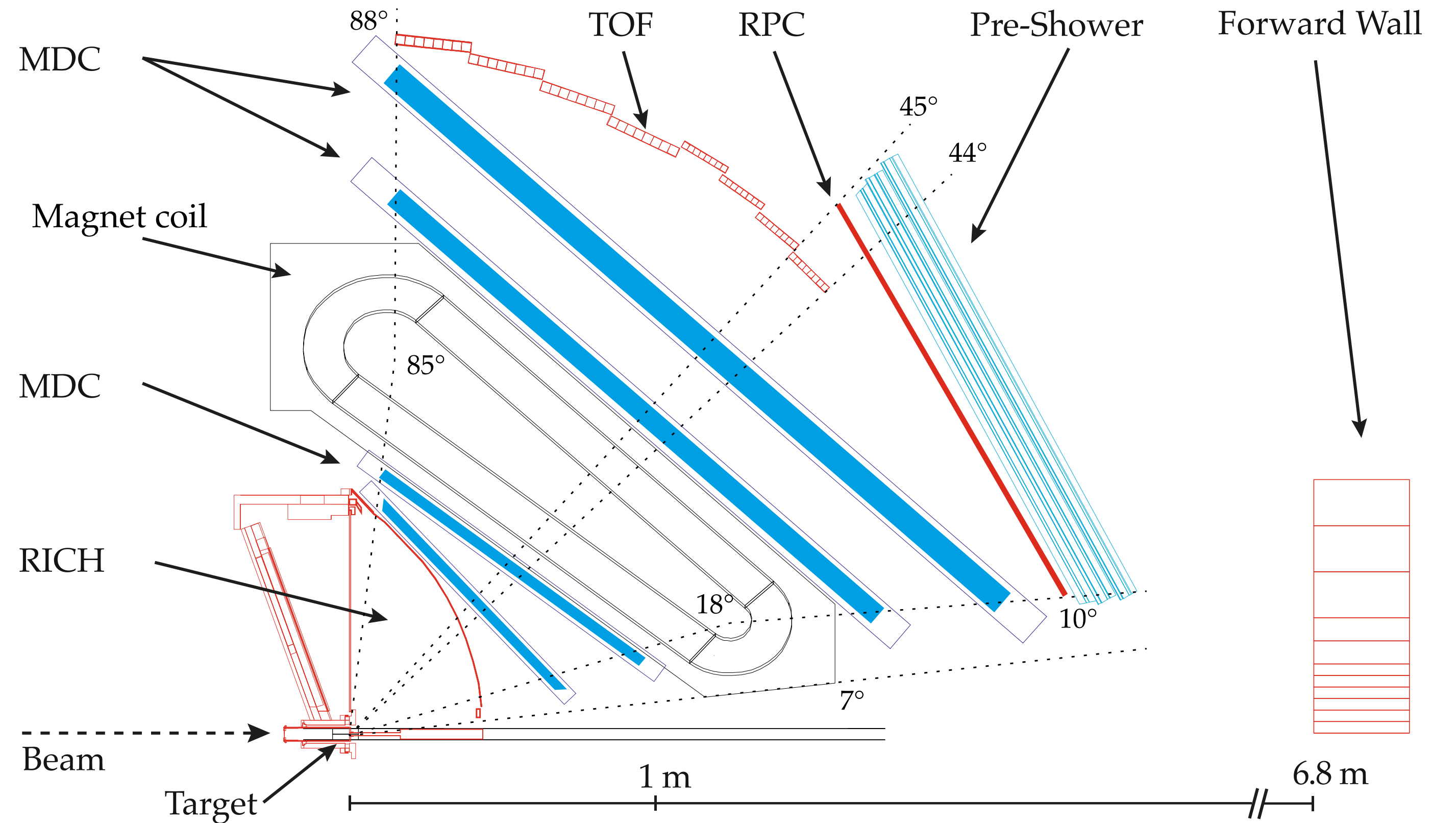


$$\begin{aligned} v_1 &= \langle \cos \phi \rangle = \langle p_x / p_t \rangle, \\ v_2 &= \langle \cos(2\phi) \rangle = \langle (p_x^2 - p_y^2) / p_t^2 \rangle, \\ v_3 &= \langle \cos(3\phi) \rangle = \langle (p_x^3 - 3p_x p_y^2) / p_t^3 \rangle, \\ v_4 &= \langle \cos(4\phi) \rangle = \langle (p_x^4 - 6p_x^2 p_y^2 + p_y^4) / p_t^4 \rangle, \\ v_5 &= \langle \cos(5\phi) \rangle = \langle (p_x^5 - 10p_x^3 p_y^2 + 5p_x p_y^4) / p_t^5 \rangle, \\ v_6 &= \langle \cos(6\phi) \rangle = \langle (p_x^6 - 15p_x^4 p_y^2 + 15p_x^2 p_y^4 - p_y^6) / p_t^6 \rangle. \end{aligned}$$



# High Acceptance Di-Electron Spectrometer

- High interaction rates and statistics
  - ▶ 5 weeks (558.3 hours) of Au+Au data taking with  $7 \times 10^9$  recorded events
  - ▶ Beam intensities  $1.2 - 2.2 \times 10^6$
- Large acceptance in 6 identical sectors
  - ▶ Symmetric azimuthal coverage
  - ▶  $18^\circ - 85^\circ$  in polar angle
- Low-mass tracking system
  - ▶ 4 Planes of multi-wire chambers with Mini-Drift Cells (MDC)
  - ▶ 6 Coils of superconducting toroidal magnets
- Particle Identification
  - ▶ Time-of-Flight (TOF and RPC)
  - ▶ Energy loss in the MDC
- Forward Wall
  - ▶ Reaction plane reconstruction

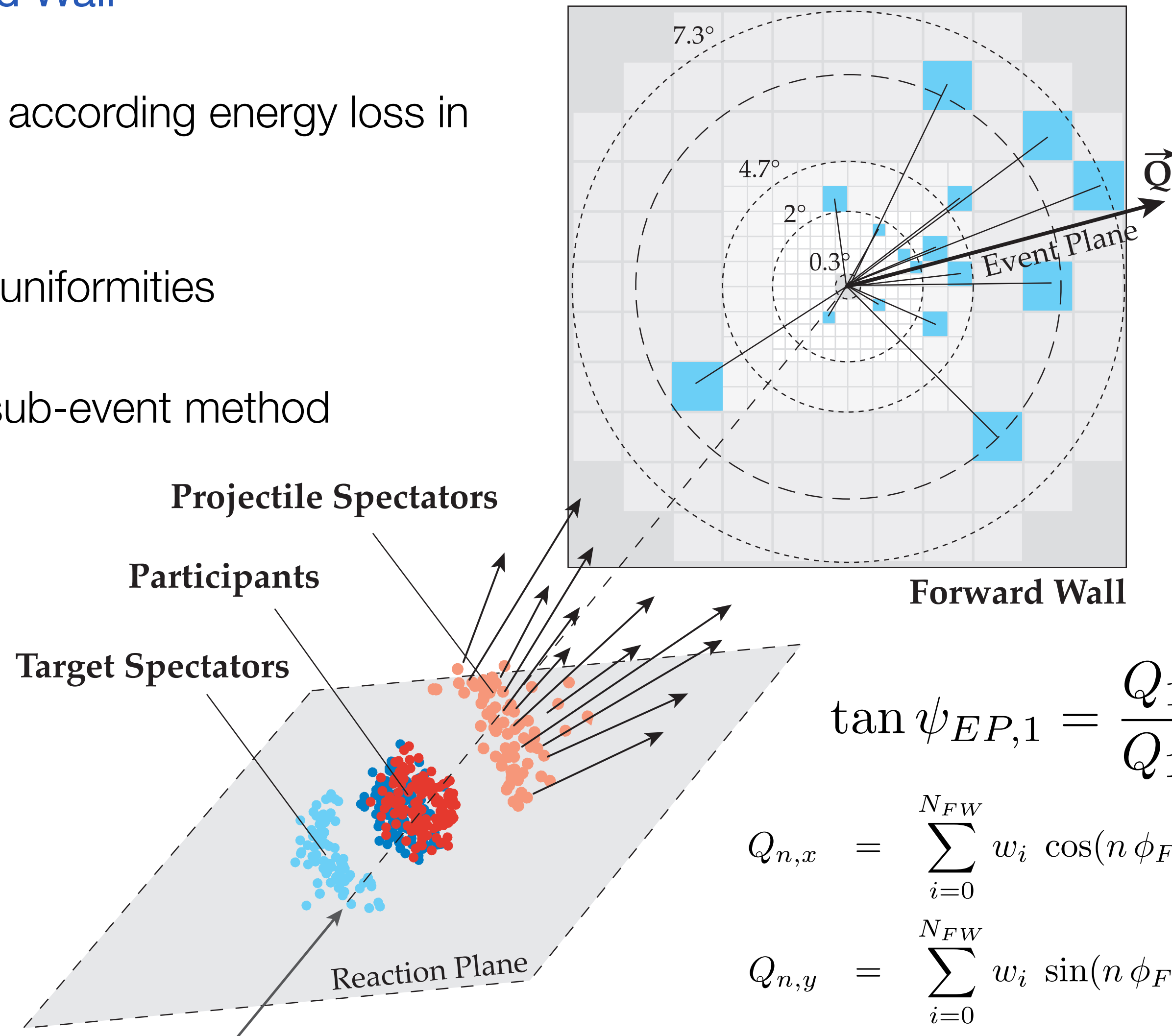




# Event Plane Reconstruction

## Event plane of 1st-Order from Projectile spectators in Forward Wall

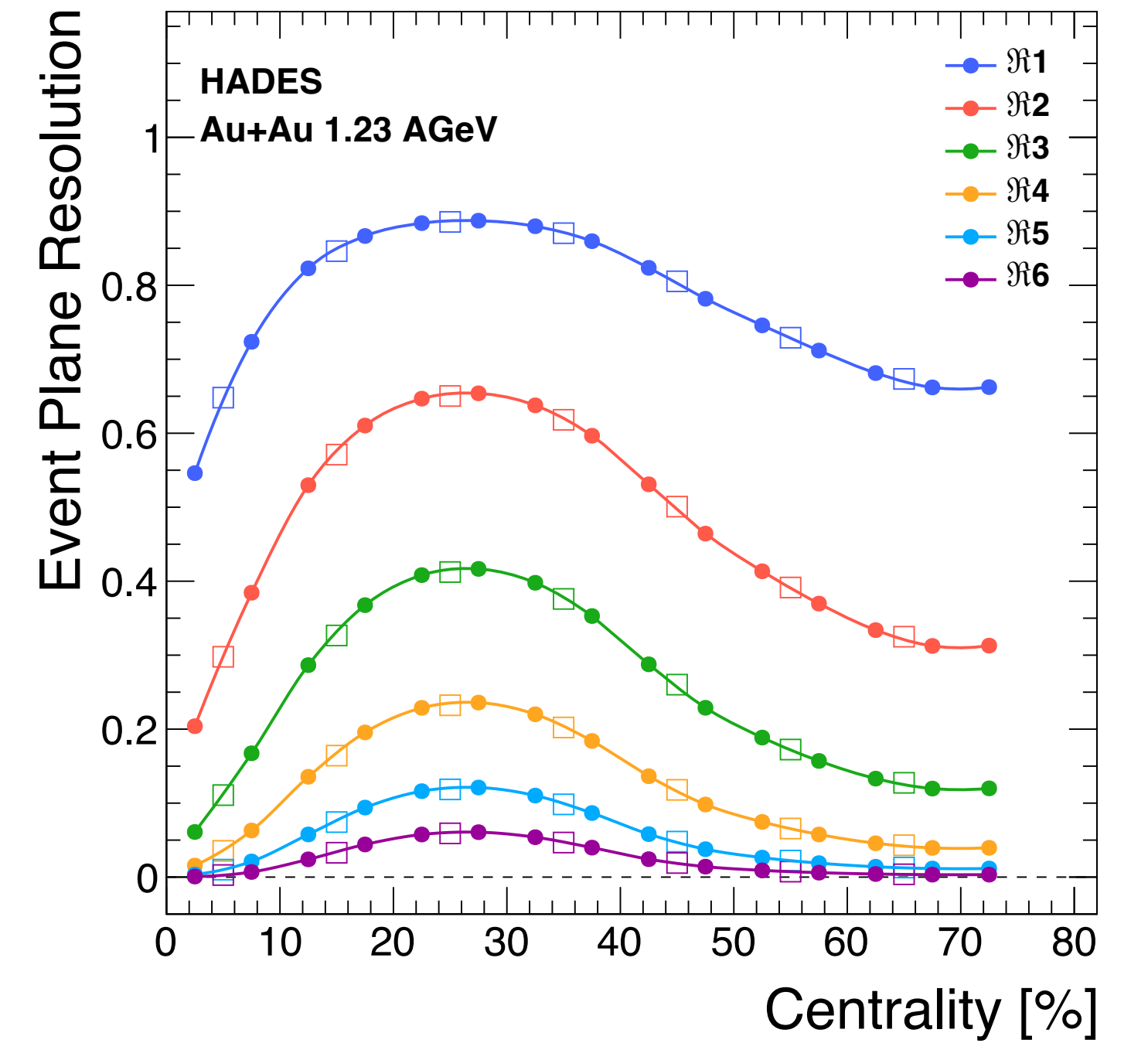
- Charge-Weighting according energy loss in scintillators
- Correction of non-uniformities
- EP-resolution via sub-event method



$$\tan \psi_{EP,1} = \frac{Q_{1,y}}{Q_{1,x}}$$

$$Q_{n,x} = \sum_{i=0}^{N_{FW}} w_i \cos(n \phi_{FW,i})$$

$$Q_{n,y} = \sum_{i=0}^{N_{FW}} w_i \sin(n \phi_{FW,i})$$



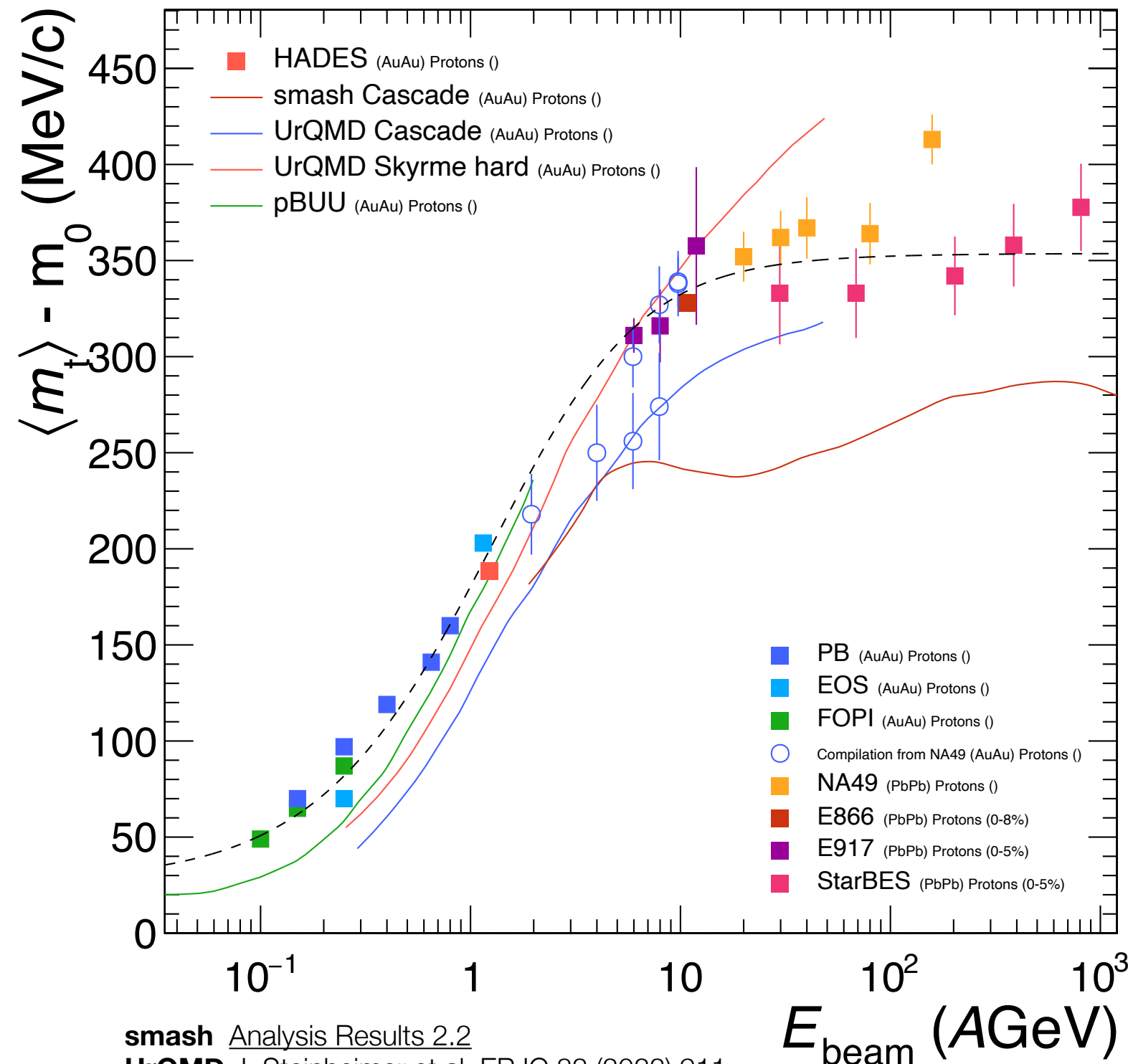
$$v_n = v_n^{obs} / \mathcal{R}_n$$

$$\mathcal{R}_n = \langle \cos[n(\Psi_n - \Psi_{RP})] \rangle$$



# Collective Effects

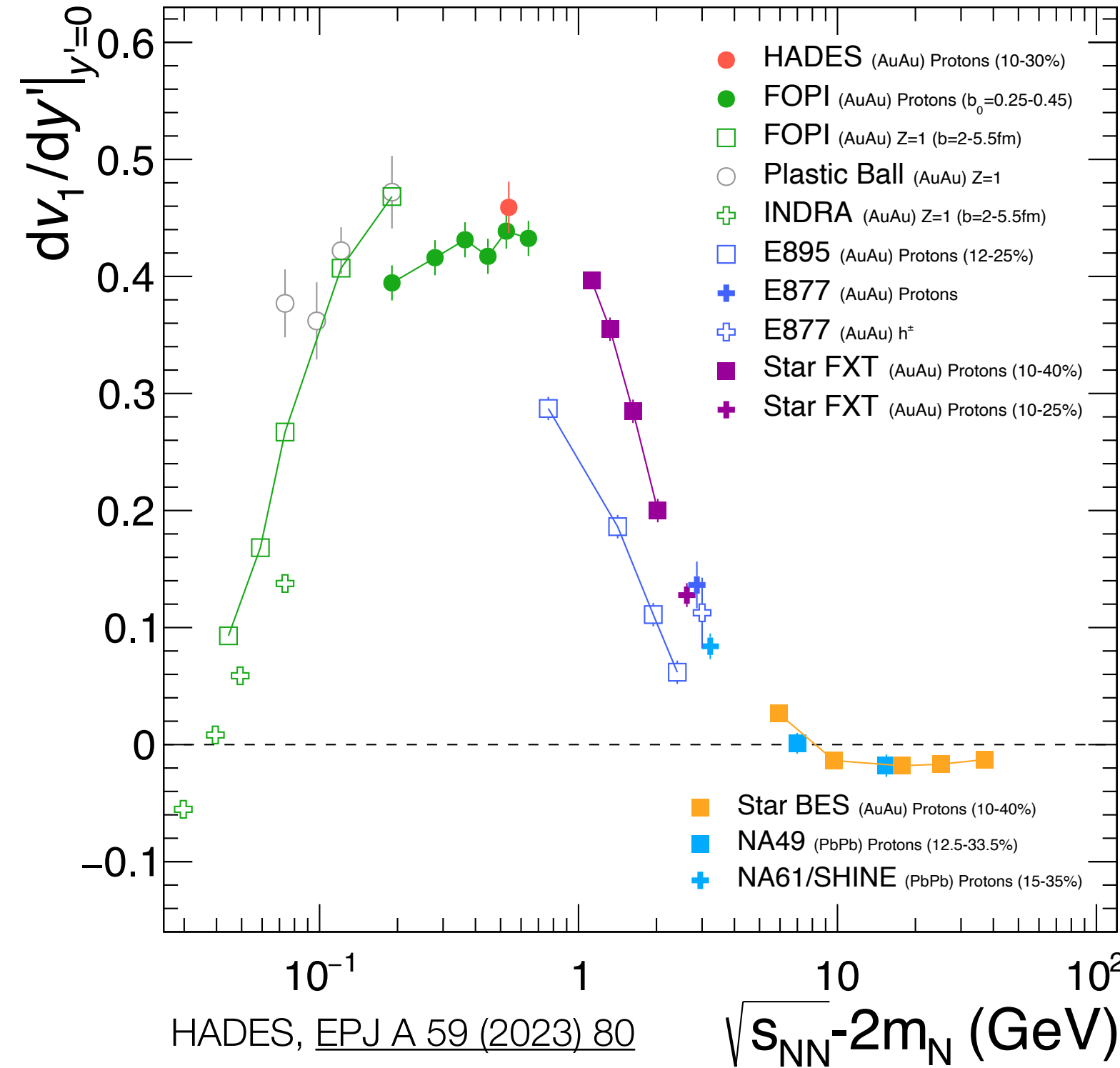
## Energy-Dependence



smash Analysis Results 2.2  
 UrQMD J. Steinheimer et al, EPJC 82 (2022) 911  
 pBUU P. Danielewicz, PRC 51 (1995) 716

### Compilation of world data

Good agreement of mean transverse mass  $\langle m_t \rangle - m_0$ , integrated directed flow  $dv_1/dy$  and elliptic flow  $v_2$

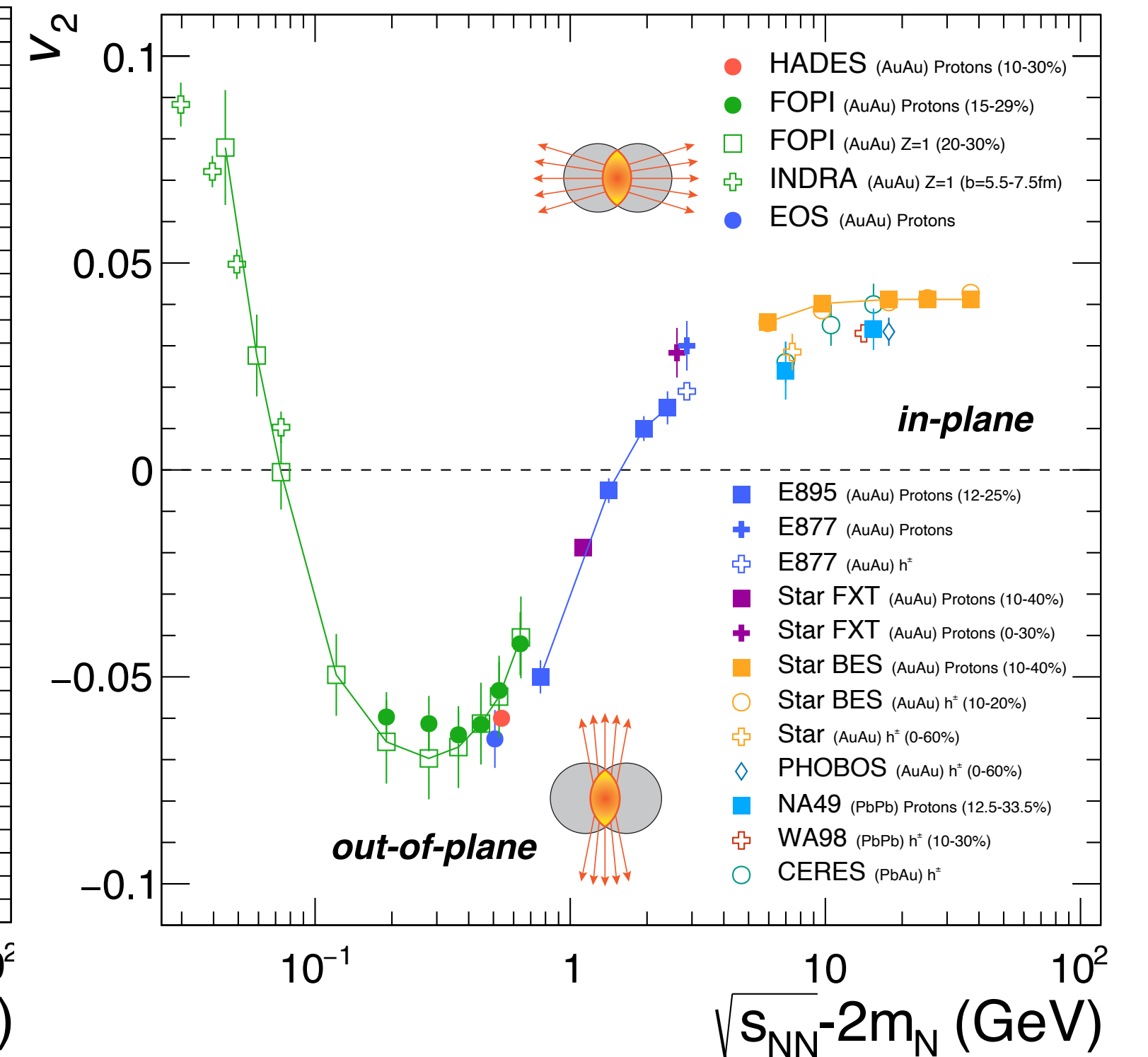


HADES, EPJ A 59 (2023) 80  
 Update Star FXT QM2023

### Out-of-Plane $v_2$

Long spectator passing time at HADES energy

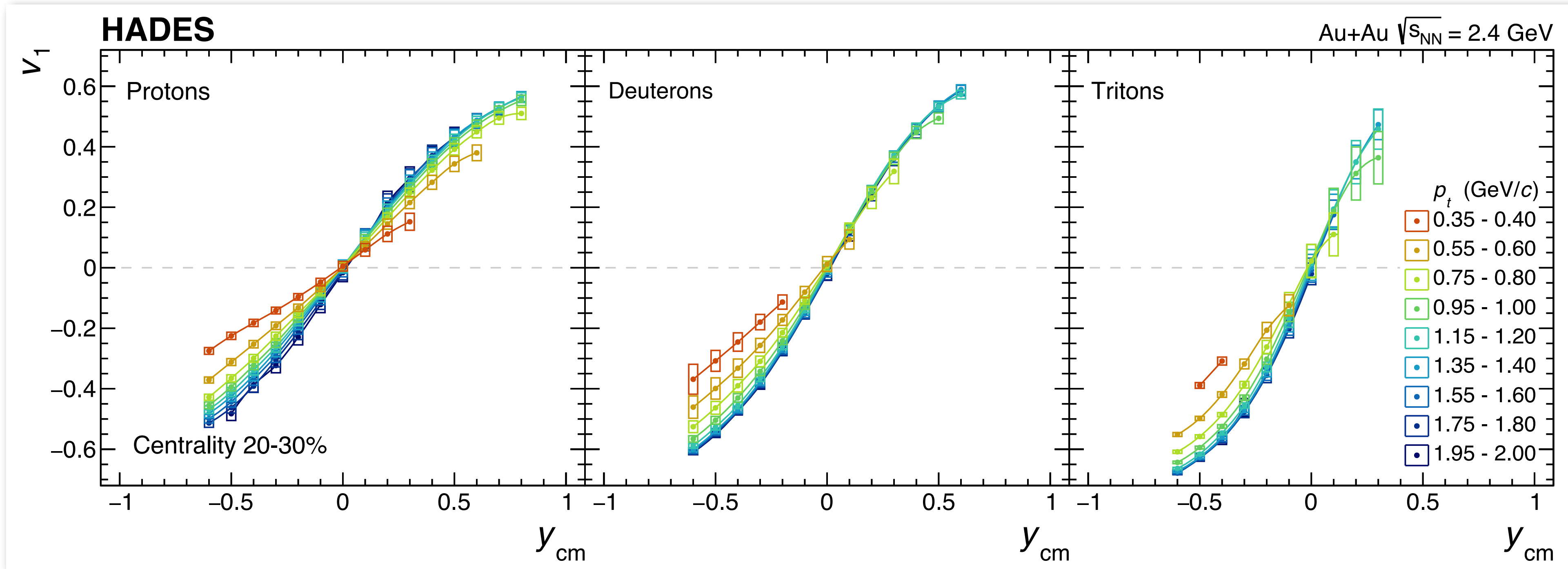
$\tau_{\text{passing}} \approx \tau_{\text{expansion}} \Rightarrow$  "squeeze-out"





# Collective Effects

Results on  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  for Protons, Deuterons and Tritons

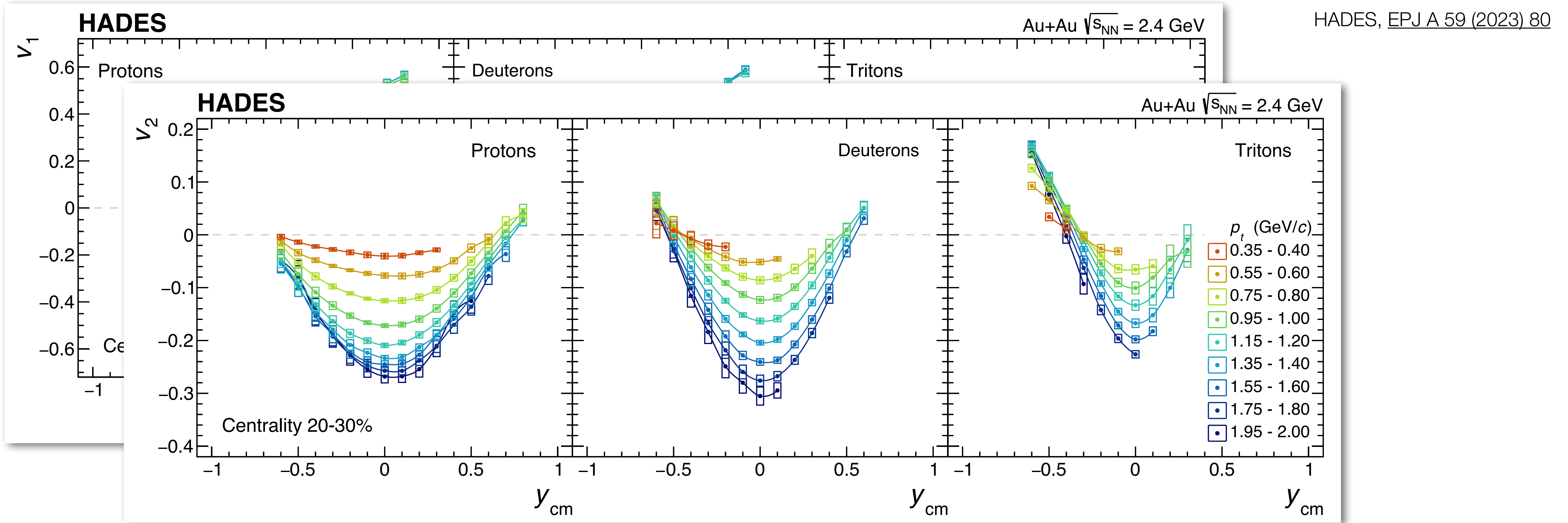


HADES, EPJ A 59 (2023) 80



# Collective Effects

Results on  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  for Protons, Deuterons and Tritons

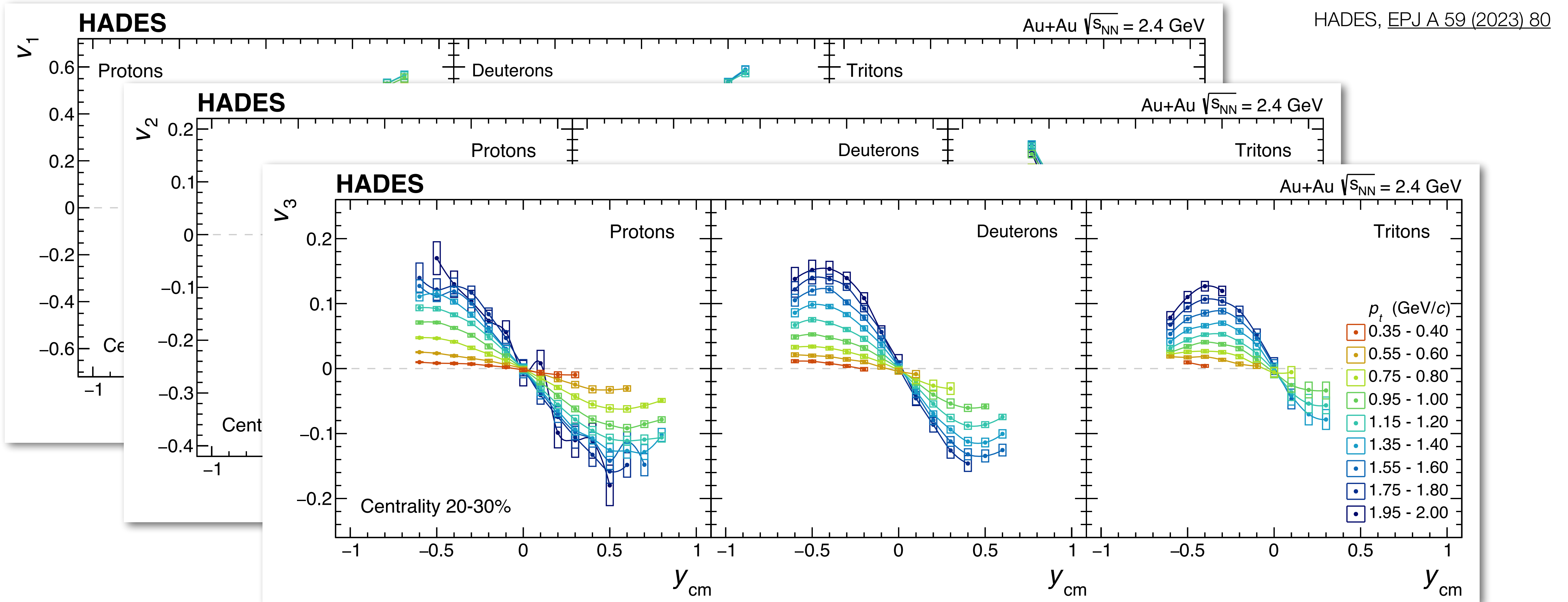


HADES, EPJ A 59 (2023) 80



# Collective Effects

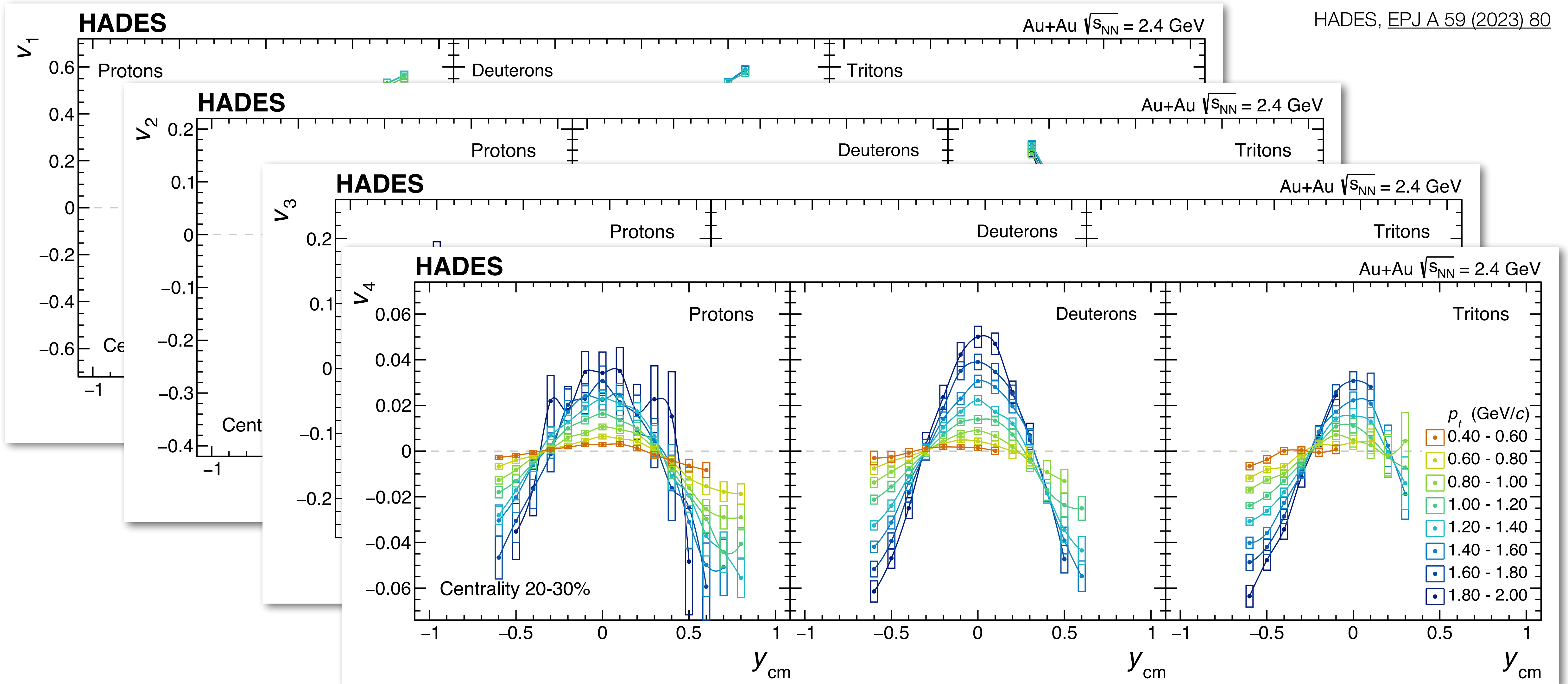
Results on  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  for Protons, Deuterons and Tritons





# Collective Effects

Results on  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  for Protons, Deuterons and Tritons



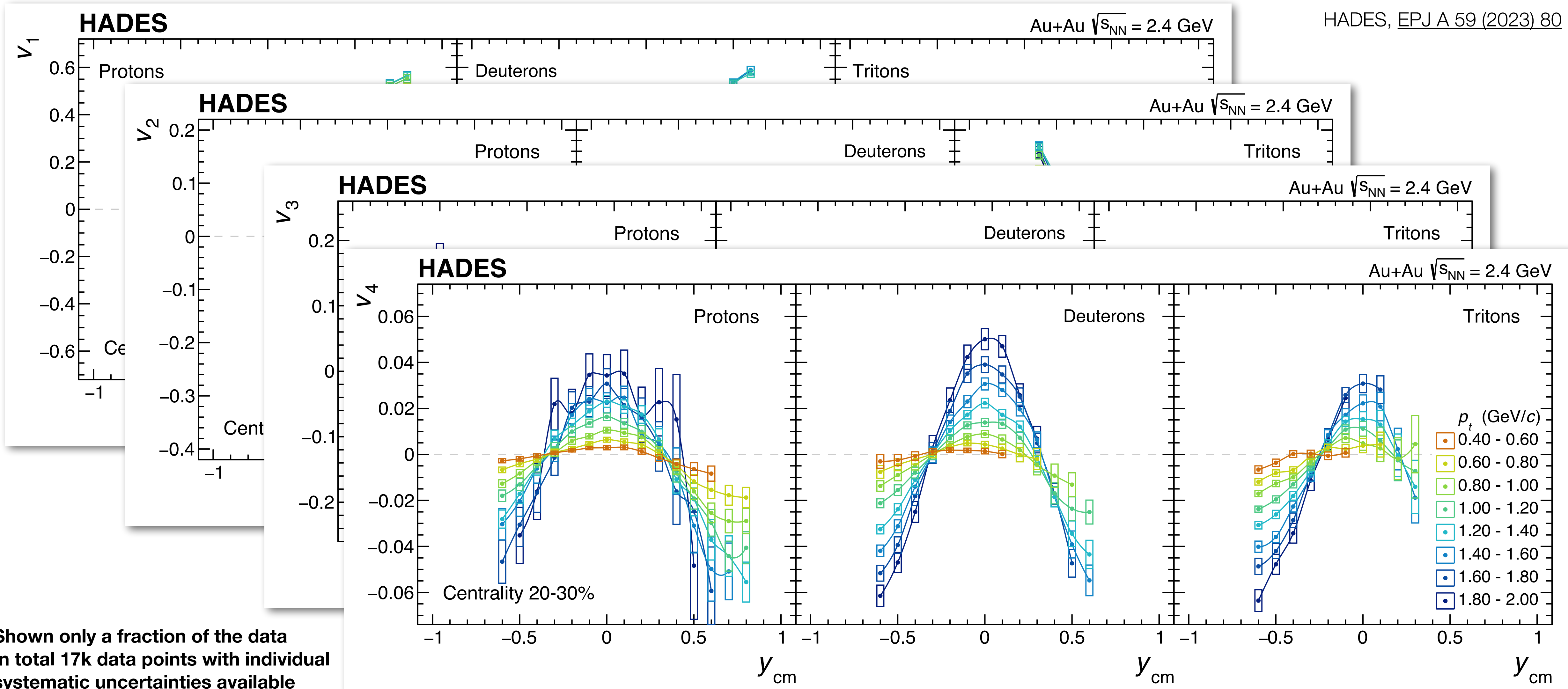
HADES, EPJ A 59 (2023) 80



# Collective Effects

Results on  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  for Protons, Deuterons and Tritons

HADES, EPJ A 59 (2023) 80



Shown only a fraction of the data  
In total 17k data points with individual  
systematic uncertainties available

# Model Comparisons to Proton Data

## Determination of EOS

New level of precision - multi differential  
Additional information from higher orders

### Models:

- JAM 1.9 NS3 (hard EOS, mom.-indep.)
- JAM 1.9 MD1 (hard EOS, mom.-dep.)
- JAM 1.9 MD4 (soft EOS, mom.dep.)
- UrQMD 3.4 (hard EOS, mom.-indep.)
- GiBUU Skyrme 12 (soft EOS)

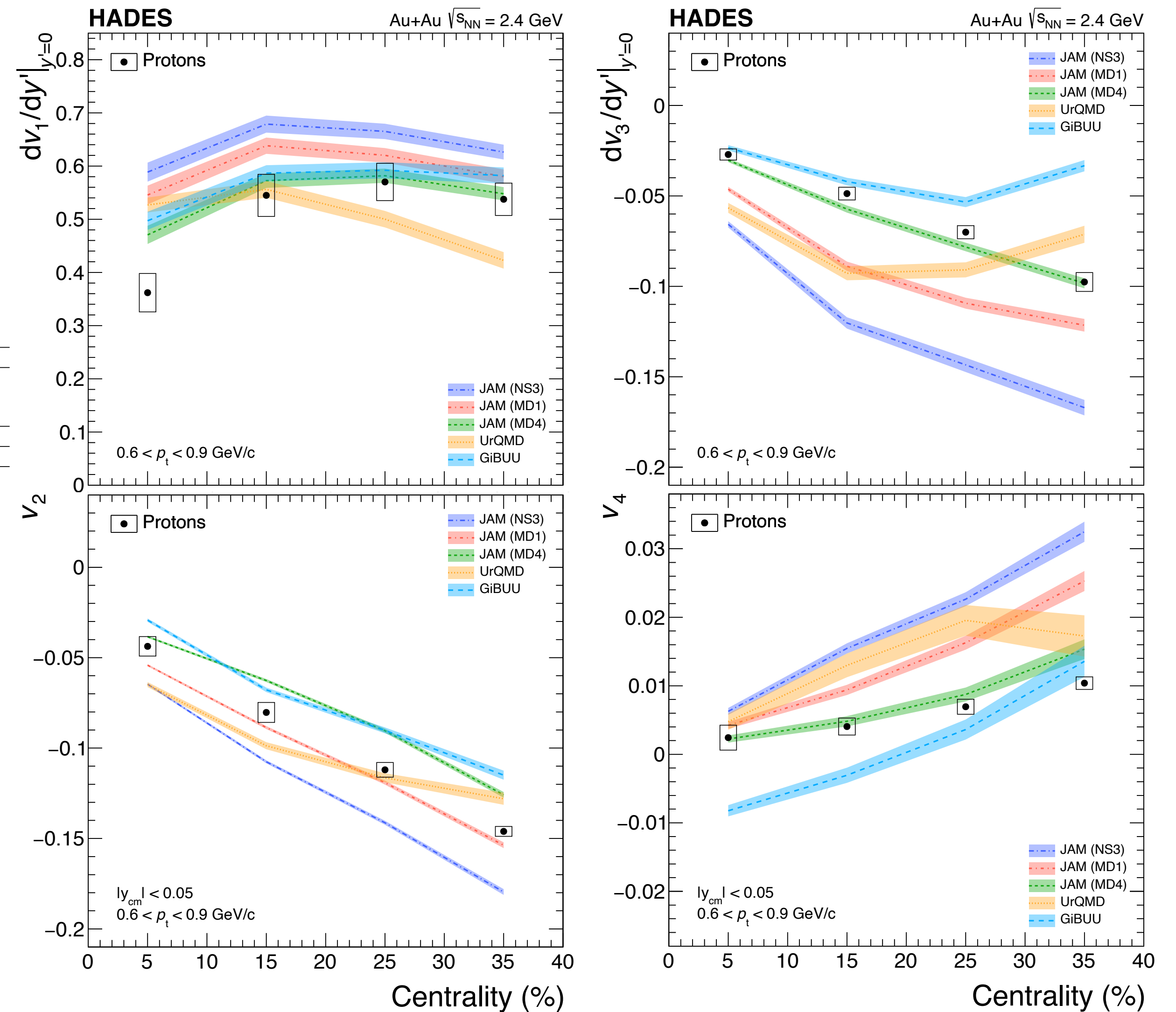
Model	EOS	$K$ (MeV)	$m^*/m$	mom-dep.
JAM 1.90591	NS1	380	0.83	no
	MD1	380	0.65	yes
	MD4	210	0.83	yes
UrQMD 3.4	Hard	380		no
GiBUU 2019 (patch7)	Skyrme 12	240	0.75	no

## Conclusions

Overall trend reasonably described,  
but no model works everywhere

Several systematic deviations

Unified description of light nuclei  
production missing





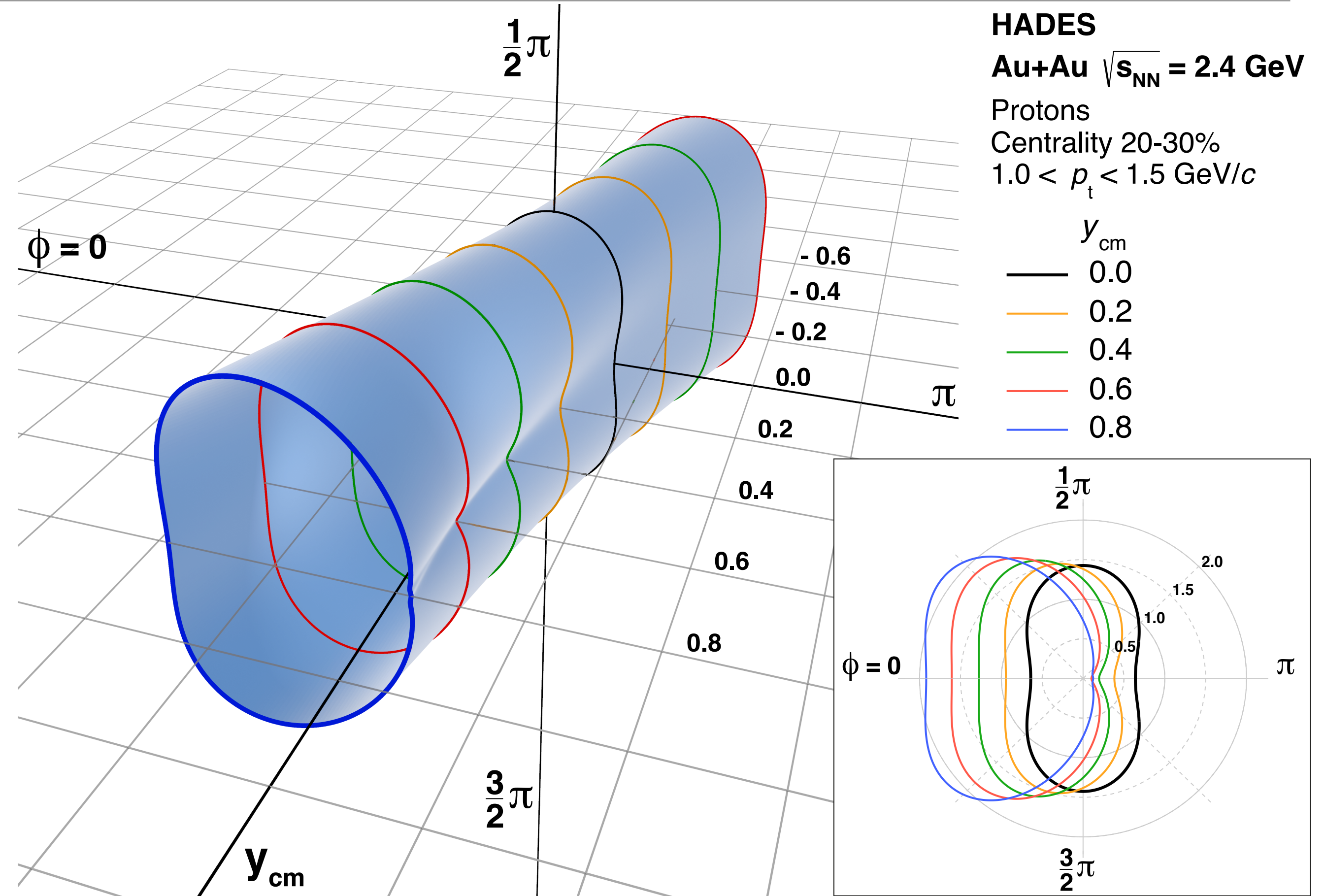
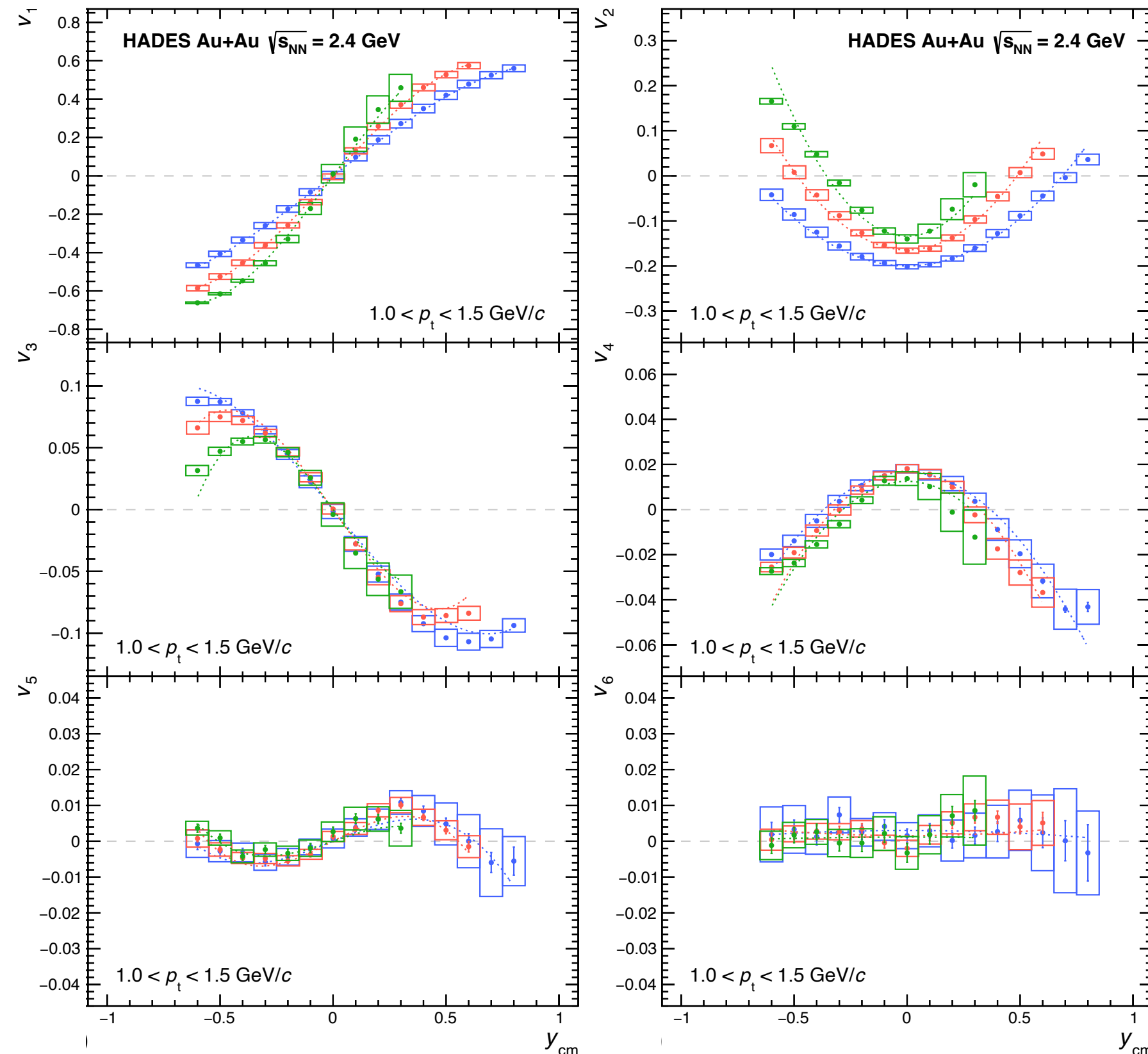
# Emission Pattern

## Protons

HADES, Phys. Rev. Lett. **125** (2020) 262301

Allows to reconstruct a full 3D-picture of the emission pattern in momentum space

Complex evolution of shape as function of rapidity determined by flow coefficients  $v_1$  —  $v_6$



$$1 + 2 \sum_{n=1}^{\infty} v_n(y_{cm}) \cos n(\phi - \psi_{RP})$$

$$v_{1,3,5}(y_{cm}) = a y_{cm} + b y_{cm}^3$$

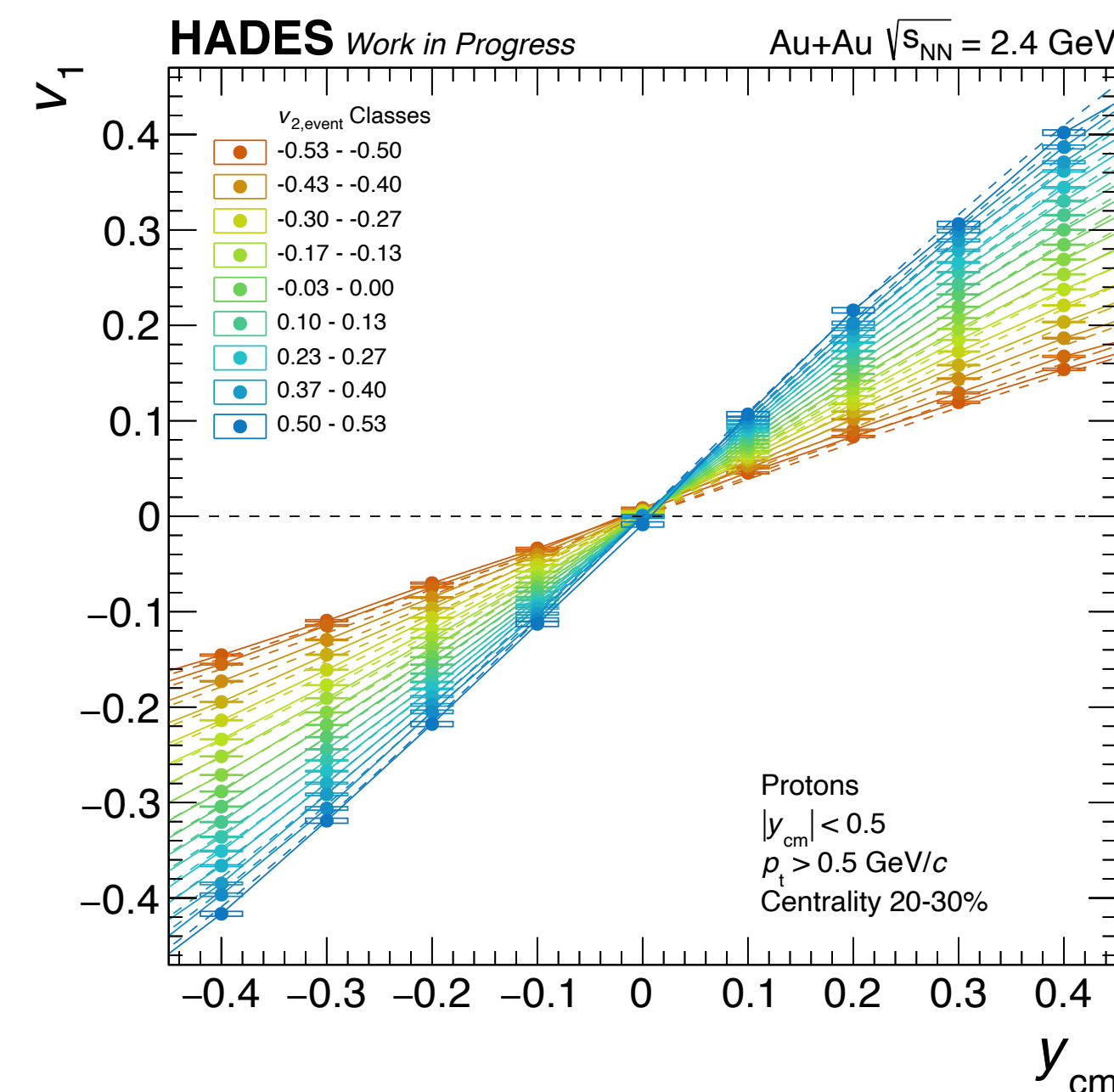
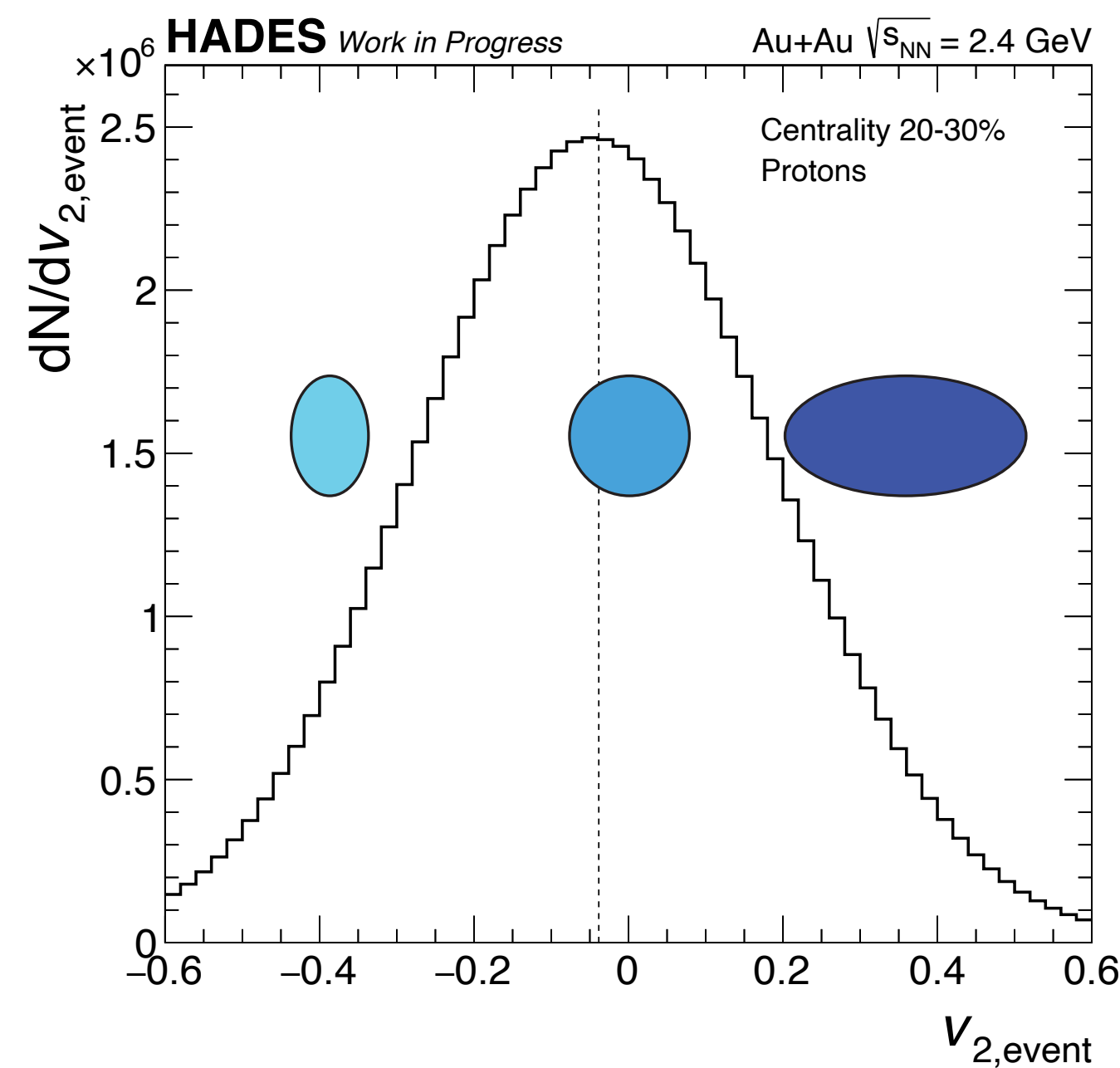
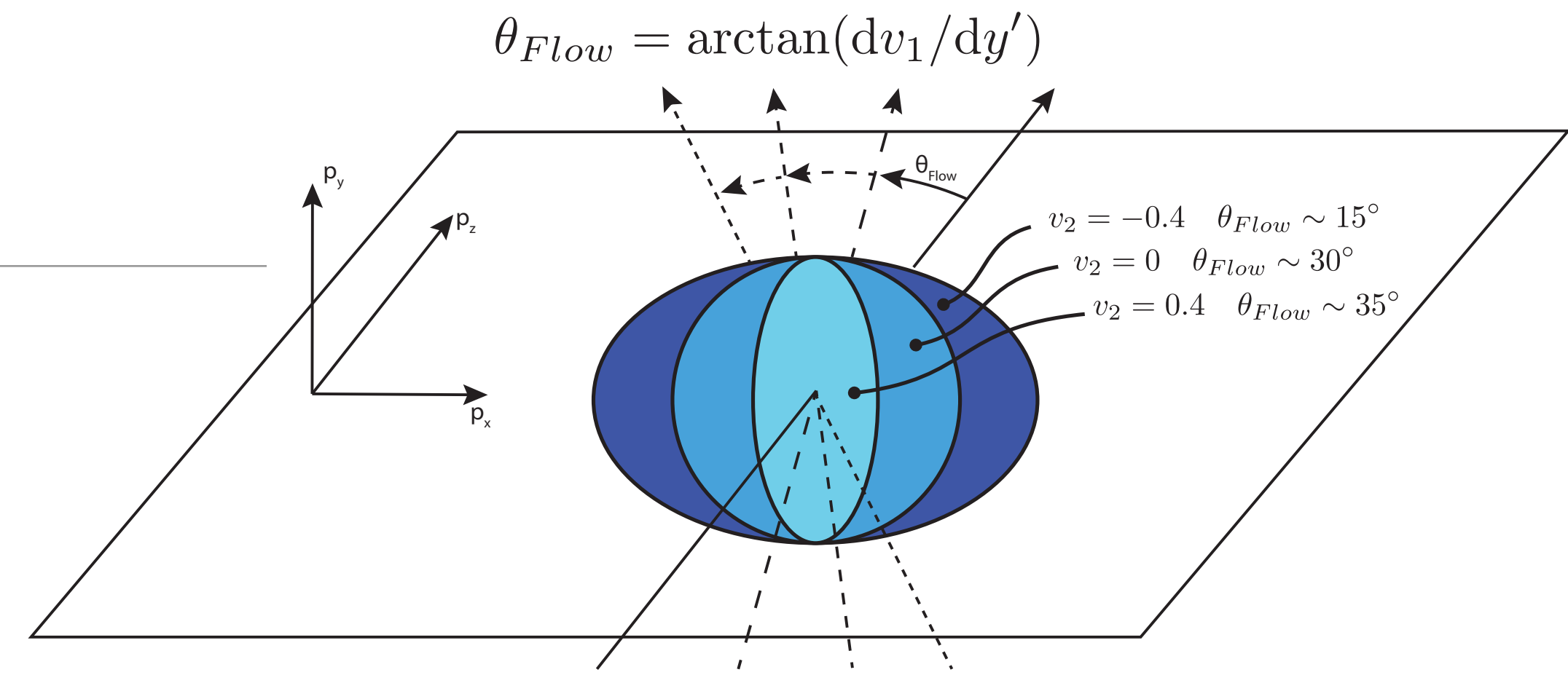
$$v_{2,4,6}(y_{cm}) = c + d y_{cm}^2$$

First Proposed in S. Voloshin and Y. Zhang  
Z.Phys. C70 (1996) 665-672

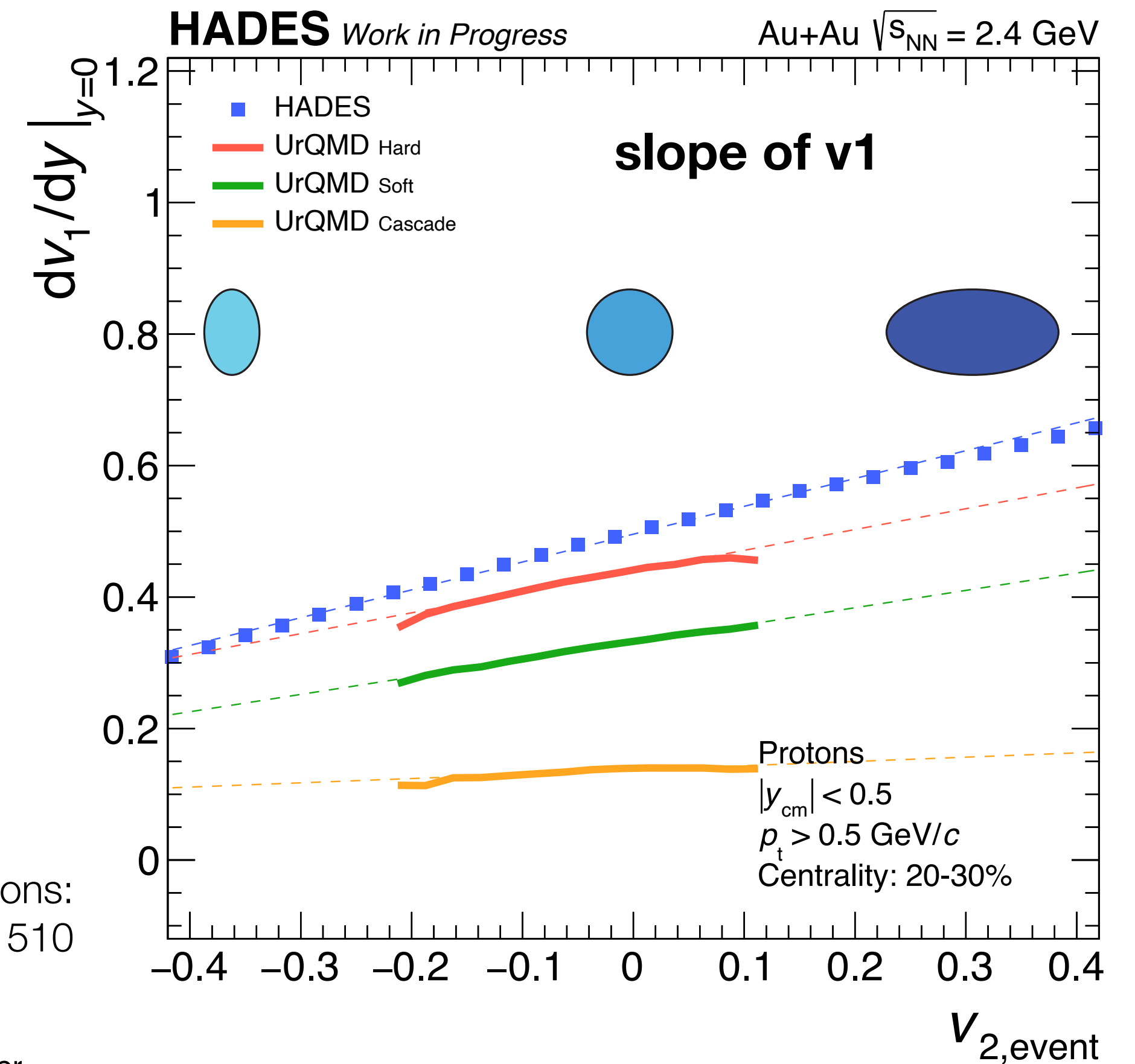
# Event-wise Flow Correlations

Events can be characterised according to the event-wise magnitude of the elliptic flow  $v_{2,event}$

Slope of directed flow  $dv_1/dy|_{y=0}$  resp. flow angle  $\theta_{Flow}$

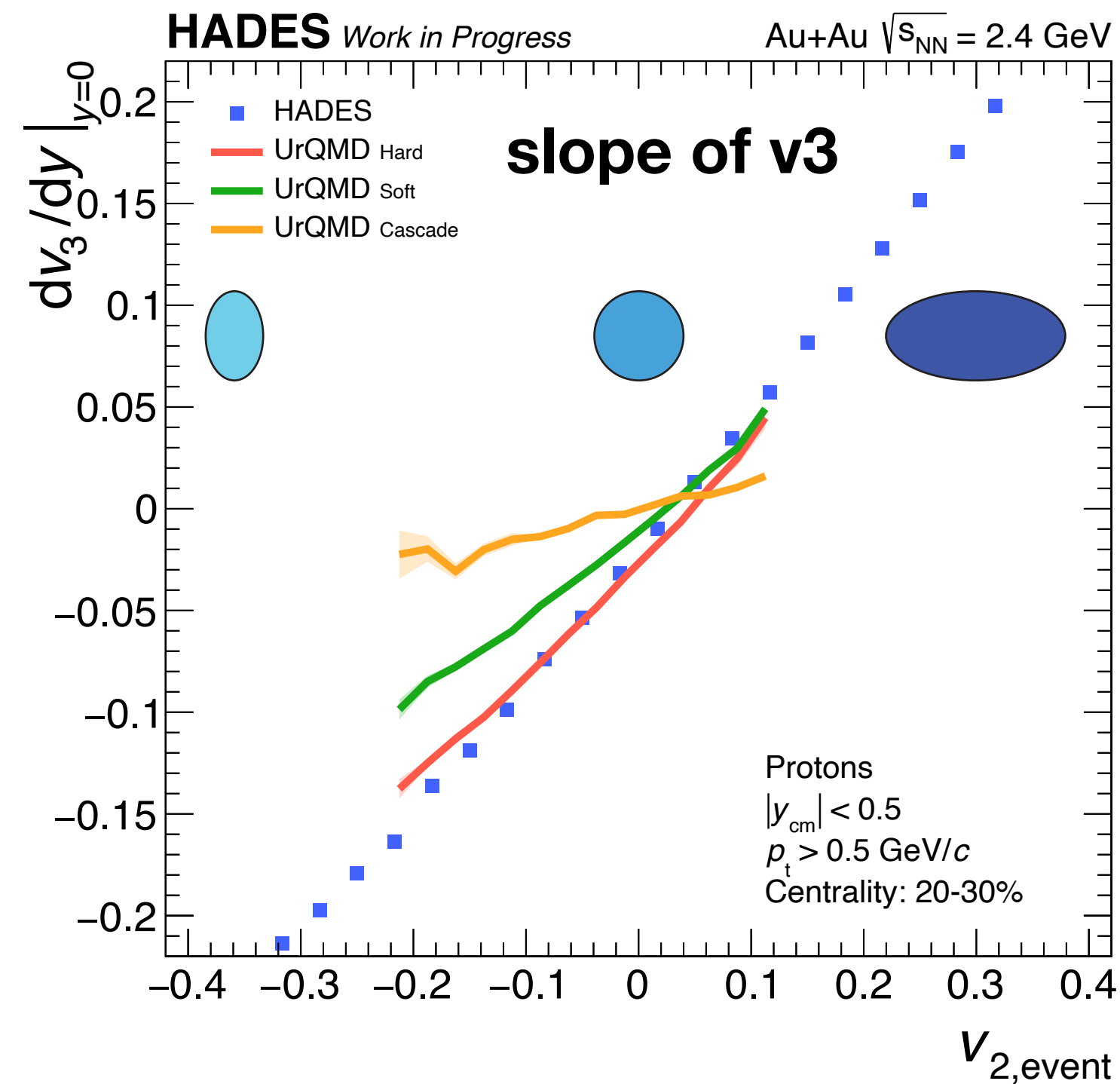


UrQMD Model Simulations:  
T. Reichert et al. EPJ C 82 (2022) 510



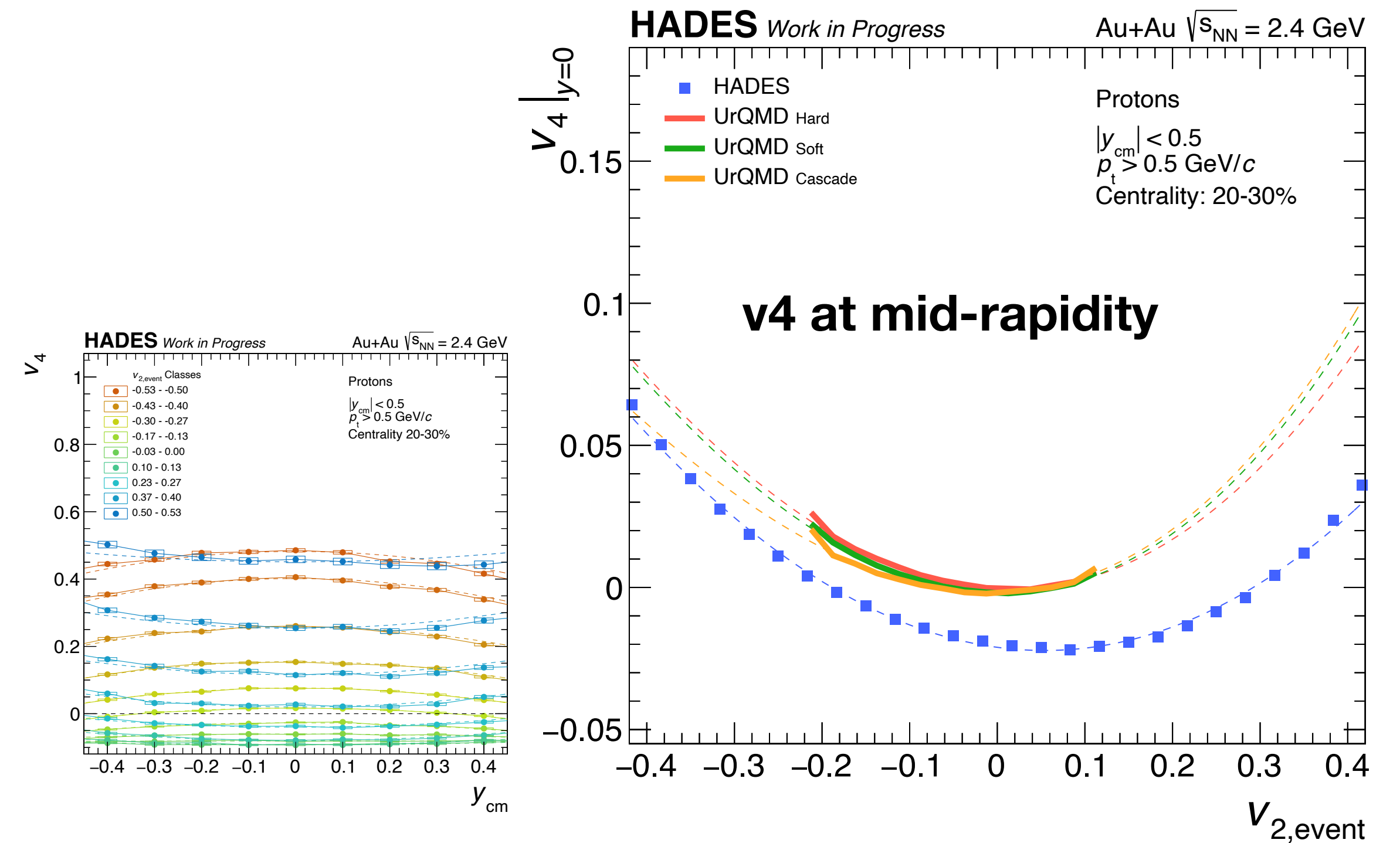


# Event-wise Flow Correlations



## Slope of the Triangular Flow $v_3$

A strong sensitivity to the EoS is seen



## Quadrangular Flow $v_4$

The magnitude of  $v_4$  seems to follow an almost quadratic dependence

**Not corrected the underlying Multiplicity Fluctuations**

UrQMD Model Simulations:  
T. Reichert et al. EPJ C 82 (2022) 510

# Conclusions and Outlook

## Flow Coefficients

Full 3D-picture of the emission pattern with complex shape determined by flow coefficients  $v_1 - v_6$

## Model Comparison

New level of precision - multi-differential  
Additional information from higher orders

Consistent modelling of light nuclei formation is essential

## Event-wise Flow Fluctuations

Correlation and Relation between Flow Coefficients

## Next Steps towards EOS

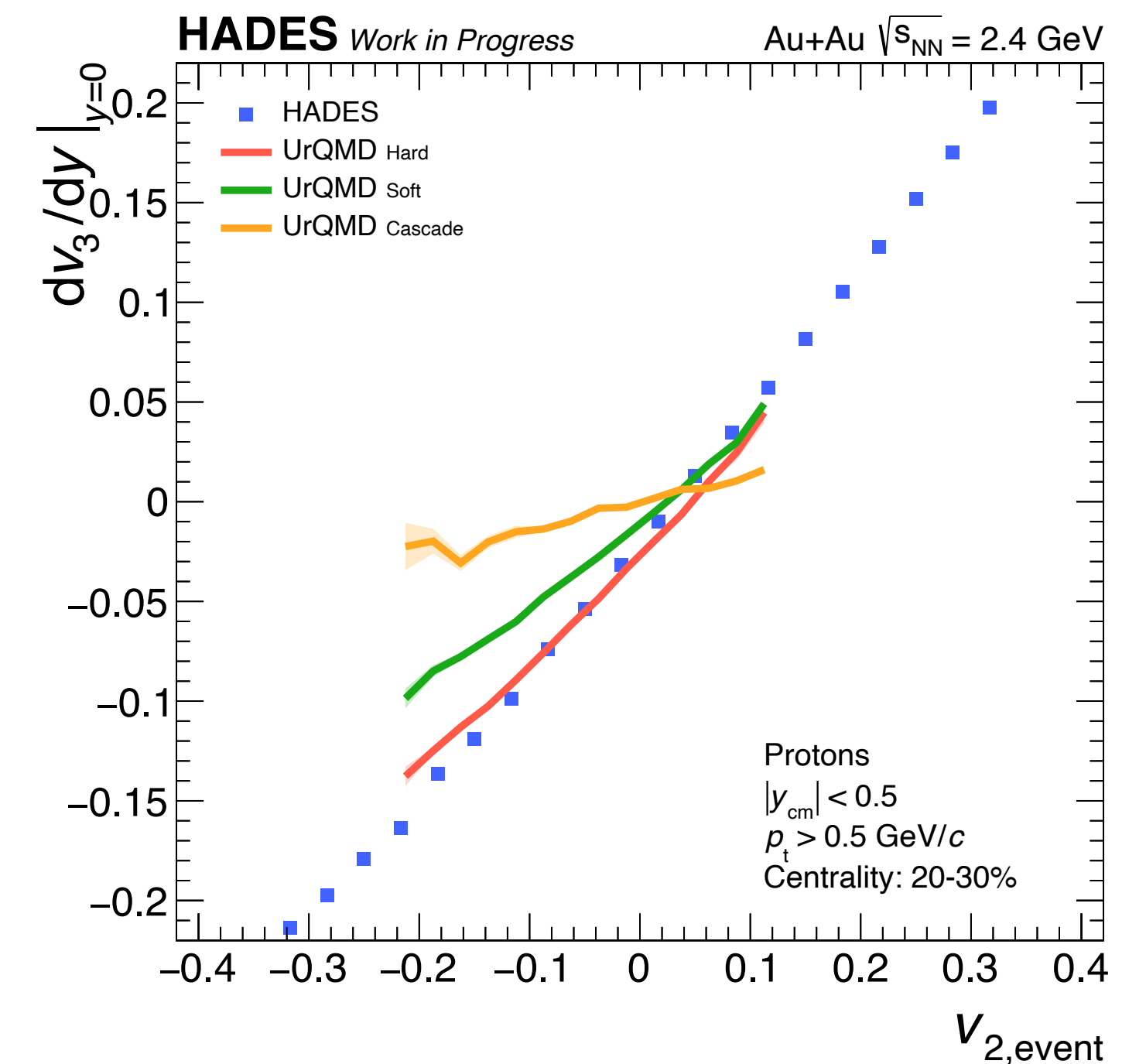
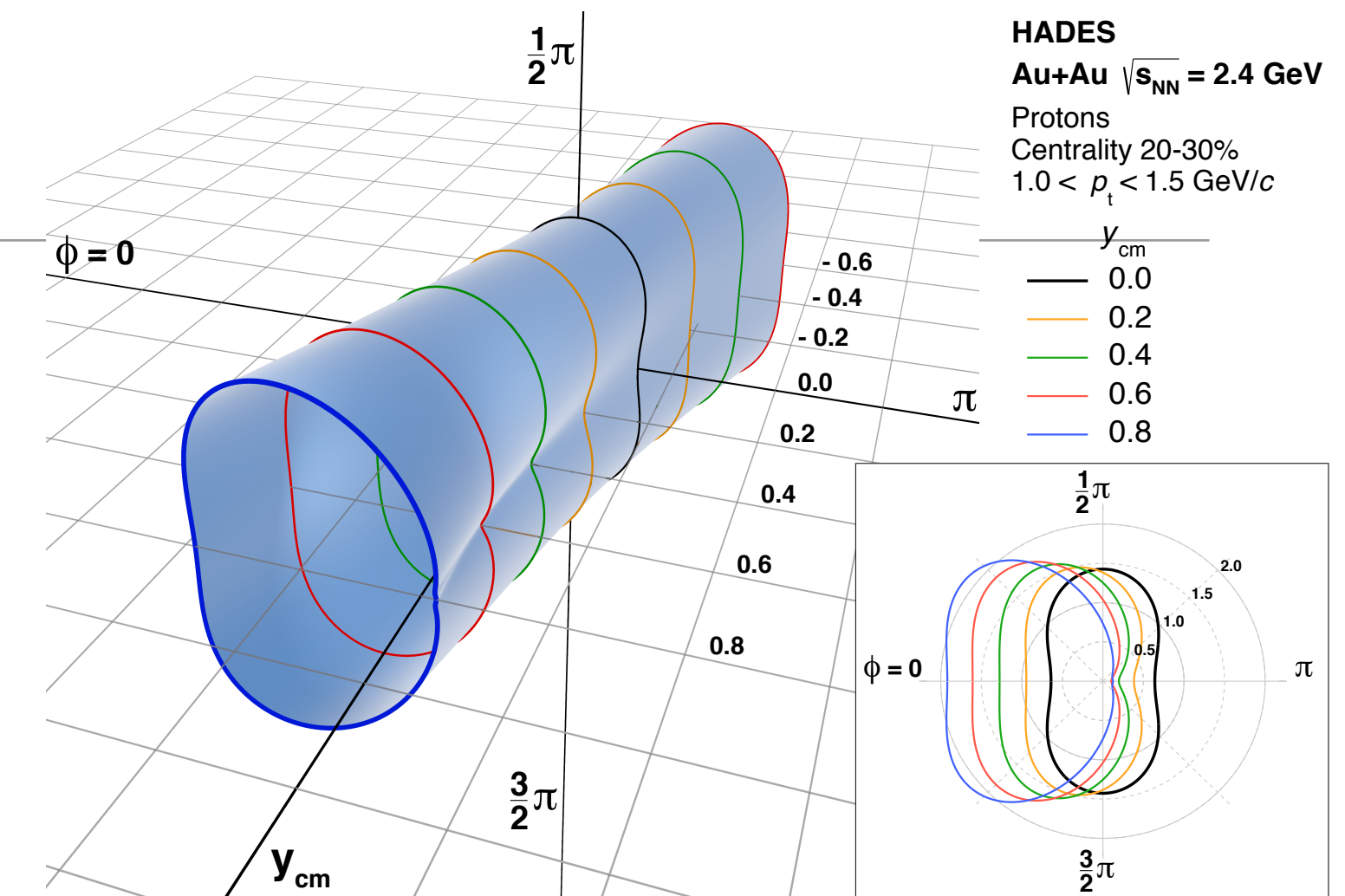
Detailed comparisons and sensitivity to model parameter space  
⇒ Bayesian analysis

## System-Size and Energy-dependence

Au+Au at 1.23 AGeV (2012)  
Ag+Ag at 1.23 and 1.58 AGeV (2019)

## SIS Beam Energy Scan

Au+Au 0.2, 0.4, 0.6 and 0.8 AGeV planned (2024)







HADES Collaboration

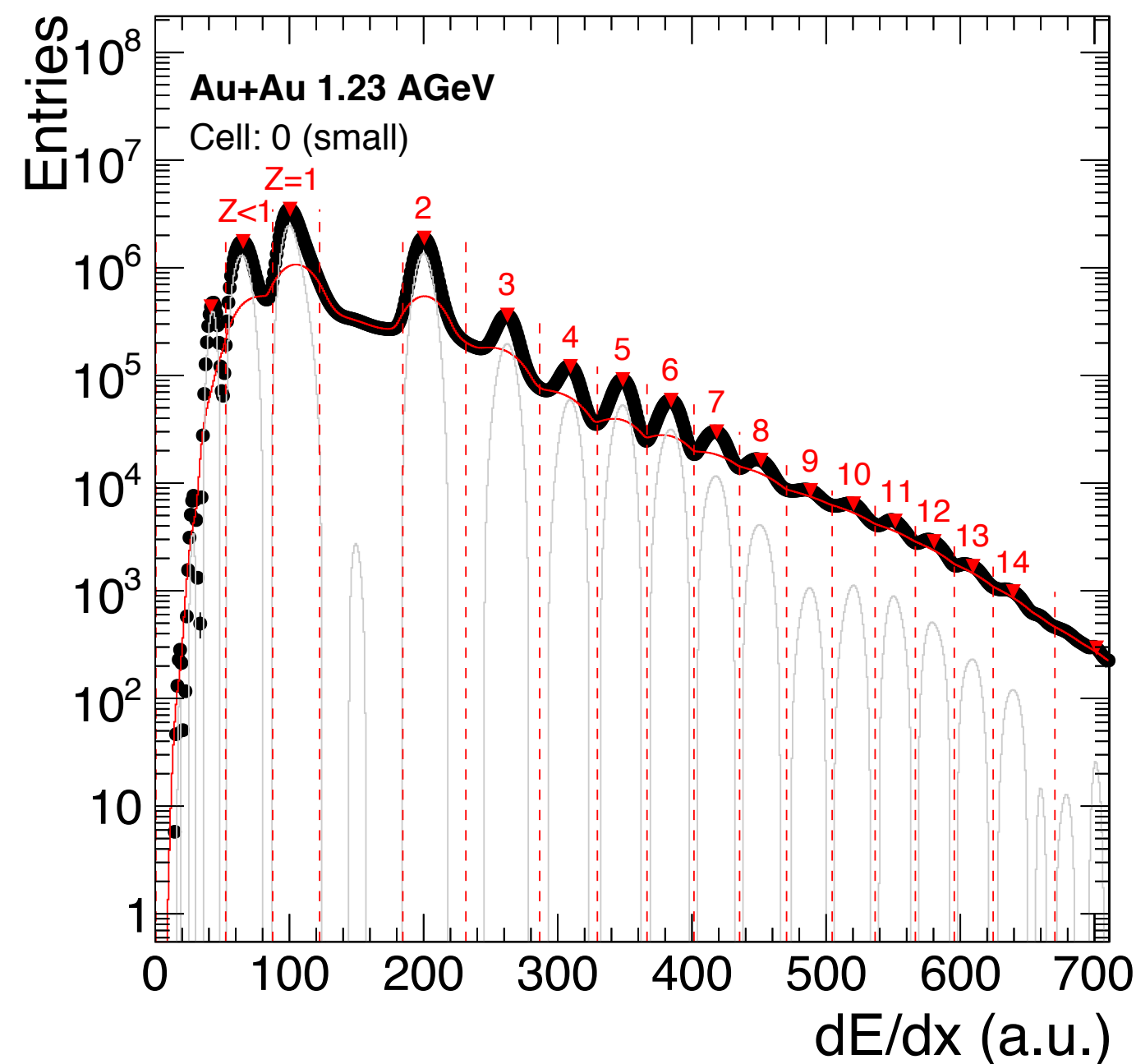
Thank you for your attention!



# Event Plane Reconstruction

1<sup>st</sup>-Order event plane from Q-Vector  
Projectile spectators in Forward Wall

Charge-Weighting of the projectile hits,  
according their energy loss in scintillators

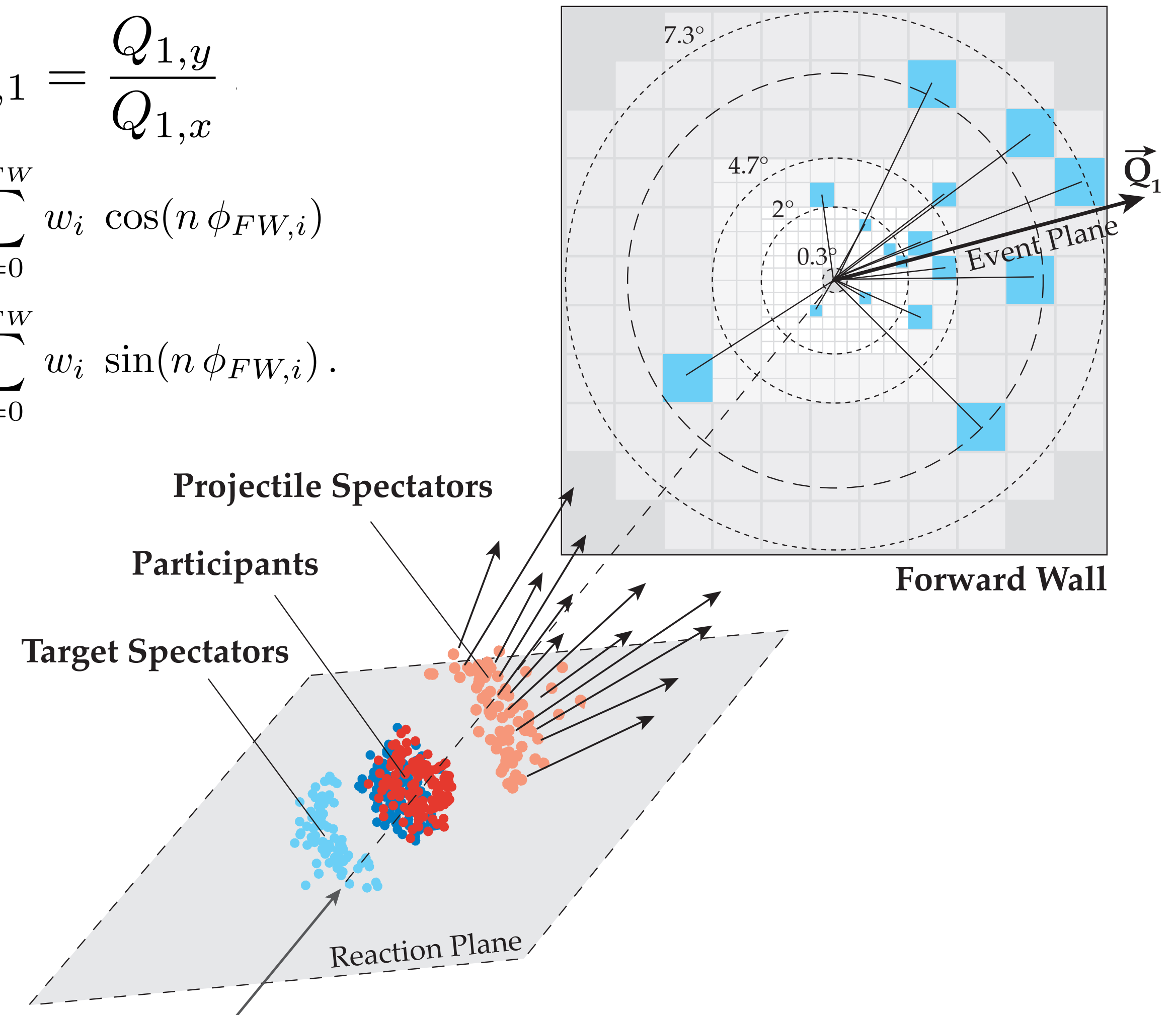


Charge Weighting

$$\tan \psi_{EP,1} = \frac{Q_{1,y}}{Q_{1,x}}$$

$$Q_{n,x} = \sum_{i=0}^{N_{FW}} w_i \cos(n \phi_{FW,i})$$

$$Q_{n,y} = \sum_{i=0}^{N_{FW}} w_i \sin(n \phi_{FW,i})$$



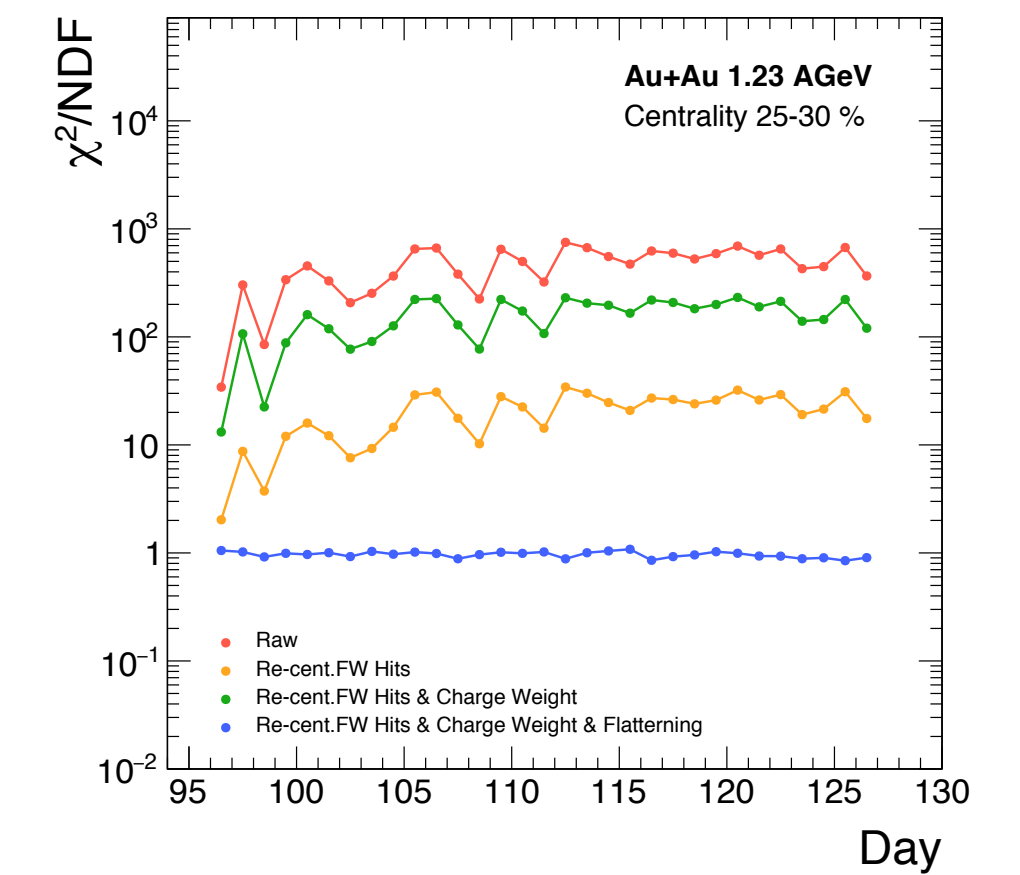
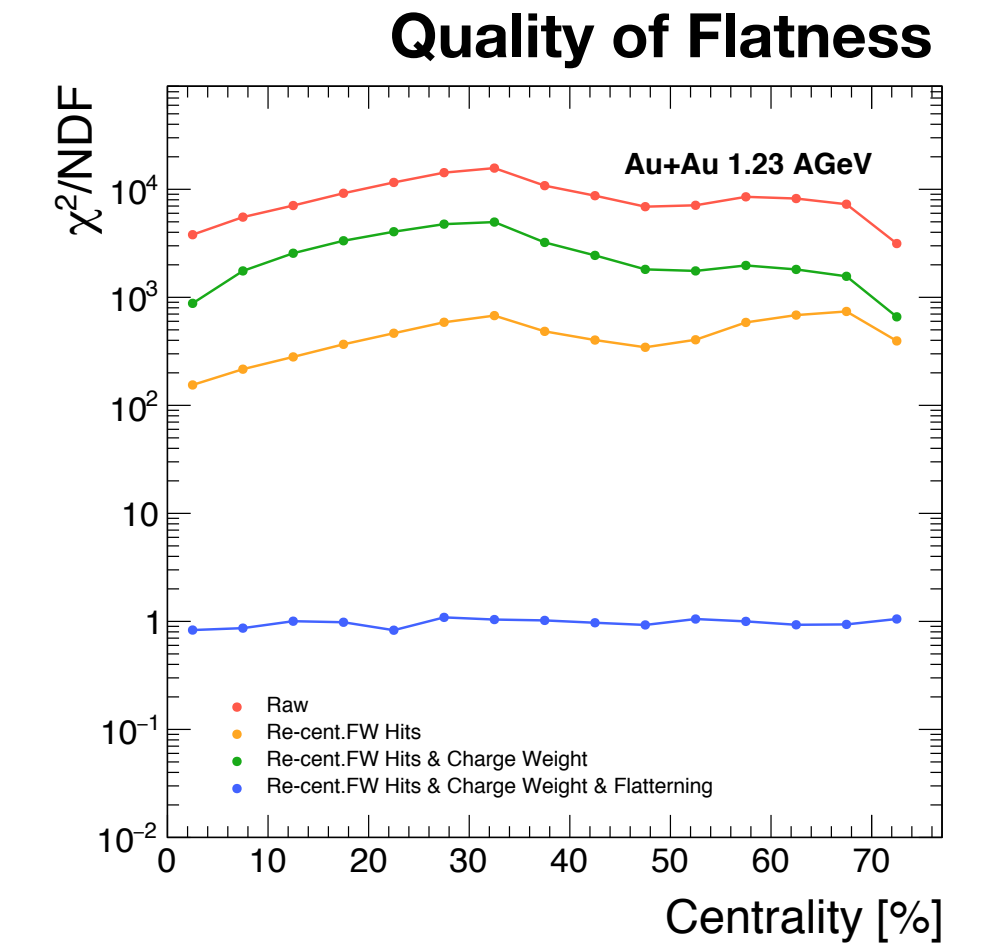
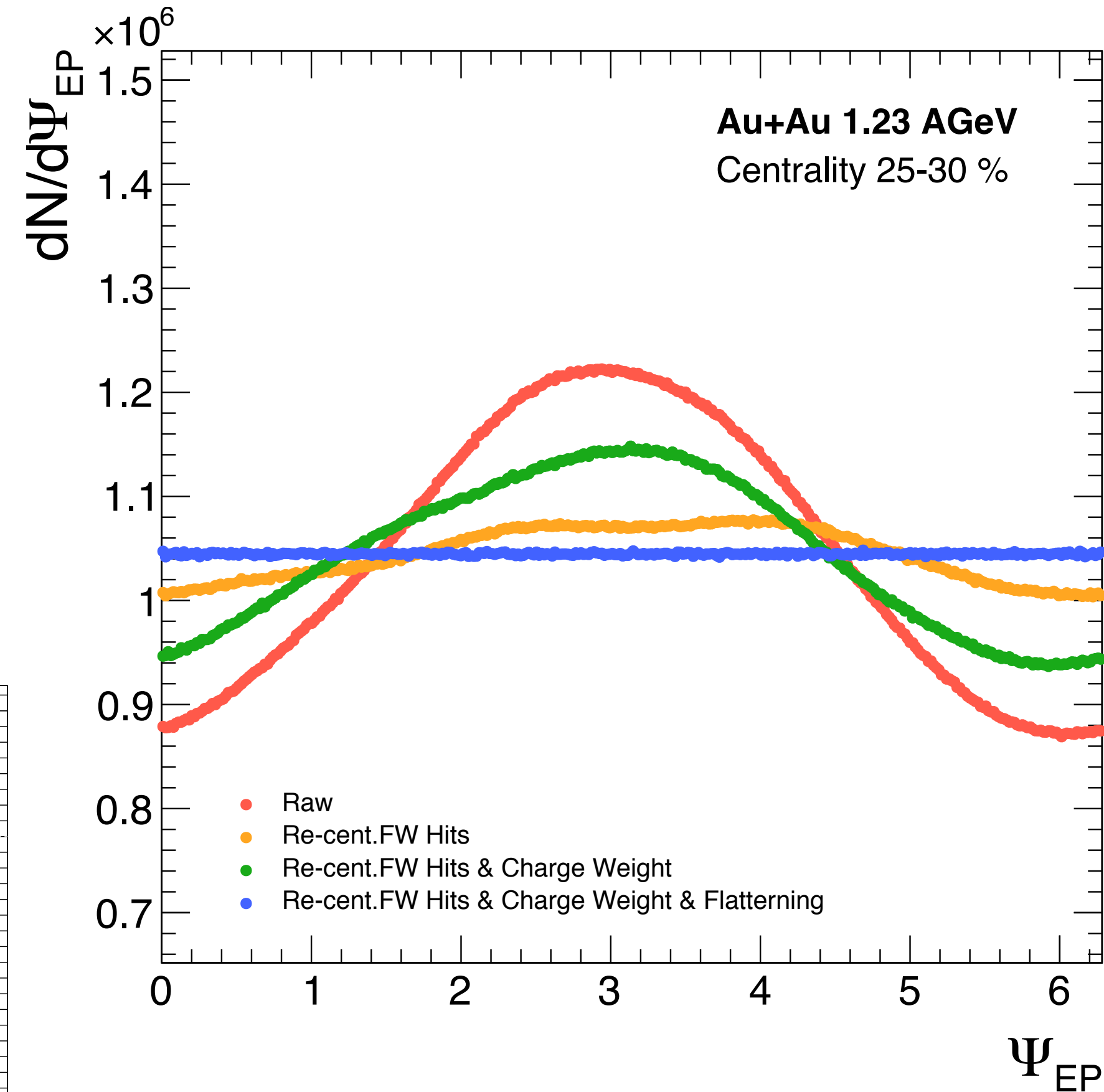
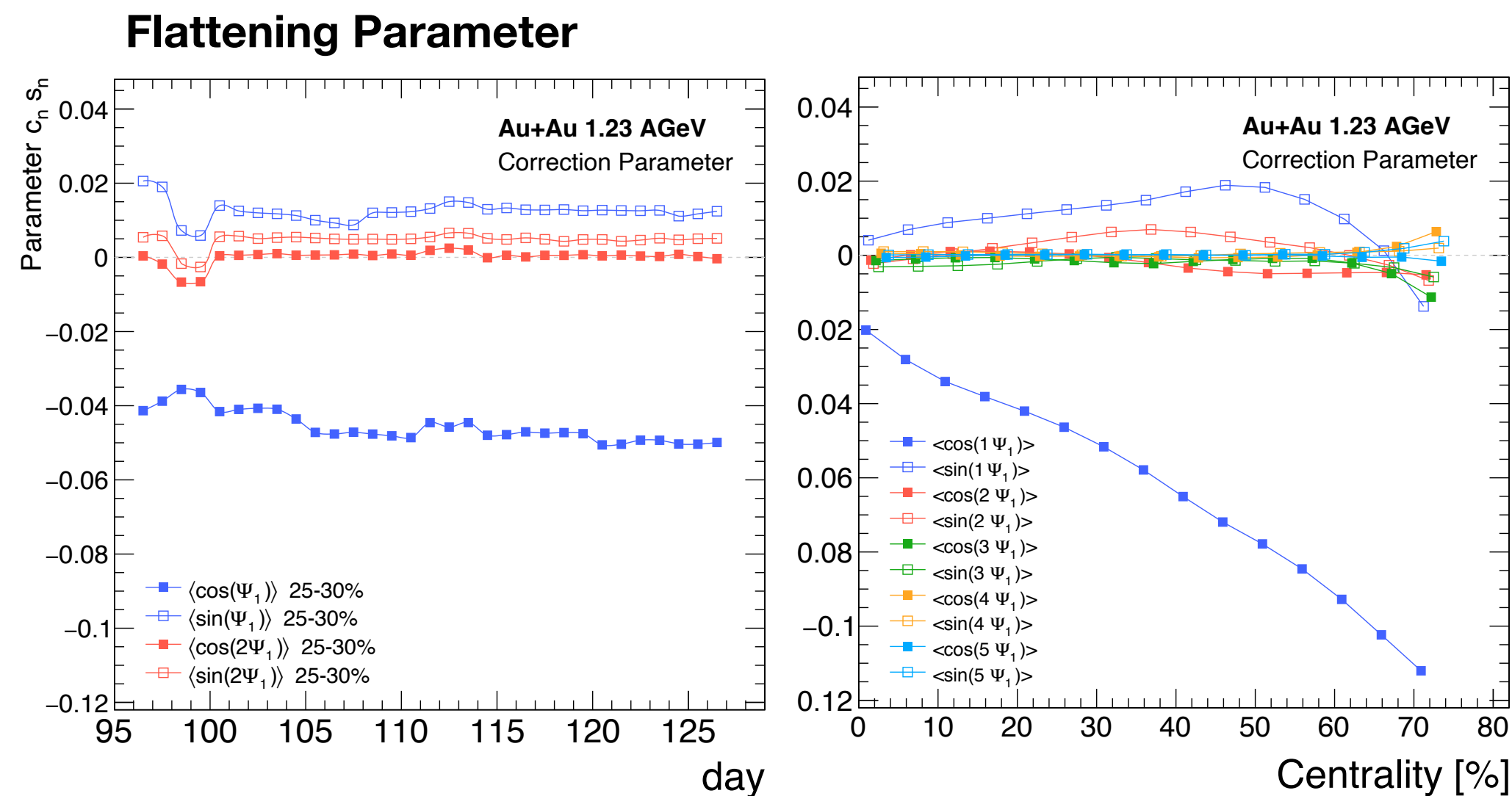


# Event Plane Determination

Correction of non-uniformities in the EP distribution (day-by-day and centrality)

Re-centering of X and Y of all FW hits

Flattening of residual Fourier components with 8 cos- and 8 sin-terms



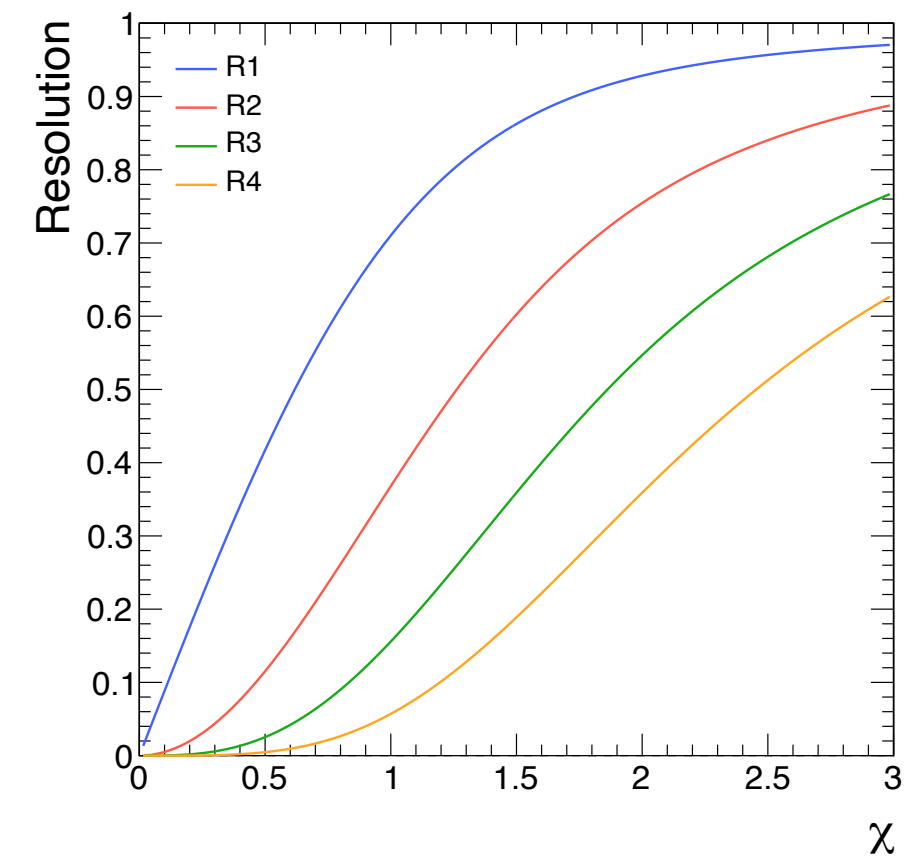
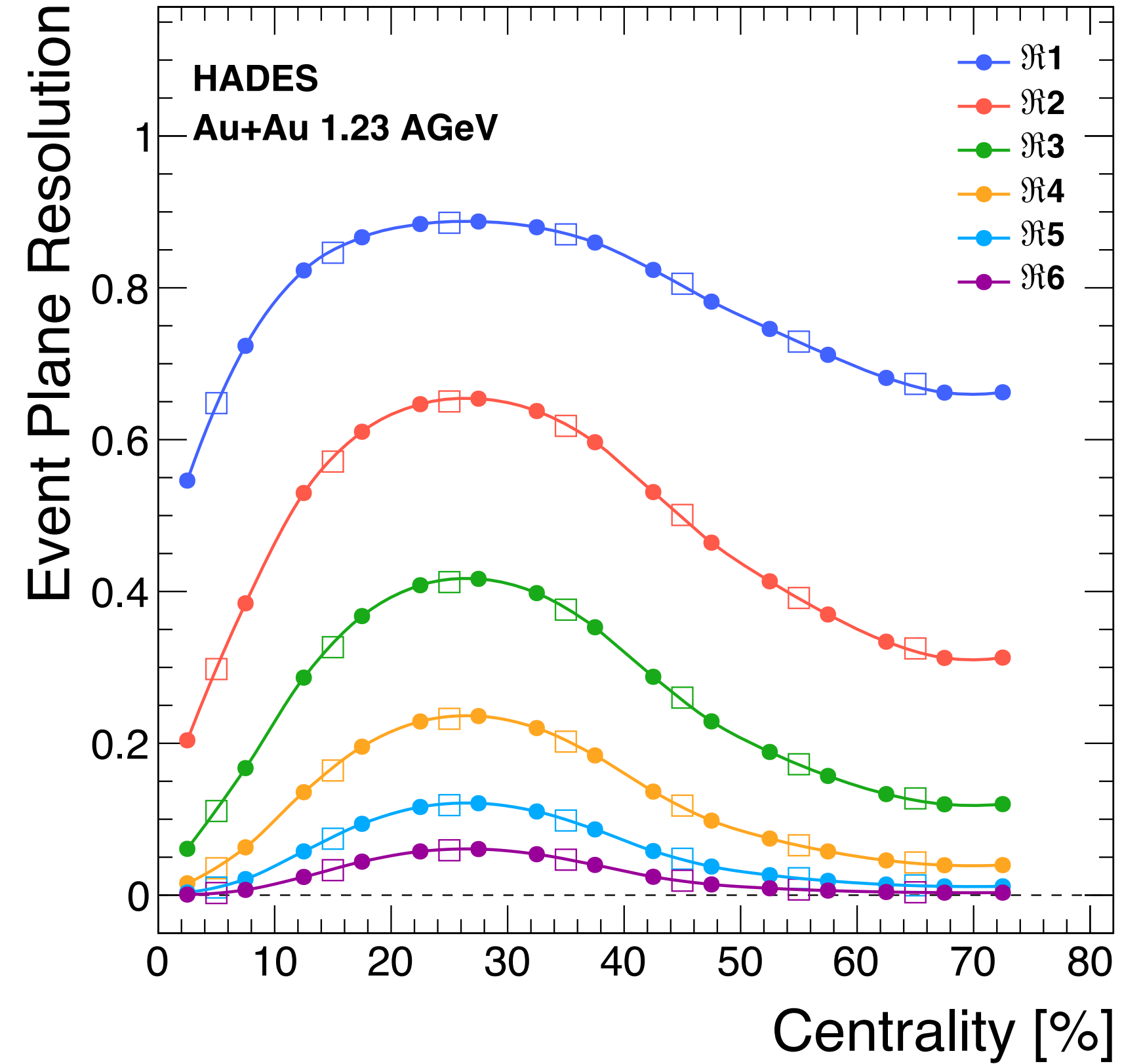
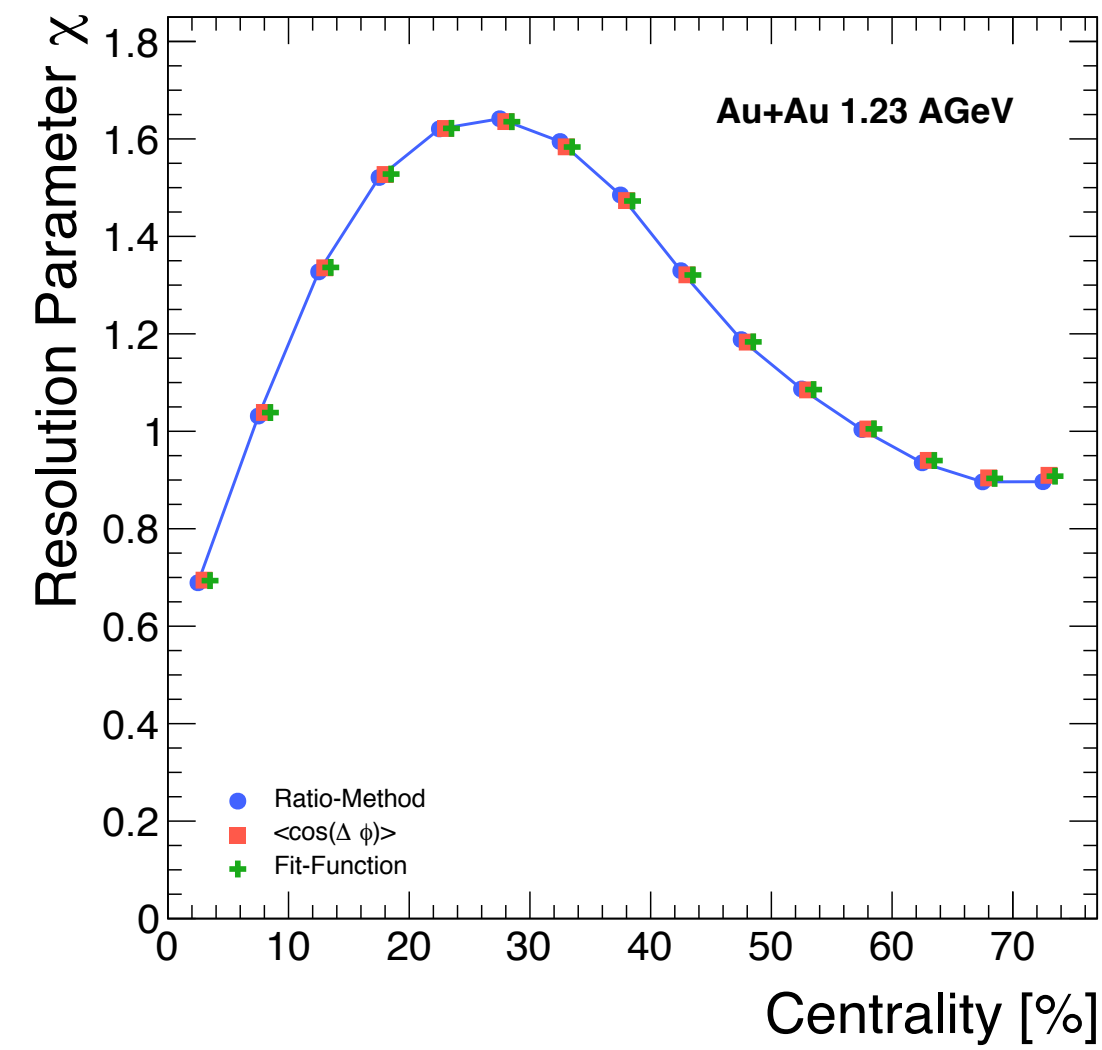
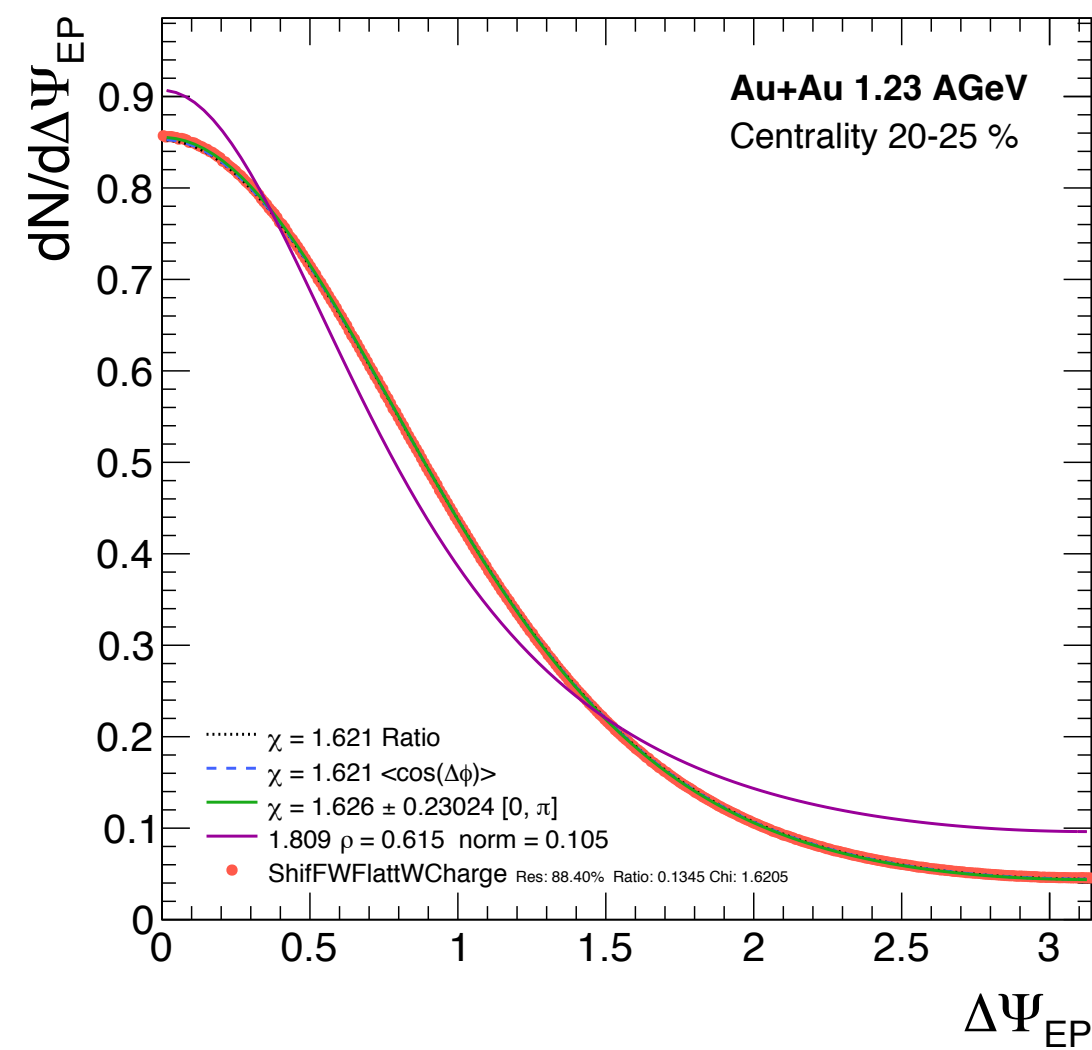
# Event Plane Resolution

## EP-resolution via sub-event method with three implementations

Determination of resolution parameter  $\chi$

- directly via  $\langle \cos(\Delta\Phi) \rangle$
- Approximation via Fraction of Events with  $\Delta\Phi > \pi/2$
- Fit-Method

Calculation of EP-Resolution of different order



$$v_n = v_n^{obs} / \mathcal{R}_n$$

$$\mathcal{R}_n = \langle \cos[n(\Psi_n - \Psi_{RP})] \rangle$$



# Systematic Uncertainties

## Validation and Consistency Checks

### Sources of uncertainties

- Track selection and PID
- Occupancy correction
- Non-uniform acceptance

### Toy MC study

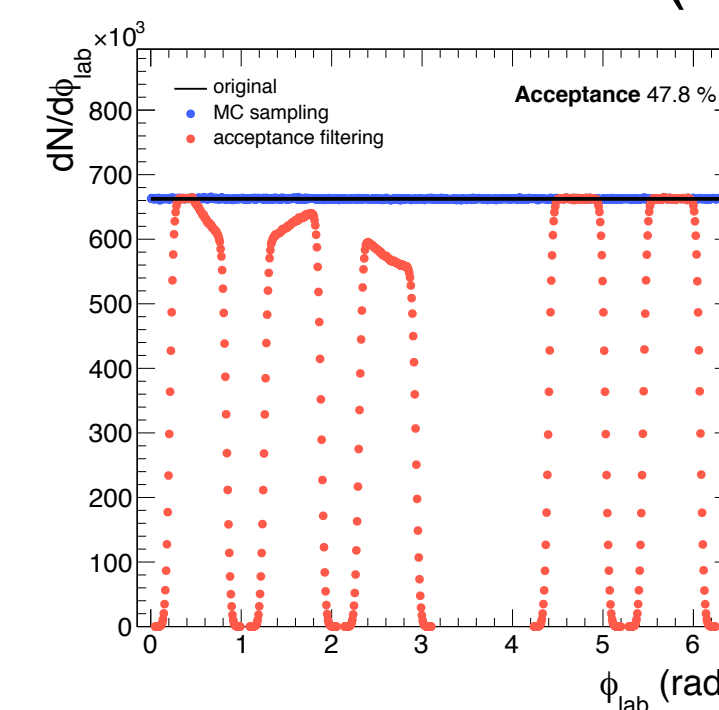
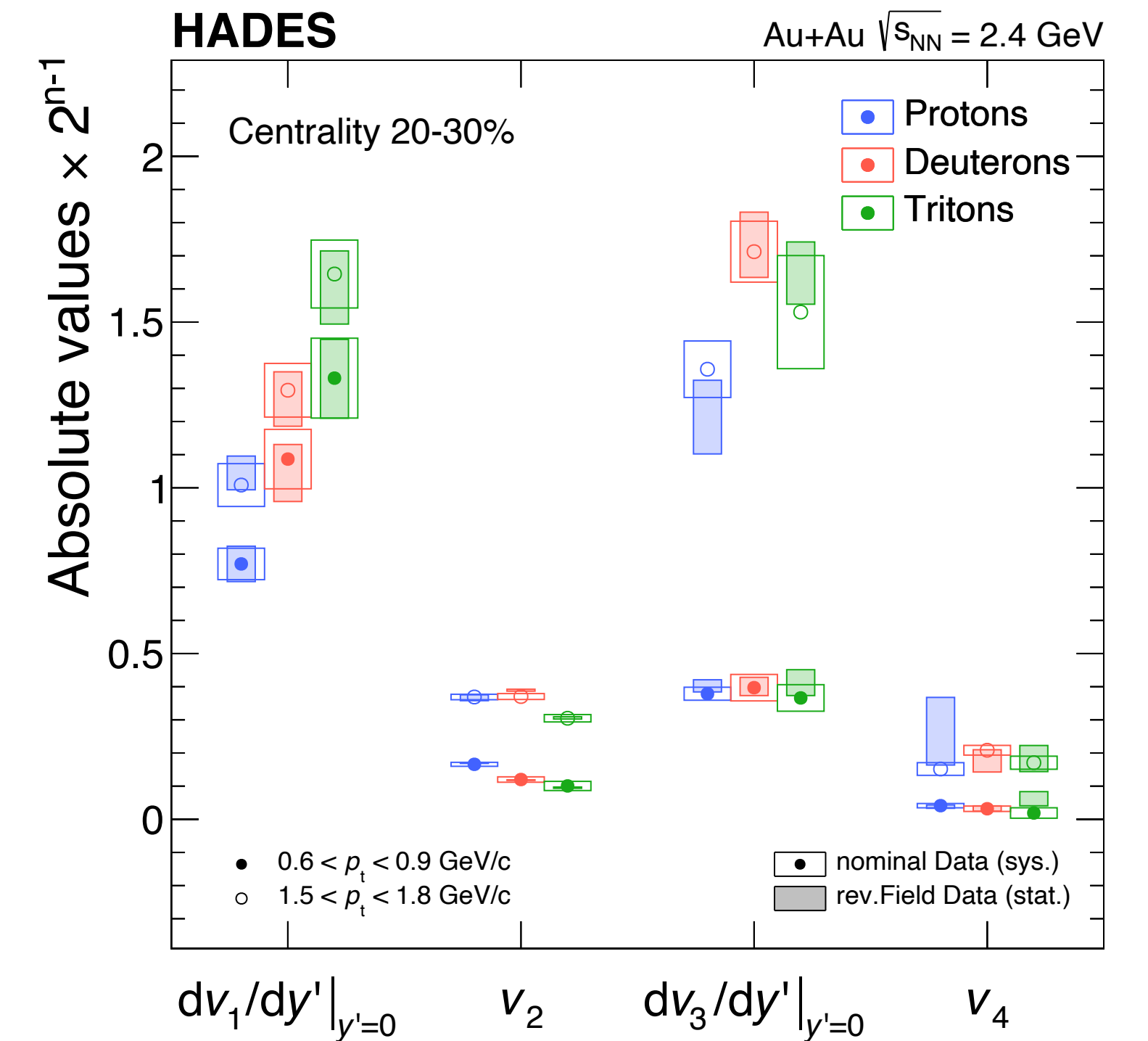
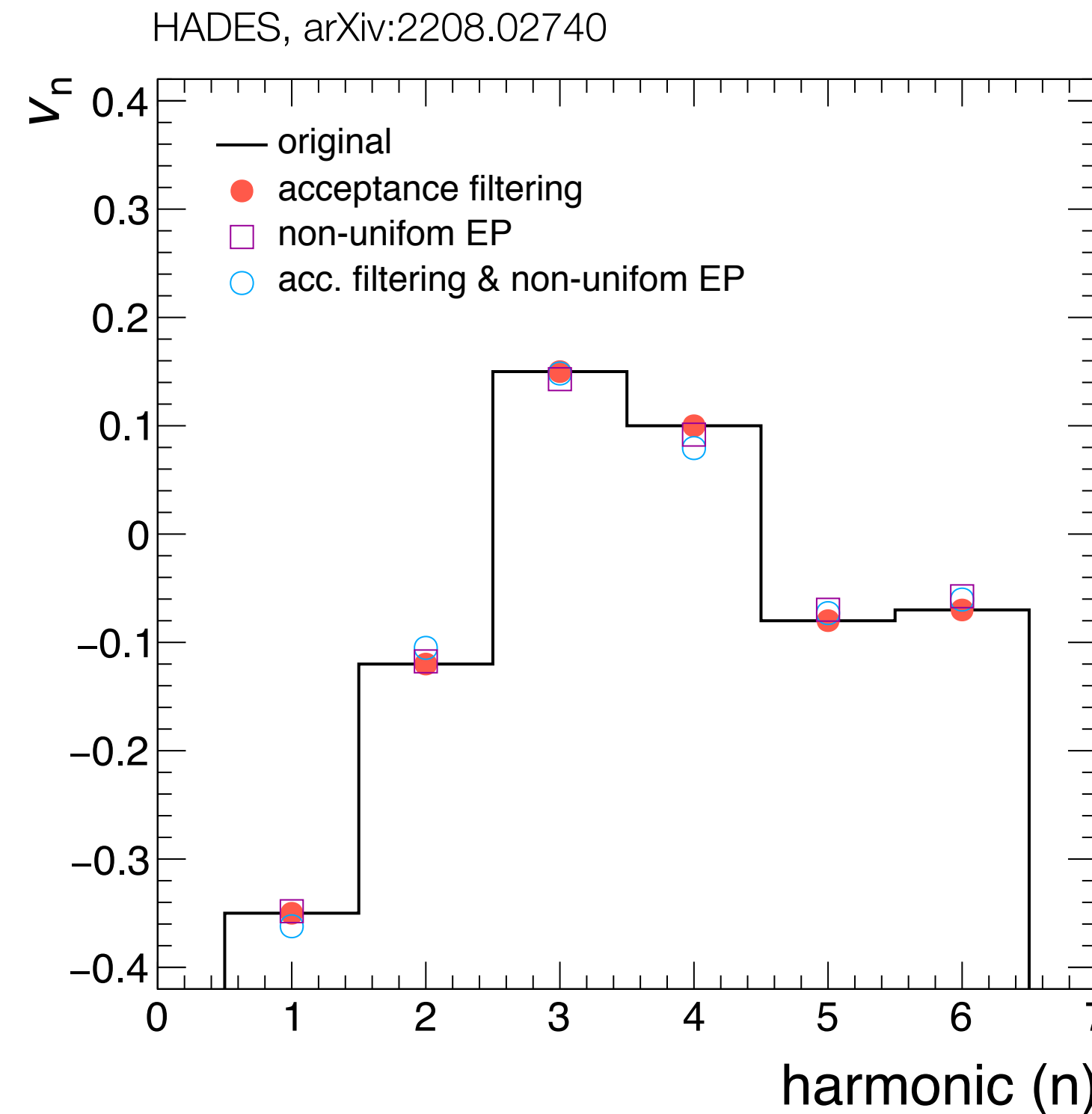
Influence of the incomplete acceptance and a non-uniform event-plane distribution

### Consistency checks:

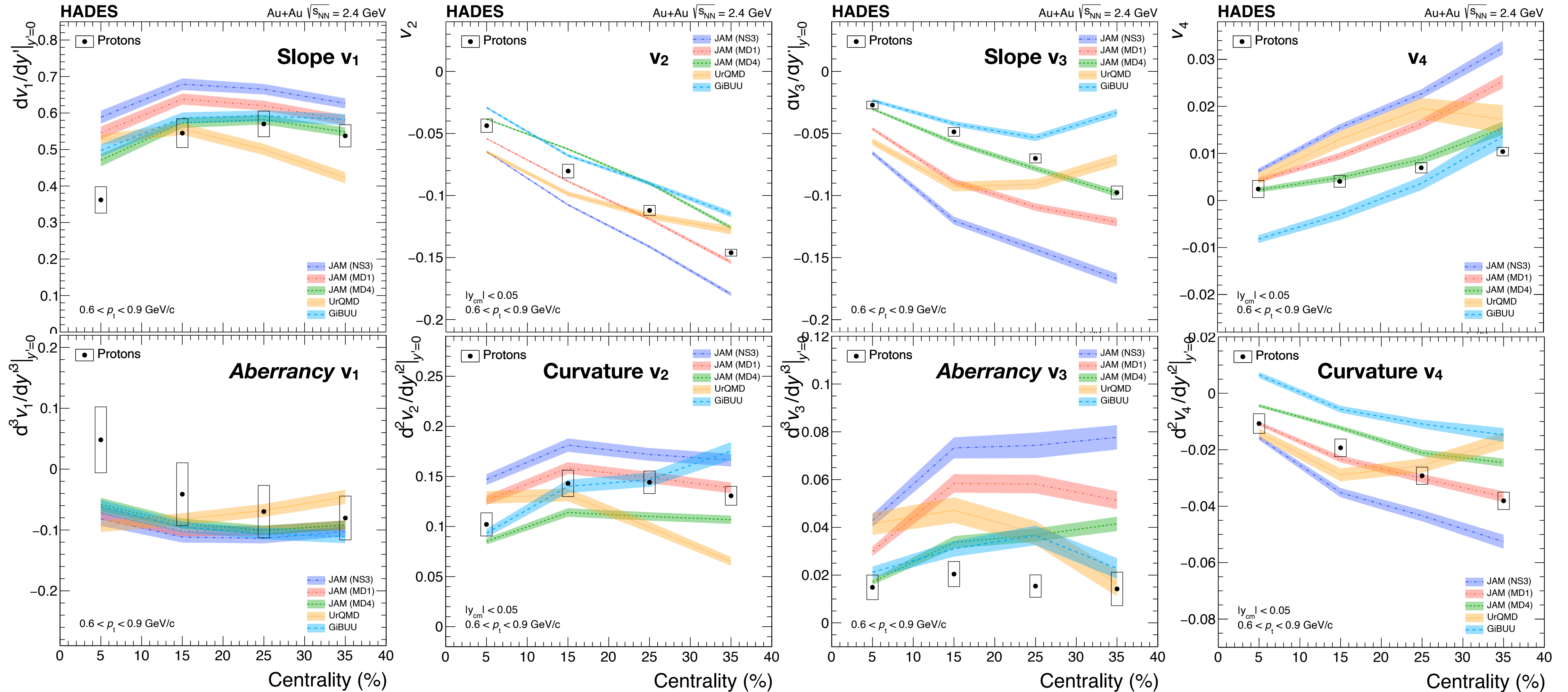
- Measurement symmetry with respect to mid-rapidity
- Zero-crossing of odd harmonics at  $y_{cm}=0$
- Vanishing residual sine-terms
- Time-dependent systematic effects

### Reversed field polarity

Comparison with flow coefficients from the full data set



# Model Comparisons to Proton Data



\* **Aberrancy**: the third derivative of a curve



# “Ideal fluid scaling”

Relation between  $v_2$  and  $v_4$

## Scaling properties

Prediction for ideal fluid:

$$v_4(p_t)/v_2^2(p_t) = 1/2$$

Slightly higher values ( $\sim 0.6$ )  
expected in more realistic scenario

## Observed ratios for p, d and t

Independent of  $p_t$  and centrality  
Close to predicted value of  $\sim 0.6$

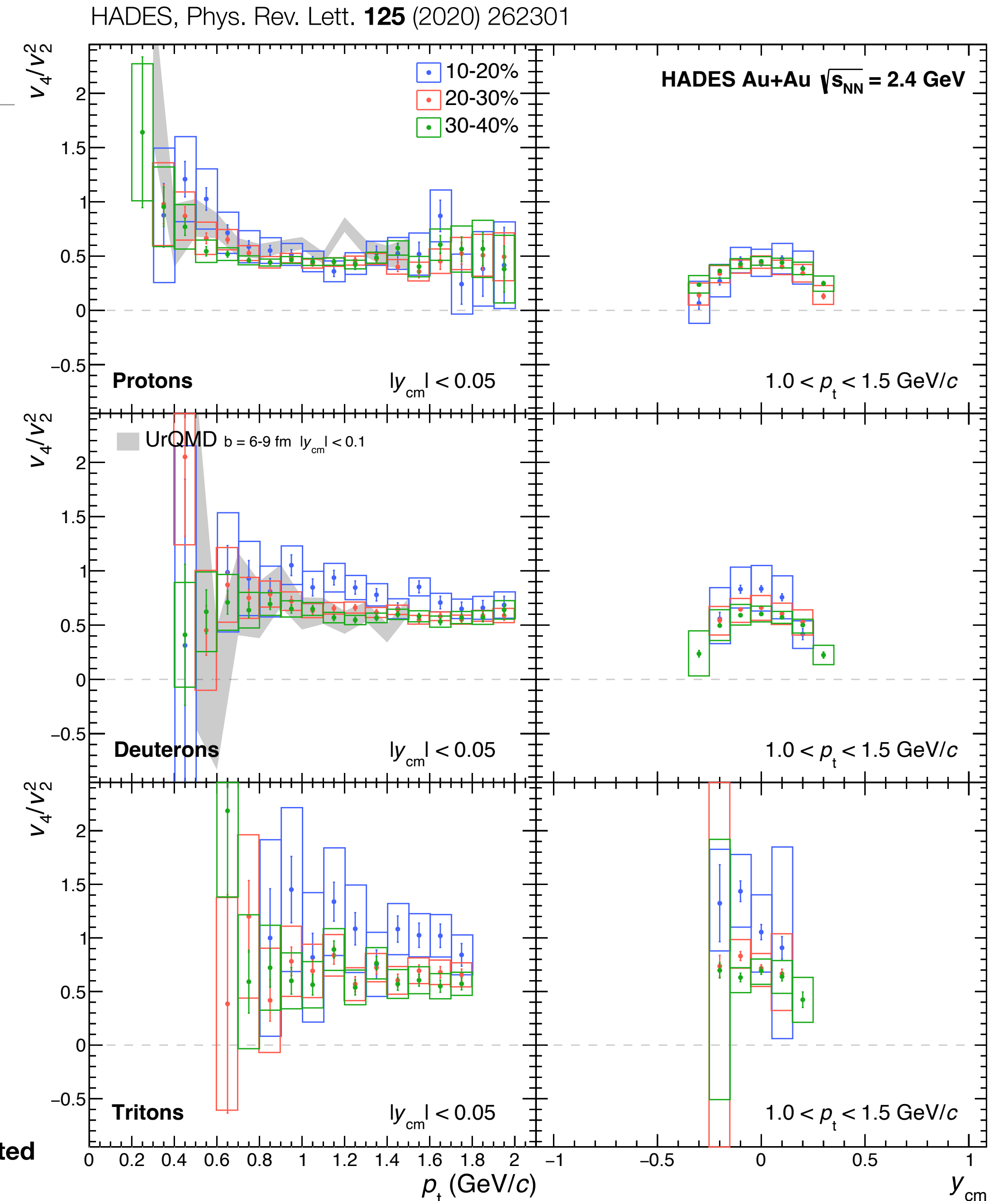
Confirmed by transport models

*Hydro-like matter at SIS energies?*

P.F. Kolb, PRC **67** (2003) 031902  
N. Borghini and J.-Y. Ollitrault, PLB **642** (2006) 227  
C. Gombeaud and J.-Y. Ollitrault, PRC **81** (2010) 014901

J. Wang et al., PRC **90** (2014) 054601 **IQMD**  
P. Hillmann et al., J.Phys. G **47** (2020) 5, 055101 **UrQMD**  
Justin Mohs et al., PRC **105** (2022) 034906 **SMASH**

Systematic Error of  $v_2$  and  $v_4$  are treated as correlated



# Nucleon Coalescence

## Scaling Properties of $v_2$ at Mid-Rapidity

Scaling of  $v_2$  and  $p_t$  with nuclear mass number  $A$  (including higher terms)

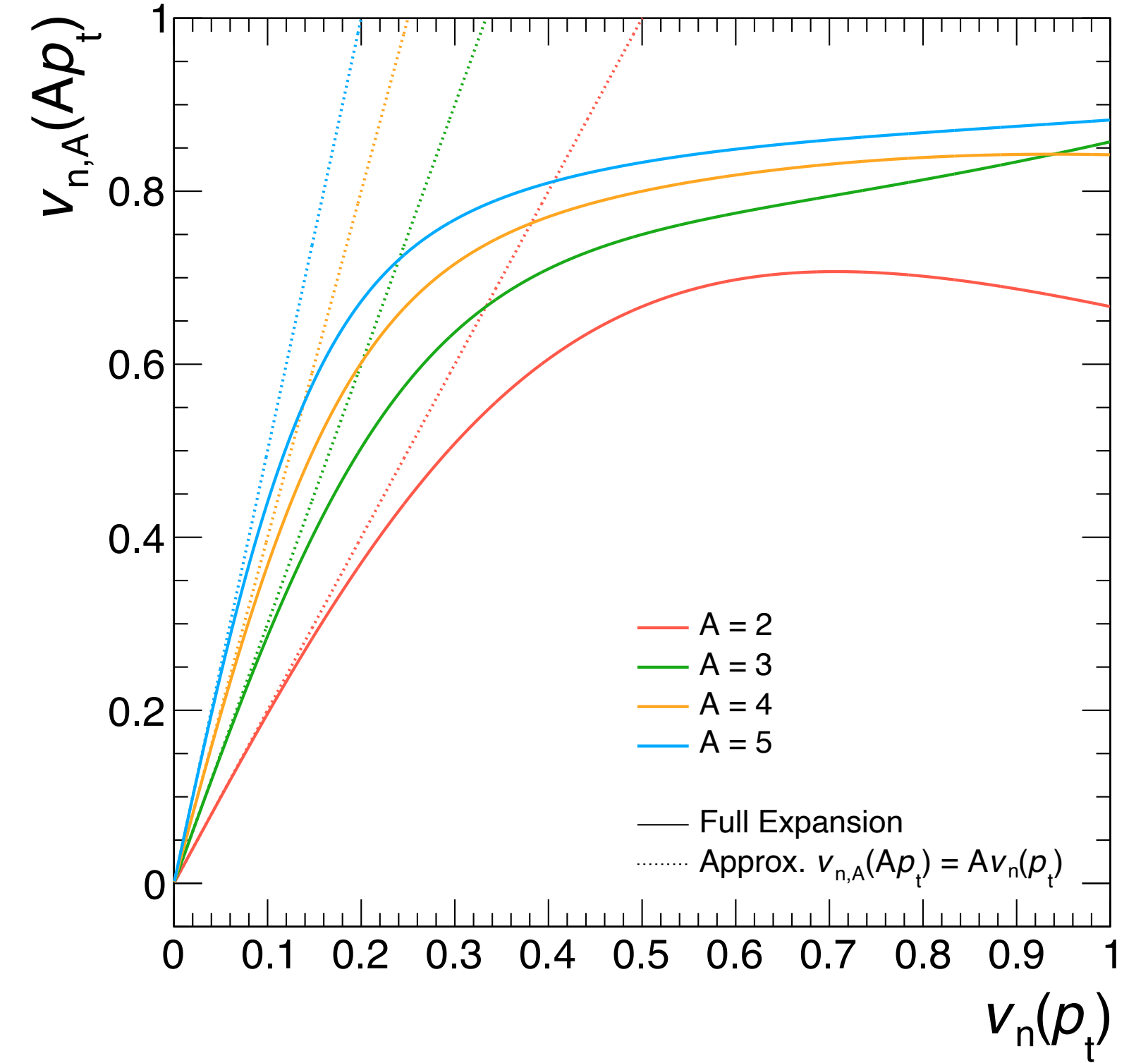
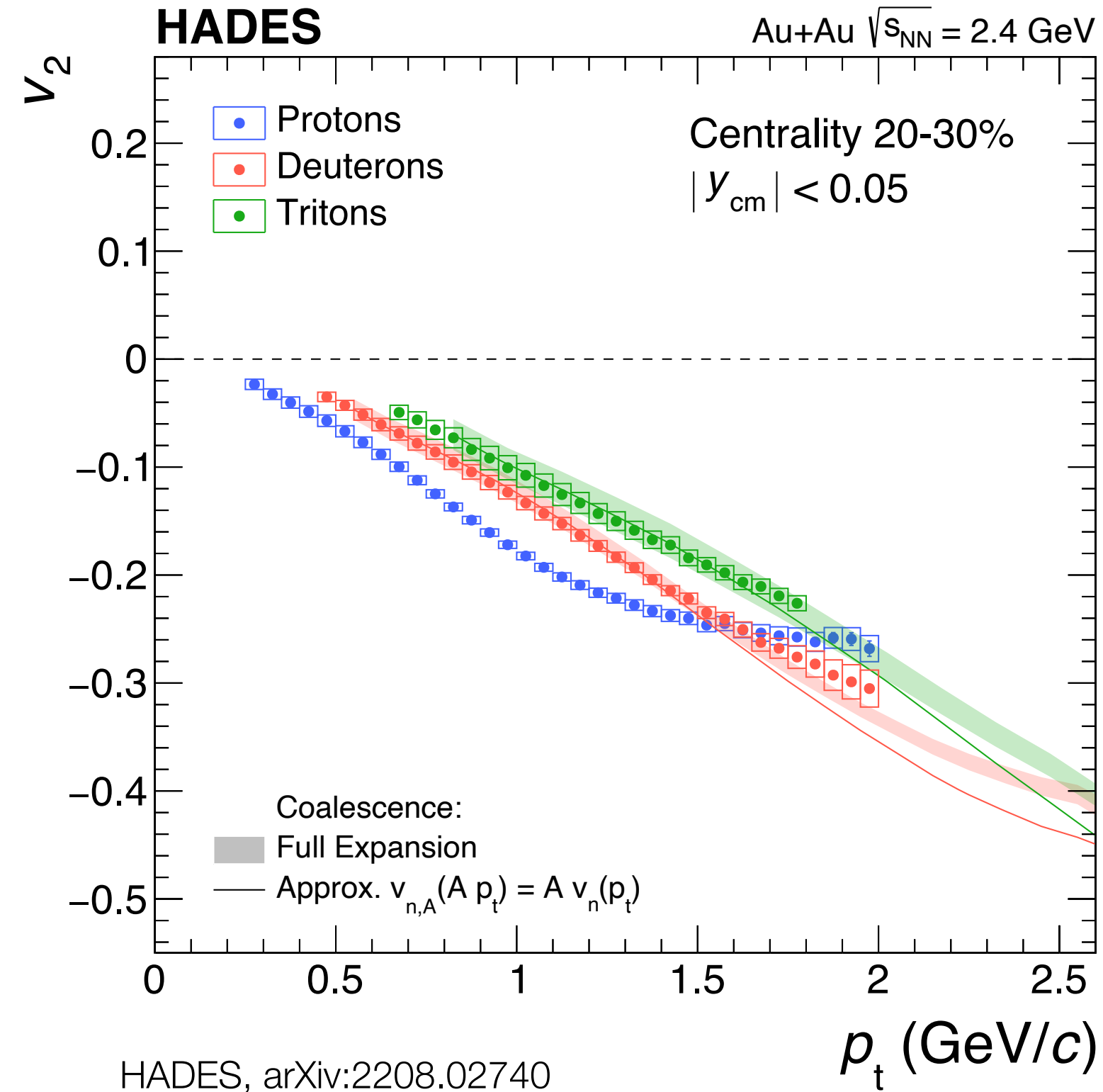
Works as expected in simple coalescence picture for the dominant flow coefficient

Odd flow coefficients vanish at mid-rapidity and  $v_4$  contribution is negligible

*Approximation for small  $v_n$*

$$v_{n,A}(A p_t) = A v_n(p_t)$$

Scaling also for  $v_4$  observed



$$v_{n,A=2}(A p_t) = 2 v_n(p_t) \frac{1}{1 + 2 v_n^2(p_t)}$$

$$v_{n,A=3}(A p_t) = 3 v_n(p_t) \frac{1 + v_n^2(p_t)}{1 + 6 v_n^2(p_t)}$$

D. Molnar and S.A. Voloshin PRL **91** (2003) 092301  
P.F. Kolb et al., PRC **69** (2004) 051901



# Nucleon Coalescence

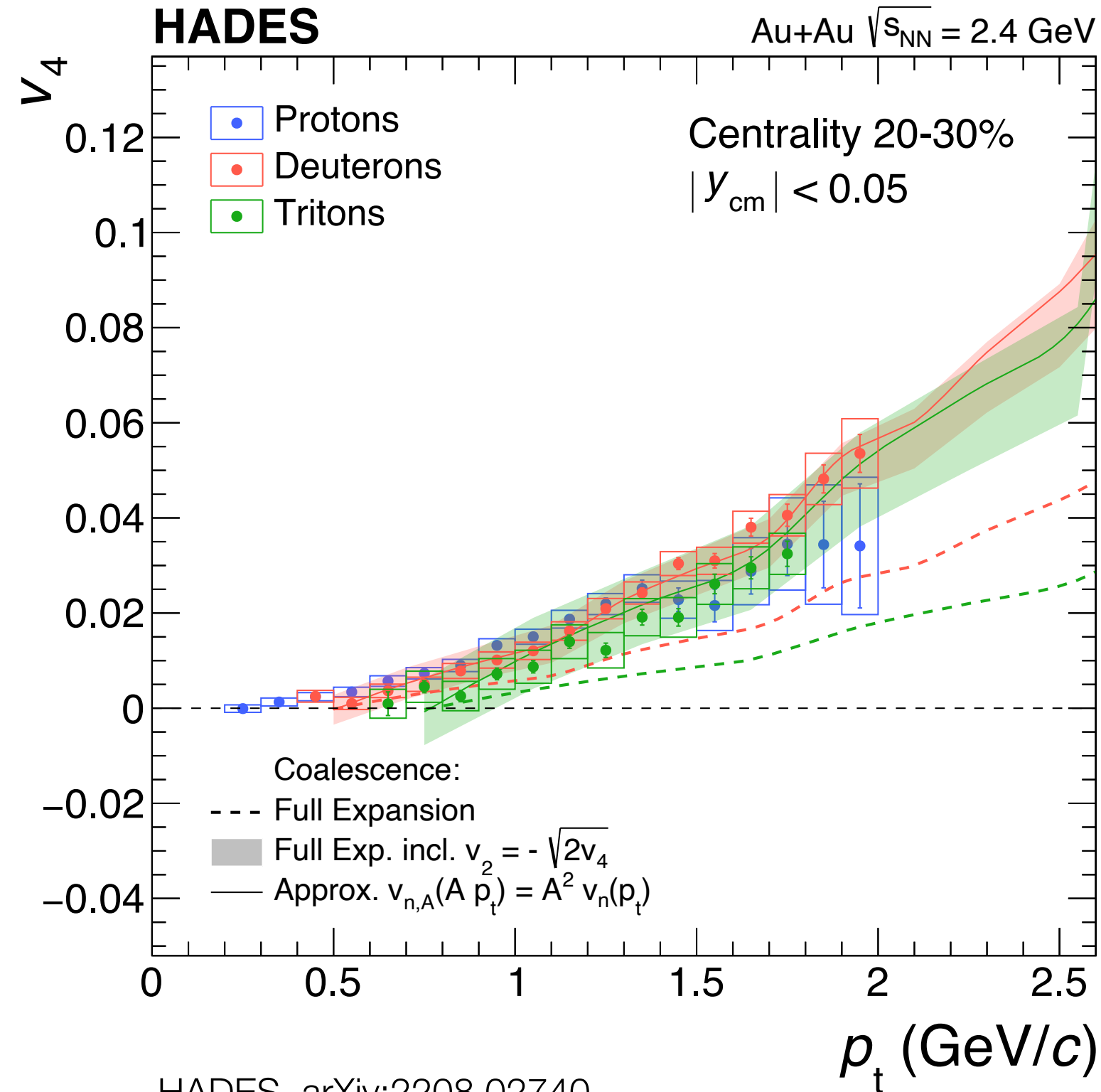
## Scaling Properties of $v_4$ at Mid-Rapidity

Scaling of  $v_4$  and  $p_t$  with nuclear mass number  $A$  (including higher terms)

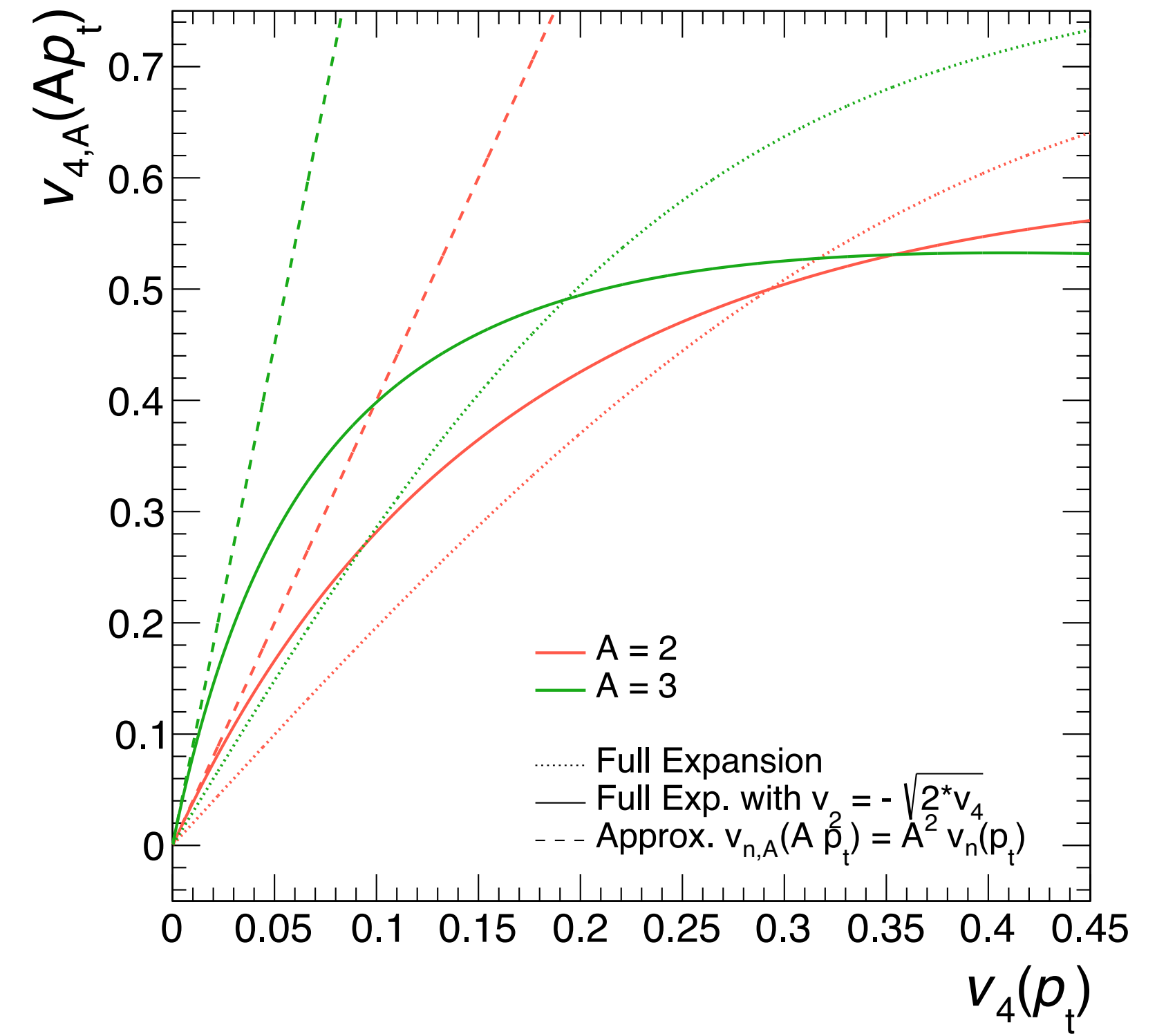
Works as expected in simple coalescence picture if contribution of dominant flow coefficient is included

Approximation for small  $v_4$  with  $v_2$  contribution:

$$v_{n,A}(A p_t) = A^2 v_n(p_t)$$



HADES, arXiv:2208.02740



$$v_{4,A=2}(A p_t) = 4 v_4(p_t) \frac{1}{1 + 4 v_4(p_t) + 2 v_4^2(p_t)}$$

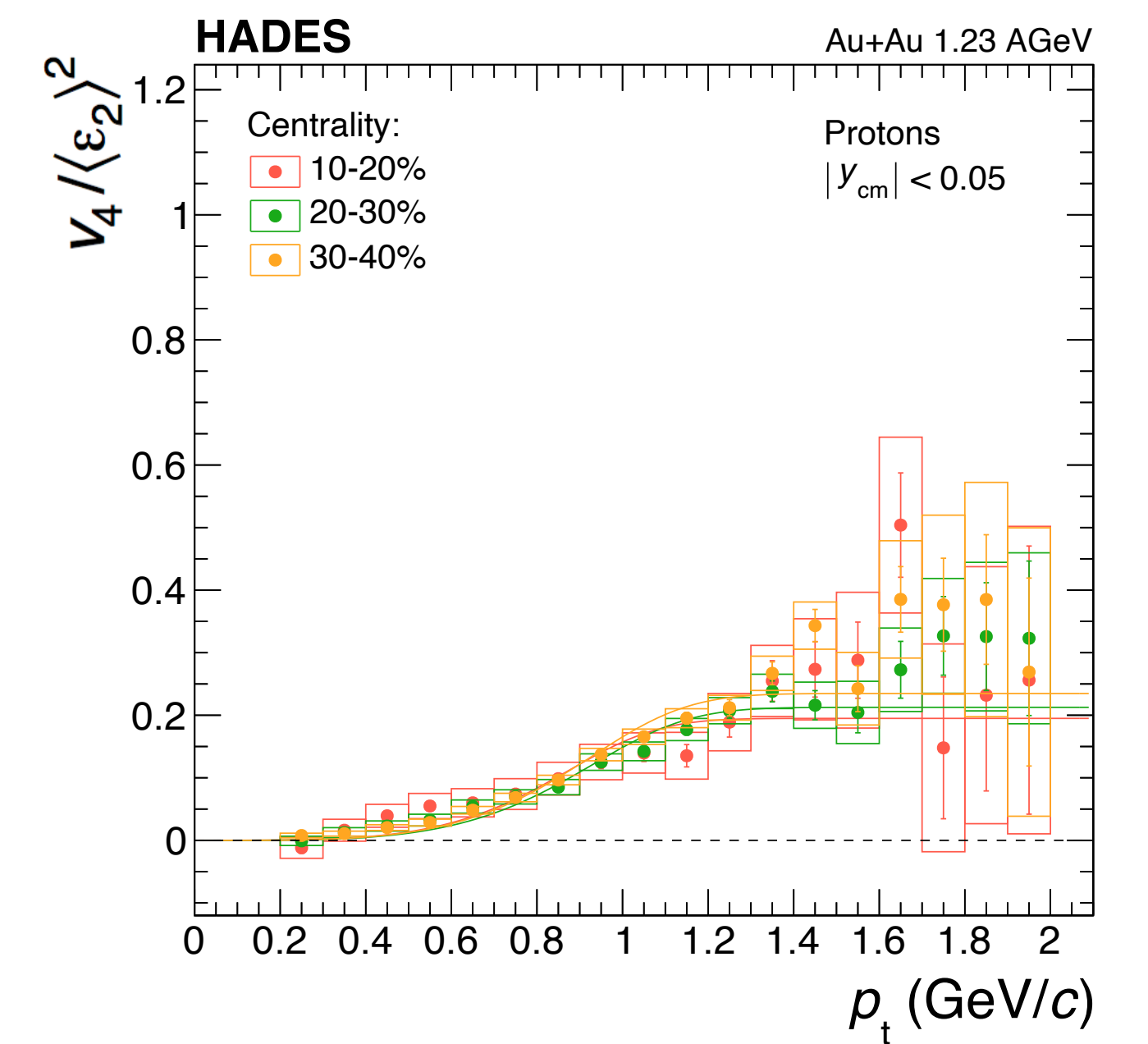
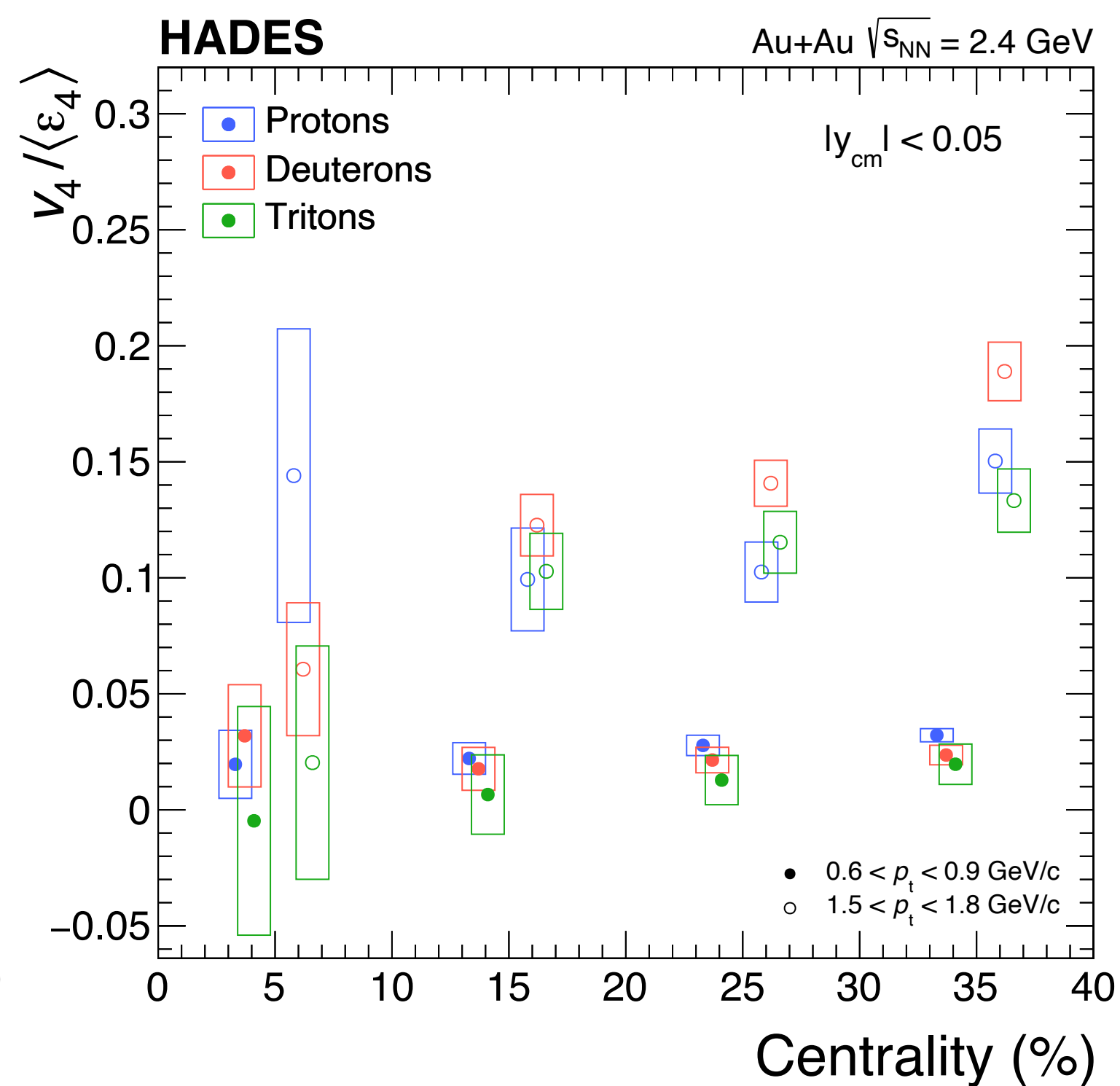
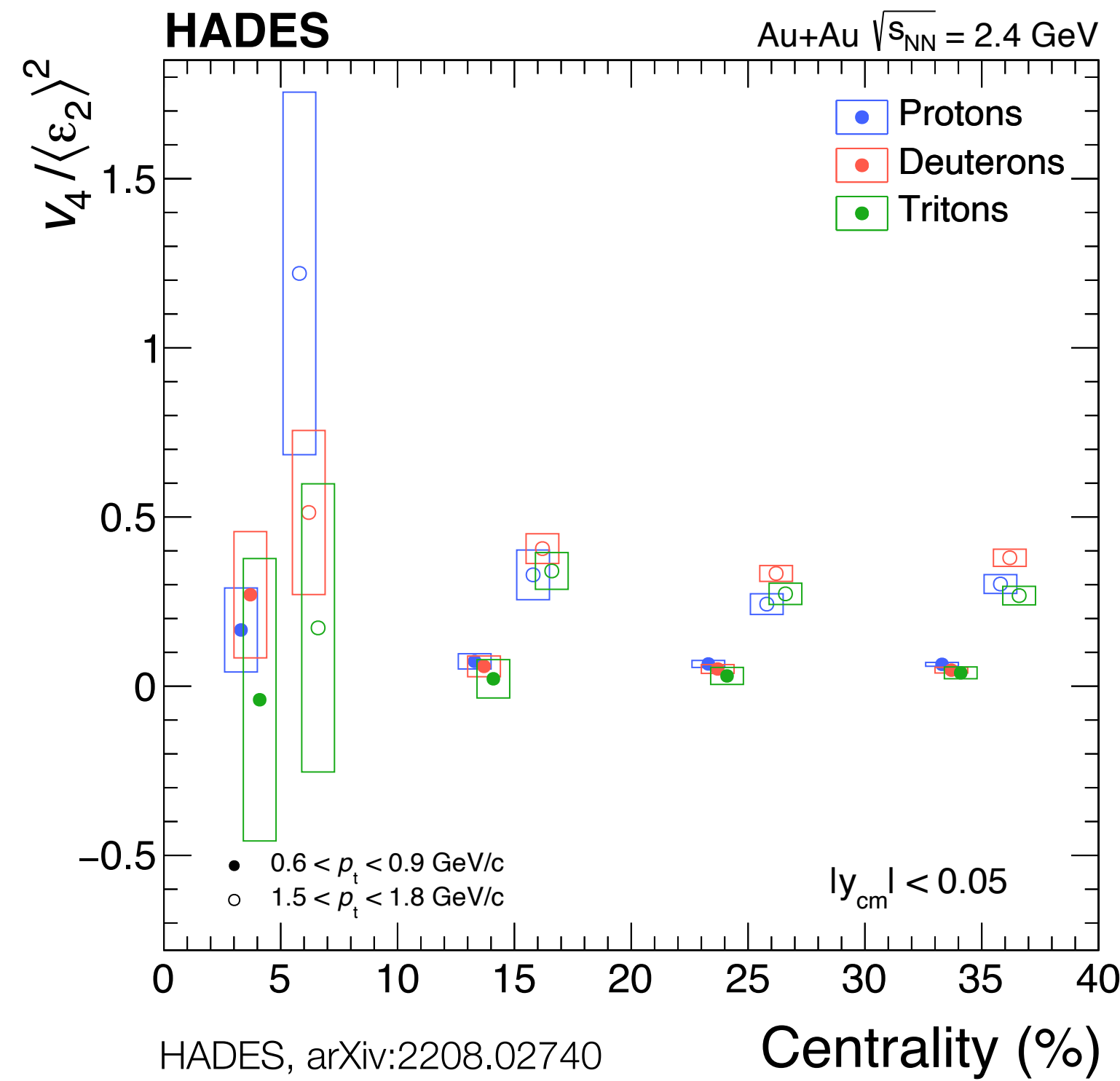
$$v_{4,A=3}(A p_t) = 9 v_4(p_t) \frac{1}{1 + 12 v_4(p_t) + 6 v_4^2(p_t)}$$

assuming:  $v_4(p_t)/v_2^2(p_t) = 1/2$

D. Molnar and S.A. Voloshin PRL **91** (2003) 092301  
P.F. Kolb et al., PRC **69** (2004) 051901

# Geometry Scaling

## Quadrangular Flow $v_4$



### Scaling with initial eccentricities

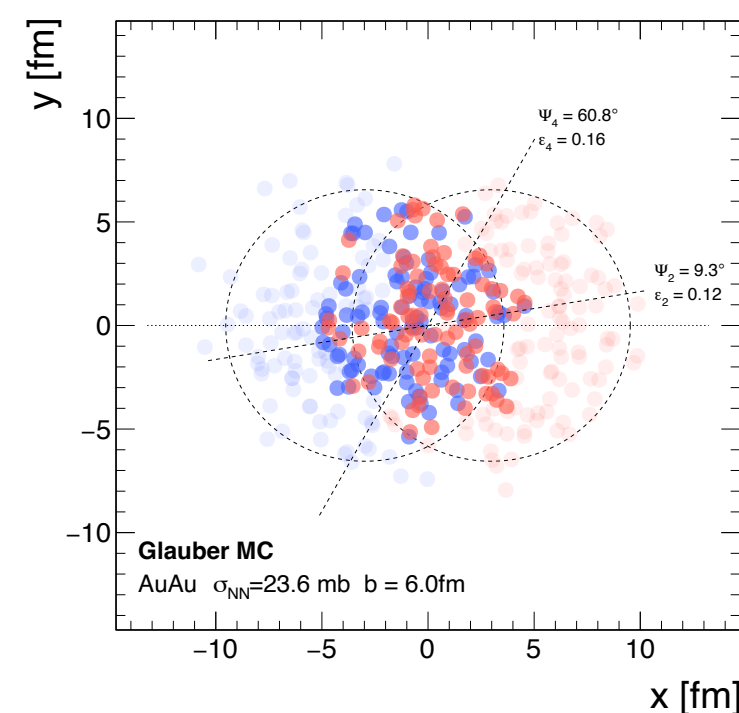
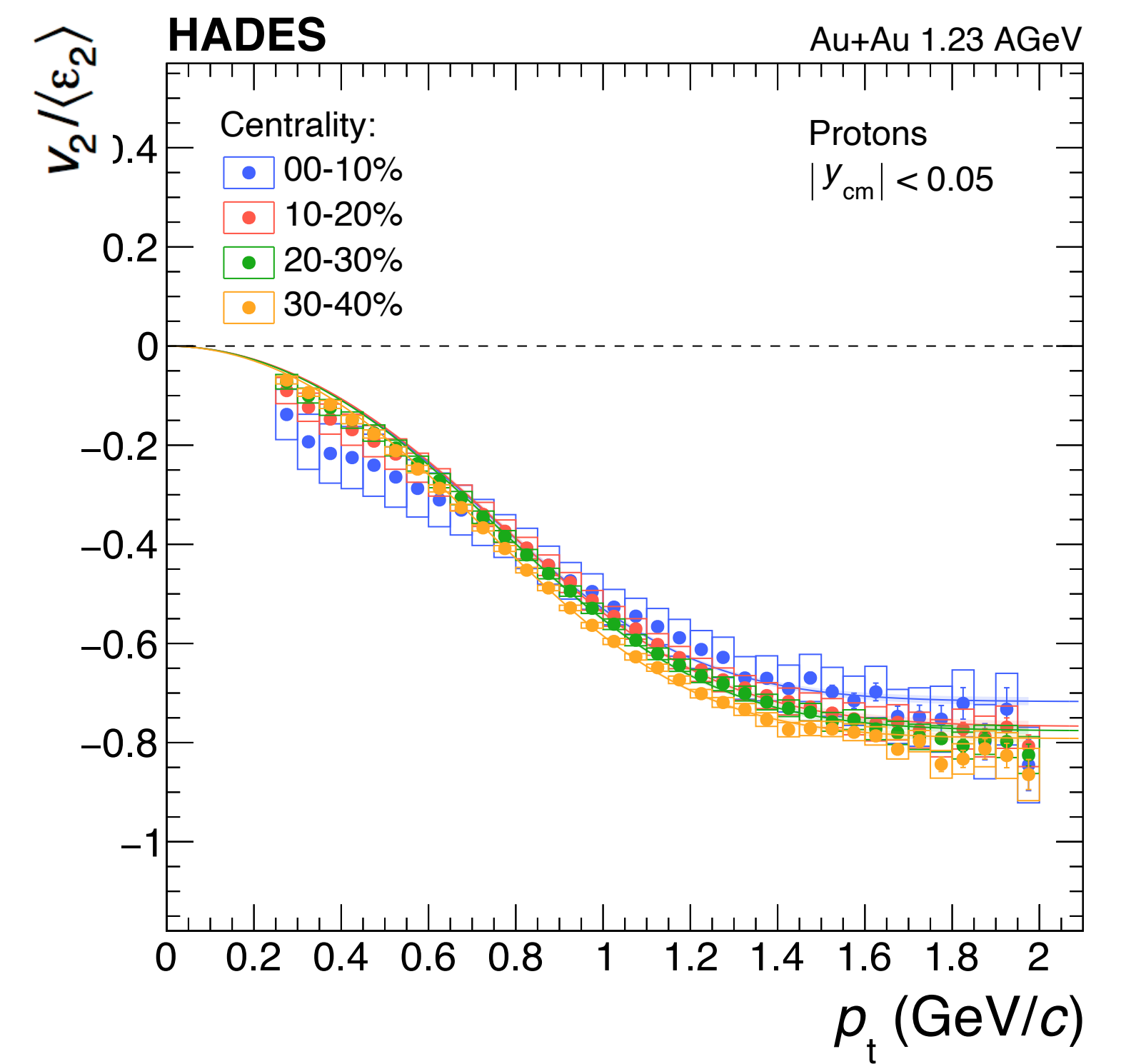
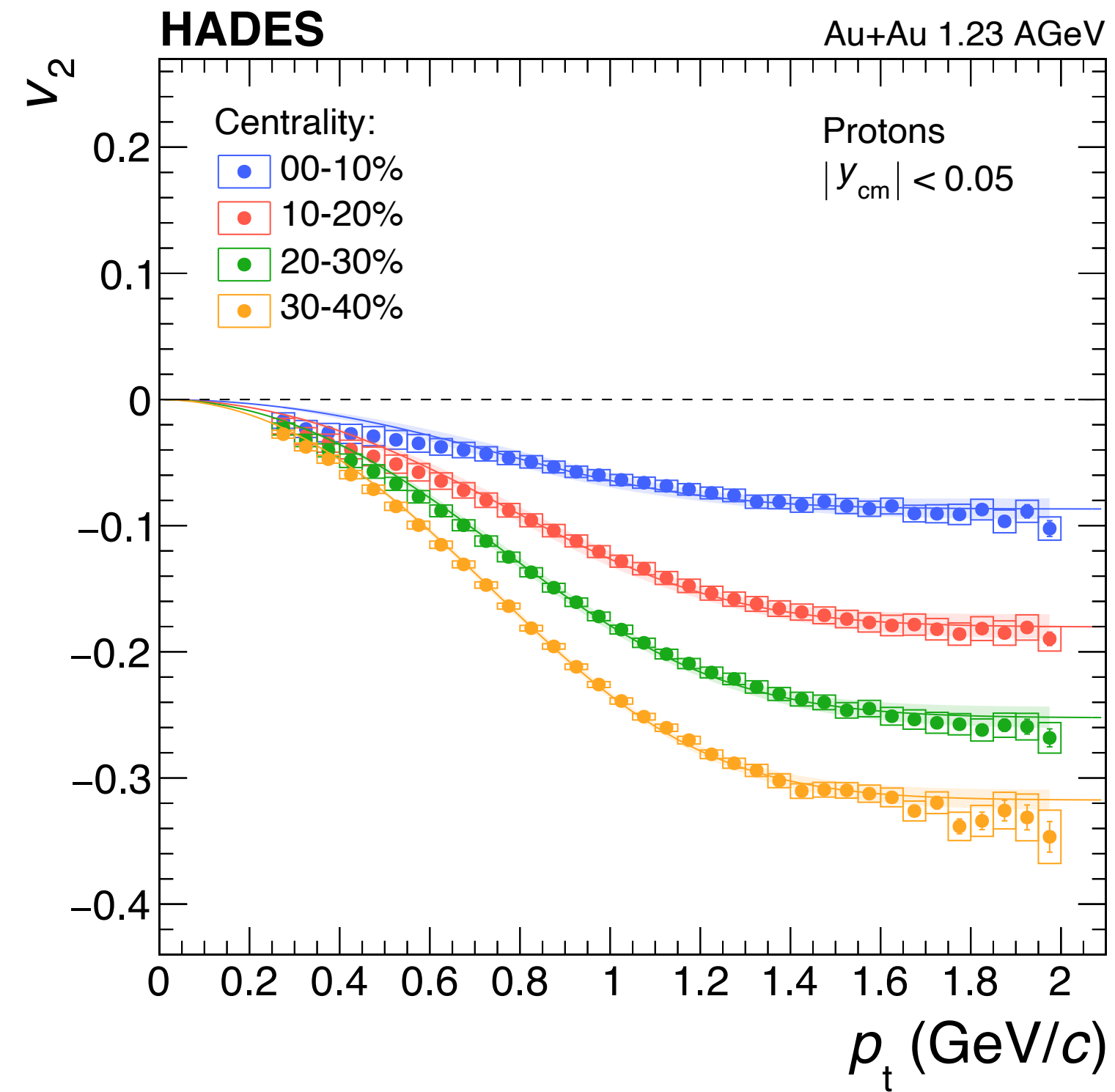
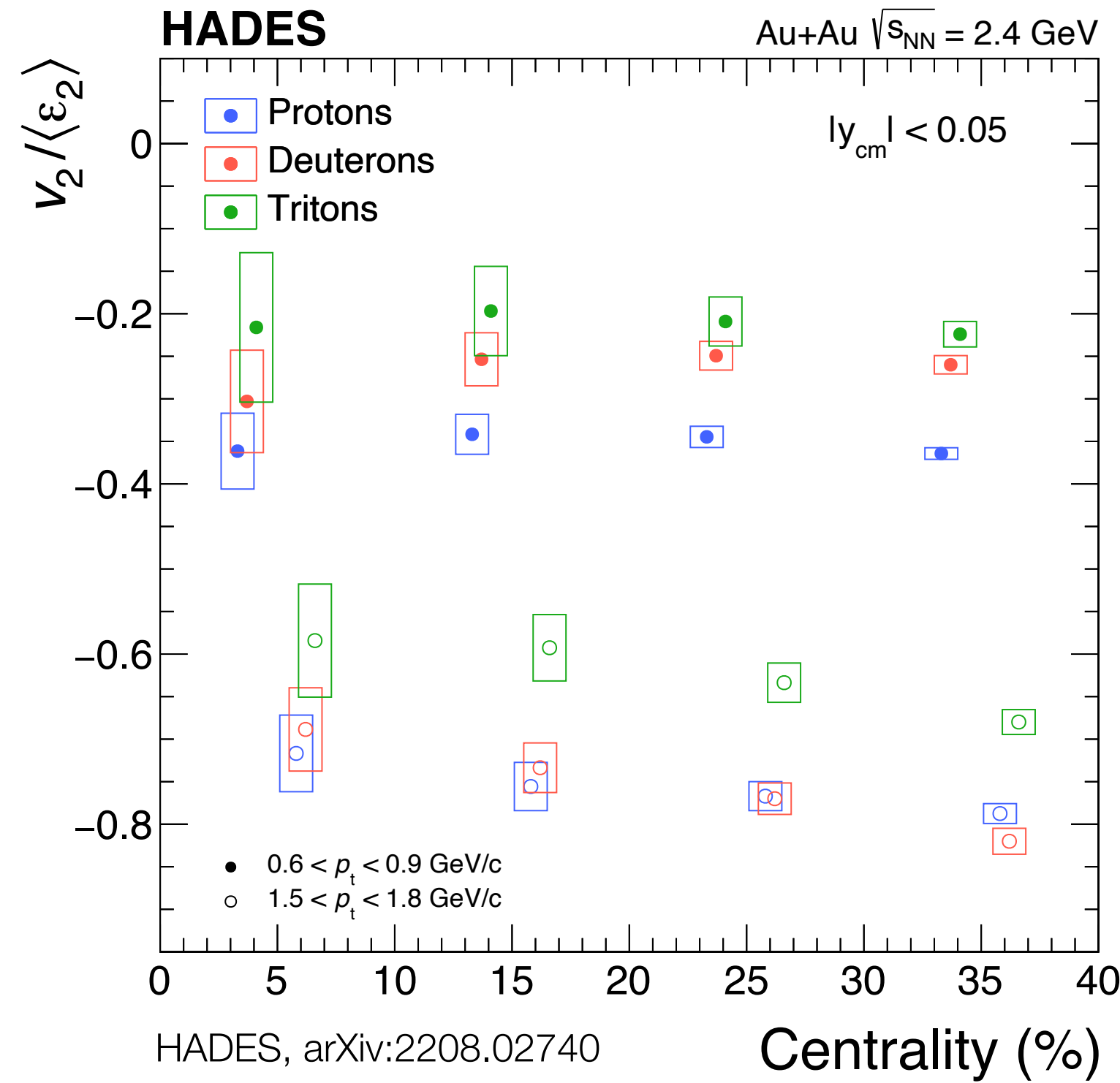
Calculated for overlap zone with Glauber MC

$v_4 / \langle \epsilon_2 \rangle^2$  almost independent of centrality and  $p_t$  ( $v_4 / \langle \epsilon_4 \rangle$  is not)  
 $\Rightarrow$  Fixed relation between  $v_2$  and  $v_4$  (different to high energies)



# Geometry Scaling

## Elliptic Flow $v_2$



### Scaling with initial eccentricities

Calculated for overlap zone with Glauber MC

$v_2/\langle \epsilon_2 \rangle$  almost independent of centrality and  $p_t$

$$\epsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2}}{\langle r^n \rangle}$$

### Orientation of symmetry-planes

Negative  $v_2/\langle \epsilon_2 \rangle$  values  $\Rightarrow v_2$  Event- and  $\epsilon_2$

Eccentricity-plane are perpendicular

Similar scaling for  $v_4$  with  $\langle \epsilon_2 \rangle^2$