Gravitational corrections to the beta functions

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elusives
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What is Unimodular Gravity

- Unimodular Gravity (UG) is a truncation of General Relativity (GR) where the space-time metric is unimodular,

$$\tilde{g} \equiv \det \tilde{g}_{\mu\nu} = -1$$

- It has the nice property that the vacuum energy does not couple to gravitation.

$$S \equiv \int d^n x \left(-\frac{1}{2\kappa^2} R[\tilde{g}] + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

- Hence, additions of the type

$$\Lambda_0 \int d^n x \sqrt{-\hat{g}}$$

are physically irrelevant.

UG Eq. of Motion

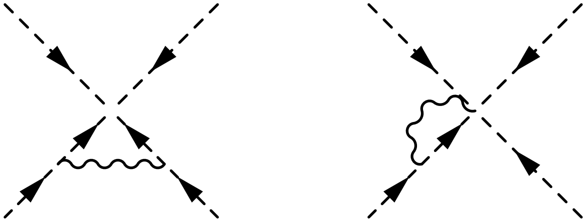
The equations of motion plus the Bianchi identity yield

$$R_{\mu\nu} - \frac{1}{2} R \hat{g}_{\mu\nu} - C \hat{g}_{\mu\nu} = M_P^{2-n} T_{\mu\nu}$$

ie, Einstein equations with a cosmological constant term BUT it is only an integration constant.

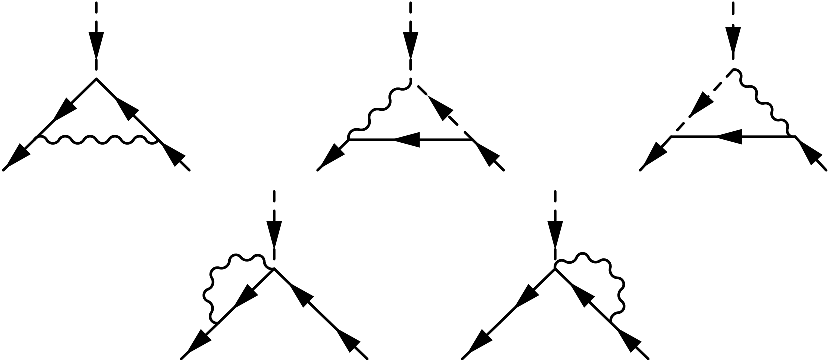
The quartic and Yukawa beta functions I

- In peturbatively renormalizable field theories the coupling constant beta functions have invaluable physical information.
- In Phys.Rev.Lett. 104 (2010) 081301 the GR corrections to the beta functions for the scalar λ and Yukawa g couplings are computed in the MS scheme in the de Donder gauge. These are given by the diagrams



Contributions to the ϕ^4 vertex.

The quartic and Yukawa beta functions I



Contributions to the Yukawa vertex.

Giving the result

$$\beta_\lambda^{\mathrm{GR}} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2 \lambda, \quad \beta_g^{\mathrm{GR}} = \frac{1}{16\pi^2}\kappa^2 \left\{ m_\phi^2 \left(\frac{1}{2}\right) + m_\Psi^2 \left(-1\right)\right\}$$

$m_\phi =$ mass of the scalar, $m_\Psi =$ mass of the fermion

The quartic and Yukawa beta functions III

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  - WRONG CONCLUSION!

The quartic and Yukawa beta functions IV

- Indeed, the beta functions defined as in Phys.Rev.Lett. 104 (2010) 081301 (ie, by a standard multiplicative renormalization) lack intrinsic physical meaning, for they turn out to be gauge dependent.
- We decided to compute the same functions –for GR– using a generalized de Donder gauge

$$\int d^n x\, \alpha \left(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h\right)^2,$$

getting

$$\beta_\lambda^{\mathrm{GR}} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2 \left(\frac{3}{2}+\alpha\right)\lambda$$

$$\beta_g^{\mathrm{GR}} = \frac{1}{16\pi^2}\kappa^2 \left\{ m_\phi^2 \left[\frac{1}{2} - \left(\frac{1}{2}+\alpha\right)\right] + m_\Psi^2 \left[-1 - \left(\frac{1}{2}+\alpha\right)\frac{85}{16}\right]\right\}$$

The quartic and Yukawa beta functions V

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The quartic and Yukawa beta functions V

- $\beta_\lambda^{\rm GR}$ and $\beta_g^{\rm GR}$ do not have intrinsic physical meaning.
- By introducing a non-multiplicative (but local) wave function renormalization (as did, in the YM case, J.Ellis & N. Mavromatos Phys.Lett. B711 (2012) 139).

$$g_0 = \mu^{-\varepsilon} Z_g Z_\psi^{-1} Z_\phi^{-1/2} g, \qquad \phi_0 = \phi + \frac{1}{2}\delta Z_\phi \phi,$$

$$\Psi_0 = \Psi + \frac{1}{2}\delta Z_\Psi \Psi + \frac{1}{2} a_1 \kappa^2 m_\Psi^2 \Psi + \frac{1}{2} b_1 \kappa^2 m_\phi^2 \Psi, \qquad m_{\Psi_0} = (1+\delta Z_{m_\Psi}) m_\Psi,$$

$$\bar\Psi_0 = \bar\Psi + \frac{1}{2}\delta Z_\Psi \bar\Psi + \frac{1}{2} a_1 \kappa^2 m_\Psi^2 \bar\Psi + \frac{1}{2} b_1 \kappa^2 m_\phi^2 \bar\Psi, \qquad m_{\phi_0} = (1+\delta Z_{m_\phi}) m_\phi .$$

one obtains that

$$\beta_g^{\rm GR} = 0 = \beta_g^{\rm UG}$$

CONCLUSIONS

- The gravitational contributions to the beta functions have no physical meaning.
- As far as we can tell there is no difference between quantum GR and quantum UG when the Cosmological Constant vanishes.
- Is UG the right low-energy theory of gravity? Does it come from String Theory? (UG and GR have the same S-matrix).