A Gauged U(1) PQ symmetry

Invisible 18 : Sep 4th 2018

Masahiro Ibe (ICRR&IPMU)

H. Fukuda (IPMU), M.Suzuki(ICRR), T.T.Yanagida (IPMU) and MI

Phys.Rev.D95(2017),9,095017

M. Suzuki(ICRR), T.T. Yanagida (IPMU) and MI

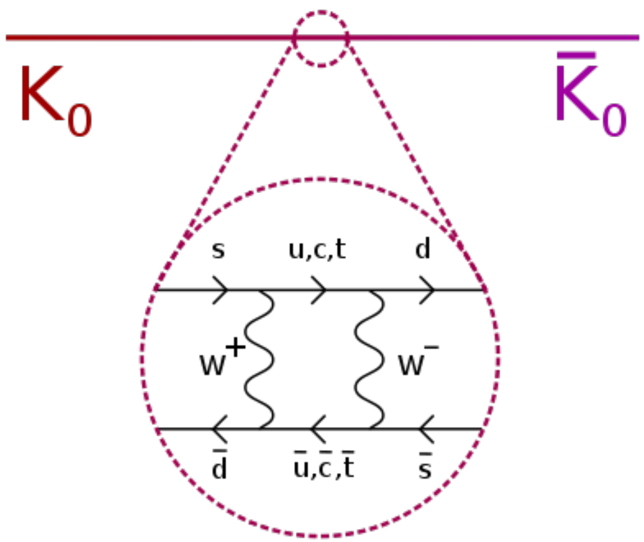
JHEP 1808 (2018) 045

Strong CP problem

Experimentally, ***QCD*** is known to preserve ***CP*** symmetry very well.

- ✓ Hadron spectrum respects ***CP*** symmetry very well.

- ✓ ***CP*** violating transitions in the ***SM*** are caused by ***CP*** violation in the weak interaction (i.e. by the ***CKM*** phase).



Picture from : https://en.wikipedia.org/wiki/Kaon

Strong CP problem

This feature is not automatically guaranteed in ***QCD*** .

- ***QCD*** has its own ***CP***-violating parameter : ***θ***

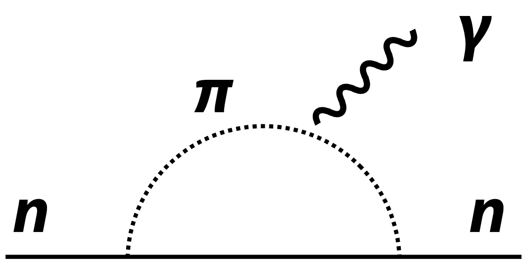
$$S_{\rm QCD} = \int d^4x \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \boxed{\frac{\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}} + \sum_{i=1}^{N_f} \bar{q}_i (D - M) q_i \right)$$

(positive valued quark mass)

- ***θ*** - term violates the ***P*** and ***CP*** symmetries

$$\int d^4x F_{\mu\nu} \tilde{F}^{\mu\nu} \to - \int d^4x F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- The ***θ*** - term is highly constrained experimentally !



Null observation of the ***neutron EDM*** :

$d_n/e < 2.9 \times 10^{-26}$ @ 90%CL

[hep-ex/0602020]

→ ***$\theta < 10^{-11}$***

$d_n/e \sim 10^{-15}\,\theta$ [1979 Crewther, Veccia, Veneziano, Witten]

Why so small ? = Strong CP Problem

Strong CP problem

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(positive valued quark mass)

- In the Standard Model, the quark mass matrix stems from the Yukawa couplings of ***3x3*** general complex matrices.

$$M_u \propto Y_u \,(\text{general complex}) \rightarrow (m_u, m_c, m_t) > 0$$
$$M_d \propto Y_d \,(\text{general complex}) \rightarrow (m_d, m_s, m_b) > 0$$

The phases of the Yukawa matrices also contribute to ***θ***.

Why the θ parameter and the phases of the Yukawa coupling conspire to be cancelling with each other ?

= Strong CP Problem

Peccei-Quinn Mechanism ['77 Peccei, Quinn]

Two Higgs doublet Model (***H_u , H_d***)

$$\mathcal{L} = y_u H_u Q_L \bar{u}_R + y_d H_d Q_L \bar{d}_R - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_u|^2 - \cdots$$

U(1) Peccei-Quinn symmetry (anomaly of ***$SU(3)_c$***)

$$H_{u,d} \to e^{i\alpha} H_{u,d} \qquad \bar{u}_R \to e^{-i\alpha} \bar{u}_R \qquad \bar{d}_R \to e^{-i\alpha} \bar{d}_R$$

By the Peccei-Quinn rotation, ***θ*** can be shifted away !

$$\theta \to \theta' = \theta - 2N_g\,\alpha \qquad (N_g=3)$$

so that the ***θ*** is unphysical (similar to ***θ_W***).

Weinberg-Wilczek Axion ['78 Weinberg, '78 Wilczek]

U(1)$_{PQ}$ is spontaneously broken at the EWSB → ***axion*** = (***CP***-odd Higgs)

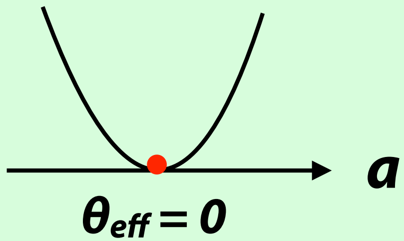
$$a = \frac{\sqrt{2} v_u v_d}{\sqrt{v_u^2 + v_d^2}} (\arg H_u + \arg H_d)$$

$$\mathcal{L}_{\rm eff} = \frac{g_s^2}{32\pi^2} \left(\theta - \frac{6a}{f_a} \right) G^{a\mu\nu} \tilde{G}^a_{\mu\nu} \qquad \left(f_a = 2\sqrt{2} v_u v_d / \sqrt{v_u^2 + v_d^2} \right)$$

Axion is massive due to the ***SU(3)$_c$*** anomaly

$$m_a = \frac{N_g \sqrt{m_u m_d}}{m_u + m_d} \frac{f_\pi}{f_a} m_\pi \quad \boldsymbol{\sim 100\ \mathrm{keV}}$$

In terms of the axion, the PQ mechanism can be interpreted as a dynamical tuning of the ***θ*** angle.

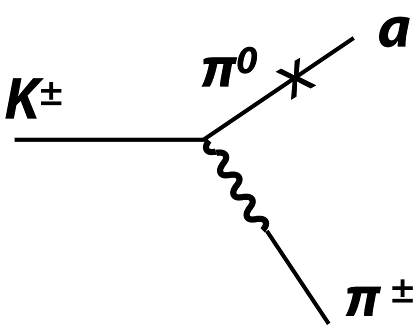


$$\mathcal{L} = \frac{1}{2} m_a^2 f_a^2 (a/f_a - \theta/6)^2 \longrightarrow \langle a/f_a \rangle = \theta/6$$

$$\theta_{\rm eff} = \theta - 6\langle a/f_a \rangle = 0$$

Weinberg-Wilczek Axion ['78 Weinberg, '78 Wilczek]

f_a is constrained by a meson decay rate into axion.



$$Br(K^\pm \to \pi^\pm + a\,(\text{invisible}))$$
$$= O(f_\pi^2 / f_a^2)\ \times Br(K^\pm \to \pi^\pm + \pi^0)$$
$$< 5 \times 10^{-11}$$ [E787 hep-ex/0403034]

$f_a > O(1)\mathrm{TeV}$

[Axion decays into two photon but the lifetime is so long for $m_a \sim 100$ keV.]

Original PQ-mechanism has been excluded !

Invisible Axion : $f_a >> v_{EW}$ ['80 Zhitnitsky, '81 Dine, Fischler, Sredniki]

ZDFS axion : Two Higgs doublet Model (***H_u , H_d***) and a Singlet ***S***

$$\mathcal{L} = y_u H_u Q_L \bar{u}_R + y_d H_d Q_L \bar{d}_R - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_u|^2 - \cdots$$

$$+ \boxed{\frac{1}{M_{PL}^{n-2}} S^n H_u H_d} + \cdots$$

U(1) Peccei-Quinn (PQ) symmetry (e.g. ***n=2***)

$H_{u,d} \to e^{i\alpha} H_{u,d}$ $\quad$ $S \to e^{i\alpha} S$ $\quad$ $\bar{u}_R \to e^{-i\alpha} \bar{u}_R$ $\quad$ $\bar{d}_R \to e^{-i\alpha} \bar{d}_R$

U(1) Peccei-Quinn symmetry is broken by QCD anomaly.

By the Peccei-Quinn rotation, θ can be shifted away !

$$\theta \to \theta' = \theta - 2N_g\,\alpha \qquad (N_g=3)$$

so that the θ is unphysical.

Invisible Axion : $f_a >> v_{EW}$ ['80 Zhitnitsky, '81 Dine, Fischler, Sredniki]

ZDFS axion : Two Higgs doublet Model (***H_u , H_d***) and a Singlet ***S***

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U(1) Peccei-Quinn (PQ) symmetry (e.g. ***n=2***)

$H_{u,d} \to e^{i\alpha} H_{u,d}$ $\quad S \to e^{i\alpha} S$ $\quad \bar{u}_R \to e^{-i\alpha} \bar{u}_R$ $\quad \bar{d}_R \to e^{-i\alpha} \bar{d}_R$

$U(1)_{PQ}$ is spontaneously broken by $\langle S \rangle = v_s \gg v$

$$a = \frac{f_a}{2} \arg S \qquad f_a = 2\sqrt{2} \langle S \rangle$$

$$m_a = \mathcal{O}(1)\mathrm{meV} \times \left(\frac{10^9 \mathrm{GeV}}{f_a} \right)$$

The axion appears as a pseudo-Goldstone boson.

Invisible Axion : $f_a >> v_{EW}$ ['79 Kim, '80 Shifman, Vainshtein, Zakharov]

KSVZ axion : ***SM*** matter field are not ***$U(1)_{PQ}$*** neutral.

$$\mathcal{L} = \mathcal{L}_{\rm SM} + S q_L \bar{q}_R - \cdots$$

Singlet ***S*** Extra colored fermions ***q_L , $\bar{q}_R$***

U(1) PQ symmetry broken by ***QCD*** anomaly

$S \to e^{i\alpha} S$ ***$q_L \to e^{-i\alpha/2} q_L$*** ***$\bar{q}_R \to e^{-i\alpha/2} \bar{q}_R$***

$U(1)_{PQ}$ is spontaneously broken by ***$< S > = v_s >> v$***

$$a = f_a \arg S \qquad f_a = \sqrt{2}\langle S \rangle$$

$$m_a = \mathcal{O}(1)\mathrm{meV} \times \left(\frac{10^9\mathrm{GeV}}{f_a}\right)$$

The axion appears as a pseudo-Goldstone boson.

Invisible Axion : $f_a >> v_{EW}$

- Invisible axion is very light :

$$\boxed{m_a = \mathcal{O}(1)\mathrm{meV} \times \left(\frac{10^9\mathrm{GeV}}{f_a}\right)}$$

Axion can be emitted in astrophysical objects leading to stringent constraint on the decay constant $\boldsymbol{f_a}$.

- Resultant constraint on the decay constant is

$$f_a > 10^9\mathrm{GeV}$$

[see e.g. a comprehensive review by Raffelt '06]

- Invisible axion is a good candidate for ***DM***

$$\Omega_a h^2 \simeq 0.18\,\theta_a^2 \left(\frac{F_A}{10^{12}\,\mathrm{GeV}}\right)^{1.19}$$

Initial angle $\boldsymbol{\theta_1 = 0\text{-}2\pi}$ [misalignment axion]

$$\Omega_a h^2 \simeq 0.035 \pm 0.012 \left(\frac{F_A}{10^{10}\,\mathrm{GeV}}\right)^{1.19}$$

[string-domain wall network axion]

[e.g. 1301.1123 Kawasaki, Nakayama]

What is the origin of the Peccei-Quinn symmetry ?

- The ***PQ*** symmetry cannot be an exact symmetry .

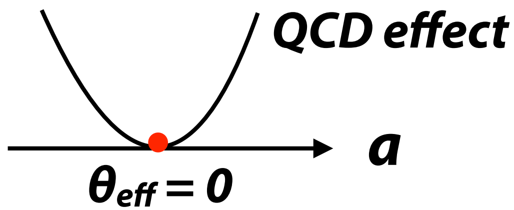
  U(1) PQ symmetry is defined to be broken by the ***QCD*** anomaly.

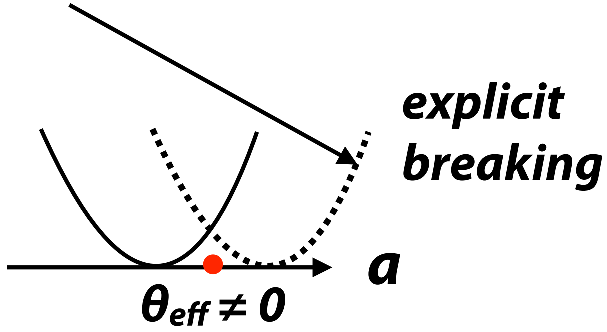
- Why is the ***PQ*** symmetry broken only by the ***QCD*** anomaly?

If the physics at the Planck scale breaks ***PQ*** symmetry we would have

$$\Delta\mathcal{L} = \frac{S^m}{M_{\rm PL}^{m-4}} + h.c.$$

which distorts the axion potential





The effective θ_{eff}-parameter is no more vanishing…

$$\Delta\theta_{\rm eff} = \frac{f_a^m}{f_\pi^2 m_\pi^2 M_{\rm PL}^{m-4}}$$

If we require $\theta_{eff} << 10^{-10}$, no term with $m < 10$ is allowed $f_a > 10^9$***GeV***.

What is the origin of the Peccei-Quinn symmetry ?

- Gauge symmetries do not suffer from explicit breaking.

  Can we make the ***PQ*** symmetry a gauge symmetry ?

  The ***PQ*** symmetry has an ***SU(3)$_c$*** anomaly…

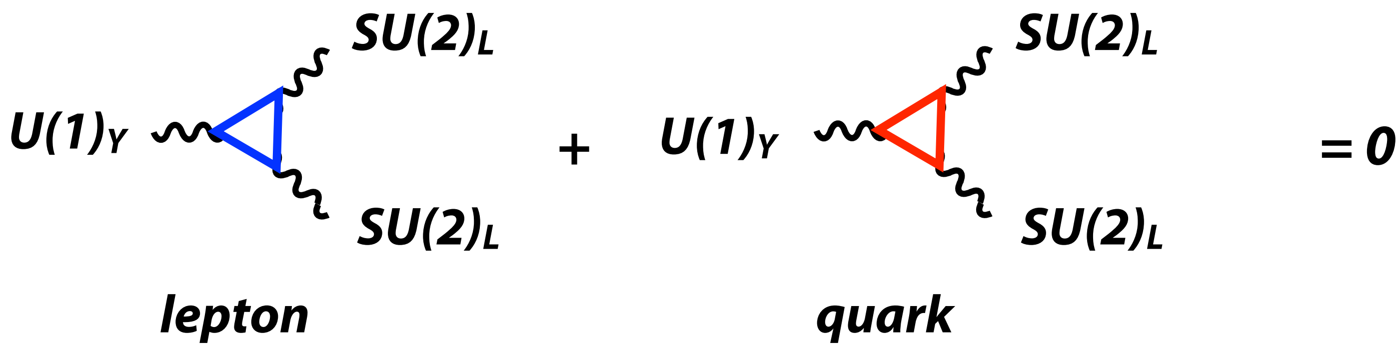
  → the ***PQ*** symmetry cannot be a gauge symmetry by itself.

- ***U(1)$_Y$*** in the Standard Model

  U(1)$_Y$ symmetry of the lepton sector has an ***SU(2)$_L$*** anomaly.

  Cannot be a gauge symmetry ? Absolutely Yes !

The ***SU(2)$_L$*** anomaly of ***U(1)$_Y$ of the lepton sector*** is cancelled by the ***SU(2)$_L$*** anomaly of ***U(1)$_Y$ of the quark sector!***



What is the origin of the Peccei-Quinn symmetry ?

- Gauge symmetries do not suffer from explicit breaking.

  Can we make the ***PQ*** symmetry a gauge symmetry ?

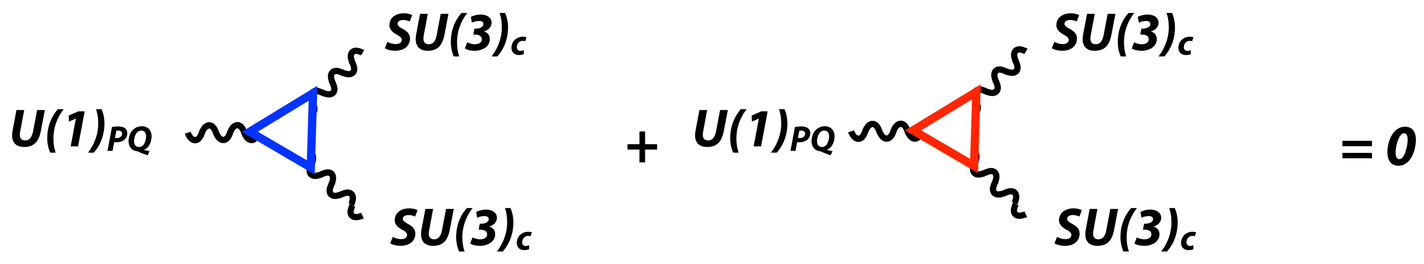
  The ***PQ*** symmetry has an ***$SU(3)_c$*** anomaly…

  → the PQ symmetry cannot be a gauge symmetry by itself.

- ***Gauged $U(1)_{PQ}$***

  We arrange the ***$U(1)_{PQ}$*** charges so that the total ***$SU(3)_c$*** anomaly is cancelled !

The ***$SU(3)_c$*** anomaly of ***$U(1)_{PQ}$ of sector 1*** is cancelled by the ***$SU(3)_c$*** anomaly of ***$U(1)_{PQ}$ of sector 2***



sector 1 ***sector 2***

General Recipe to Construct gauged PQ models

Let us bring any "two" invisible axion models :

sector 1

$\boxed{U(1)_{PQ1}}$

cf. ***DSFZ*** model

cf. ***KSVZ*** model

sector 2

$\boxed{U(1)_{PQ2}}$

cf. ***DSFZ*** model

cf. ***KSVZ*** model

$$\partial j_{\mathrm{PQ}_1} = \frac{g_s^2}{32\pi^2} N_1 F^a \tilde{F}^a \qquad \partial j_{\mathrm{PQ}_2} = \frac{g_s^2}{32\pi^2} N_2 F^a \tilde{F}^a$$

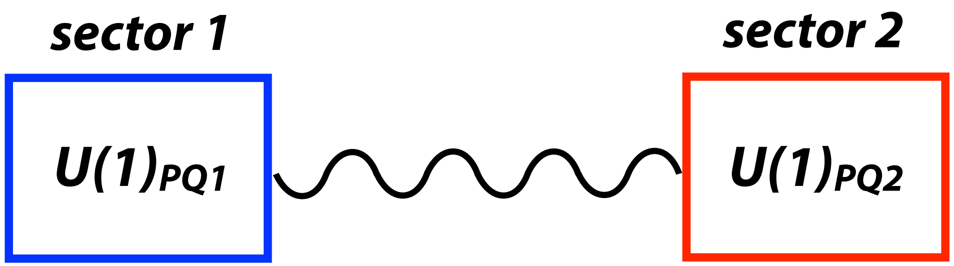
An anomaly free combination,

$$j^\mu_{PQ} \propto N_2\ j^\mu_{PQ_1} - N_1\ j^\mu_{PQ_2}$$

can be a gauge symmetry !

→ Gauged ***U(1)***$_{PQ}$ Symmetry !

General Recipe to Construct gauged PQ models



- No ***gauged*** ***U(1)***$_{PQ}$ breaking term is allowed since ***U(1)***$_{PQ}$ is an exact symmetry !

- No ***global*** ***U(1)***$_{PQ1}$ breaking term consisting of fields in the sector 1 due to the ***gauged*** ***U(1)***$_{PQ}$ symmetry.

  U(1)$_{PQ1}$ breaking term = ***U(1)***$_{PQ}$ breaking term

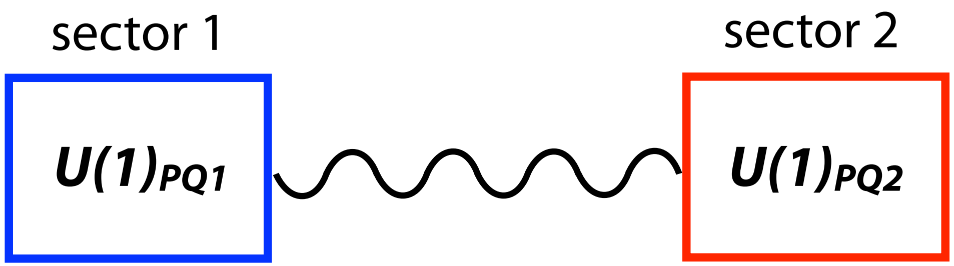
$$L = \Phi_1\cancel{(x)}^n + h.c.$$

- No ***gauged*** ***U(1)***$_{PQ2}$ breaking term consisting of fields in the sector 2 due to the ***global*** ***U(1)***$_{PQ2}$ symmetry.

  U(1)$_{PQ2}$ breaking term = ***U(1)***$_{PQ}$ breaking term

$$L = \Phi_2\cancel{(x)}^m + h.c.$$

General Recipe to Construct gauged PQ models



✔ No ***gauged*** $U(1)_{PQ}$ breaking term is allowed since $U(1)_{PQ}$ is an exact symmetry !

✔ Only dangerous operators to break $U(1)_{PQ1}$ and $U(1)_{PQ2}$ symmetries are

$$L = M_{PL}{}^{4-(\dim O1+\dim O2)}\, O_1\, O_2 + h.c.$$

$U(1)_{PQ1}$ of $O_1 \neq 0$ $\qquad$ $U(1)_{PQ2}$ of $O_2 \neq 0$

while $O_1 O_2$ ***is gauge invariant.***

✔ If PQ_1 and PQ_2 breaking scales are $O(10^9)\mathrm{GeV}$, the resultant breaking of either PQ_1 or PQ_2 is suppressed by arranging the charge assignment so that

$$\dim O_1 + \dim O_2 > 10$$

→ well-protected ***global PQ*** symmetry by the ***gauged PQ*** symmetry !

General Recipe to Construct gauged PQ models

✓ Example : Barr-Seckel Model ['92 Barr-Seckel]

Bring two independent ***KSVZ*** axion models

$$L = y_1 S_1\, q_{1L}\, \bar{q}_{1R} + y_2 S_2\, q_{2L}\, \bar{q}_{2R} + h.c.$$

KSVZ fermions : N_1 flavors of q_1 , N_2 flavors of q_2

✓ $U(1)_{PQ1}$ symmetry

$$S_1 \to e^{ia} S_1 \qquad q_{1L} \to e^{-ia} q_{1L} \qquad \partial j_{\mathrm{PQ}_1} = \frac{g_s^2}{32\pi^2} N_1 F^a \tilde{F}^a$$

✓ $U(1)_{PQ2}$ symmetry

$$S_2 \to e^{ia} S_2 \qquad q_{2L} \to e^{-ia} q_{2L} \qquad \partial j_{\mathrm{PQ}_2} = \frac{g_s^2}{32\pi^2} N_2 F^a \tilde{F}^a$$

General Recipe to Construct gauged PQ models

✓ Example : Barr-Seckel Model ['92 Barr-Seckel]

Gauged $U(1)_{PQ}$ symmetry

$$S_1(q_1) \quad S_2(q_2) \qquad\qquad q_1 : q_2 = N_2 : -N_1$$

$$\rightarrow \partial j_{PQ} = 0$$

$|q_1|$ and $|q_2|$ are taken to be relatively prime integers

✓ The lowest dimensional $U(1)_{PQ1,PQ2}$ breaking operators

$$L = M_{PL}^{4-(|q1|+|q2|)} S_1^{|q1|} S_2^{|q2|} + h.c.$$

To obtain high quality ***global PQ*** symmetry : $|q_1| + |q_2| > 10$

ex) $N_1 = 1,\ N_2 = 9$

General Recipe to Construct gauged PQ models

✔ Example 2 : Application to the Composite Axion Model

SU(N$_c$) gauge theory [1985 Kim]

	$SU(N_c)$	$SU(3)$	$U(1)_{PQ}$
Q_L	N_c	3	1
$\bar{Q}_R$	$\bar{N}_c$	$\bar{3}$	1
q_L	N_c	1	-3
$\bar{q}_R$	$\bar{N}_c$	1	-3

$\subset SU(4)$

$U(1)_{PQ}$ is free from $SU(N_c)$ anomaly but is broken by ***QCD*** anomaly !

Assume no quark mass terms.

✔ Strong dynamics of $SU(N_c)$ exhibits the chiral symmetry breaking.

15 Goldstone Modes

$SU(4)_L \times SU(4)_R \to SU(4)_V$

$$\left\{ \begin{array}{l} SU(3) : \text{Octet} + 3 + \bar{3} = \text{Massive} \ (\sim g_s \Lambda_{N_c}) \\ U(1)_{PQ} : \text{singlet} = \text{axion} \end{array} \right.$$

$$\mathcal{L}_{QCD} = \frac{g_s^2}{32\pi^2} N_c \frac{a}{f_a} F^a \tilde{F}^a$$

General Recipe to Construct gauged PQ models

Example 2 : Application to the Composite Axion Model

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	$SU(N_c)$	$SU(3)$	$U(1)_{PQ}$
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$\bar{Q}_R$	$\bar{N}_c$	$\bar{3}$	1
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$\bar{q}_R$	$\bar{N}_c$	1	-3

$\subset SU(4)$

$U(1)_{PQ}$ is free from ***SU(N$_c$)*** anomaly but is broken by ***QCD*** anomaly !

Assume no quark mass terms.

There are ***PQ*** breaking operators ;

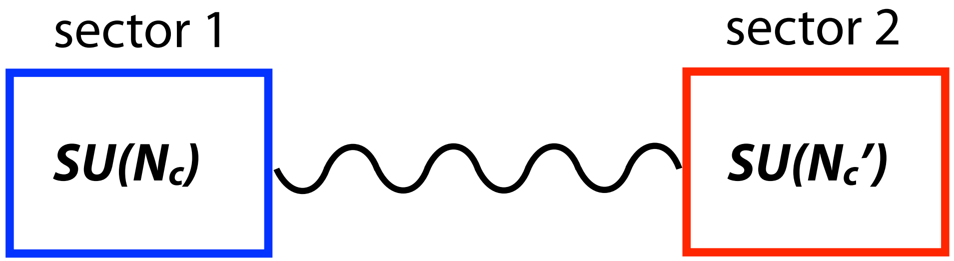
$$L = m\,(Q_L\,\bar{Q}_R) + (Q_L\,\bar{Q}_R)^2/M_{PL}^2 + \ldots$$

which should be suppressed by hand.

Gauged PQ mechanism suppresses those operators !

General Recipe to Construct gauged PQ models

✔ Example 2 : Application to the Composite Axion Model



Bring two composite axion models and consider gauged $U(1)_{PQ}$ with the charge normalization

$$Q_L(q)\,,\ \bar{Q}_R(q),\ Q_L'(q')\,,\ \bar{Q}_R'(q') \qquad q : q' = N_c' : -N_c$$

$$\rightarrow\ \partial j_{PQ} = 0$$

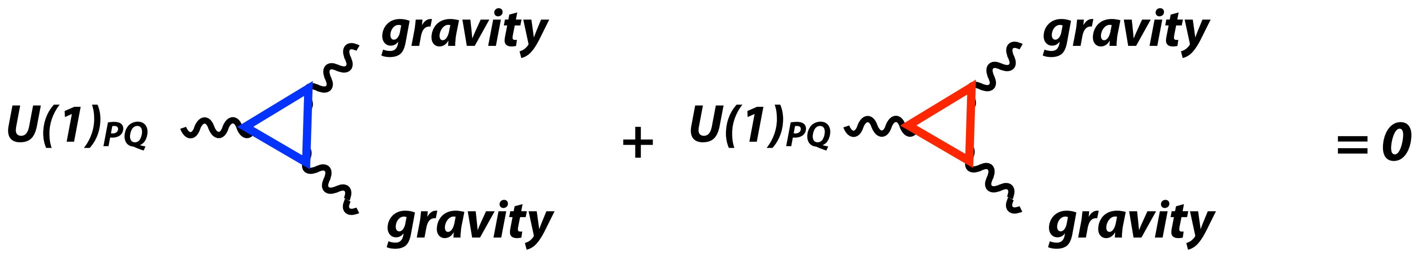
The lowest dimensional global ***PQ*** breaking operators

$$L = (Q_L\,\bar{Q}_R)^{|q'|}\,(Q'_L\,\bar{Q}'_R)^{|q|} / M_{PL}{}^{3|q| + 3|q'| - 4}$$

→ $N_c = 2,\ N_c' = 5$ model is good enough to obtain the high quality global PQ symmetry !

B-L as a gauged PQ symmetry

- In the above examples, we assumed that ***$[U(1)_{PQ}]^3$*** and ***$U(1)_{PQ}$-gravitational*** anomalies are cancelled by the ***PQ*** charged but ***SM*** singlet fields.



SM-charged fields

SM singlet fields

→ The ***PQ***-charged ***SM*** singlet fields tend to be light and have rather long lifetime.

Those long-lived particles sometime cause cosmological problems.

- We can construct a good ***gauged PQ symmetry*** without stable ***SM*** singlets based on the ***B-L*** gauge symmetry !

B-L as a gauged PQ symmetry

- In the ***SU(5)*** GUT, ***B-L*** is achieved as the ***U(1) fiveness = 5(B-L) - 4 Y :***

$$\underline{\boldsymbol{10_{SM}(+1)\,,\quad \bar{5}_{SM}(-3)\,,\quad \bar{N}_R(+5)}}$$

- The seesaw mechanism is realized by introducing ***ϕ(-10)*** ;

$$\mathcal{L} = -\frac{1}{2} y_N \phi \bar{N}_R \bar{N}_R + h.c.$$

$$\rightarrow M_R = y_N < \phi >$$

- The fiveness in the ***SM*** and ***the right-handed neutrino sector*** is anomaly free.
[***$SU(5) \times U(1)_5 \subset SO(10)$ without new fermions***]

B-L as a gauged PQ symmetry

- Introduce an extra pair of ***(5_K,$\bar{5}_K$)*** coupling to ***ϕ(-10)*** as in the ***KSVZ*** model

$$\mathcal{L} = y_K \phi^* \underbrace{\mathbf{5}_K \bar{\mathbf{5}}_K}_{(-10)} + h.c. \ ,$$

→ ***U(1) fiveness*** has ***QCD*** anomaly

$$\partial j_5 \big|_{\mathrm{SM+N+K}} = -\frac{g_a^2}{32\pi^2} 10 F^a \tilde{F}^a$$

- For the gauged ***PQ*** mechanism, introduce, for example ***ϕ'(+1)*** and ***10*** pairs of ***(5_K', $\bar{5}_K$')*** .

$$\mathcal{L} = y'_K \phi'^* \underbrace{\mathbf{5}'_K \bar{\mathbf{5}}'_K}_{(-1)} + h.c. \ ,$$

→ ***U(1) fiveness*** is free from ***QCD*** anomaly

$$\partial j_5 \big|_{\mathrm{SM+N+K+K'}} = 0$$

B-L as a gauged PQ symmetry

✔ We assign $\bar{\mathbf{5}}$ **'s** the same quantum numbers with $\bar{\mathbf{5}}_{SM}$.

$$\underline{\mathbf{5}_K(-7),\ \bar{\mathbf{5}}_K(-3),\ \mathbf{5}_K'(+4),\ \bar{\mathbf{5}}_K'(-3)} \longleftarrow \underline{\bar{\mathbf{5}}_{SM}(-3)}$$

The ***11*** out of ***14*** indistinguishable $\bar{\mathbf{5}}$**'s** become mass partners of $\mathbf{5}_K$ and $\mathbf{5}_K'$

All the extra matter particles decay into the ***SM*** particles.

✔ With this charge assignment, $[U(1)_{PQ}]^3$ and gravitational anomalies are cancelled without ***SM*** singlets !

$$[U(1)_{gPQ}]^3 \propto \left((-10-\bar{q}_K)^3 + (\bar{q}_K)^3\right) + 10\left((1-\bar{q}'_K)^3 + (\bar{q}'_K)^3\right) \quad \boldsymbol{= 0}$$

$$[\text{gravitational}] \propto ((-10-\bar{q}_K) + (\bar{q}_K)) + 10\,((1-\bar{q}'_K) + (\bar{q}'_K)) \quad \boldsymbol{= 0}$$

Satisfying these conditions is not easy than it looks.

Unlike the B-L of ***SM* + *right-handed neutrinos***, there is no larger GUT group like ***SO(10)*** which guarantees the anomaly cancellation…

B-L as a gauged PQ symmetry

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The ***11*** out of ***14*** indistinguishable $\bar{\mathbf{5}}$'s become mass partners of $\mathbf{5}_K$ and $\mathbf{5}_K'$

All the extra matter particles decay into the ***SM*** particles.

✔ With this charge assignment, $[U(1)_{PQ}]^3$ and gravitational anomalies are cancelled without ***SM*** singlets !

$$[U(1)_{gPQ}]^3 \propto \left((-10-\bar{q}_K)^3 + (\bar{q}_K)^3\right) + 10\left((1-\bar{q}_K')^3 + (\bar{q}_K')^3\right) \quad \boldsymbol{= 0}$$

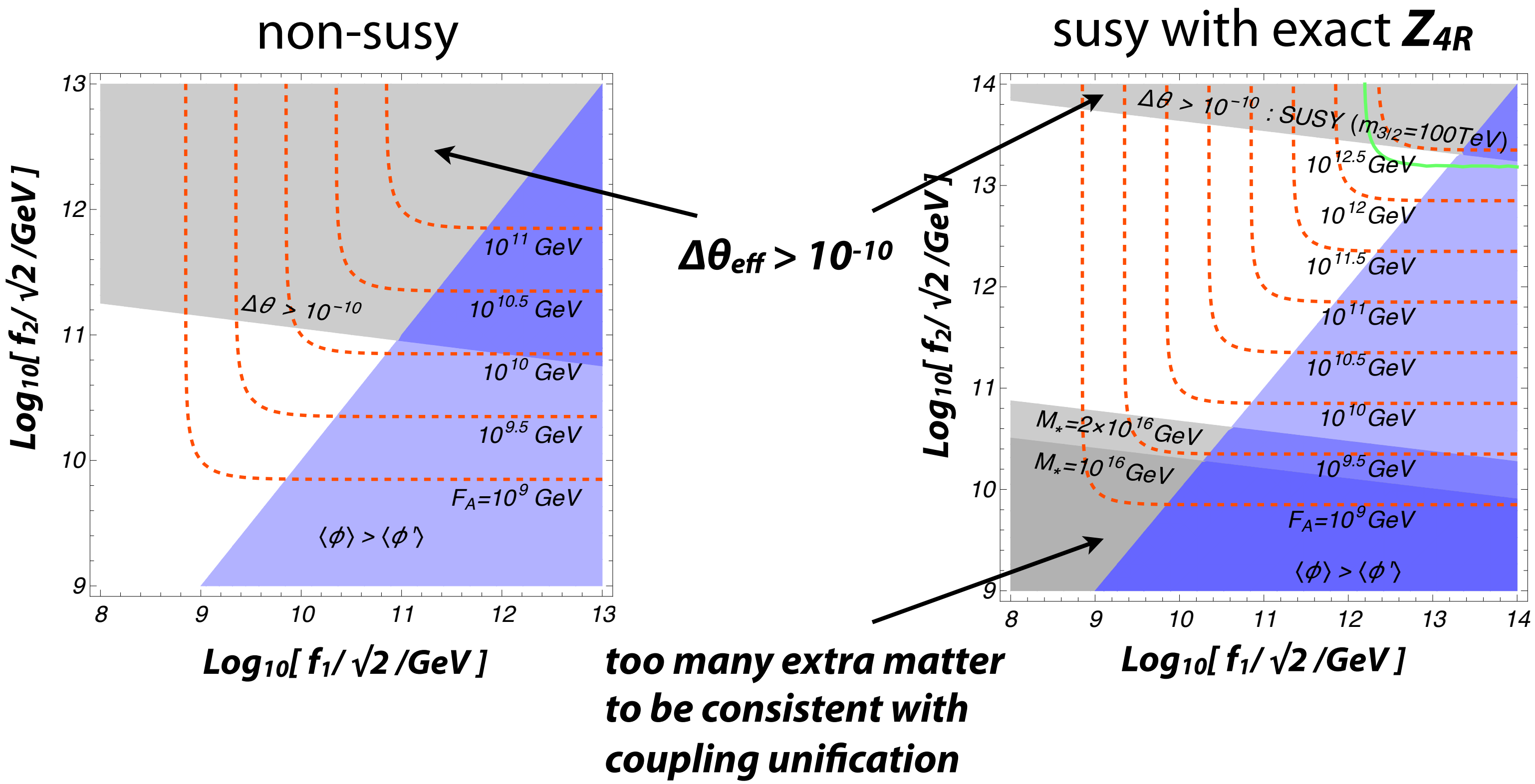
$$[\text{gravitational}] \propto \left((-10-\bar{q}_K) + (\bar{q}_K)\right) + 10\left((1-\bar{q}_K') + (\bar{q}_K')\right) \quad \boldsymbol{= 0}$$

✔ Global ***PQ*** breaking operator :

$$\mathcal{L}_{\cancel{PQ}} \sim \frac{1}{10!}\frac{\phi\phi'^{10}}{M_{\rm PL}^7} + h.c.\ ,$$

Global ***PQ (φ'(+1), 5***$_K$***'(+1), others neutral)*** symmetry is well protected !

B-L as a gauged PQ symmetry



✔ ***B-L (fiveness)*** works as the gauged ***PQ*** symmetry for a wide range of axion decay constant

$$F_a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}}$$

✔ Right handed neutrino mass is given by ***O(f_1)***

B-L does not only provide frameworks for seesaw mechanism and thermal leptogenesis but also solves the strong ***CP*** problem !

Remarks on exact gauged discrete symmetry

✔ Exact discrete symmetry also protects the global ***PQ*** symmetry.

cf. In the exact gauged $\boldsymbol{Z_{10}}$ symmetric model,

$$\Delta\mathcal{L} = \frac{S^{10}}{M_{\rm PL}^6} + h.c.$$

→ a ***global and continuous PQ*** symmetry is well protected and leads to successful ***PQ*** mechanism.

✔ The gauged ***PQ*** mechanism converges to the discrete symmetry model when we take $\boldsymbol{\langle \phi(+10) \rangle = O(M_{PL})}$.

$$\Delta\mathcal{L} = \frac{\phi\phi'^{10}}{M_{\rm PL}^7} + h.c. \quad \longrightarrow \quad \Delta\mathcal{L} = \frac{\phi'^{10}}{M_{\rm PL}^6} + h.c.$$

$\boldsymbol{\langle \phi(+10) \rangle}$ breaks the gauge PQ symmetry down to gauged $\boldsymbol{Z_{10}}$ symmetry.

Remarks on exact gauged discrete symmetry

- In the discrete symmetry model, the axion domain wall problem is serious.

$$\mathcal{L}_{QCD} = \frac{g_s^2}{32\pi^2} N \frac{a}{f_a} F^a \tilde{F}^a$$

N should be the multiple of ***10*** due to the ***exact Z₁₀*** symmetry.

- The axion spontaneously breaks the symmetry below the ***QCD*** scale if the ***PQ*** symmetry takes place after inflation.

  → Stable domain wall dominates the universe below the ***QCD*** scale.

- If the ***PQ*** symmetry breaking takes place before inflation, no domain wall problem, but the ***axion iso-curvature constraints*** put stringent upper limit on the inflation scale.

$$H_{\rm inf} \lesssim 10^{-5} \times \frac{\Omega_{DM}}{\Omega_{\rm axion}} \times F_a$$

- The ***gauged PQ*** mechanism, on the other hand, can evade both the domain wall problem even for a large $\boldsymbol{H_{inf}}$.

Summary

- ***PQ*** axion models are one of the most successful solution to the strong ***CP*** problem.

- By definition, the ***PQ*** symmetry is quite puzzling…

- The ***gauged PQ*** mechanism provides a simple way to provide a well-protected ***global PQ*** symmetry.

- ***Gauged B-L*** symmetry can solve the strong ***CP*** problem in addition to providing a good framework for the seesaw mechanism and thermal leptogenesis !

Backup Slides

Radiative generation of the global PQ breaking terms ?

- We assume that the effective Lagrangian is given by local operators with a cutoff around the Planck scale, ***M_{PL}*** .

- By the ***gauged PQ*** symmetry, the ***global PQ*** symmetry is broken only by higher dimensional terms.

$$\mathcal{L} = \mathcal{L}_{PQ\,\mathrm{symmetric}} + \frac{\kappa_1}{M_{\mathrm{PL}}^7}\phi\phi'^{10} + \frac{\kappa_2}{M_{\mathrm{PL}}^{18}}\left(\phi\phi'^{10}\right)^2 + \cdots + h.c.$$

(c.f. a model with ***ϕ(+10)*** and ***ϕ' (-10)***)

[Here, we show operators relevant for the axion potential.]

- After performing path-integration, ***κ_2*** term can be, for example, overlaid by

$$\frac{\kappa_1^2}{M_{\mathrm{PL}}^{14} m^4}\left(\phi\phi'^{10}\right)^2$$

where m is some mass scale which couples to ***ϕ*** and ***ϕ'***.

(When ***m*** << < ***ϕ*** > or < ***ϕ'*** > , it means that the coupling is highly suppressed, and hence, it is good enough to think of ***m*** ~ < ***ϕ*** > or < ***ϕ'*** >)

Radiative generation of the global PQ breaking terms ?

- We assume that the effective Lagrangian is given by local operators with a cutoff around the Planck scale, M_{PL} .

- By the ***gauged PQ*** symmetry, the ***global PQ*** symmetry is broken only by higher dimensional terms.

$$\mathcal{L} = \mathcal{L}_{PQ\,\mathrm{symmetric}} + \frac{\kappa_1}{M_{\mathrm{PL}}^7}\phi\phi'^{10} + \frac{\kappa_2}{M_{\mathrm{PL}}^{18}}\left(\phi\phi'^{10}\right)^2 + \cdots + h.c.$$

(c.f. a model with $\boldsymbol{\phi(+10)}$ and $\boldsymbol{\phi'(-10)}$)

- ***The lowest dimensional breaking term*** is, on the other hand, not overlaid by

$$\frac{\kappa_1}{M_{\mathrm{PL}}^{7-n}m^n}\phi\phi'^{10} \qquad\qquad (n>0)$$

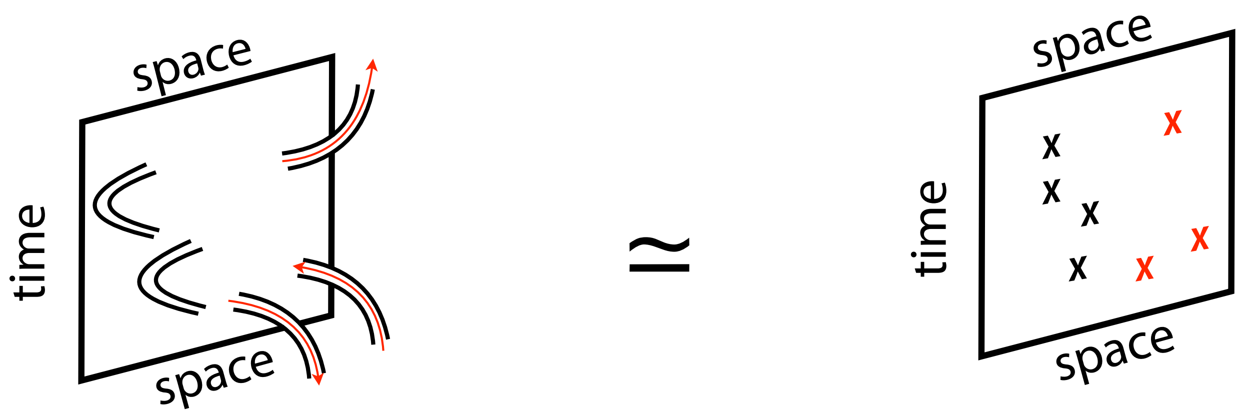
since the ***UV*** divergences are renormalized by local operators.

EX), seesaw $M_R N_R N_R$ (Dim3) —> $(LH)(LH)/M_R$ (Dim5)
but $M_R N_R N_R$ itself is not enhanced.

What is the origin of the Peccei-Quinn symmetry ?

- ✔ Why is the ***PQ*** symmetry broken only by the QCD anomaly?

The wormhole transitions may make things worse…



The charged particle under global symmetries can go through the wormhole leaving symmetry breaking terms

$$L = g_n\, \Phi(x)^n + h.c.$$

$$g_n \sim (8\pi M_{PL})^{4-n} \quad (\text{for a large } n)$$

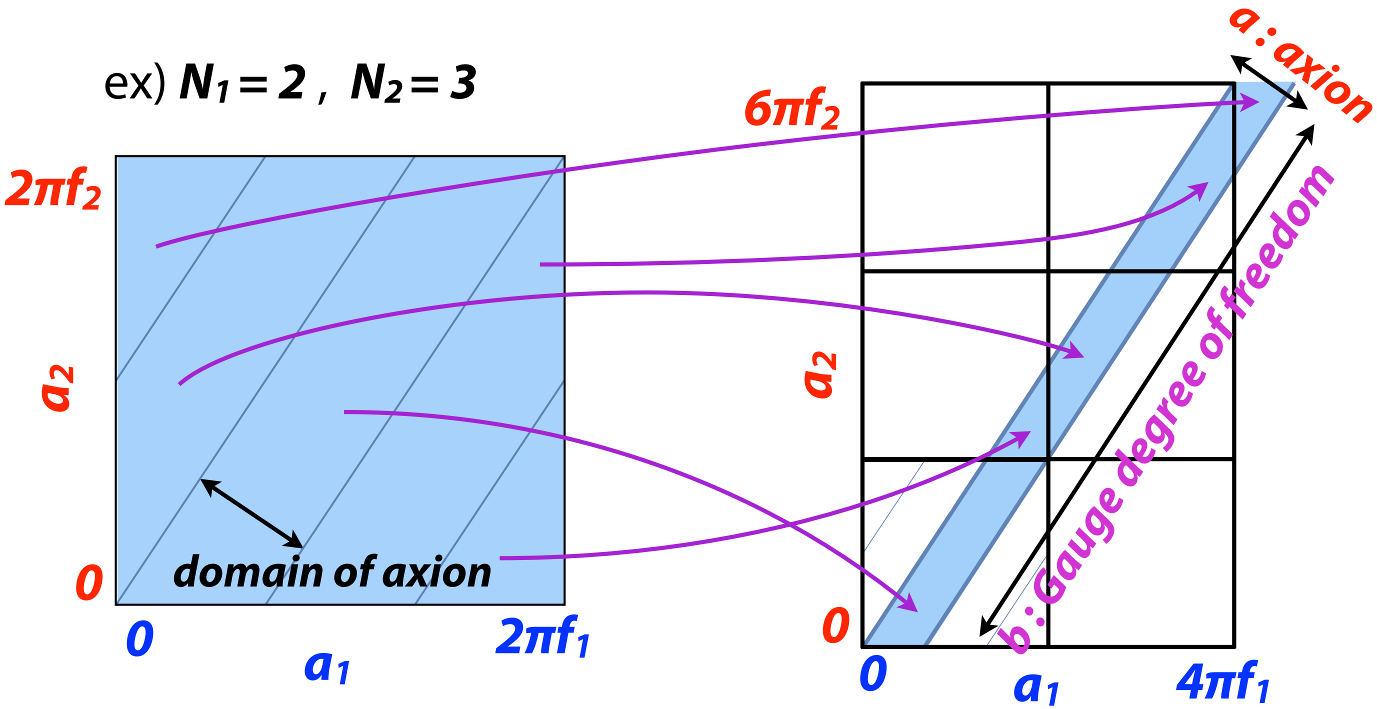
[1989 Abott, Wise, 1995 Kallosh, Linde, Linda, Susskind 2017 Aronso Urbano]

The existence of the global symmetries is quite unnatural.

General Recipe to Construct gauged PQ models

- Axion appears when both ***$U(1)_{PQ}$'s*** are spontaneously broken

ex) $N_1 = 2$, $N_2 = 3$



$$L = (\partial_\mu a)^2/2 + m_A^2 (A_\mu - \partial_\mu b/m_A)^2/2$$

$$[m_A^2 = g^2(q_1^2 f_1^2 + q_2^2 f_2^2)]$$

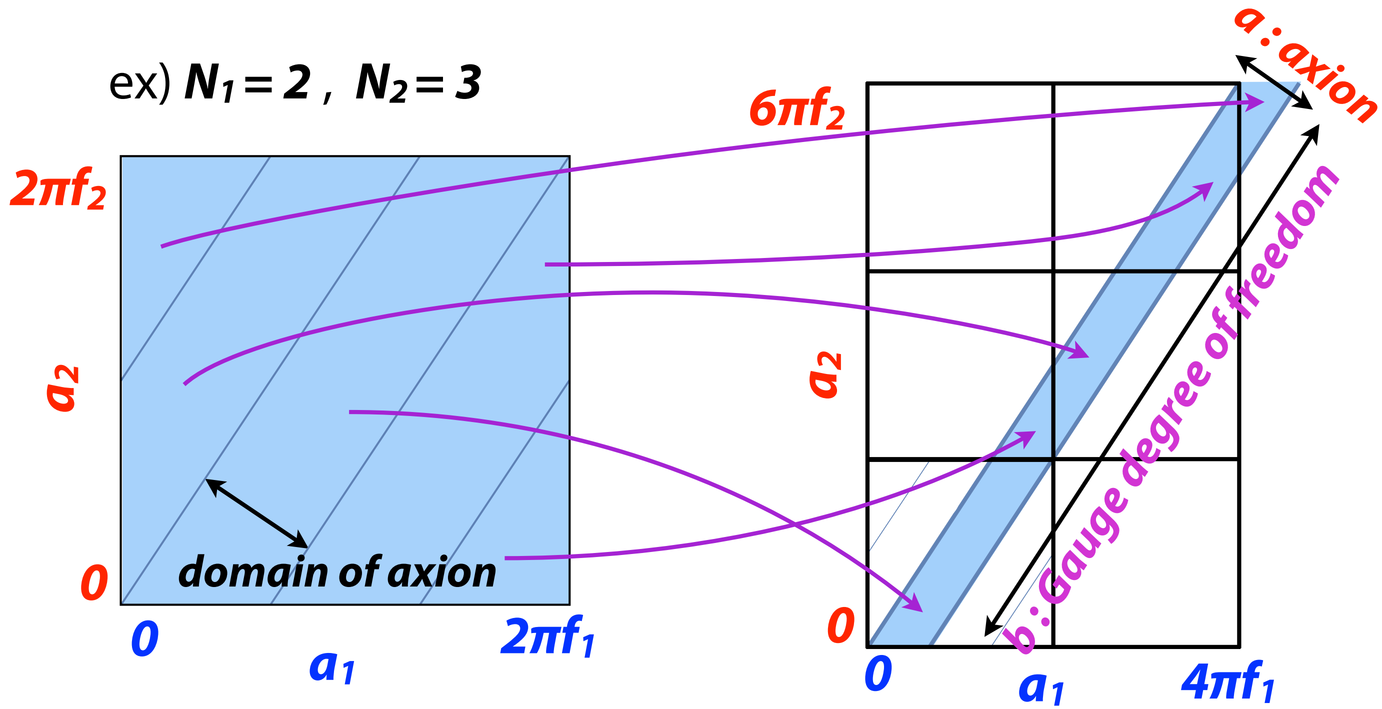
a : gauge invariant axion

b : would-be goldstone boson of gauged $U(1)_{PQ}$

General Recipe to Construct gauged PQ models

- Axion appears when both $U(1)_{PQ}$'s are spontaneously broken

ex) $N_1 = 2\ ,\ N_2 = 3$



Only the gauge invariant axion has an anomalous coupling

$$\mathcal{L}_{QCD} = \frac{g_s^2}{32\pi^2} n_{GCD} \left(\frac{q_2 a_1}{f_1} - \frac{q_1 a_2}{f_2} \right) F^a \tilde{F}^a \;=\; \frac{g_s^2}{32\pi^2} n_{GCD} \frac{a}{F_a} F^a \tilde{F}^a$$

$$\frac{a}{F_a} = [0, 2\pi) \qquad F_a = \frac{f_1 f_2}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}}$$

domain wall number

n_{GCD} : the greatest common divisor of N_1 and N_2 ($N_1 = n_{GCD}|q_2|$, $N2 = n_{GCD}|q_1|$)

Weak θ phase ?

✔ Why don't we care the weak $\boldsymbol{\theta}$ phase ?

$$\frac{\theta_W}{32\pi^2} W_{\mu\nu} W^{\mu\nu}$$

The weak $\boldsymbol{\theta_W}$ shifts by

$$\boldsymbol{\theta_W \to \theta_W + 6\,\alpha}$$

under the baryon rotation

$$\boldsymbol{Q_L \to e^{i\alpha} Q_L}$$

$$\boldsymbol{u_R, d_R \to e^{-i\alpha} u_R, d_R}$$

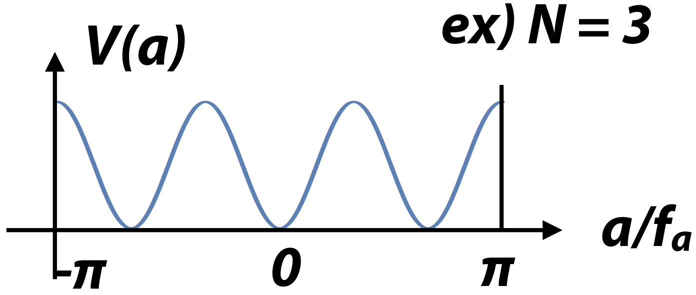
while other parameters in the theory intact.

→ There is no weak $\boldsymbol{\theta_W}$ problem.

Domain Wall Problem in conventional PQ models

- In the conventional ***PQ*** model, ***$U(1)_{PQ}$*** is explicitly broken down to ***Z_N*** symmetry by the ***QCD*** anomaly.

$$\mathcal{L}_{QCD} = \frac{g_s^2}{32\pi^2} N \frac{a}{f_a} F^a \tilde{F}^a$$



- ***Z_N*** is eventually broken spontaneously by the VEV of the axion .

→ Domain walls are formed when the axion gets VEV!

$$\rho_{DW} \sim \sigma \times H \propto T^2 \qquad (\sigma \sim f_a \Lambda_{QCD}^2)$$

[scaling solution 1990 Ryden, Press, Spergel]

Domain wall dominates over the energy density of the Universe for ***N>1*** !

What happens in the gauged ***PQ*** models ?

Domain Wall Problem in the conventional PQ model

Closer look at the domain wall problem :

(1) In the conventional ***PQ*** model, $U(1)_{PQ}$ is spontaneously broken at f_a.

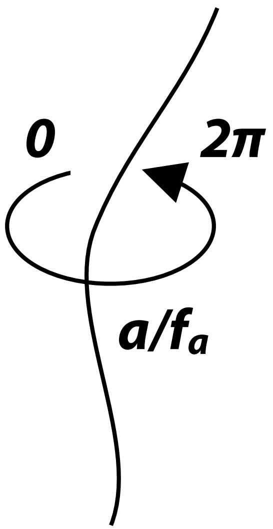
One global strings are formed in each Hubble volume in average.

Tension : $\mu^2 \sim 2\pi f_a^2 \log[f_a/H] \sim 2\pi f_a^2 \log[M_{PL}/f_a]$

Energy density of the strings do not cause problem due to its scaling nature:

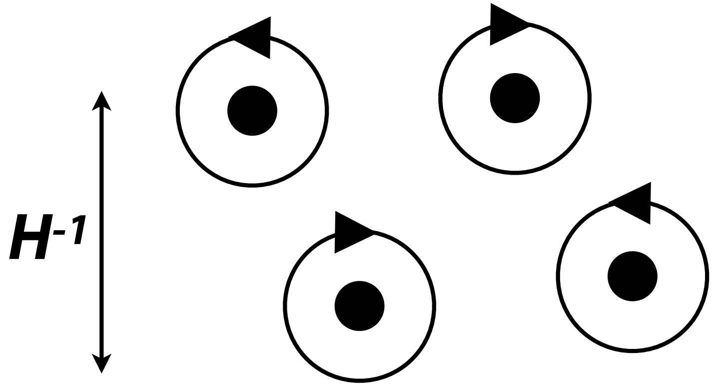
$\rho_{string} \sim \mu^2 H^2 \propto T^4$ [e.g. 1012.5502 Hiramatsu et.al.]

Around the cosmic strings, the axion field takes nontrivial configuration.



Winding number = 1

Top view of the strings

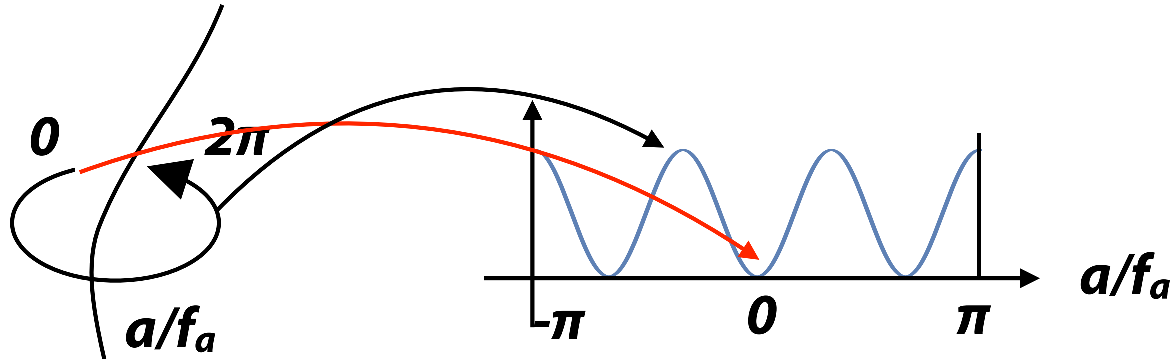


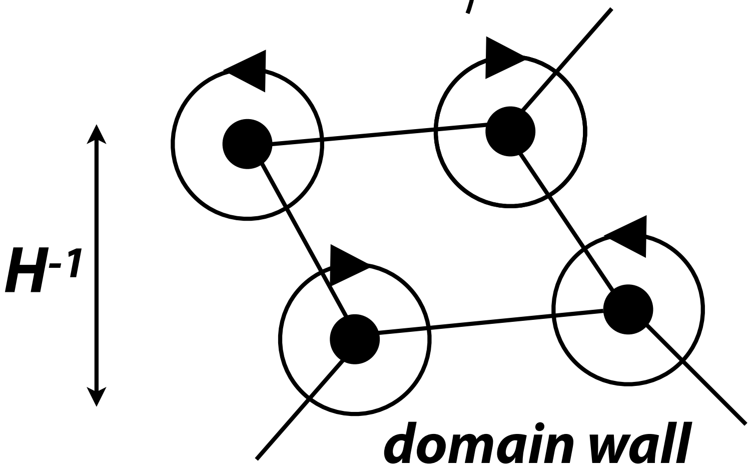
Domain Wall Problem in the conventional PQ model

Closer look at the domain wall problem :

(2) Below the QCD scale, the axion feels its axion potential.

Non-trivial axion field values around the strings causes non-uniformity of the energy density around the cosmic strings.





Non-uniform energy density is concentrated on the domain walls stretching between the cosmic strings

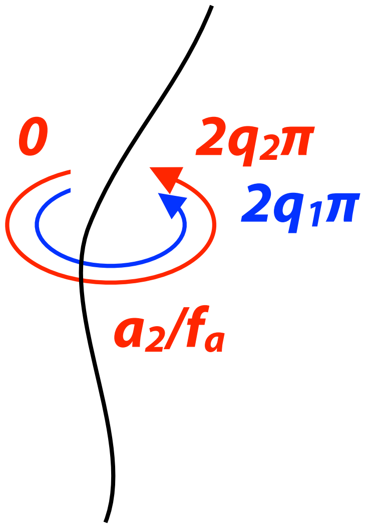
→ ***domain wall problems.***

[1012.4558 Hiramatsu et. al.]

Domain Wall Problem in the gauged PQ model

In the gauged ***PQ*** model, the genuine symmetry is not $\boldsymbol{Z_N}$ but $\boldsymbol{U(1)_{PQ}}$.

✔ Does it mean that there is no domain wall problem ?



Winding number $= (q_1, q_2)$

Around the local string, only the would-be goldstone mode winds, and hence, the axion is trivial.

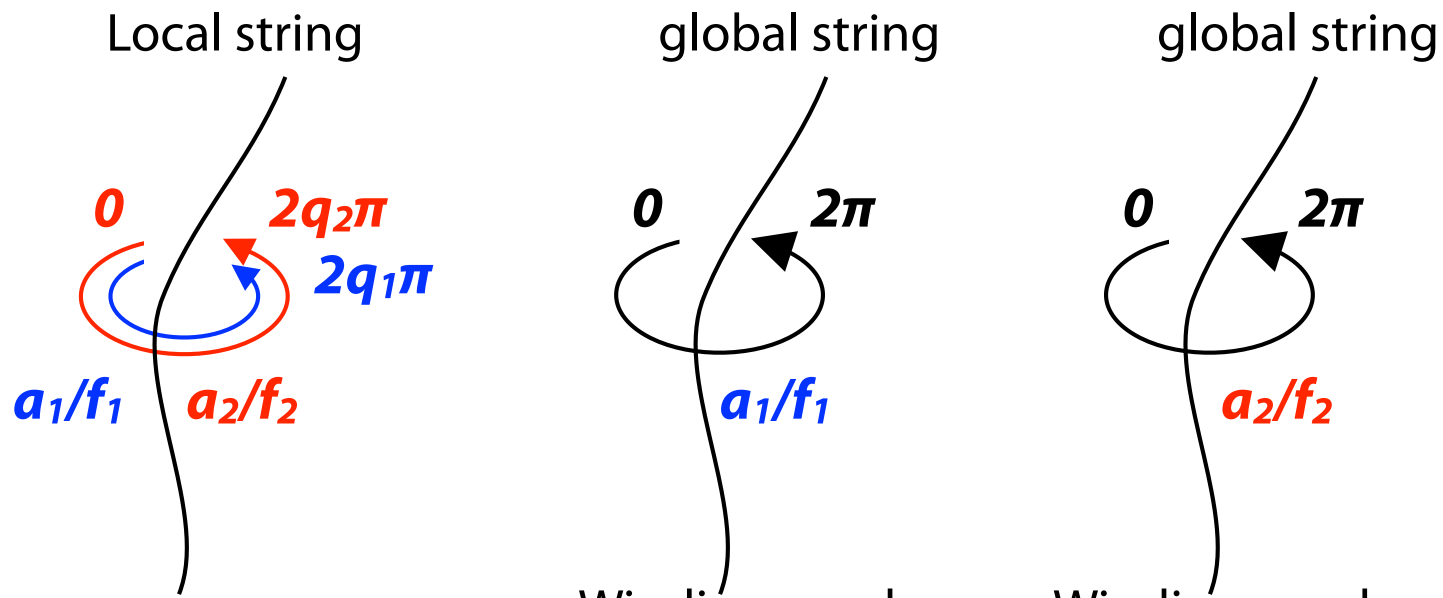
$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \begin{pmatrix} q_2 f_2 & -q_1 f_1 \\ q_1 f_1 & q_2 f_2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

→ Around the local string, ***no domain walls are formed*** even below the ***QCD*** scale !

Domain walls problems are solved … No Unfortunately…

Domain Wall Problem in the gauged PQ model

Even in the gauged PQ model, we could have global strings…



Winding number $= (q_1, q_2)$	Winding number $= (1, 0)$	Winding number $= (0, 1)$
$\delta a/F_a = 0$	$\delta a/F_a = q_2$	$\delta a/F_a = -q_1$

Once the global strings are formed in the universe, the domain walls are formed below the ***QCD*** scale unless global strings disappear which is unlikely due to the suppressed interaction between two sectors.

[The lifetime of the domain wall between the global strings are very long…]

Domain Wall Problem in the gauged PQ model

Cosmologically Safe Scenarios ?

(1) Trivial solution : ***PQ*** breaking before inflation.

✔ In this case, the axion field value is fixed to a single value and hence, no domain wall problem happens.

✔ Such scenarios predict the isocurvature fluctuation which is constrained by CMB observations.

$$\frac{\delta\rho_a}{\rho_a} \sim \frac{H_{\rm INF}}{\pi F_a} < 10^{-5} \times \frac{\Omega_{\rm DM} h^2}{\Omega_a h^2}$$ [isocurvature constraint]

$$\Omega_a h^2 = 0.18\ \theta_1^2 \left(\frac{F_a}{10^{12}{\rm GeV}}\right)^{1.19} \left(\frac{\Lambda}{400{\rm MeV}}\right)$$

Initial angle
θ_1 = 0-2π

$$\Omega_{\rm DM} h^2 \simeq 0.12$$ [e.g. 1301.1123 Kawasaki, Nakayama]

This scenario requires rather low scale inflation : e.g. $\boldsymbol{H_{INF} \sim 10^7 GeV}$ for $\boldsymbol{\Omega_a = \Omega_{DM}}$.

Domain Wall Problem in the gauged PQ model

Cosmologically Safe Scenarios ?

(2) Less trivial solution : ***PQ*** breaking before inflation.

Bring two independent ***KSVZ*** axion models

$$L = y_1 S_1\, q_{1L}\, \bar{q}_{1R} + y_2 S_2\, q_{2L}\, \bar{q}_{2R} + h.c.$$

KSVZ fermions : N_1 flavor of ***1*** , N_2 flavor of ***9***

Gauged $U(1)_{PQ}$ ***charge :*** $S_1(9)$, $S_2(-1)$

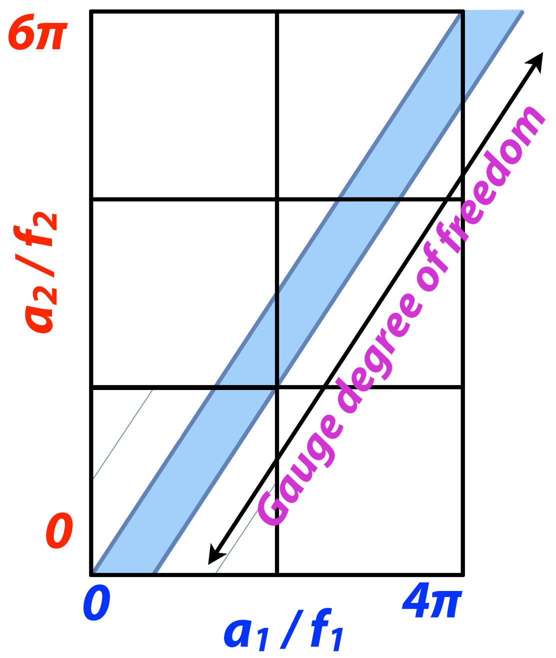
S_1-global string = axion winding number 1

S_2-global string = axion winding number 9

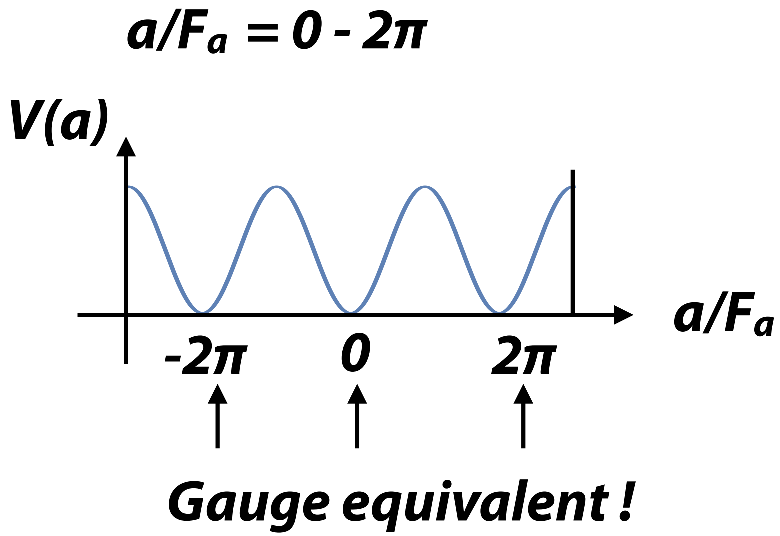
Arrange $\langle S_2 \rangle \gg T_R$ so that the gauged $U(1)_{PQ}$ is not restored after inflation while the global $U(1)_{PQ}$ breaking takes place after inflation.

- ✓ All the non-trivial axion winding strings are inflated away.
- ✓ No isocurvature fluctuation is generated since there is no massless mode during inflation.

Domain Wall Problem in the gauged PQ model



For a model with $\boldsymbol{n_{GCD}}$ **= 1**, the bottoms of the axion potential are gauge equivalent !



There is no absolutely stable domain wall !

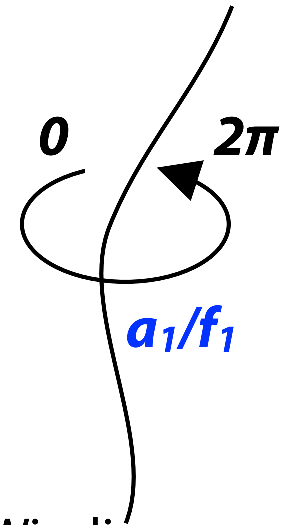
Practically, however, we have domain wall problem…

Domain Wall Problem in the gauged PQ model

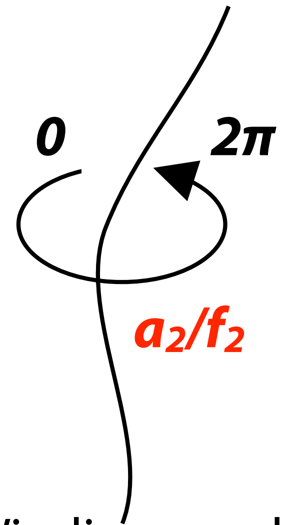
ex) ***Gauged $U(1)_{PQ}$ charge : $S_1(3)$, $S_2(-1)$***

S_1-global string = axion winding number 1

S_2-global string = axion winding number 3

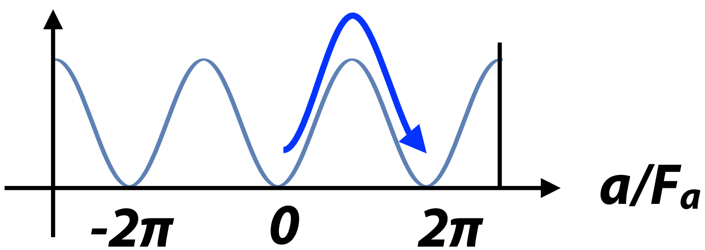


Winding number
= (1, 0)

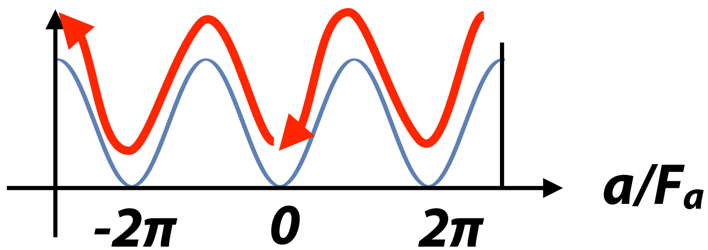


Winding number
= (0, 1)

$\delta a/F_a = 1$



$\delta a/F_a = -3$



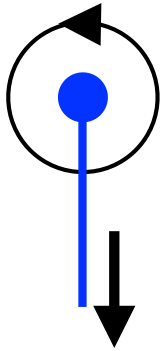
Domain Wall Problem in the gauged PQ model

ex) ***Gauged $U(1)_{PQ}$ charge :*** ***$S_1(3)$*** , ***$S_2(-1)$***

Below the ***QCD*** scale, the non-uniformity of the energy density leads to the domain wall formation.

S_1-domain wall

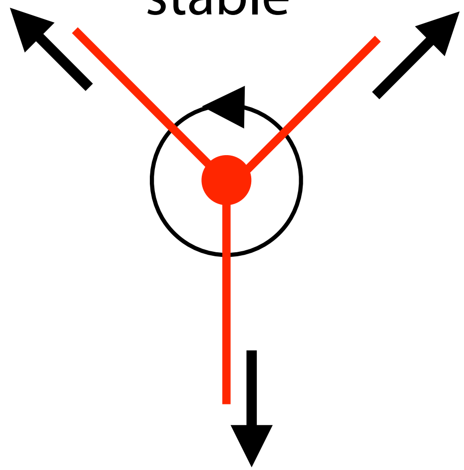
unstable



Pulled by other strings
in another Hubble patch

S_2-domain wall

stable



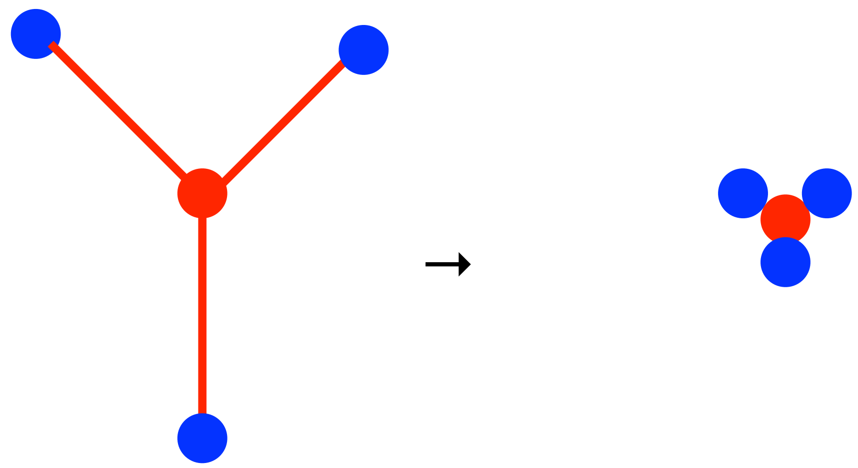
Pulled by other strings
in another Hubble patch

Since the domain wall formation is at very low energy, the walls do not know whether there were gauge bosons!

Domain Wall Problem in the gauged PQ model

ex) ***Gauged $U(1)_{PQ}$ charge : $S_1(3)$, $S_2(-1)$***

Some lucky domain walls can annihilate into composite strings

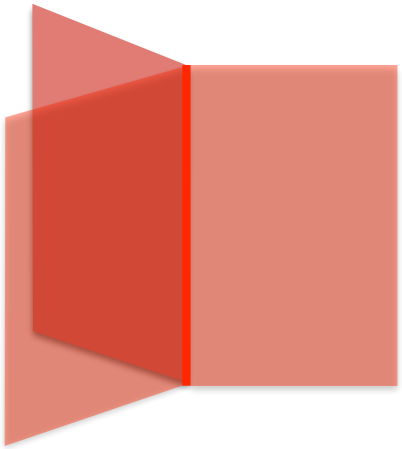


It is however difficult to imagine that all the walls annihilates away successfully, since the strings are typically separated by the Hubble length…

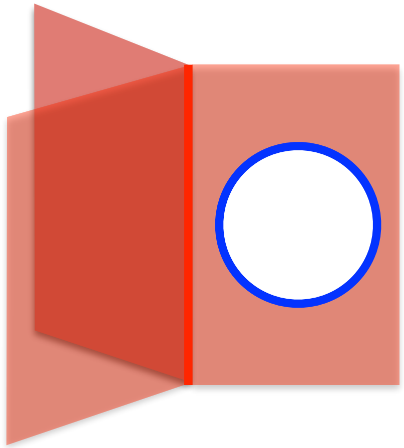
Domain Wall Problem in the gauged PQ model

ex) ***Gauged $U(1)_{PQ}$ charge : $S_1(3)$, $S_2(-1)$***

S_2 domain wall can be pierced by the S_1 string.



→



The tunneling rate is quite low…

$$\Gamma \propto Exp[\,-F_a^3/\Lambda_{QCD}^2\,T\,] \sim Exp[-10^{10}]$$

[1982, Kibble, Lazarides, Shafi]

The domain walls are almost stable…