



The Path to Predict the Axion Mass with large String Tension

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Klaer and Moore, [arXiv:1708.07521](https://arxiv.org/abs/1708.07521) and [arXiv:1707.05566](https://arxiv.org/abs/1707.05566)

The Axion Quest!

Dark matter is still a mystery, it is

- ▶ **matter** : makes up 25% of the density of the Universe
- ▶ **dark** : interaction is feeble (except gravitationally)
- ▶ **cold** : almost pressureless

The Axion could be a likely candidate, would you like to investigate the Axion?

Yes

No



The Axion Quest!

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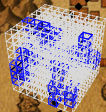
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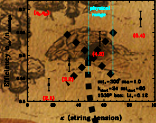
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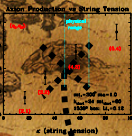
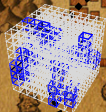
$$L_a = \partial^\mu \phi^* \partial_\mu \phi + \frac{\lambda}{8} (2\phi^* \phi - f^2)^2 + \chi(T) \text{Re} \phi$$

Action Production of String Tension



$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2 + \frac{\lambda}{8} (2\phi_1^* \phi_1 - f^2 + 2\phi_2^* \phi_2 - f^2)^2 + |\partial_\mu - i q_1 e A_\mu \phi_1|^2 + |\partial_\mu - i q_2 e A_\mu \phi_2|^2$$

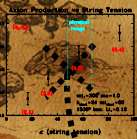
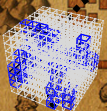




$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\mu - \partial_\nu A_\mu)^2 + \frac{\lambda}{8} [(2\phi_1^2 - f^2) + (2\phi_2^2 - f^2)] + \frac{1}{2} (\partial_\mu - ieA_\mu)\phi_1^2 + \frac{1}{2} (\partial_\mu - iqeA_\mu)\phi_2^2$$

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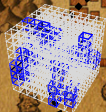




$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2 + \frac{\lambda}{8} [(2\phi_1^2 \phi_2 - \phi_2^3) - (2\phi_2^2 \phi_1 - \phi_1^3)] + \frac{1}{2} [(\partial_\mu - i g e A_\mu) \phi_1]^2 + [(\partial_\mu - i q z e A_\mu) \phi_2]^2$$

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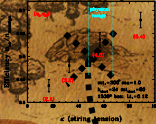




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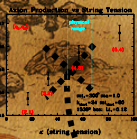
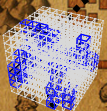


Action Production of String Tension



$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2 + \frac{\lambda}{8} [(2\phi_1^* \phi_1 - f^2) + (2\phi_2^* \phi_2 - f^2)] + |\partial_\mu - i q_1 e A_\mu \phi_1|^2 + |\partial_\mu - i q_2 e A_\mu \phi_2|^2$$

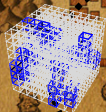




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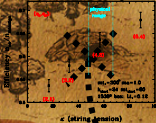
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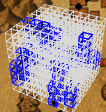
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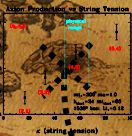


$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2 + \frac{\lambda}{8} (2\phi_1^a \phi_1^a - f^2 + 2\phi_2^a \phi_2^a - f^2)^2 + \frac{1}{2} (\partial_\mu - i g_e A_\mu) \phi_1^a \phi_1^a + \frac{1}{2} (\partial_\mu - i g_e A_\mu) \phi_2^a \phi_2^a$$



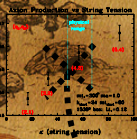
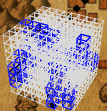


$$L_a = \partial^\mu \phi^a \partial_\mu \left[\frac{\lambda}{8} (2\phi^a \phi^a - f^2)^2 + \chi(T) \text{Re}\phi \right]$$



$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{\lambda}{8} [(2\phi_1^2 \phi_2^2 - f^4) + (2\phi_2^2 \phi_1^2 - f^4)] + \frac{1}{2} [(\partial_\mu - i q_1 e A_\mu) \phi_1]^2 + [(\partial_\mu - i q_2 e A_\mu) \phi_2]^2$$

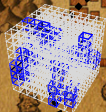




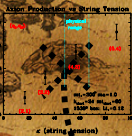
$$L(\phi_1, \phi_2, A_{\phi}) = \frac{1}{4} (\partial_t A_{\phi_1} - \partial_t A_{\phi_2})^2 + \frac{\lambda}{8} [(2\phi_1^2 \phi_2 - \phi_1^3) + (2\phi_2^2 \phi_1 - \phi_2^3)] + \frac{1}{2} (\partial_{\mu} - q_1 e A_{\mu}) \phi_1^2 + \frac{1}{2} (\partial_{\mu} - i q_2 e A_{\mu}) \phi_2^2$$

$$L_a = \partial^{\mu} \phi^{\nu} \partial_{\mu} \phi_{\nu} - \frac{1}{2} (\partial^{\mu} \phi - \dot{\phi})^2 + \chi(T) \text{Re} \phi$$





$$L_a = \partial^\mu \phi^a \partial_\mu \phi^b + \lambda (2\phi^a \phi^b - f^2) + \chi(T) \text{Re} \phi$$



$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{\lambda}{8} [(2\phi_1^* \phi_1 - f^2) + (2\phi_2^* \phi_2 - f^2)] + |\partial_\mu - i q_1 e A_\mu \phi_1|^2 + |\partial_\mu - i q_2 e A_\mu \phi_2|^2$$

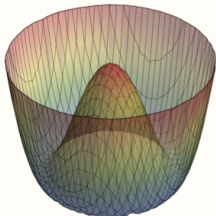


The Axion Lagrangian

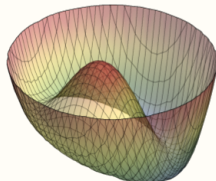
You found a Lagrangian

$$\mathcal{L}_a = \partial^\mu \phi^* \partial_\mu \phi + \frac{\lambda}{8} (2\phi^* \phi - f_a^2)^2 + \chi(T) \text{Re}\phi$$

$$T_{PQ} > T > T_C$$



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Would you like to take this Lagrangian to describe your Axion dynamics?

Yes

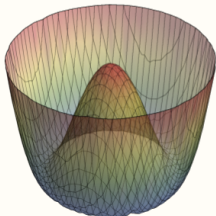
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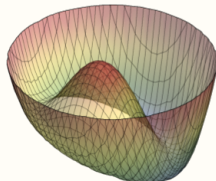
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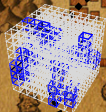
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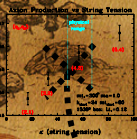
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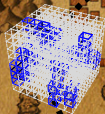


$$L_a = \partial^\mu \phi \frac{\lambda}{8} (2\phi^* \psi - \psi^* \phi)^2 + \chi(T) \text{Re} \phi$$

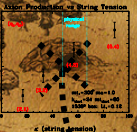


$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2 + \frac{\lambda}{8} [(2\phi_1^* \phi_1 - \phi_2^* \phi_2 - f^2)]^2 + |\partial_\mu - i q e A_\mu \phi_1|^2 + |\partial_\mu - i q e A_\mu \phi_2|^2$$



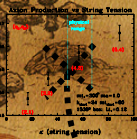
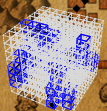


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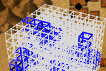




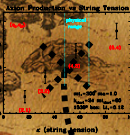
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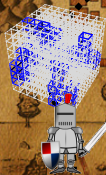


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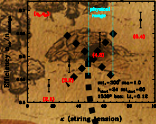
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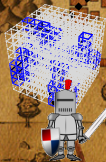
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Action Production of String Tension

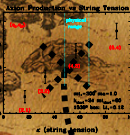


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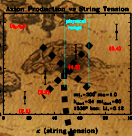
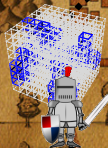


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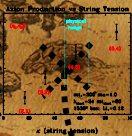
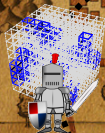




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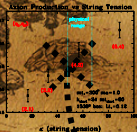




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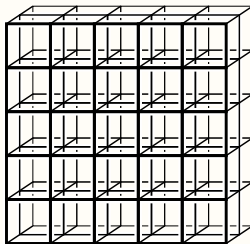
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Axion Cosmology solving on Lattice

You found a tool to calculate the EQM

- ▶ Put \mathcal{L}_a as classical field theory on **lattice**
- ▶ Simulation starts **after Inflation**
- ▶ ϕ is a complex field
- ▶ θ is chosen randomly
- ▶ Topological defects will arise
- ▶ Simulate with **Hubble drag**
- ▶ Count axions at the end



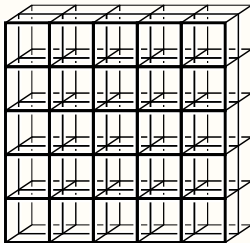
Solve it by hand

Use a computer

Axion Cosmology solving on Lattice

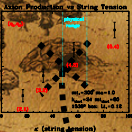
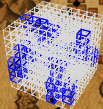
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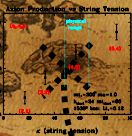
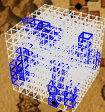
Use a computer



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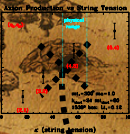
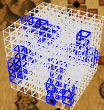
$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

$$+ \frac{\lambda}{8} [(2\phi_1^* \phi_1 - f^2) + (2\phi_2^* \phi_2 - f^2)]$$

$$+ |\partial_\mu - i q e A_\mu \phi_1|^2 + |\partial_\mu - i q e A_\mu \phi_2|^2$$

$$L_a = \partial^\mu \phi^* \partial_\mu \phi + \frac{\lambda}{8} (2\phi^* \phi - f^2)^2 + \chi(T) \text{Re} \phi$$

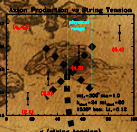
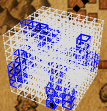




$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{\lambda}{8} [(2\phi_1^2 \phi_2 - \phi_1^2) + (2\phi_2^2 \phi_1 - \phi_2^2)] + \frac{1}{2} [(\partial_\mu - i q e A_\mu) \phi_1]^2 + [(\partial_\mu - i q e A_\mu) \phi_2]^2$$

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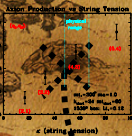
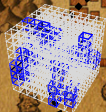




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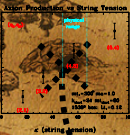
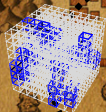




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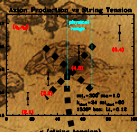
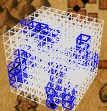




$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\mu - \partial_\nu A_\mu)^2 + \frac{\lambda}{8} [(2\phi_1^2 - r^2) + (2\phi_2^2 - r^2)] + \frac{1}{2} (g_1 e A_\mu \phi_1)^2 + \frac{1}{2} (g_2 e A_\mu \phi_2)^2$$

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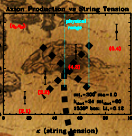
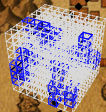




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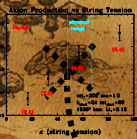
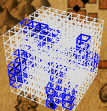




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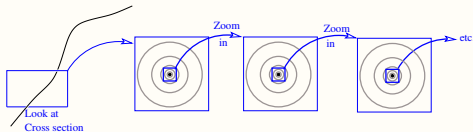
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Physical range not achievable!

$$E_{str} = \iiint r dr dz d\phi (\nabla\phi^* \nabla\phi \simeq f_a^2/2r^2) \simeq \pi l f_a^2 \int_{\sim f_a^{-1}}^{\sim H^{-1}} \frac{r dr}{r^2}$$



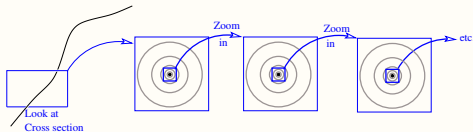
- ▶ **Log-large** string tension $T_{str} = \pi f_a^2 \ln(f_a/H)$
- ▶ $f_a/H \simeq 10^{30}$
- ▶ Equal energy in each x2 scale
- ▶ Scale range is $10^{30} \rightarrow \kappa = \ln(10^{30}) \simeq 70$, **achievable** is $\kappa = 6$

Find new methodes

Quit job

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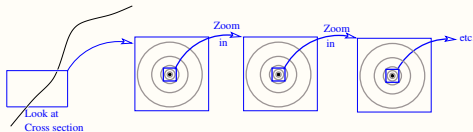
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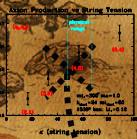
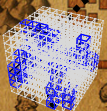
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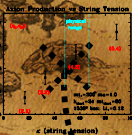
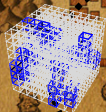
Quit job



$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\mu - \partial_\nu A_\mu)^2 + \frac{\lambda}{8} [(2\phi_1^2 - r^2) + (2\phi_2^2 - r^2)] + \frac{1}{2} [(\partial_\mu - ieA_\mu)\phi_1]^2 + [(\partial_\mu - iqeA_\mu)\phi_2]^2$$

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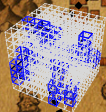


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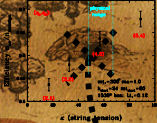
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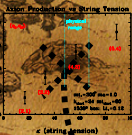
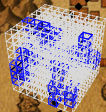
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Action Production of String Tension



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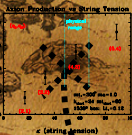
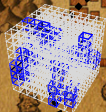




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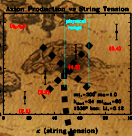
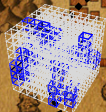




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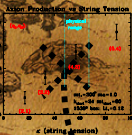
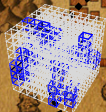
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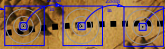
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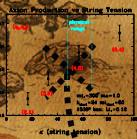
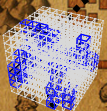




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$$+ \frac{\lambda}{8} [(2\phi^* \psi - 1)^2 + \psi^* \psi]$$

$$+ \frac{1}{2} (\partial_\mu - ieA_\mu)\phi \cdot (\partial_\mu - ieA_\mu)\psi^2$$



$$L = \partial^\mu \phi^* \partial_\mu \psi + \frac{\lambda}{8} [(2\phi^* \psi - 1)^2 + \psi^* \psi] + \chi(T) \text{Re} \phi$$



Use a effective theory

Found an effective Lagrangian using two complex scalars and A_μ

$$\begin{aligned}\mathcal{L}(\phi_1, \phi_2, A_\mu) = & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ & + \frac{\lambda}{8}[(2\phi_1^* \phi_1 - f^2) + (2\phi_2^* \phi_2 - f^2)] \\ & + |(\partial_\mu - iq_1 e A_\mu)\phi_1|^2 + |(\partial_\mu - iq_2 e A_\mu)\phi_2|^2\end{aligned}$$

- ▶ Pick $q_1 \neq q_2$
- ▶ $T \simeq \pi f_a^2 (\kappa_{\text{eff}} + \kappa)$ with $\kappa_{\text{eff}} \simeq 2 (q_1^2 + q_2^2)$
- ▶ $q_1 \theta_1 + q_2 \theta_2$ gauged, $q_2 \theta_1 - q_1 \theta_2$ global

Use new methode

Do nothing

Use a effective theory

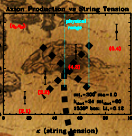
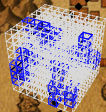
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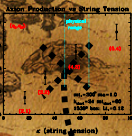
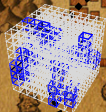
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$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\mu A_\nu - A_\mu \partial_\nu)^2 + \frac{\lambda}{8} (2\phi_1^2 - r^2)^2 + \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2 - i q e A_\mu \phi_2)^2$$

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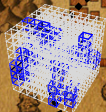




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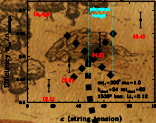
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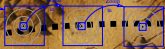


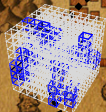
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Action Production of String Tension



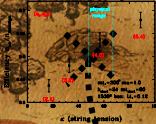
$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\mu - \partial_\nu A_\mu)^2 + \frac{\lambda}{8} (2\phi_1^* \phi_1 - f^2 + 2\phi_2^* \phi_2 - f^2)^2 + |\partial_\mu - i q_1 e A_\mu \phi_1|^2 + |\partial_\mu - i q_2 e A_\mu \phi_2|^2$$





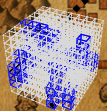
$$L_a = \partial^\mu \phi^* \partial_\mu \phi + \frac{\lambda}{8} (2\phi^* \phi - r^2)^2 + \chi(T) \text{Re} \phi$$

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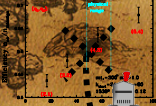


$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2 + \frac{\lambda}{8} (2\phi_1^* \phi_1 - r^2 + 2\phi_2^* \phi_2 - r^2)^2 + |\partial_\mu - i q_1 e A_\mu \phi_1|^2 + |\partial_\mu - i q_2 e A_\mu \phi_2|^2$$





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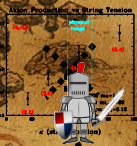
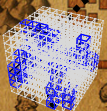
$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2$$

$$+ \frac{\lambda}{8} [(2\phi_1^* \phi_1 - f^2) + (2\phi_2^* \phi_2 - f^2)]$$

$$+ |\partial_\mu - ieA_\mu \phi_1|^2 + |\partial_\mu - iqeA_\mu \phi_2|^2$$

$$L = \partial^\mu \phi^* \partial_\mu \phi + \frac{\lambda}{8} (2\phi^* \phi - f^2)^2 + \chi(T) \text{Re} \phi$$



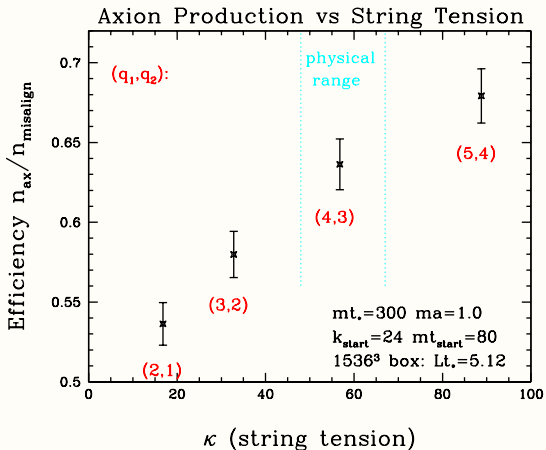


$$L(\phi_1, \phi_2, A_\mu) = \frac{1}{4} (\partial_\nu A_\nu - \partial_\nu A_\nu)^2 + \frac{\lambda}{8} [(2\phi_1^2 - r^2) + (2\phi_2^2 - r^2)] + \frac{1}{2} (\partial_\mu - ieA_\mu)\phi_1^2 + \frac{1}{2} (\partial_\mu - iqeA_\mu)\phi_2^2$$

$$L = \partial^\mu \phi^* \partial_\mu \phi + \frac{\lambda}{8} (2\phi^* \phi - r^2)^2 + \chi(T) \text{Re} \phi$$



Congratulation you did it!



- ▶ Axion production changes **only a little**
- ▶ $f_a = (2.21 \pm 0.29) \times 10^{11}$ GeV
- ▶ $m_a = 26.2 \pm 3.4 \mu\text{eV}$

- ▶ Axion can explain the **CP-problem** and the **dark matter**
- ▶ In early Universe the Axion dynamics are **string defects**
- ▶ With the **two-field-model** we get the string defect physics right
- ▶ We find **Axion mass** $m_a = 26.2 \pm 3.4 \mu\text{eV}$
 - ▶ Assuming Axions make all DM
 - ▶ θ is chosen randomly

Thank you for your Attention!