Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

Higgs Production with Full Quark-Mass dependence

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Institute for Theoretical Particle Physics and Cosmology

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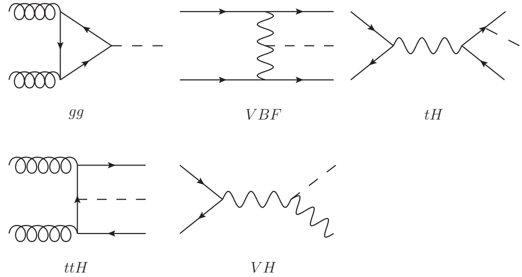
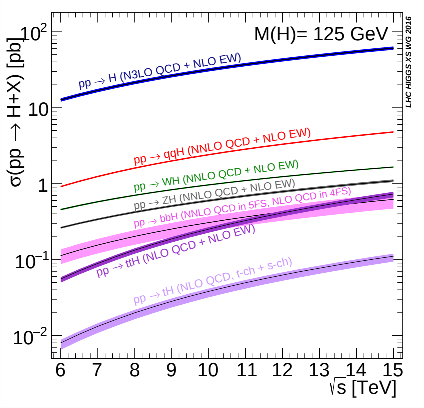
Higgs Production

Motivation

- With the discovery of the Higgs in 2012 we have entered a new era of precision physics.
- We need to know properties of the Higgs very accurately to be able to search for new Physics.
- An important observable here is the Higgs **production cross section**.

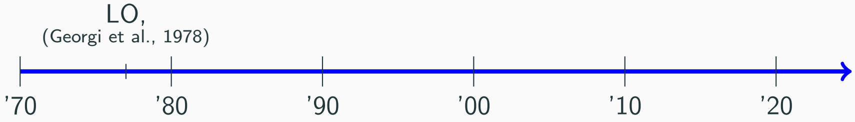
- The gluon fusion channel is the most dominant production channel. It is therefore the channel we must determine most precisely.





(Florian et al., 1610.07922)

The Gluon Fusion Channel



	LO	NLO(HEFT)	NLO	NNLO(HEFT)	N3LO(HEFT)	NNLO
$\sigma(13\,\text{TeV})$ [pb]	16.30					

(Czakon et al., 2105.04436),(Anastasiou et al., 1602.00695)

- Very large loop corrections and quite small mass corrections

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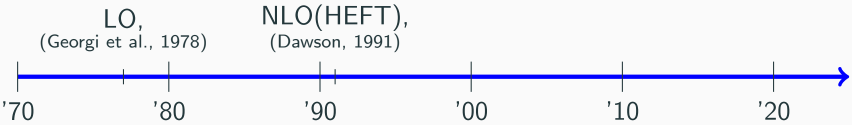

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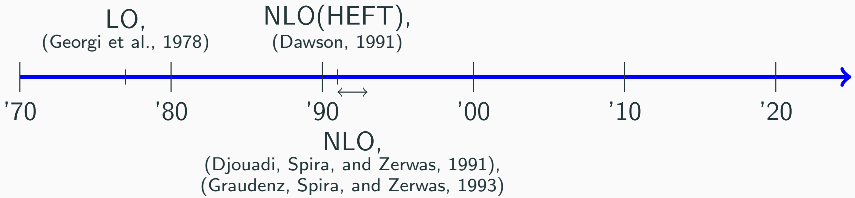

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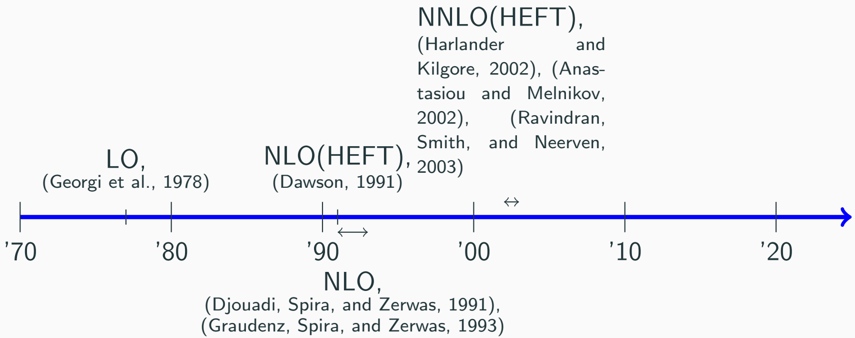

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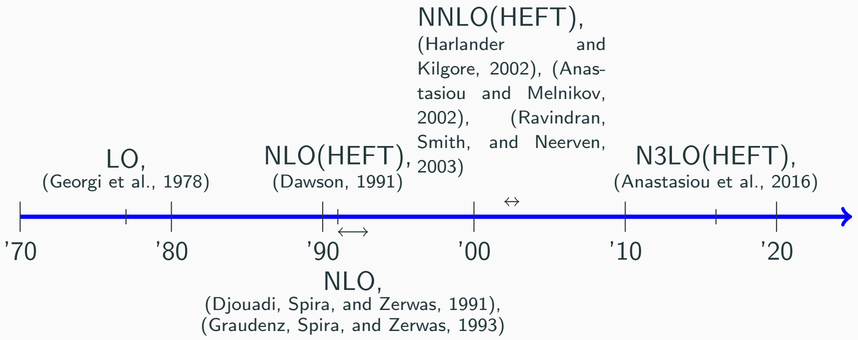

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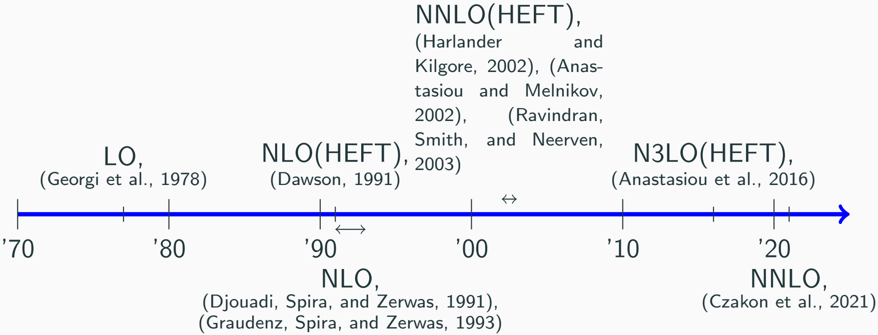

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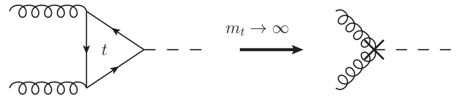
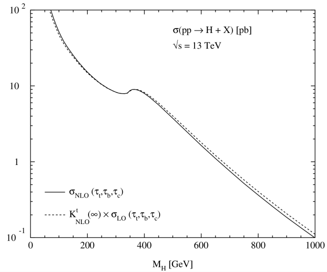
Higgs Effective Field Theory (HEFT)

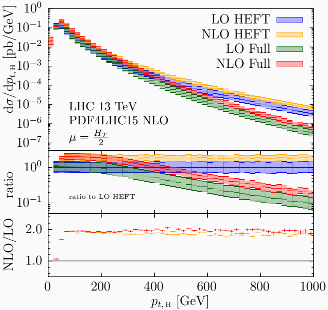
- Typically the top-quark is integrated out: Heavy Top Limit (HTL)



- Number of loops is reduced by one
- HEFT works remarkably well for gluon fusion given that the approximation $\frac{m_H^2}{m_t^2} = 0.52 \ll 1$ is rather bad
- Qualitative explanation: suppression of large-s region by the PDFs



(Spira, 1612.07651)

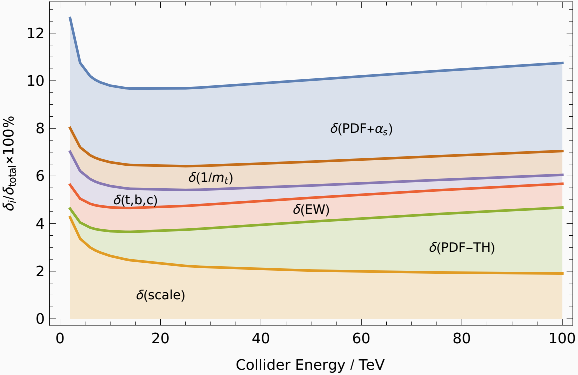


(Jones, Kerner, and Luisoni, 1802.00349)

The Gluon Fusion Channel

- Current state of the art for gluon fusion in HEFT is **N3LO** (Anastasiou et al., 1602.00695).

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}(+4.56\%)}_{-3.27\,\text{pb}(-6.72\%)}\,(\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\,(\text{PDF} + \alpha_s)$$



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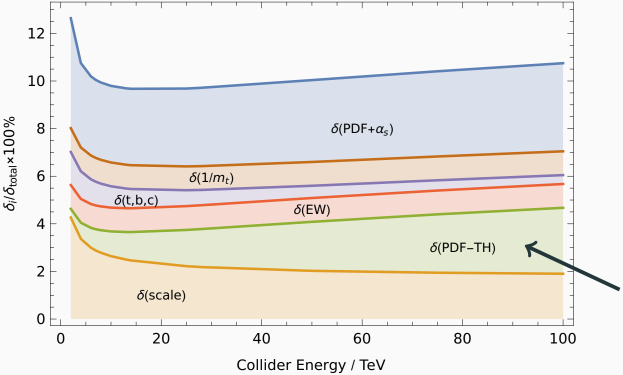

PDFs and their evolution only known at NNLO

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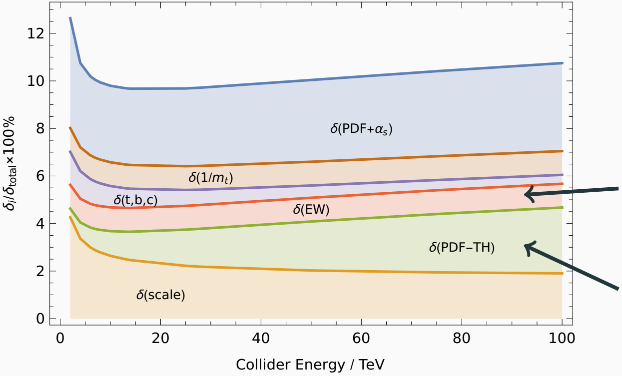

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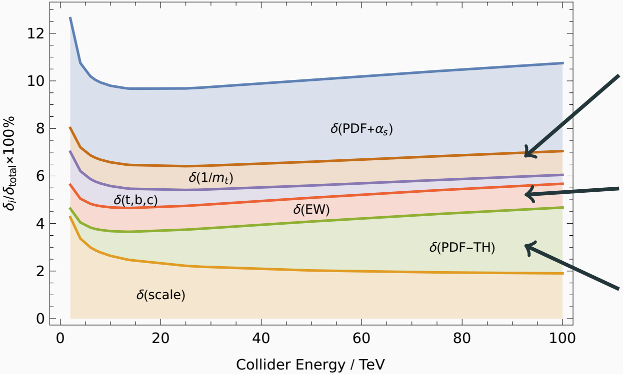

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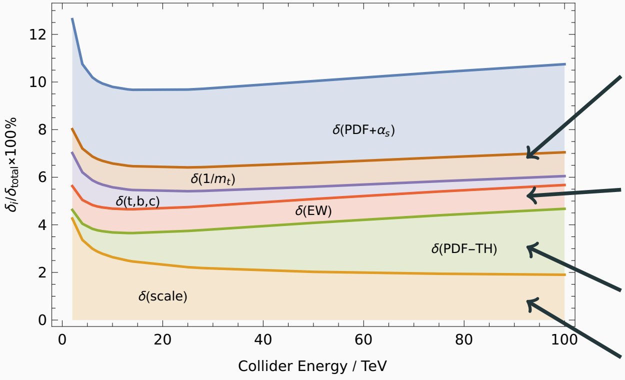

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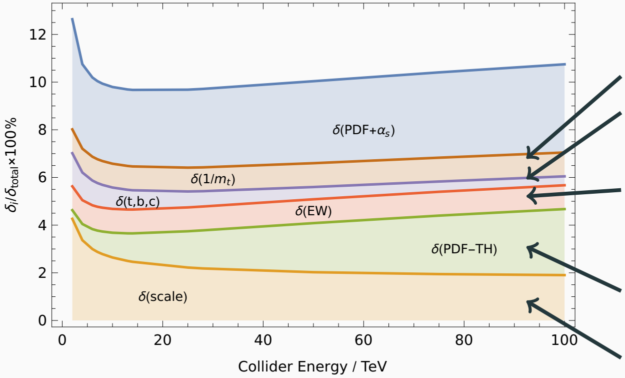

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Our Goal!

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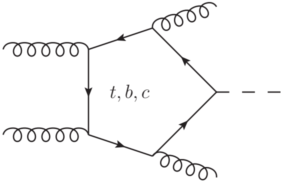
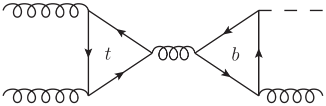
Computational Details

Ingredients

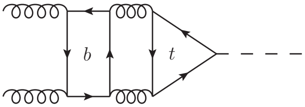
- Double-real corrections



- 2-loop real-virtual corrections

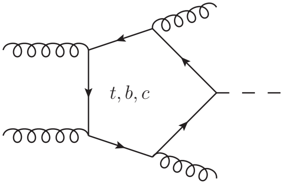


- 3-loop virtual corrections

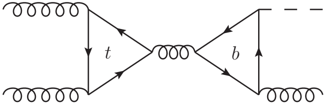


Ingredients

- Double-real corrections
    - Easy, since one-loop calculation. We use MCFM (Budge et al., 2002.04018). But computationally most expensive.



- 2-loop real-virtual corrections



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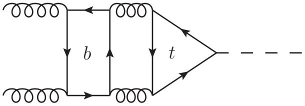

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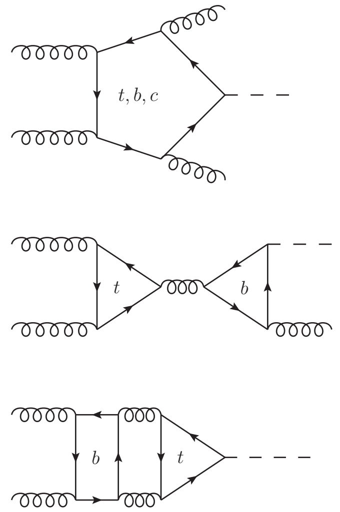
  - Same diagrams as in the t-quark case (Czakon et al., 2105.04436) apart from mixed contributions, which are however factorizable.
  - Integrals with b-quarks now exhibit additional poles.
- 3-loop virtual corrections



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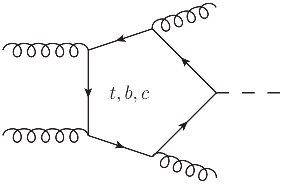
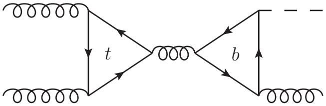
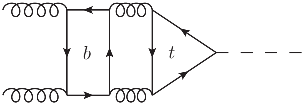
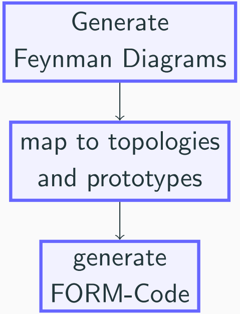
- 3-loop virtual corrections
  - Contains truly new contributions that need to be computed







Amplitude calculation



Generate Feynman Diagrams

map to topologies and prototypes

generate FORM-Code

DiaGen

(Czakon, unpublished)

Amplitude calculation

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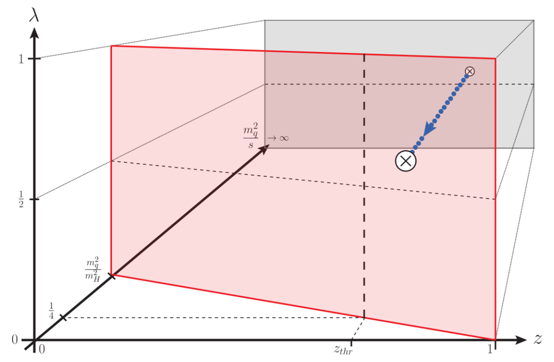
- Solve 3-Point and 4-Point integrals numerically with differential equations

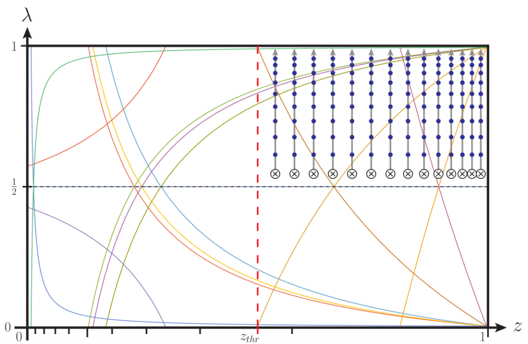


(Niggetiedt, PhD thesis)

$$z = 1 - \frac{m_H^2}{s}, \quad \lambda = \frac{t}{t+u}$$

Solving the Master Integrals

- Afterwards solve differential equation in λ to map out points in λ, z plane.
- Poles of the differential equation (thin lines) are avoided with complex contour



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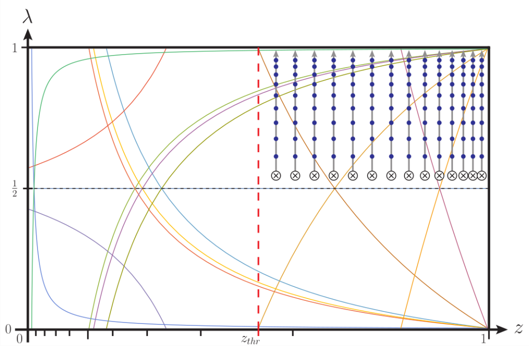

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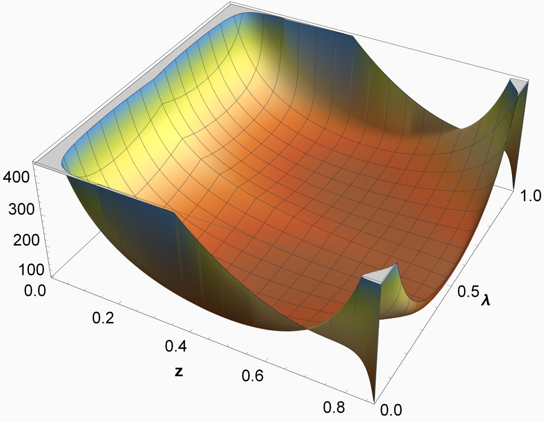
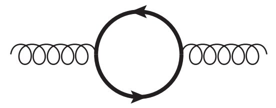
- This way the shape of the amplitude can be mapped out and used to compute the cross section



(Czakon et al., 2105.04436)

Renormalization

- We work in the **5 flavor scheme**.
- The quark masses are renormalized in the **On-shell scheme**.
- The field renormalization constants contain heavy quark contributions, while the LSZ constants do not, therefore the heavy contributions must be considered extra



$$a_s^b = \frac{\alpha_s^b}{2\pi} \sum_i \left(\frac{\mu^2}{m_i^2}\right)^\epsilon (4\pi)^\epsilon \Gamma(1+\epsilon)$$

$$Z_3^{\text{OS}} = a_s^b T_F n_h \left(-\frac{2}{3\epsilon}\right) + \left(a_s^b\right)^2 n_h T_F \left(n_h T_F \left(\frac{4}{9\epsilon^2}\right) + C_F \frac{4\epsilon^3 - 7\epsilon - 1}{\epsilon(4\epsilon^3 - 8\epsilon^2 - \epsilon + 2)} + C_A \frac{-4\epsilon^5 + 15\epsilon^3 + \epsilon^2 - 11\epsilon - 3}{2\epsilon^2(4\epsilon^4 - 4\epsilon^3 - 13\epsilon^2 + 7\epsilon + 6)}\right)$$

(Czakon, Mitov, and Moch, 0707.4139)

- Furthermore, we need Z_m, Z_ξ and Z_g in the presence of additional massive quarks (Gray et al., 1990), (Bernreuther and Wetzel, 1982)

Infrared Divergences

- The infrared divergences are handled with the sector improved residue subtraction scheme (Czakon, 1005.0274)
- The infrared structure of the amplitudes can be greatly simplified by subtracting a rescaled version of the HTL.

$$\sigma^{\mathrm{HO}}_{\mathrm{EFT}} = \sigma^{\mathrm{HO}}_{\mathrm{HTL}} \frac{\sigma^{\mathrm{LO}}}{\sigma^{\mathrm{LO}}_{\mathrm{HTL}}}$$

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- The EFT exhibits the same infrared behavior as the exact amplitudes
- Example Real-Virtual corrections:

$$\langle M^{(1)}_{\text{exact}} | M^{(2)}_{\text{exact}} \rangle - \left[\langle M^{(1)}_{\text{EFT}} | M^{(2)}_{\text{EFT}} \rangle + \frac{8\pi\alpha_s}{t} \langle P^{(0)}_{gg} (\frac{s}{s+u}) \rangle \langle F^{(1)} | (F^{(2)}_{\text{exact}} - F^{(2)}_{\text{EFT}}) \rangle \right]$$

is IR finite. **No one-loop splitting needed**

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- The EFT exhibits the same infrared behavior as the exact amplitudes
- Example Real-Virtual corrections:

$$\langle M^{(1)}_{\text{exact}} | M^{(2)}_{\text{exact}} \rangle - \left[\langle M^{(1)}_{\text{EFT}} | M^{(2)}_{\text{EFT}} \rangle + \frac{8\pi\alpha_s}{t} \langle P^{(0)}_{gg} (\frac{s}{s+u}) \rangle \langle F^{(1)} | (F^{(2)}_{\text{exact}} - F^{(2)}_{\text{EFT}}) \rangle \right]$$

  is IR finite. **No one-loop splitting needed**
- For t-quarks the rescaling is a small correction that has physical meaning. For b-quarks **this is only a computational trick.**

Results

Results

Preliminary!

channel	Top-Bottom Interference [pb]			$(\sigma^{\text{NNLO}}/\sigma^{\text{NNLO}}_{\text{HEFT}} - 1)[\%]$
	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	
$\sqrt{s} = 13\,\text{TeV}$				
gg	-1.975	$-0.8546(36)$	$+0.121(14)$	1.4
qg		$+0.4077(5)$	$+0.2798(27)$	33
qq		-0.00039	$-0.0083(1)$	-8.3
total	-1.975	$-0.4473(36)$	$+0.393(14)$	$+4.1$

- $m_H = 125\,\text{GeV}, m_t^2/m_H^2 = 23/12, m_b^2/m_H^2 = 1/684, \mu = m_H/2$, PDF-set = NNPDF3.1
- Missing are (expected to be small due to color suppression):
  - the 3-loop mixed quark contributions,
  - the 2-loop real virtual mixed corrections
- If you are interested in bottom-bottom, top-charm or bottom-charm effects please ask

Conclusions and Outlook

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- We computed the Higgs production cross section in the gluon fusion channel with full quark mass dependence at NNLO.
- The associated uncertainty in the gluon fusion channel is now almost completely diminished (Theory uncertainty below 4% at $13\,\text{TeV}$)
- Investigate the effects and uncertainties of different top, bottom and charm masses and the choice of the renormalization scheme.
- Stay tuned...

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Thank You!

channel	Top-Bottom Interference [pb]			Bottom^2 [pb]		
	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$
$\sqrt{s} = 13\,\text{TeV}$						
gg	-1.975	$-0.8546(36)$	$+0.121(14)$	$+0.182$	$+0.1256(4)$	$-0.0109(45)$
qg		$+0.4077(5)$	$+0.2798(27)$		$-0.02100(4)$	$-0.02371(23)$
qq		-0.00039	$-0.0083(1)$		$+2.5 \times 10^{-6}$	$+0.00014$
total	-1.975	$-0.4473(36)$	$+0.393(14)$	$+0.182$	$+0.1466(6)$	$-0.0345(45)$

channel	Top-Charm Interference [pb]			Bottom-Charm [pb]		Charm^2 [pb]	
	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$
$\sqrt{s} = 13\,\text{TeV}$							
gg	-0.512	$-0.1891(9)$	$+0.121(4)$	$+0.0707$	$+0.0463(2)$	$+0.0072$	$+0.0041$
qg		$+0.1142(1)$	$+0.0805(7)$		-0.0104		-0.0013
qq		-6.8×10^{-5}	-0.0030		$+8 \times 10^{-7}$		$+6 \times 10^{-8}$
total	-0.512	$-0.0750(9)$	$+0.199(4)$	$+0.0707$	$+0.0359(2)$	$+0.0072$	$+0.0028$