Linear power corrections to hadron collider processes with top-quarks

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Motivation

When working with the hadronic final states, we use the quark and gluon degrees of freedom. This induces a difference between the partonic and hadronic cross sections:

$$\mathrm{d}\sigma_{\mathrm{x}} = \sum_{ij} \int \mathrm{d}x_i \mathrm{d}x_j f_i(x_i) f_j(x_j) \mathrm{d}\sigma^{\mathrm{part}}_{ij\to\mathrm{x}} \times \left[1 + \mathcal{O}\left(\frac{\Lambda^p_{\mathrm{QCD}}}{Q^p} \right) \right]$$

There's no theory of power corrections, determining the parameter *p* is not possible.

$$Q \sim 30\mathrm{GeV} \qquad \Lambda_{\mathrm{QCD}} \sim 300\mathrm{MeV} \qquad \frac{\Lambda_{\mathrm{QCD}}}{Q} \sim 10^{-2}$$

The precision of the perturbative calculations reached the level of 1-10%. The further increase would require taking into account the non-perturbative effects. These corrections are significant for the high precision physics (top quark mass measurements, strong coupling constant, etc).

Renormalon model

Yet a particular model can be employed

Assumption – all the corrections come from the landau pole of the running α_s

$$\int \mathrm{d}k\; k^{p-1}\alpha_s(\mu)F(k) \to \int \mathrm{d}k\; k^{p-1}\alpha_s(k)F(k)$$

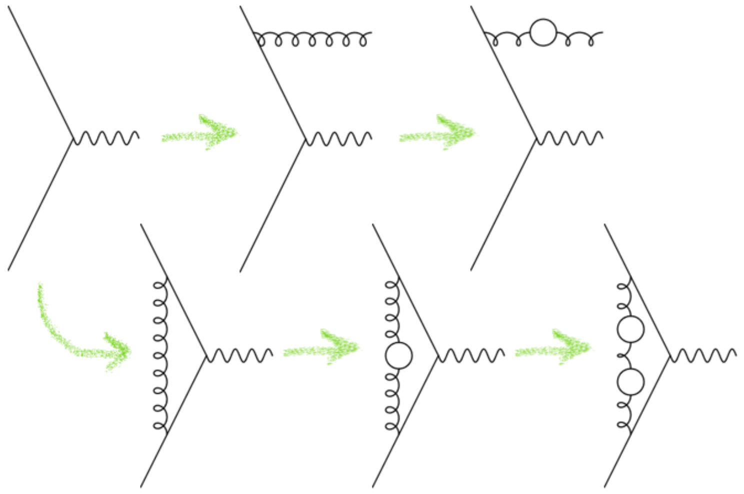
$$\alpha_s(k) = \frac{\alpha_s(\mu)}{1-\alpha_s(\mu)\beta_0 \ln\frac{k^2}{\mu^2}} \equiv \frac{1}{-\beta_0\ln\frac{k^2}{\Lambda^2_{\mathrm{QCD}}}} \approx \frac{1}{-\beta_0}\frac{\Lambda^2_{\mathrm{QCD}}}{k^2-\Lambda^2_{\mathrm{QCD}}}$$

The idea is to do the resummation
of the fermion bubbles with $N_f \to -\infty$

It effectively works as calculations
with the non-zero gluon mass $\lambda \leftrightarrow \Lambda_{\mathrm{QCD}}$

We then would need to consider the $\mathcal{O}(\alpha_s)$
corrections in the soft limit $k \sim \lambda$

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_s \Delta\sigma(\lambda)$$



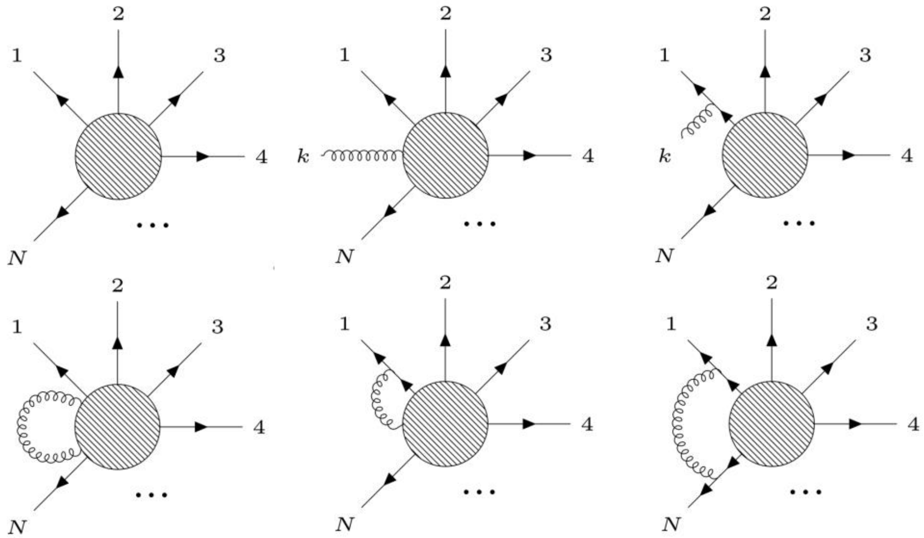
The problem

$$d\sigma = \mathrm{dLips}_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}_{\mathcal{O}(k)}|^2 \times \mathcal{O}_{\mathcal{O}(\lambda,k)}$$

1) Amplitudes and the LBK theorem

2) Phase space and cross-section

3) Kinematic distributions



Specific processes

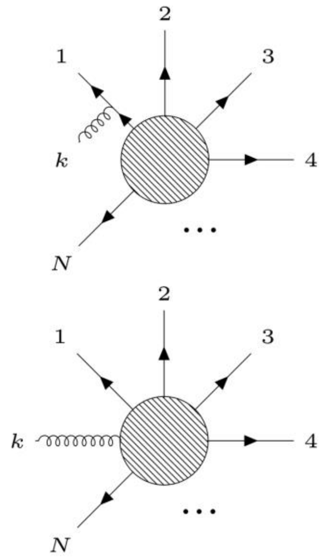
$$u(p_u) + b(p_b) \to d(q_d) + t(q_t) + X(p_X)$$

$$q(p_q) + \bar{q}(p_{\bar{q}}) \to t(q_t) + \bar{t}(q_{\bar{t}}) + X(p_X)$$

Real emission and the Low-Burnett-Kroll theorem

Acquire the amplitude of a radiative process in terms of the amplitude of an elastic process and factor out the k-dependence

Gauge invariance allows us to express the structural radiation in terms of the elastic amplitude



$$k_\mu \mathcal{M}^{a,\mu} = 0 \qquad \mathcal{M}^{a,\mu}_{\text{reg}}|_{k=0} = -g_s \sum_{i=1}^{N} \eta_i D_i^\mu T_i^a \mathcal{M}_0$$

$$|\mathcal{A}_{\text{real}}|^2 = -g_s^2 \sum_{i,j\in N} \eta_i \eta_j W_i^\mu W_{j,\mu} F_{\text{LO}}^{ij}$$

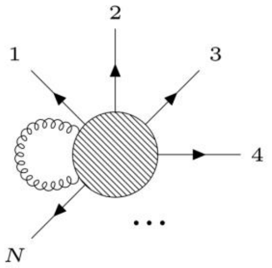
$$W_i^\mu = J_i^\mu + \frac{1}{2} L_i^\mu \qquad L_i^\mu = J_i^\mu k^\nu D_{i,\nu} - D_i^\mu$$

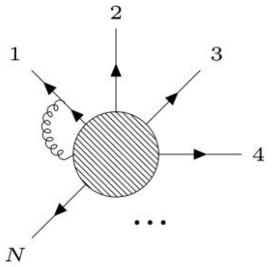
$$J_i^\mu = \frac{2p_i^\mu + k^\mu}{d_i} \qquad d_i = (p_i + k)^2 - m_i^2$$

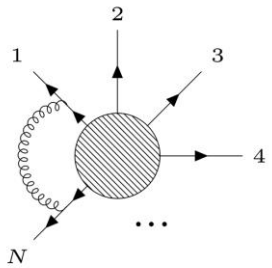
Virtual contribution

The case of the virtual gluon exchange is very similar to the
real emission calculations. Both contributions are built out of similar objects

$$\mathcal{A}_{V_i} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{-i}{k^2 - \lambda^2} \mathcal{M}_{V_i}$$







$$|\mathcal{A}_{\mathrm{real}}|^2 = -g_s^2 \sum_{i,j \in N} \eta_i \eta_j W_i^{\mu} W_{j,\mu} F_{\mathrm{LO}}^{ij}$$

$$|\mathcal{M}_V|^2 = g_s^2 \mathcal{T}_\lambda \Bigg[\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{-i}{k^2 - \lambda^2} \Bigg\{ \sum_{i \neq j \in N} \eta_i \eta_j \; W_i^{\mu}(k) W_{j,\mu}(-k) F_{\mathrm{LO}}^{ij} - C_F \sum_{i=1}^{N} \left(J_i^{\mu} D_{i,\mu} F_{\mathrm{LO}} - \frac{2}{d_i} F_{\mathrm{LO}} + \frac{2\bar{\eta}_i}{d_i} F_{\mathrm{LO}}|_{\hat{\rho}_i \to m_i \mathbf{1}} \right) \Bigg\} \Bigg]$$

Renormalisation

Renormalization computed with a massive gluon introduces additional *k* terms

$$Z_m = 1 + \frac{C_F g_s^2 m_t^{-2\epsilon}\Gamma(1+\epsilon)}{(4\pi)^{d/2}}\left[-\frac{3}{\epsilon} - 4 + \frac{2\pi\lambda}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right)\right],$$

$$Z_2 = 1 + \frac{C_F g_s^2 m_t^{-2\epsilon}\Gamma(1+\epsilon)}{(4\pi)^{d/2}}\left[-\frac{1}{\epsilon} - 4 + 4\ln\frac{m_t}{\lambda} + \frac{3\lambda\pi}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right)\right]$$

The wave function renormalisation contribution is simply added to the virtual corrections, while the mass counter-term, on the other hand, is inserted through the internal top quark lines

$$\mathcal{T}_\lambda\left[\sigma_{\rm ren}\right] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips}_{\rm LO}\left[3F_{\rm LO} + m_t{\rm Tr}\left[\hat{\rho}_t\frac{\partial\mathbf{N}}{\partial m_t}\hat{\rho}_{\bar{t}}\bar{\mathbf{N}}\right] + m_t{\rm Tr}\left[\hat{\rho}_t\mathbf{N}\hat{\rho}_{\bar{t}}\frac{\partial\bar{\mathbf{N}}}{\partial m_t}\right]\right]$$

Phase space corrections

Lorentz-Invariant Phase-Space Measure is affected by this change and produces additional terms

$$\mathrm{dLips}(p_i, q_i, k) = \frac{\mathrm{d}^4 k}{(2\pi)^4}\delta(k^2 - \lambda^2)\delta^{(4)}\left(\sum_{\text{initial}} p_i - \left(\sum_{\text{final}} q_i + k\right)\right)\prod_{\text{final}} \frac{\mathrm{d}^4 q_i}{(2\pi)^4}\delta(q_i^2 - m_i^2)$$

The delta-function enforces the energy-momentum conservation.
Thus, we redefine the final momenta up to the order O(k)

$$q_j^\mu = p_j^\mu + K_j^{\mu\nu} k_\nu$$

Jacobian and the on-shell condition produce additional terms as well

And this induces terms from the amplitudes

$$F_{\mathrm{LO}}(q_i) = \left[1 + \sum_i k^\mu \frac{\partial q_i^\nu}{\partial k^\mu} D_{i,\nu}\right] F_{\mathrm{LO}}(p_i)$$

The final phase space contribution:

$$\mathrm{dLips}(p_i, q_i, k) = \mathrm{dLips}_{\mathrm{LO}}(p_i) \times \frac{\mathrm{d}^4 k}{(2\pi)^4}\delta(k^2 - \lambda^2) \times (1 + k^\mu V_\mu(p_i))$$

Mass redefinition

The use of the pole quark mass in physical predictions is one of the sources of linear power corrections. These corrections are artificial and can be removed by employing one of the many short-distance mass schemes. We want to reverse the renormalization and get rid of the linear power corrections in the physical quantities.

There are 2 distinct reasons of why the dependencies on the top mass arise:

1) The explicit dependence of the matrix element squared on the top-mass

$$F_{\mathrm{LO}}\left(m_t\right)=F_{\mathrm{LO}}\left(\tilde{m}_t+\delta m_t\right)$$

2) The implicit dependence of the energies of the final state particles on it

$$p_t^2=m_t^2$$

In the first case, we simply expand the matrix element up to the first power of δm_t

In the second, a redefinition of the momenta is needed to achieve $\tilde{p}_t^2=\tilde{m}_t^2=m_t^2-\delta m_t^2$

δm_t is determined from the renormalization contribution

$$\delta m_t=-m_t \frac{C_F \alpha_s}{2 \pi} \frac{\pi \lambda}{m_t}$$

Single top cross-section results

$$\mathcal{T}_\lambda\left[\delta\sigma_{\rm mass}\right] = \frac{C_F\alpha_s}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips_{LO}} \times \left[\frac{m_t^2}{p_d p_t}\left[1 + p_d^\mu\left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu}\right)\right]F_{\rm LO} - m_t {\rm Tr}\left[\mathbf{1}\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right]\right.$$
$$\left. - m_t {\rm Tr}\left[(\not{p}_t + m_t)\left(\frac{\partial \mathbf{N}}{\partial m_t}\not{p}_b\bar{\mathbf{N}} + \mathbf{N}\not{p}_b\frac{\partial\bar{\mathbf{N}}}{\partial m_t}\right)\right]\right],$$

$$\mathcal{T}_\lambda\left[\sigma_{\rm R}\right] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips_{LO}}\left[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b}\right) - \frac{m_t^2}{p_d p_t}p_d^\mu\left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu}\right) - \frac{m_t^2}{p_t p_b}p_b^\mu D_{p,\mu}\right]F_{\rm LO},$$

$$\mathcal{T}_\lambda\left[\sigma_V\right] = -\frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips_{LO}}\left[{\rm Tr}\left[\not{p}_t\mathbf{N}\not{p}_b\bar{\mathbf{N}}\right] + \left(\frac{(2p_t p_b - m_t^2)}{p_t p_b} - \frac{m_t^2}{p_t p_b}\,p_b^\mu D_{p,\mu}\right)F_{\rm LO}\right],$$

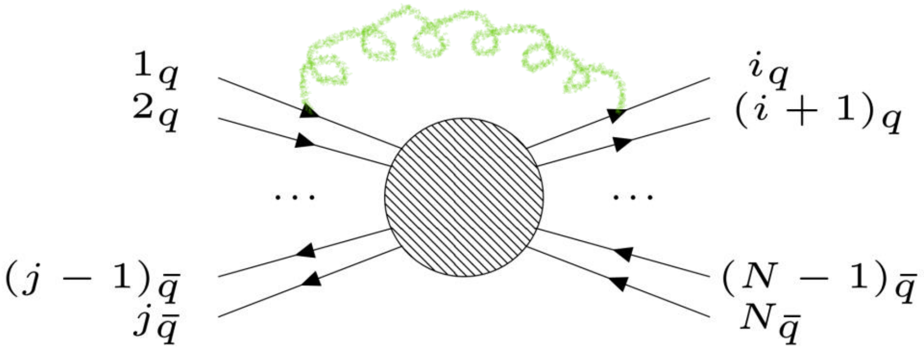
$$\mathcal{T}_\lambda\left[\sigma_{\rm ren}\right] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int {\rm dLips_{LO}}\left[\frac{3}{2}F_{\rm LO} + m_t{\rm Tr}\left[(\not{p}_t + m_t)\frac{\partial\mathbf{N}}{\partial m_t}\not{p}_b\bar{\mathbf{N}}\right] + m_t{\rm Tr}\left[(\not{p}_t + m_t)\mathbf{N}\not{p}_b\frac{\partial\bar{\mathbf{N}}}{\partial m_t}\right]\right]$$

$$\delta\sigma_{\rm NLO} = {\rm d}\sigma_R + {\rm d}\sigma_V + {\rm d}\sigma_{\rm ren} + \delta\sigma_{\rm mass}^{\rm expl} + \delta\sigma_{\rm mass}^{\rm impl} = 0$$

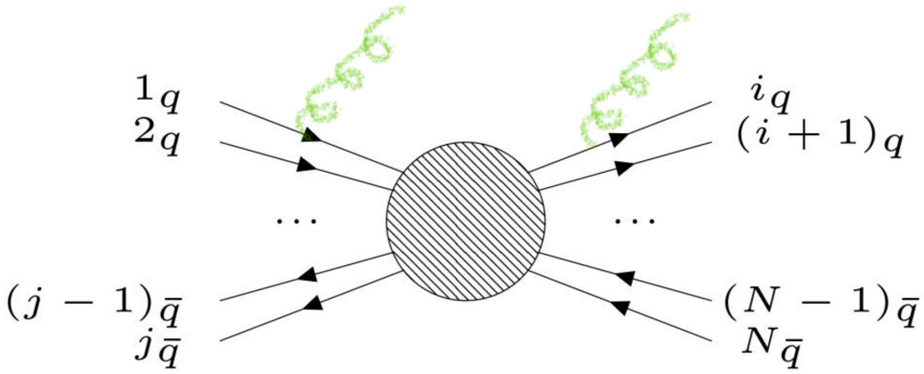
$t\bar{t}$ cross-section results

Previously the cancellations took place when everything is summed up. Now we can see, that they happen on the dipole level.

$$\mathcal{T}_\lambda\left[\mathrm{d}\sigma_V^{ij}\right]+\mathcal{T}_\lambda\left[\mathrm{d}\sigma_R^{ij}\right]=0$$



The renormalization and the mass redefinition corrections then correspond to the monopole contribution (*i* = *j*) and cancel together with the real and virtual tt and $\overline{tt}$ contribution



$$\delta\sigma_{\mathrm{mass}}^{\mathrm{expl}}+\delta\sigma_{\mathrm{mass}}^{\mathrm{impl}}+\mathcal{T}_\lambda\left[\sigma_{\mathrm{ren}}\right]+\mathcal{T}_\lambda\left[\mathrm{d}\sigma_V^{tt}+\mathrm{d}\sigma_V^{\overline{tt}}\right]+\mathcal{T}_\lambda\left[\mathrm{d}\sigma_R^{tt}+\mathrm{d}\sigma_R^{\overline{tt}}\right]=0$$

Kinematic distributions

The momentum mapping produces shifts in observables.

$$O_X = \int d\sigma\, X(q_t) \Rightarrow O_X = \bar{O}_X^{\mathrm{LO}} + \int d\sigma_{\mathrm{LO}} \frac{\partial X(p_t)}{\partial p_t^\mu} \delta^{\mathrm{mass}} p_t^\mu + \sum_{\mathfrak{a}} \int d\sigma_R^{(\mathfrak{a})}(p_t, \ldots) \frac{\partial X(p_t)}{\partial p_t^\mu} \delta^{(\mathfrak{a})} p_t^\mu$$

This sum can be expressed as an expanded quantity with a shifted variable

$$d\sigma_{\mathrm{LO}}^{\mathfrak{a}} = C^{\mathfrak{a}}\, d\sigma_{\mathrm{LO}}$$

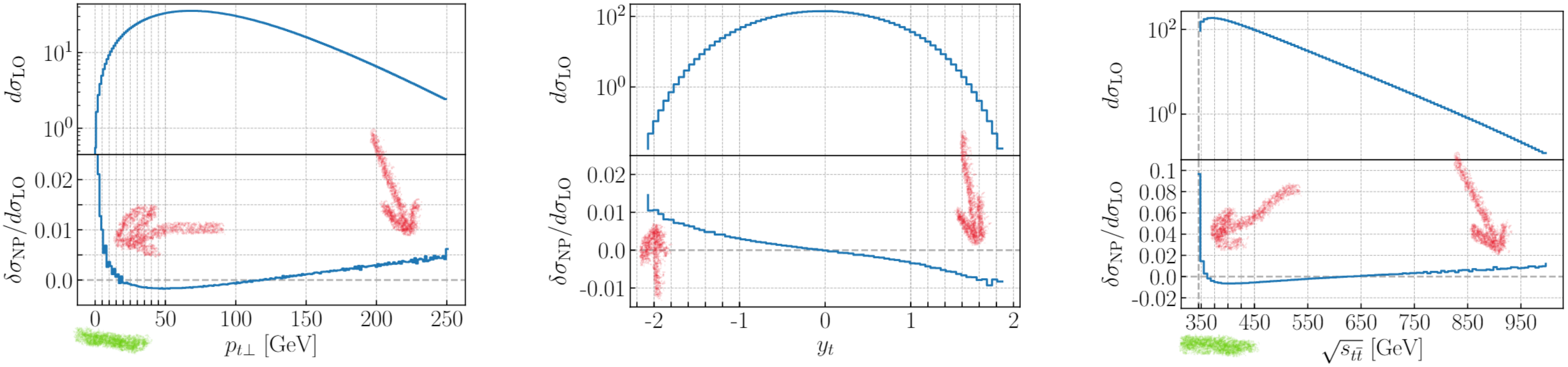
$$\mathcal{O}_X = \int d\sigma_{\mathrm{LO}}\, X\left(p_t + \frac{\alpha_s}{2\pi} \sum_{\mathfrak{a}} C^{\mathfrak{a}}\, \delta p_{t,\mathfrak{a}}\right)$$

$$\delta p_{t,\mathfrak{a}} = \frac{\pi\lambda}{m_t} l_{\mathfrak{a}}(p_i)$$

In order to pick a value for the parameter λ, we assume that the non-perturbative shift in the top-quark mass would correspond to the recent estimations on the ambiguity of the pole mass m_t, which is we take as 200MeV

$$\alpha_s \lambda = \frac{0.4\ \mathrm{GeV}}{C_F} = 0.3\ \mathrm{GeV}$$

The effect on the differential cross-sections



The relative change can reach the order of $\mathcal{O}(1\%)$ in the threshold regions for the invariant mass and the transverse momentum. The effect is also significant in the regions where the leading order distributions start to decrease rapidly.

$$\frac{\delta_{\rm NP}\,[p_{t\perp}]}{p_{t\perp}} = \frac{\alpha_s}{2\pi}\frac{\pi\lambda}{m_t}\frac{(2C_F - C_A\tau)}{2(1-\tau)} \qquad \tau = 4m_t^2/s_{t\bar{t}}$$

Outlook

1) Modern high precision measurements may find the non-perturbative calculations important for some processes
2) The discussed method allows one to calculate the linear power corrections from all necessary sources (amplitude, phase space, observables) in a process independent way*
3) The total cross-section is not affected by the linear power corrections, while the kinematic observables experience a shift within certain mass schemes
4) This shift may cause corrections of order 1% near the threshold regions or where the cross-section is small.

*Except for some processes with gluons in the leading order