Diagrammatic resummation of double logarithms in $B_c \to \eta_c$ form factors

Dennis Horstmann

based on work in progress with Guido Bell, Philipp Böer, Thorsten Feldmann and Vladyslav Shtabovenko [2309.08410, PoS]

Theoretische Physik I, Universität Siegen

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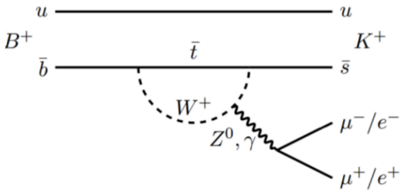
B-physics

- Allows to test CP violation and the CKM matrix
- High sensitivity to new physics contributions
- More precision necessary in view on future experimental developments



[fig taken from 1609.02015]

Exclusive decays at large recoil

$$\left\langle \pi(p) \middle| \bar{q}\gamma^\mu b \middle| \bar{B}(p_B) \right\rangle = F_+(q^2)(p_B^\mu + p^\mu) + F_-(q^2) q^\mu$$

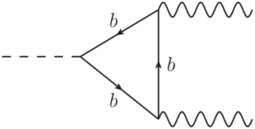
$$F_i(q^2) = C_i \xi_{B\to\pi}(q^2) + \phi_B \otimes T_i \otimes \phi_\pi$$

$$\xi_{B\to\pi}(q^2) = \sum_i \int_0^\infty \mathrm{d}\omega \int_0^1 \mathrm{d}u \;\; \phi_B^i(\omega) T^i(u,\omega) \phi_\pi^i(u)$$

- Complicated interplay of soft and collinear dynamics
- Naively amplitude factorises according to SCET / QCD factorisation [Beneke/Feldmann '00]
- Factorisation theorem is plagued by endpoint divergences
- Factorisation of scales is spoilt
- Generic problem of SCET at subleading power

Endpoint divergences in $h \to \gamma\gamma$

$$l^\mu = l_- \frac{n^\mu}{2} + l_+ \frac{\bar{n}^\mu}{2} + l_\perp^\mu$$



- Scale hierarchy $M_h \gg m_b$ demands resummation of large logs
- Factorisation theorem has endpoint divergences [Liu/Neubert '19]

$$\mathcal{M}_b(h \to \gamma\gamma) = H_1 \langle \gamma\gamma | O_1 | h \rangle + 4 \int_0^1 \frac{\mathrm{d}z}{z} \bar{H}_2(z) \langle \gamma\gamma | O_2(z) | h \rangle$$
$$+ H_3 \int_0^\infty \frac{\mathrm{d}l_-}{l_-} \int_0^{l_-} \frac{\mathrm{d}l_+}{l_+} S(l_+ l_-) J(l_+) J(l_-)$$

- Rearrange using refactorisation identities [Böer '18, Liu/Neubert '19]

$$4[[\bar{H}_2]] \otimes [[\langle O_2 \rangle]] = 2H_3 \int_0^\infty \frac{\mathrm{d}\omega}{\omega} S(\omega) \int_0^{M_h} \frac{\mathrm{d}l_-}{l_-} J(\omega/l_-) J(l_-)$$

Endpoint divergences in $h \to \gamma\gamma$

- Rearranged factorisation theorem has no endpoint divergences:

$$\begin{aligned}\mathcal{M}_b(h \to \gamma\gamma) =& (H_1 + \Delta H_1) \langle \gamma\gamma | O_1 | h \rangle \\ &+ 4 \int_0^1 \frac{\mathrm{d}z}{z} \Big[\bar{H}_2(z) \langle \gamma\gamma | O_2(z) | h \rangle - [[\bar{H}_2(z)]][[\langle \gamma\gamma | O_2(z) | h \rangle]] \Big] \\ &+ H_3 \int_0^{M_h} \frac{\mathrm{d}l_-}{l_-} \int_0^{M_h} \frac{\mathrm{d}l_+}{l_+} J(l_-) J(l_+) S(l_+ l_-)\end{aligned}$$

- Leading double logs are given by last term:

$$\mathcal{M}_b(h \to \gamma\gamma)\Big|_{DL} = \mathcal{M}_0 \int\limits_{m_b^2/M_h}^{M_h} \frac{\mathrm{d}l_+}{l_+} \int\limits_{m_b^2/M_h}^{M_h} \frac{\mathrm{d}l_-}{l_-} \theta(l_+ l_- - m_b^2)$$

with $\quad L = \ln \frac{M_h^2}{m_b^2} \quad$ and $\quad S(x,y) = \frac{\alpha_s C_F}{2\pi} \ln x \ln y$

[Liu/Neubert '19]

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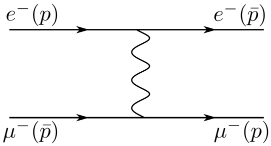
with $\quad L = \ln \frac{M_h^2}{m_b^2} \quad$ and $\quad S(x, y) = \frac{\alpha_s C_F}{2\pi} \ln x \ln y$

[Liu/Neubert '19]

Endpoint divergences in μe-backward scattering

- Electrons and muons scatter exactly backwards ($m_e = m_\mu \equiv m \ll \sqrt{s}$)
- Leading logs are generated by soft lepton configurations



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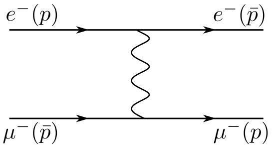
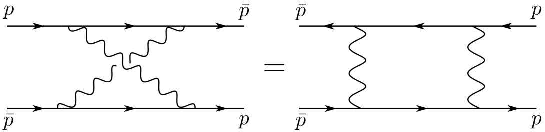


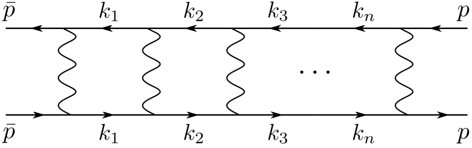
- Twisted ladder diagrams give leading logs



$$\mathcal{M}^{(1)} \simeq \mathcal{M}^{(0)} \int_{\frac{m^2}{\sqrt{s}}}^{\sqrt{s}} \frac{\mathrm{d}k_+}{k_+} \int_{\frac{m^2}{\sqrt{s}}}^{\sqrt{s}} \frac{\mathrm{d}k_-}{k_-}\, \theta(k_+k_- - m^2) = \frac{L^2}{2} \quad \text{with} \quad L = \ln\frac{m^2}{s}$$

Endpoint divergences in μe-backward scattering

To all orders double logs are generated by crossed diagrams with strongly ordered longitudinal momenta



$$\frac{m^2}{\sqrt{s}} \approx \bar{p}_- \ll k_{1-} \ll \cdots \ll k_{n-} \ll p_- \approx \sqrt{s}$$
$$\frac{m^2}{\sqrt{s}} \approx p_+ \ll k_{n+} \ll \cdots \ll k_{1+} \ll \bar{p}_+ \approx \sqrt{s}$$

$$\mathcal{M}^{(n)} = \mathcal{M}^{(0)} \int_{\bar{p}_-}^{p_-} \frac{\mathrm{d}k_{1-}}{k_{1-}} \int_{k_{1-}}^{p_-} \frac{\mathrm{d}k_{2-}}{k_{2-}} \cdots \int_{k_{n-1,-}}^{p_-} \frac{\mathrm{d}k_{n-}}{k_{n-}}$$
$$\times \int_{\frac{m^2}{k_{1-}}}^{\bar{p}_+} \frac{\mathrm{d}k_{1+}}{k_{1+}} \int_{\frac{m^2}{k_{2-}}}^{k_{1+}} \frac{\mathrm{d}k_{2+}}{k_{2+}} \cdots \int_{\frac{m^2}{k_{n-}}}^{k_{n-1,+}} \frac{\mathrm{d}k_{n+}}{k_{n+}} = \mathcal{M}^{(0)} \frac{L^{2n}}{n!(n+1)!}$$

Endpoint divergences in μe-backward scattering

- Integral equations can be recast into recursive form ($\mathcal{M} \simeq F\mathcal{M}^{(0)}$):

$$F(l_-, l_+) = 1 + \frac{\alpha_{\text{em}}}{2\pi} \int_{l_-}^{p_-} \frac{\mathrm{d}k_-}{k_-} \int_{\frac{m^2}{k_-}}^{l_+} \frac{\mathrm{d}k_+}{k_+} F(k_-, k_+)$$

$$\Longrightarrow \quad \mathcal{M} \simeq \mathcal{M}^{(0)} F(\bar{p}_-, \bar{p}_+) = \mathcal{M}^{(0)} \frac{I_1(2\sqrt{z})}{\sqrt{z}} \quad \text{with} \quad z = \frac{\alpha_{\text{em}}}{2\pi} L^2$$

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- Recursive structure can be understood by involved refactorisation in SCET [Bell,Böer,Feldmann '22]

$$[[f_c(x)]] \simeq \int_0^1 \frac{\mathrm{d}x'}{x'} f_c(x') \int \frac{\mathrm{d}\rho}{\rho} J_{hc}(\rho x') S(x, \rho)$$

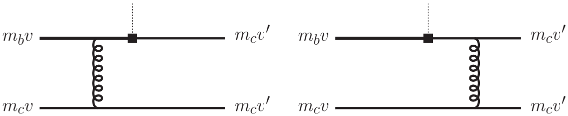
Compared to refactorisation in $h \to \gamma\gamma$

$$4[[\bar{H}_2]] \otimes [[\langle O_2 \rangle]] = 2H_3 \int_0^\infty \frac{\mathrm{d}\omega}{\omega} S(\omega) \int_0^{M_h} \frac{\mathrm{d}l_-}{l_-} J(\omega/l_-) J(l_-)$$

Non-relativistic heavy-to-light form factors

- Process considered:
  - $B_c \to \eta_c$ at large recoil $\gamma \equiv v \cdot v' = \mathcal{O}(m_b/m_c)$
  - non-relativistic approximation with $m_b \gg m_c \gg \Lambda_{\mathrm{QCD}}$
  - perturbative toy example for $B \to \pi$ form factors
- Factorisation theorems for heavy-to-light form factors have factorisable and non-factorisable terms
- Non-factorisable part is called "soft overlap" contribution

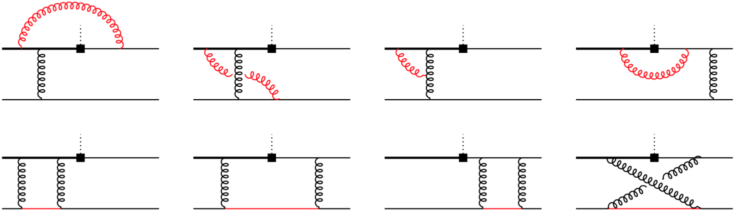
$$F(\gamma) \equiv \frac{1}{2E_\eta} \langle \eta_c(p_\eta) | (\bar{c}\,\Gamma\, b)(0) | B_c(p_B) \rangle \qquad \text{with} \qquad \Gamma = \frac{\bar{\not{n}}\not{n}}{4}$$



Non-relativistic heavy-to-light form factors

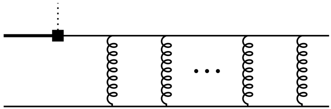
Goal of this work:

Diagrammatic resummation of leading double logs of the soft overlap contribution



Soft quark logs in $B_c \to \eta_c$ form factor

- In light-cone gauge and Abelian limit all soft-quark logs are given by ladder diagrams:



- Soft-quark logs are governed by recursive integral equations

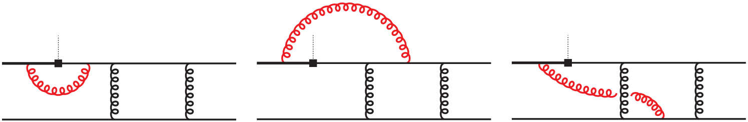
$$F(\gamma)\Big|_{\text{soft quark}} \propto 6f(\gamma) - 1 \qquad \text{with} \qquad f(\gamma) \equiv f(m_c, m_c)$$

$$f(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta-}} \frac{\mathrm{d}k_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{\mathrm{d}k_+}{k_+} \left(f(k_+, k_-) + \frac{1}{2} f_{m_c}(k_+, k_-) \right)$$

$$f_{m_c}(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta-}} \frac{\mathrm{d}k_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{\mathrm{d}k_+}{k_+} f_{m_c}(k_+, k_-)$$

Interplay with soft gluons

Sudakov logs account for soft gluon effects



$$F(\gamma) \propto \exp\left\{-\frac{\alpha_s C_F}{4\pi} L^2\right\} \times (6f(\gamma) - 1) \quad \text{with} \quad f(\gamma) \equiv f(m_c, m_c)$$

$$f(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta -}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{dk_+}{k_+} e^{-S(l_+/k_+, p_{\eta -}/k_-)} \left(f(k_+, k_-) + \frac{1}{2} f_{m_c}(k_+, k_-) \right)$$

$$f_{m_c}(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta -}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{dk_+}{k_+} e^{-S(l_+/k_+, p_{\eta -}/k_-)} f_{m_c}(k_+, k_-)$$

$$S(x, y) = \frac{\alpha_s C_F}{2\pi} \ln x \ln y$$

Order-by-order solution

- Solving this integral equation in a closed form is work in progress
- First few terms are given by:

$$f(\gamma) = 1 + \frac{3}{2}\left(\frac{\alpha_s C_F}{4\pi}L^2\right) + \frac{5}{12}\left(\frac{\alpha_s C_F}{4\pi}L^2\right)^2 - \frac{1}{180}\left(\frac{\alpha_s C_F}{4\pi}L^2\right)^3 + \dots$$

- Independently checked up to 2-loops by means of pole cancellation arguments and explicit calculation of the only unknown pole in hard-collinear region
- Typical tool chain (`qgraf`, `Form`, `Fire`, `pySecDec`, ...)

Summary and outlook

- $h \to \gamma\gamma$

$$\mathcal{M}_b(h \to \gamma\gamma)\Big|_{DL} = \mathcal{M}_0 \int_{m_b^2/M_h}^{M_h} \frac{\mathrm{d}l_+}{l_+} \int_{m_b^2/l_+}^{M_h} \frac{\mathrm{d}l_-}{l_-}\, e^{-S(M_h/l_+,\, M_h/l_-)}$$

- μe backward scattering

$$F(l_-, l_+) = 1 + \frac{\alpha_{\text{em}}}{2\pi} \int_{l_-}^{p_-} \frac{\mathrm{d}k_-}{k_-} \int_{m^2/k_-}^{l_+} \frac{\mathrm{d}k_+}{k_+} F(k_-, k_+)$$

- $B_c \to \eta_c$ form factor at large recoil

$$f(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta-}} \frac{\mathrm{d}k_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{\mathrm{d}k_+}{k_+} e^{-S(l_+/k_+, p_{\eta-}/k_-)} \left(f(k_+, k_-) + \frac{1}{2} f_{m_c}(k_+, k_-) \right)$$

- Cross check at α_s^3
- Non-Abelian colour structures
- Develop EFT language