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Progress in N3LO 0-jettiness soft function calculation

Andrey Pikelner | CRC YSM 2023, Siegen

in collaboration with: Daniel Baranowski, Maximilian Delto, Kirill Melnikov and Chen-Yu Wang

Plan of the talk

1. Introduction and motivation

2. Soft function for 0-jettiness slicing

3. IBP reduction of integrals with θ-functions

4. Techniques for master integrals calculation
   - RRR
   - RRV

5. Results and conclusion

Motivation

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- Differential calculation require a good handle on IR divergences, many schemes at NNLO
- Slicing scheme seems feasible at N3LO due to complexity of subtraction schemes

$$\sigma(O) = \int_0 \mathrm{d}\tau \frac{\mathrm{d}\sigma(O)}{\mathrm{d}\tau} = \int_0^{\tau_0} \mathrm{d}\tau \frac{\mathrm{d}\sigma(O)}{\mathrm{d}\tau} + \int_{\tau_0} \mathrm{d}\tau \frac{\mathrm{d}\sigma(O)}{\mathrm{d}\tau}$$

  - q_T subtraction scheme [Catani,Grazzini'07]
  - N-jettiness subtraction scheme [Boughezal et al.'15][Gaunt et al.'15]
- SCET factorization theorem motivates us to consider jettiness as convenient slicing variable

$$\lim_{\tau \to 0} \mathrm{d}\sigma(O) = B_\tau \otimes B_\tau \otimes S_\tau \otimes H_\tau \otimes \mathrm{d}\sigma_{\mathrm{LO}}$$

Difficulties with 0-jettiness variable

- 0-jettiness for the colorless final state is complicated due to the presence of min-function

$$\tau = \sum_{i=1}^{m} \min_{q \in \{n,\bar{n}\}} \left[\frac{2q \cdot k_i}{n \cdot \bar{n}} \right] = \sum_{i=1}^{m} \min\{\alpha_i, \beta_i\}$$

- Definition which is more friendly for PS integration generates different configurations

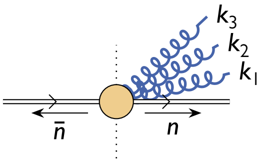
$$\delta\left(\tau - \sum_{i=1}^{m} \min\{\alpha_i, \beta_i\}\right) = \delta(\tau - \alpha_1 - \alpha_2 - \ldots)\theta(\beta_1 - \alpha_1)\theta(\beta_2 - \alpha_2)\ldots$$
$$+ \delta(\tau - \alpha_1 - \beta_2 - \ldots)\theta(\beta_1 - \alpha_1)\theta(\alpha_2 - \beta_2)\ldots$$

Sudakov decomposition

$$k_i = \frac{\alpha_i}{2} n + \frac{\beta_i}{2} \bar{n} + k_{i,\perp}, \quad k_i \cdot n = \beta_i, \quad k_i \cdot \bar{n} = \alpha_i, \quad n \cdot \bar{n} = 2$$

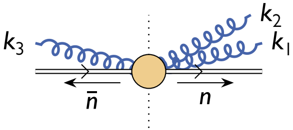
Unique emission configurations and present status

Same hemisphere



$$\delta(\tau - \beta_1 - \beta_2 - \beta_3)$$

Different hemispheres

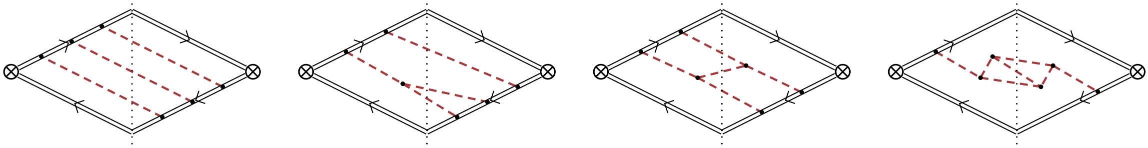


$$\delta(\tau - \beta_1 - \beta_2 - \alpha_3)$$

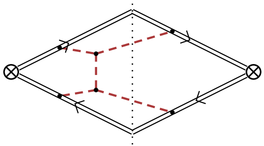
- Same hemisphere calculated with modern techniques for multi-loop calculation [Baranowski et al.'22]
- Our goal is to calculate different hemispheres contribution $\Longrightarrow$ this talk
- Emission of two gluons with one-loop correction $\Longrightarrow$ this talk

Eikonal factors and soft amplitudes squared

- RRR - split according to soft-gluon propagator structure [Catani,Colferai,Torrini'19]



S_a $\quad$ $S_b \sim \frac{1}{k_1 \cdot k_2}$ $\quad$ $S_c \sim \frac{1}{(k_1 \cdot k_2)(k_1 \cdot k_3)}$ $\quad$ $S_d \sim \frac{1}{(k_1 + k_2 + k_3)^2}$



- RRV squared amplitudes generated from scratch
- Results for one-loop soft current are known [Zhu'20][Czakon et al.'22]
- Calculated with different technique RRV result is known [Chen,Feng,Jia,Liue'22]

Configuration specific measurement functions

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Same hemisphere

- RRR: $\alpha_1, \alpha_2, \alpha_3$ unconstrained

$$\mathrm{d}\Phi^{nnn}_{\theta\theta\theta} = \delta(\tau - \beta_1 - \beta_2 - \beta_3) \\ \theta(\alpha_1 - \beta_1)\theta(\alpha_2 - \beta_2)\theta(\alpha_3 - \beta_3)$$

- RRV: α_1, α_2 unconstrained

$$\mathrm{d}\Phi^{nn}_{\theta\theta} = \delta(\tau - \beta_1 - \beta_2) \\ \theta(\alpha_1 - \beta_1)\theta(\alpha_2 - \beta_2)$$

Different hemispheres

- RRR: $\alpha_1, \alpha_2, \beta_3$ unconstrained

$$\mathrm{d}\Phi^{nn\bar{n}}_{\theta\theta\theta} = \delta(\tau - \beta_1 - \beta_2 - \alpha_3) \\ \theta(\alpha_1 - \beta_1)\theta(\alpha_2 - \beta_2)\theta(\beta_3 - \alpha_3)$$

- RRV: α_1, β_2 unconstrained

$$\mathrm{d}\Phi^{n\bar{n}}_{\theta\theta} = \delta(\tau - \beta_1 - \alpha_2) \\ \theta(\alpha_1 - \beta_1)\theta(\beta_2 - \alpha_2)$$

- Well known technique: reduction of integrals with δ-functions [Anastasiou,Melnikov'02]
- New technique: reduction of integrals with θ-functions [Baranowski,Delto,Melnikov,Wang'21]

IBP for integrals with θ functions

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- In dimensional regularisation system of IBP equation can be constructed by integration under integral sign

$$\int \mathrm{d}^d l \frac{\partial}{\partial l_\mu}\left[v_\mu \cdot f(\{l\})\right], \qquad \frac{\partial}{\partial k\cdot\bar{n}}\theta(k\cdot\bar{n}-k\cdot n) = \delta(k\cdot\bar{n}-k\cdot n)$$

- IBP for integrals with θ-functions generate new auxiliary topologies, partial fractioning required

$$\frac{\theta(k\cdot\bar{n}-k\cdot n)}{(k\cdot\bar{n})^a(k\cdot n)^b} \to \frac{\delta(k\cdot\bar{n}-k\cdot n)}{(k\cdot\bar{n})^a(k\cdot n)^b}$$

  - RRR $\underbrace{\theta\theta\theta}_{\text{Level 3}} \to \underbrace{\delta\theta\theta+\theta\delta\theta+\theta\theta\delta}_{\text{Level 2}} \to \underbrace{\delta\delta\theta+\delta\theta\delta+\theta\delta\delta}_{\text{Level 1}} \to \underbrace{\delta\delta\delta}_{\text{Level 0}}$
  - RRV $\underbrace{\theta\theta}_{\text{Level 2}} \to \underbrace{\delta\theta+\theta\delta}_{\text{Level 1}} \to \underbrace{\delta\delta}_{\text{Level 0}}$

Details of IBP reduction

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- Many integrals can be mapped on a small unique set, same symmetries as loop integral [Pak'11]

$$\theta(A) \to \frac{1}{A - m_\theta^2}, \quad \delta(A) \to \frac{1}{A - m_\delta^2}$$

- User-defined system reduction option available in `Kira` [Klappert et al.'21]
- Complexity of master integrals after IBP reduction
  - RRR: number of θ-functions, complexity of gluon propagators
  - RRV: number of θ-functions, complexity of one-loop integral
- No $\theta \ldots \theta$ master integrals for both RRR and RRV same hemisphere configurations

DE for RRR integrals with auxiliary mass

- Integrals for both nnn and $nn\bar{n}$ configurations with denominator $1/k_{123}^2$ are difficult to calculate
- Integrals are single scale, auxiliary parameter needed to construct DE system $I \to J(m^2)$
- Our solution is to make the most complicated propagator massive $\frac{1}{(k_1+k_2+k_3)^2} \to \frac{1}{(k_1+k_2+k_3)^2+m^2}$
- Calculation of boundary conditions simplifies in the limit $m^2 \to \infty$
- Result for integrals of our interest from the solution for $J(m^2)$ in the limit $m^2 \to 0$

Difficulties of the chosen strategy:

- Both points $m^2 \to 0$ and $m^2 \to \infty$ are singular points of the DE system
- Solution of the DE for integrals with massive denominator possible only numerically

Details of the DE solution

- Larger DE system size with 670 equations for $nn\bar{n}$ configuration compared to 176 for nnn
- Needed to calculate all contributing regions into boundary conditions in the $m^2 \to \infty$ limit

$\sim (m^2)^0$	$\sim (m^2)^{-\varepsilon}$	$\sim (m^2)^{-2\varepsilon}$
$1/m^2$	$\alpha_i \sim m^2$	$\alpha_i, \alpha_j \sim m^2$

- For each large parameter $\alpha_i \sim m^2$ we remove $\theta \implies$ additional IBP reduction of BC integrals possible
- Numerical solution of the DE system as a sequence of series expansions [Liu et al.'18][Chen et al.'22]

Unregularized singularities from IBP reduction

- Not all integrals appearing during IBP reduction are regulated dimensionally, initial integrals are regular

- Possible solution is to introduce additional regulator: $\mathrm{d}\Phi^{nn\bar{n}}_{f_1 f_2 f_3} \to \mathrm{d}\Phi^{nn\bar{n}}_{f_1 f_2 f_3} (k_1 \cdot n)^{\nu} (k_2 \cdot n)^{\nu} (k_3 \cdot \bar{n})^{\nu}$

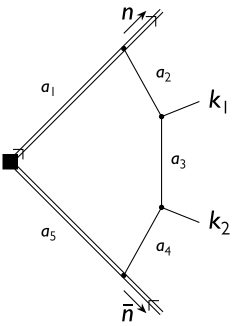
$$I \sim \int_0^1 \mathrm{d}z \frac{1}{z} z^{\varepsilon} z^{-\varepsilon} (\dots) \quad \longrightarrow \quad I(\nu) \sim \int_0^1 \mathrm{d}z \frac{1}{z^{1+\nu}} (\dots)$$

- IBP reduction with additional regulator possible but more complicated, especially for auxiliary integrals $J(m^2)$

- Special choice of MIs in ν-dependent IBP reduction with minimal number of $1/\nu$ divergent integrals $\bar{I}^{\nu}_{\alpha}$

$$S = \sum_{\alpha} c_{\alpha} I_{\alpha} \quad \rightarrow \quad S^{\nu} = \sum_{\alpha} c_{\alpha}(\nu) I^{\nu}_{\alpha} + \nu \sum_{\alpha} c_{\alpha}(\nu) \bar{I}^{\nu}_{\alpha}$$

RRV master integrals calculation



- Number of MIs after IBP reduction of both configurations in RRV case

$\delta\delta$	$\delta\theta + \theta\delta$	$\theta\theta$
8	13	17

- Direct integration, except pentagon loop part and box with $a_3 = 0$
- DE in auxiliary parameters for most complicated integrals

Original integrals from DE solution

Different strategy compared to RRR case: instead of $I = \lim_{z\to z_0} J(z)$ now $I = \int \mathrm{d}z\, J(z)$

RRV master integrals from differential equations

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- For $\delta\delta$ integrals we introduce auxiliary paremeter x and solve DE system $\partial_x J(x) = M(\varepsilon, x) J(x)$

$$I_{\delta\delta} = \int \mathrm{d}(k_1 \cdot k_2) f(k_1 \cdot k_2) = \int_0^1 \mathrm{d}x \int \mathrm{d}(k_1 \cdot k_2)\, \delta(k_1 \cdot k_2 - \frac{x}{2}) f(k_1 \cdot k_2) = \int_0^1 J(x)\mathrm{d}x$$

- For $\delta\theta$ and $\theta\delta$ we use integral representation for θ-function and solve DE system $\partial_z J(z) = M(\varepsilon, z) J(z)$

$$\theta(b-a) = \int_0^1 b\delta(zb-a)\mathrm{d}z, \quad I_{\delta\theta} = \int_0^1 J(z)\mathrm{d}z$$

- For $\theta\theta$ integrals PDE system in two variables z_1, z_2, no IBP reduction with θ-functions needed

$$I_{\theta\theta} = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 J(z_1, z_2)$$

Differential equations in canonical form

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- For all auxiliary integrals it is possible to find alternative basis of integrals, such ε dependence of the DE system matrix factorizes completely: $M(\varepsilon) \to \varepsilon A$
- Straightforward solution for integrals in canonicaal basis in terms of GPLs
- Simpler boundary conditions fixing due to known general form of expansion near singular points

$$g(z) = z^{a_1+b_1\varepsilon}\left(c_1 + \mathcal{O}(z)\right) + z^{a_2+b_2\varepsilon}\left(c_2 + \mathcal{O}(z)\right) + \ldots$$

- Construction of subtraction terms to remove endpoint singularities in final integration

$$\int_0^1 J(z)\mathrm{d}z = \int_0^1 \underbrace{\left[J(z) - z^{a_i+b_i\varepsilon} j_0(z) - (1-z)^{a_k+b_k\varepsilon} j_1(z)\right]}_{\varepsilon-\text{expanded}} \mathrm{d}z + \int_0^1 \underbrace{\left(z^{a_i+b_i\varepsilon} j_0(z) - (1-z)^{a_k+b_k\varepsilon} j_1(z)\right)}_{\varepsilon-\text{exact}} \mathrm{d}z$$

Conclusion

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- Triple-real part
  - Same hemisphere result is known, and many new techniques have developed - now successfully applied in the more complicated calculation of different hemispheres contribution
  - For different hemispheres case parts free from $1/k_{123}^2$ denominator are calculated
  - Calculated all boundaries and DEs solved for auxiliary integrals for most complicated part with $1/k_{123}^2$ denominator
  - Careful analysis of contributions from integral with potentially unregularized divergencies is ongoing
- Real-real-virtual part
  - For same hemisphere contribution all needed integrals are calculated, checks of obtained result are in progress
  - Calculating remaining $\theta\theta$ integrals not entering the same-hemisphere part is in progress

Thank you for attention!