Universität Siegen

Naturwissenschaftlich-Technische Fakultät

Sum Rules for Beyond the Standard Model

Young Scientists Meeting of the CRC TRR 257

Based on a work in progress in collaboration with Alex Lenz and Matthew Black

Zachary Wüthrich - 16.10.2023

Outline

Goal: Determine non-perturbative effects in B-lifetimes

- Theoretical Foundations
- Heavy Quark Effective Theory
- Sum Rules

Theoretical Foundations

B-Meson Mixing and Lifetimes

Standard Formalism

- Hamiltonian describing meson/anti-meson system:

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \equiv \begin{pmatrix} \langle B_q|H|B_q\rangle & \langle B_q|H|\overline{B_q}\rangle \\ \langle\overline{B_q}|H|B_q\rangle & \langle\overline{B_q}|H|\overline{B_q}\rangle \end{pmatrix} \equiv \hat{M} - \frac{i}{2}\hat{\Gamma}$$

- Schrödinger equation:

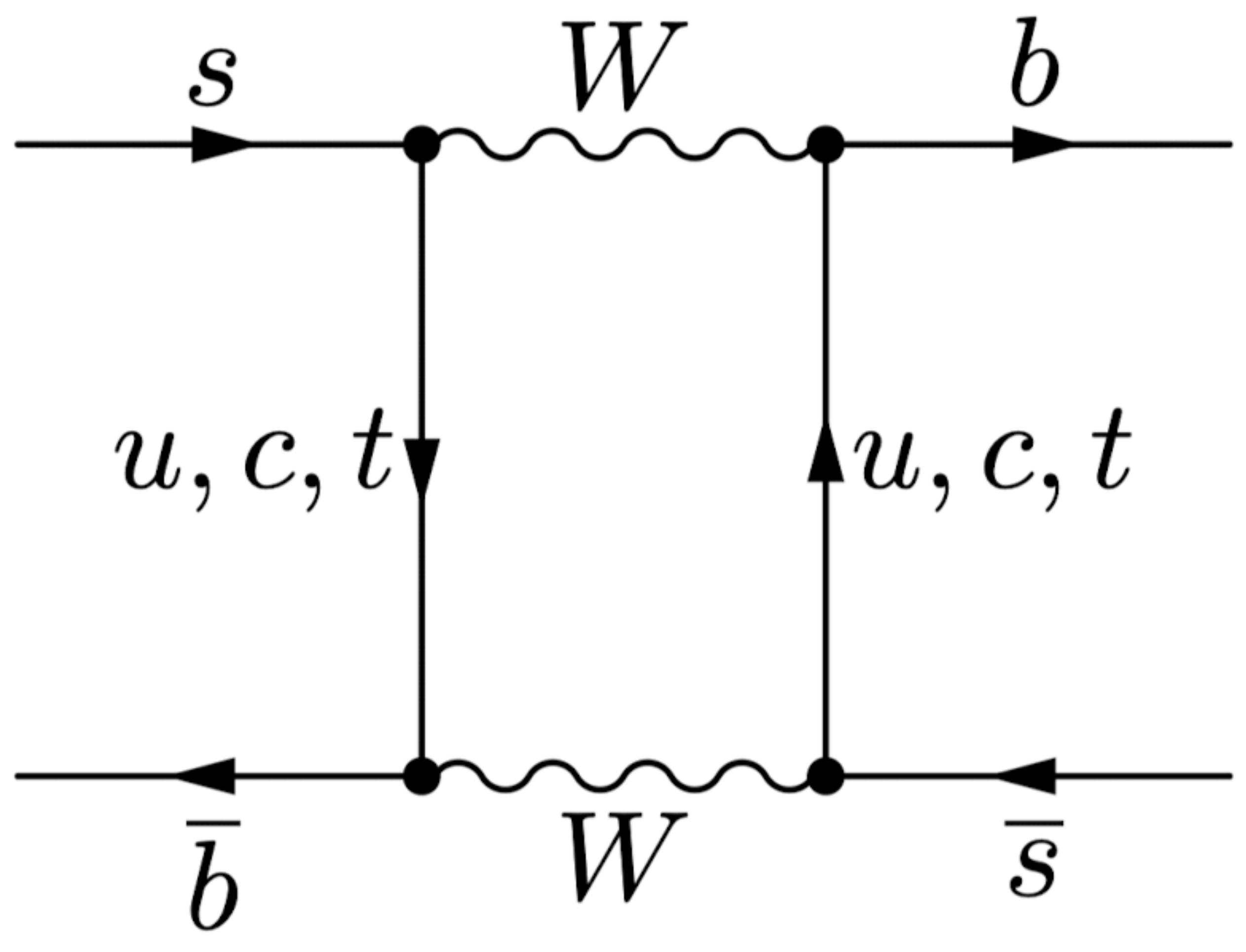
$$i\frac{d}{dt}\begin{pmatrix} |B_q(t)\rangle \\ |\overline{B_q}(t)\rangle \end{pmatrix} = \left(\hat{M}^q - \frac{i}{2}\hat{\Gamma}^q\right)\begin{pmatrix} |B_q(t)\rangle \\ |\overline{B_q}(t)\rangle \end{pmatrix}$$

- Calculate observables:

$$\Delta M_q \equiv M_H^q - M_L^q,\ \Delta\Gamma_q \equiv \Gamma_H^q - \Gamma_L^q$$

B-Meson Mixing and Lifetimes

Standard Model Case



- Diagram leading to B-Mixing:
  - GIM suppressed
- Leads to an effective Hamiltonian:

$$\mathcal{H}_{eff}^{\Delta B=2} = C_1 Q_1 + \text{ h.c.}, \qquad Q_1 = \bar{b}_i \gamma_\mu \frac{(1-\gamma^5)}{2} q_i \bar{b}_j \gamma^\mu \frac{(1-\gamma^5)}{2} q_j$$

- Apply Vacuum Insertion Approximation (VIA) and the correction to VIA is given by the Bag parameter $B_1(\mu)$

$$\langle \bar{B}_s | Q_1 | B_s \rangle = \left(2 + \frac{2}{N_c}\right) M_{B_s}^2 f_{B_s}^2 B_1(\mu)$$

Vacuum Insertion Approximation

And the Bag Parameter

- Input a complete set of states, but assume the main contribution comes from inserting the vacuum

$$\langle \overline{B} | Q(\mu) | B \rangle = \langle \overline{B} \big| J^1_\mu \big| 0 \rangle \cdot \langle 0 \big| J^{2,\mu} \big| B \rangle B(\mu) = A_Q f_B^2 M_B^2 B(\mu)$$

$$\left\langle 0 \middle| \bar{b} \gamma^\mu \gamma^5 q \middle| B(p) \right\rangle = -i f_B p^\mu$$

- The Bag Parameter parametrizes how good this approximation is
  - $B(\mu) = 1$ would imply the VIA is exact

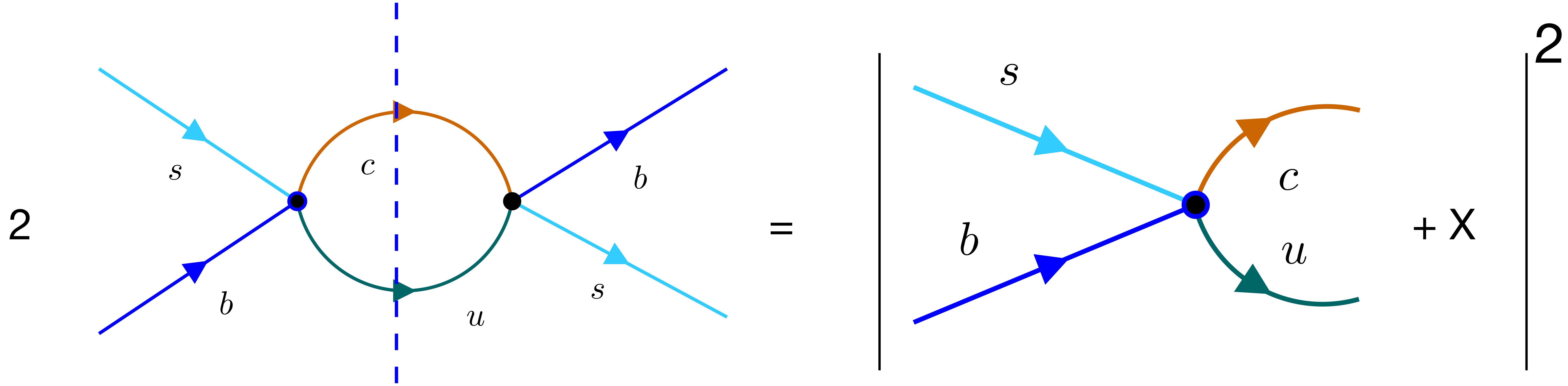
Optical Theorem

Inclusive Calculations

$$S_{fi} \equiv \langle f|S|i\rangle = \delta_{fi} + iT_{fi}$$

$$T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i)\mathcal{M}_{fi}$$

$$2\,\mathrm{Im}\,\mathcal{M}_{ii} = \sum_n \int_n (2\pi)^4 \delta^{(4)}\left(\sum_{j=1}^{n} p_j - p_i\right) |\mathcal{M}_{ni}|^2$$

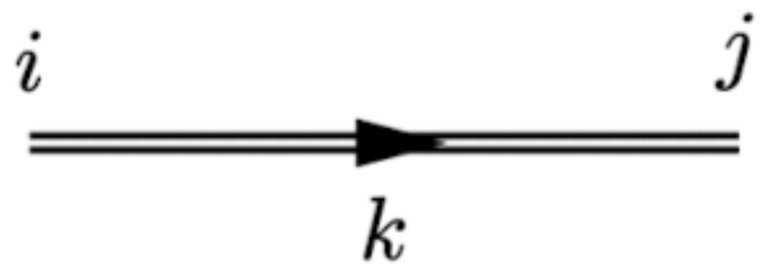


Heavy Quark Effective Theory (HQET)

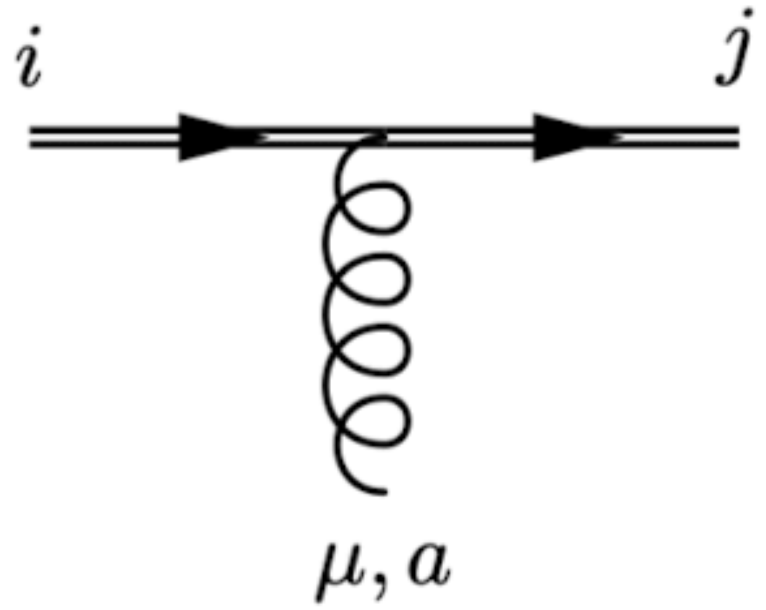
HQET

The Basics

- Expansion in the <u>heavy quark mass</u>: $\Lambda_{QCD} \ll m_Q$
- Parametrize heavy quark momentum: $p_Q^\mu = p^\mu + m_Q v^\mu$
- Parametrize heavy quark field: $Q(x) = e^{-im_Q v\cdot x} h(x) + \mathcal{O}\left(\frac{1}{m_Q}\right)$
- Basic **Feynman rules**:
  - Propagator: $$i\frac{\not{p}_Q + m_Q}{p_Q^2 - m_Q^2} = i\frac{\not{p} + m_Q(1+\not{v})}{p^2 + 2m_Q p\cdot v}, \\ = i\frac{(1+\not{v})}{2\omega} + \mathcal{O}\left(\frac{1}{m_Q}\right)$$



$$i\,\delta^{ij}\,\frac{(1+\not{v})}{2(k\cdot v)}$$



$$i\,g\,t^a_{ij}\,v^\mu$$

Heavy Quark Expansion

The Total Decay Rate

- Using the **optical theorem**:

$$\Gamma(B) = \frac{1}{2M_B}\langle B|\mathcal{T}|B\rangle$$

$$\mathcal{T} \equiv \mathrm{Im}\, i \int d^4x \, \mathrm{T}\{\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\}$$

- Separating the short- and long-distance dynamics with an **OPE**:

$$\Gamma(B) = \Gamma_3\langle\mathcal{O}_3\rangle + \Gamma_5\frac{\langle\mathcal{O}_5\rangle}{m_b^2} + \Gamma_6\frac{\langle\mathcal{O}_6\rangle}{m_b^3} + \ldots + 16\pi^2\left[\tilde{\Gamma}_6\frac{\left\langle\tilde{\mathcal{O}}_6\right\rangle}{m_b^3} + \tilde{\Gamma}_7\frac{\left\langle\tilde{\mathcal{O}}_7\right\rangle}{m_b^4} + \ldots\right]$$

$$\Gamma_{12}^s = \frac{\Lambda^3}{m_b^3}\left[\Gamma_3^{s(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{s(1)} + \ldots\right] + \frac{\Lambda^4}{m_b^4}\left[\Gamma_4^{s(0)} + \ldots\right] + \ldots$$

QCD-HQET Matching for Mixing

Matching Operators

QCD

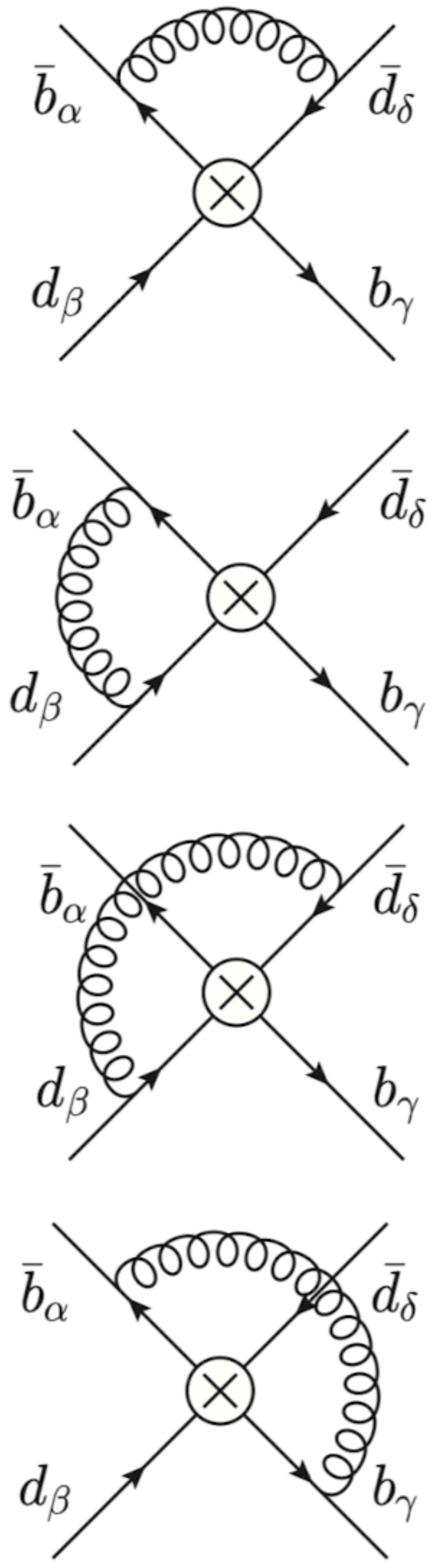
$$Q_1 = \bar{b}_i \gamma_\mu (1-\gamma^5) q_i \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

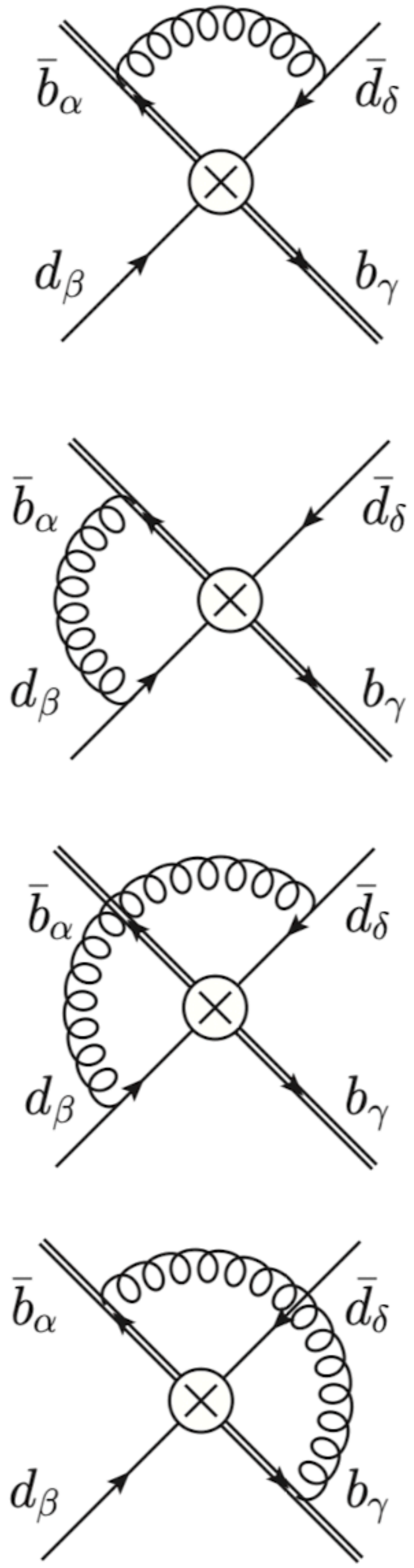
$$Q_2 = \bar{b}_i (1-\gamma^5) q_i \bar{b}_j (1-\gamma^5) q_j$$

$$Q_3 = \bar{b}_i (1-\gamma^5) q_j \bar{b}_j (1-\gamma^5) q_i$$

$$Q_4 = \bar{b}_i (1-\gamma^5) q_i \bar{b}_j (1+\gamma^5) q_j$$

$$Q_5 = \bar{b}_i (1-\gamma^5) q_j \bar{b}_j (1+\gamma^5) q_i$$





HQET

$$\mathbf{Q}_1 = \bar{h}_i^{\{(+)} \gamma_\mu (1-\gamma^5) q_i \bar{h}_j^{(-)\}} \gamma^\mu (1-\gamma^5) q_j$$

$$\mathbf{Q}_2 = \bar{h}_i^{\{(+)} (1-\gamma^5) q_i \bar{h}_j^{(-)\}} (1-\gamma^5) q_j$$

$$\mathbf{Q}_4 = \bar{h}_i^{\{(+)} (1-\gamma^5) q_i \bar{h}_j^{(-)\}} (1+\gamma^5) q_j$$

$$\mathbf{Q}_5 = \bar{h}_i^{\{(+)} (1-\gamma^5) q_j \bar{h}_j^{(-)\}} (1+\gamma^5) q_i$$

QCD-HQET Matching for Lifetimes

Matching Operators

QCD

$$Q_1^q = \bar{b}\gamma_\mu(1-\gamma^5)q\ \bar{q}\gamma^\mu(1-\gamma^5)b,$$
$$Q_2^q = \bar{b}(1-\gamma^5)q\ \bar{q}(1+\gamma^5)b,$$
$$T_1^q = \bar{b}\gamma_\mu(1-\gamma^5)T^A q\ \bar{q}\gamma^\mu(1-\gamma^5)T^A b,$$
$$T_2^q = \bar{b}(1-\gamma^5)T^A q\ \bar{q}(1+\gamma^5)T^A b.$$

HQET

$$\tilde{Q}_1^q = \bar{h}\gamma_\mu(1-\gamma^5)q\ \bar{q}\gamma^\mu(1-\gamma^5)h,$$
$$\tilde{Q}_2^q = \bar{h}(1-\gamma^5)q\ \bar{q}(1+\gamma^5)h,$$
$$\tilde{T}_1^q = \bar{h}\gamma_\mu(1-\gamma^5)T^A q\ \bar{q}\gamma^\mu(1-\gamma^5)T^A h$$
$$\tilde{T}_2^q = \bar{h}(1-\gamma^5)T^A q\ \bar{q}(1+\gamma^5)T^A h.$$

Must extend this to include all possible BSM operators

Sum Rules

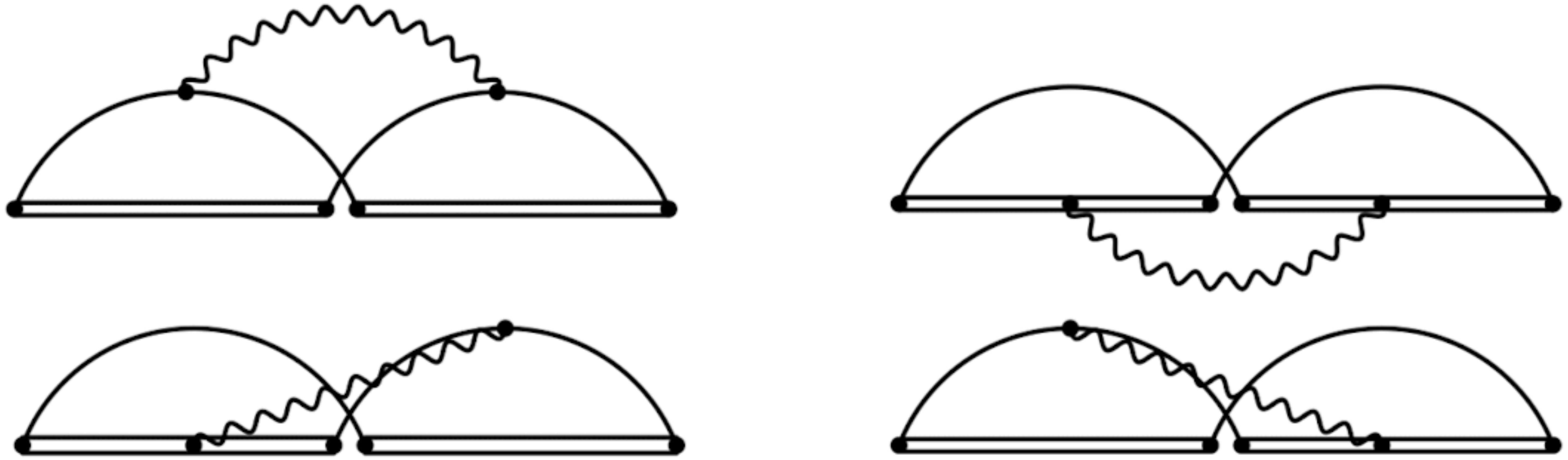
HQET Sum Rules

Three-point Correlator

$$K_{\mathbf{Q}}(\omega_1, \omega_2) = \int d^d x_1 d^d x_2 e^{ip_1 \cdot x_1 - ip_2 \cdot x_2} \left\langle 0 \right| \mathrm{T}\left[\tilde{j}_+(x_2)\mathbf{Q}(0)\tilde{j}_-(x_1)\right] \left| 0 \right\rangle$$

$$\tilde{j}_+ = \bar{q}\gamma^5 h^{(+)} \qquad \tilde{j}_- = \bar{q}\gamma^5 h^{(-)}$$

This depends on the heavy quark's residual energy: $\omega = p \cdot v$



Nonfactorizable diagrams

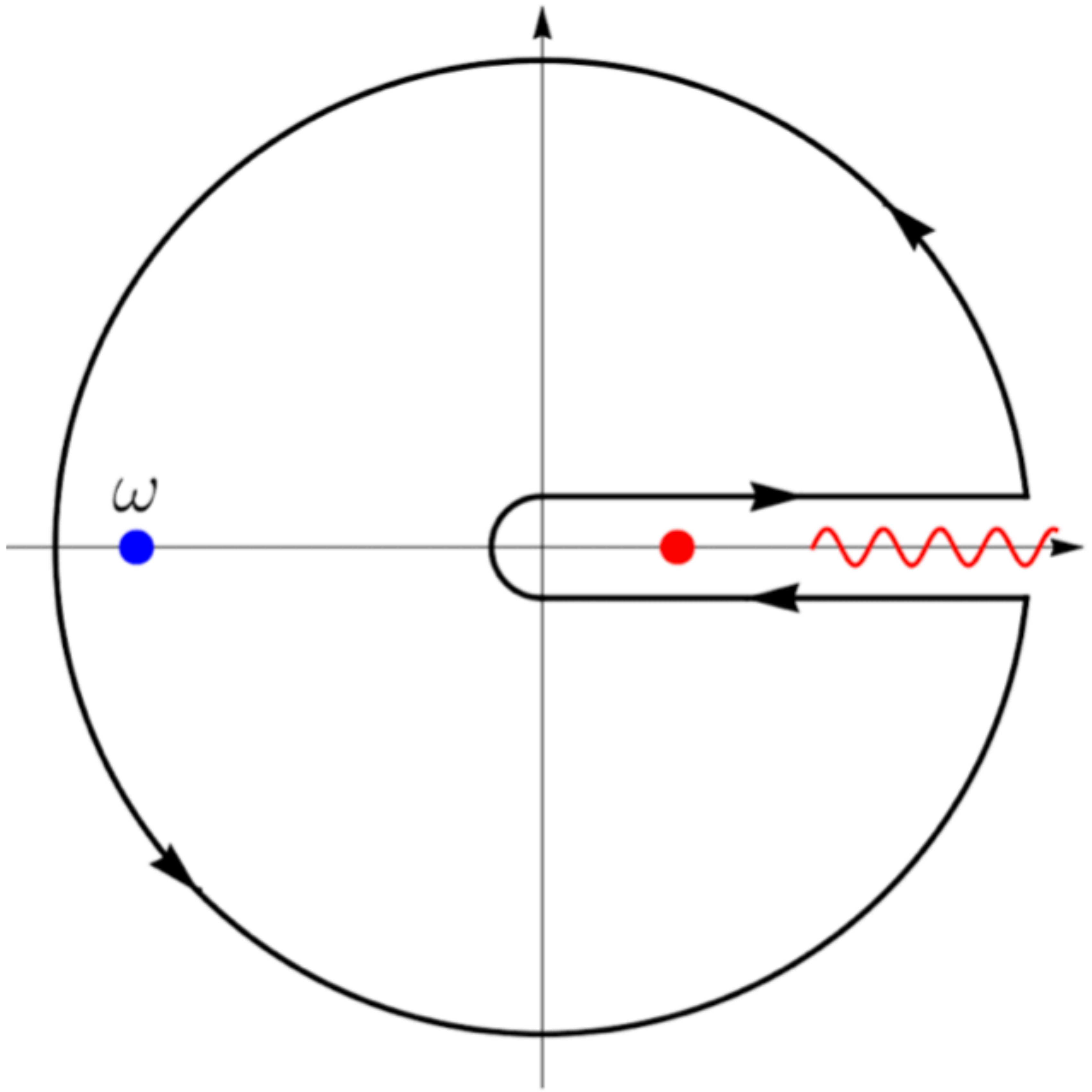
HQET Sum Rules

Perturbative vs Non-perturbative

What is calculable?

If the heavy quark has a large negative residual energy, the light quarks must have very large momenta.

QCD is perturbative at high energies, so the **large negative residual energy limit** is calculable in perturbation theory.



Contour chosen to isolate the pole at ω. The red marker indicates the lowest single state hadronic resonance. The squiggly line shows the continuum of multi-particle and excited states.

HQET Sum Rules

Analyticity

We relate the perturbative part of the correlator to the non-perturbative bound states via a dispersion relation:

$$
\begin{aligned}
\Pi(\omega) &= \frac{1}{2\pi i}\oint_C ds\,\frac{\Pi(s)}{(s-\omega)} \\
&= \frac{1}{2\pi i}\lim_{\epsilon\to 0}\int_0^\infty ds\,\frac{\Pi(s+i\epsilon)-\Pi(s-i\epsilon)}{(s-\omega)} + \frac{1}{2\pi i}\int_R ds\,\frac{\Pi(s)}{(s-\omega)} \\
&= \int_0^\infty ds\,\frac{\rho(s)}{(s-\omega)}
\end{aligned}
$$

Dispersion Relation: $K_{\mathbf{Q}}(\omega_1,\omega_2) = \int_0^\infty d\eta_1 d\eta_2\,\frac{\rho_{\mathbf{Q}}(\eta_1,\eta_2)}{(\eta_1-\omega_1)(\eta_2-\omega_2)} + [\,\text{subtraction terms}\,]$

HQET Sum Rules

Hadronic Spectral Function

$$K_{\mathbf{Q}}(\omega_1, \omega_2) = \int_0^\infty d\eta_1 d\eta_2 \frac{\rho_{\mathbf{Q}}(\eta_1, \eta_2)}{(\eta_1 - \omega_1)(\eta_2 - \omega_2)} + [\,\text{subtraction terms}\,]$$

$$\rho_{\mathbf{Q}}^{\text{had}}(\omega_1, \omega_2) = F^2(\mu)\langle \mathbf{Q}(\mu)\rangle \delta\left(\omega_1 - \bar{\Lambda}\right)\delta\left(\omega_2 - \bar{\Lambda}\right) + \rho_{\mathbf{Q}}^{\text{cont}}(\omega_1, \omega_2)$$

$$\bar{\Lambda} = M_B - m_b$$

Quark-Hadron duality: $\rho_{\mathbf{Q}}^{\text{cont}}(\omega_1, \omega_2) = \rho_{\mathbf{Q}}^{\text{OPE}}(\omega_1, \omega_2)[1 - \theta(\omega_c - \omega_1)\theta(\omega_c - \omega_2)]$

Borel Sum Rule:

$$F^2(\mu)\langle \mathbf{Q}(\mu)\rangle e^{-\frac{\bar{\Lambda}}{t_1} - \frac{\bar{\Lambda}}{t_2}} = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1} - \frac{\omega_2}{t_2}} \rho_{\mathbf{Q}}^{\text{OPE}}(\omega_1, \omega_2)$$

HQET Sum Rules

Borel Transform

$$\Pi(t) \equiv \mathcal{B}_t \Pi(\omega) = \lim_{\substack{-\omega, n \to \infty \\ -\omega/n \to t}} \frac{(-\omega)^{n+1}}{n!} \left[\frac{d}{d\omega} \right]^n \Pi(\omega)$$

$$\mathcal{B}_t \left[\omega^i \right] = 0$$

Removes subtraction terms

$$\mathcal{B}_t \left[\frac{1}{(s-\omega)^i} \right] = \lim_{\substack{-\omega, n \to \infty \\ -\omega/n \to t}} \frac{(-\omega)^{n+1}}{n!} \left[\frac{d}{d\omega} \right]^n (s-\omega)^{-i}$$

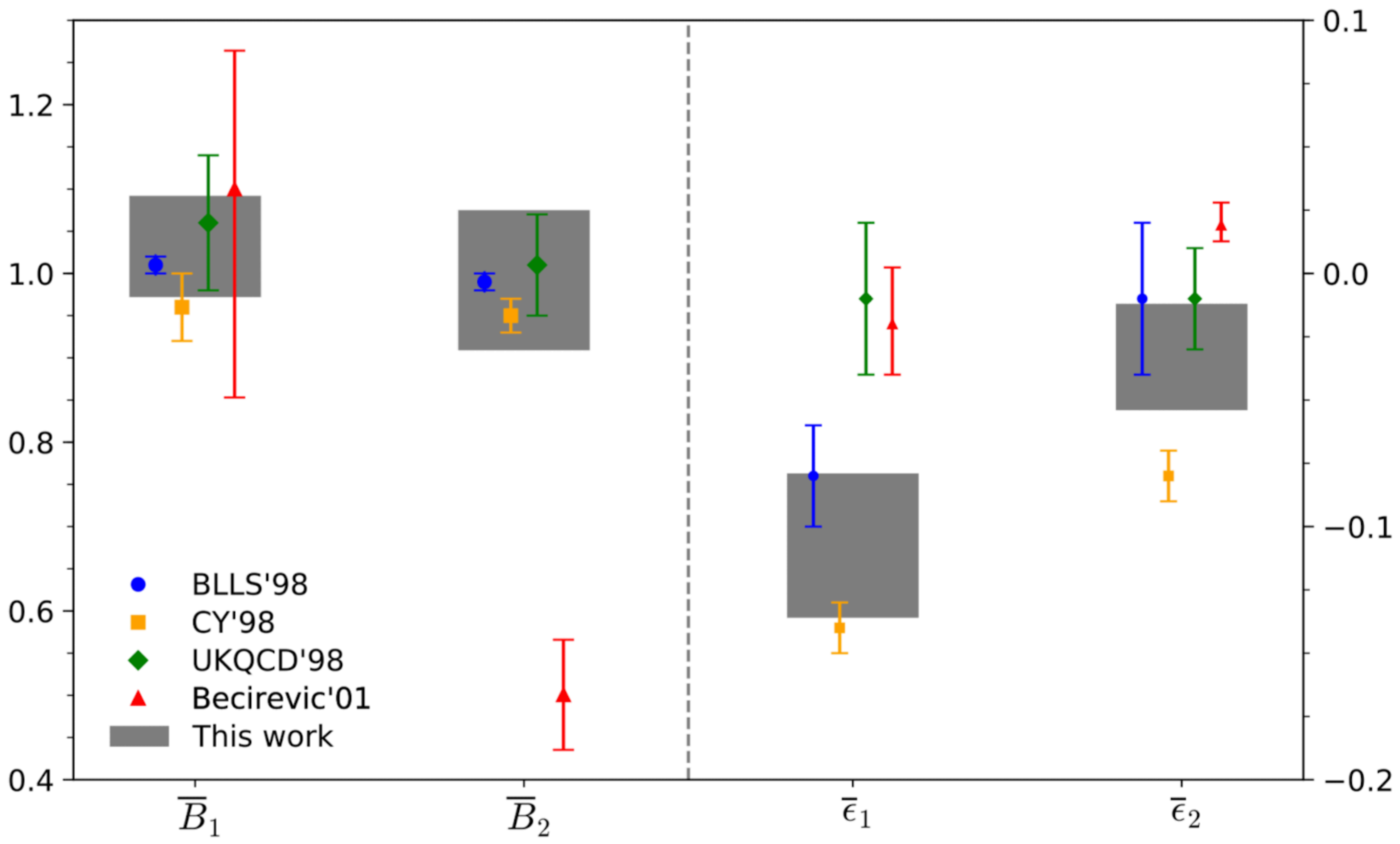
$$= \lim_{n \to \infty} \frac{1}{(i-1)! t^{(i-1)}} \frac{(i+n-1)!}{(n-1)! n^i} \left(1 + \frac{s}{nt} \right)^{-(i+n)}$$

$$= \frac{e^{\frac{-s}{t}}}{(i-1)! t^{(i-1)}}$$

Exponential suppresses the continuum tail, improving the QHD assumption

Previous Results in Lifetimes

SM Operators Only



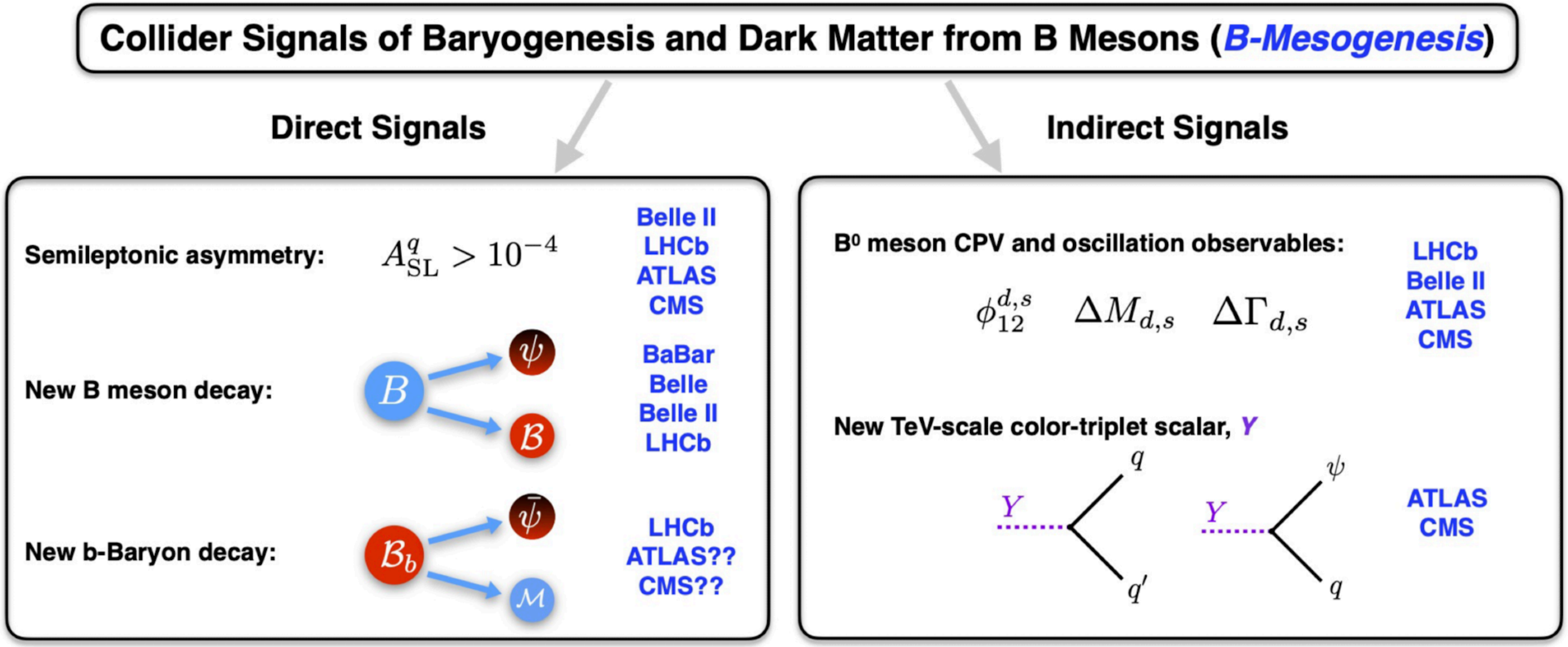
Next, perform full BSM analysis including all BSM operators

M. Kirk, A. Lenz, and T. Rauh, *Dimension-six matrix elements for meson mixing and lifetimes from sum rules*, JHEP **12** (2017) 068, arXiv:1711.02100.

Where is this needed?

B-Mesogenesis

- Will be presented tomorrow by Ali



G. Alonso-Álvarez, G. Elor and M. Escudero, Phys. Rev. D **104**, no.3, 035028 (2021) doi:10.1103/PhysRevD.104.035028 [arXiv:2101.02706 [hep-ph]].

Thank you!

Backup Slides

B-Meson Mixing and Lifetimes

Observable parameters

$$\begin{aligned}\Delta M_s &\equiv M_H^s - M_L^s \\ &= 2|M_{12}^s|\left(1 - \frac{|\Gamma_{12}^s|\sin^2\phi_{12}^s}{8|M_{12}^s|^2} + \ldots\right) \\ &\approx 2|M_{12}^s|,\end{aligned}$$

$$\phi_{12}^s \equiv \left(-\frac{M_{12}^s}{\Gamma_{12}^s}\right).$$

$$\begin{aligned}\Delta\Gamma_s &\equiv \Gamma_H^s - \Gamma_L^s \\ &= 2|\Gamma_{12}^s|\cos\phi_{12}^s\left(1 - \frac{|\Gamma_{12}^s|\sin^2\phi_{12}^s}{8|M_{12}^s|^2} + \ldots\right) \\ &\approx 2|\Gamma_{12}^s|\cos\phi_{12}^s,\end{aligned}$$

HQET

Hadronic Matrix Elements

$$|B(p)\rangle = \sqrt{2M_B}|\mathbf{B}(v)\rangle + \mathcal{O}(1/m_b),$$

$$\left\langle \mathbf{B}\left(v'\right) \mid \mathbf{B}(v)\right\rangle = \frac{v^0}{M_B^3}(2\pi)^3\delta^{(3)}\left(\mathbf{v}' - \mathbf{v}\right).$$

$$f_B = \sqrt{\frac{2}{M_B}}C(\mu)F(\mu) + \mathcal{O}(1/m_b)$$

$$C(\mu) = 1 - 2C_F\frac{\alpha_s(\mu)}{4\pi} + \mathcal{O}\left(\alpha_s^2\right)$$

QCD-HQET Matching For Mixing

Matching Bag Parameters

QCD	HQET
$\langle Q(\mu)\rangle = A_Q f_B^2 M_B^2 B_Q(\mu)$	$\langle \mathbf{Q}(\mu)\rangle = A_{\mathbf{Q}} F^2(\mu) B_{\mathbf{Q}}(\mu)$
$\left\langle 0 \middle| \bar{b}\gamma^\mu\gamma^5 q \middle| B(p)\right\rangle = -i f_B p^\mu$	$\left\langle 0 \middle| \bar{h}^{(-)}\gamma^\mu\gamma^5 q \middle| \mathbf{B}(v)\right\rangle = -iF(\mu) v^\mu$

$$\langle Q_i\rangle(\mu) = \sum C_{Q_i\mathbf{Q_j}}(\mu)\langle \mathbf{Q_j}\rangle(\mu) + \mathcal{O}\left(\frac{1}{m_b}\right)$$

$$B_{Q_i}(\mu) = \sum_j \frac{A_{\mathbf{Q}_j}}{A_{Q_i}} \frac{C_{Q_i\mathbf{Q}_j}(\mu)}{C^2(\mu)} B_{\mathbf{Q}_j}(\mu) + \mathcal{O}(1/m_b)$$

HQET Sum Rules

Nonfactorizable Contribution

$$K_{\tilde{Q}}^{\text{pert}}(\omega_1,\omega_2) = K_{\tilde{Q}}^{(0)}(\omega_1,\omega_2) + \frac{\alpha_s}{4\pi} K_{\tilde{Q}}^{(1)}(\omega_1,\omega_2) + \dots$$

$$\rho_{\tilde{Q}_i}^{\text{pert}}(\omega_1,\omega_2) = A_{\tilde{Q}_i}\rho_\Pi(\omega_1)\rho_\Pi(\omega_2) + \Delta\rho_{\tilde{Q}_i}$$

$$\Delta B_{\tilde{Q}_i} = \frac{1}{A_{\tilde{Q}_i} F(\mu)^4} \int_0^{\omega_c} d\omega_1 d\omega_2 e^{\frac{\bar{\Lambda}-\omega_1}{t_1}+\frac{\bar{\Lambda}-\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1,\omega_2)$$

$$= \frac{1}{A_{\tilde{Q}_i}} \frac{\int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1,\omega_2)}{\left(\int_0^{\omega_c} d\omega_1 e^{-\frac{\omega_1}{t_1}}\rho_\Pi(\omega_1)\right)\left(\int_0^{\omega_c} d\omega_2 e^{-\frac{\omega_2}{t_2}}\rho_\Pi(\omega_2)\right)}$$

HQET Sum Rules

Utilizing a Weight Function

$$F^4(\mu)e^{-\frac{\bar{\Lambda}}{t_1}-\frac{\bar{\Lambda}}{t_2}}w(\bar{\Lambda},\bar{\Lambda}) = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}} w(\omega_1,\omega_2)\rho_\Pi(\omega_1)\rho_\Pi(\omega_2) + \ldots$$

$$w_{\tilde{Q}_i}(\omega_1,\omega_2) = \frac{\Delta\rho^{\text{pert}}_{\tilde{Q}_i}(\omega_1,\omega_2)}{\rho^{\text{pert}}_\Pi(\omega_1)\rho^{\text{pert}}_\Pi(\omega_2)} = \frac{C_F}{N_c}\frac{\alpha_s}{4\pi} r_{\tilde{Q}_i}(x, L_\omega)$$

$$\Delta B^{\text{pert}}_{\tilde{Q}_i}(\mu_\rho) = \frac{C_F}{N_c A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} r_{\tilde{Q}_i}\left(1, \log\frac{\mu_\rho^2}{4\bar{\Lambda}^2}\right)$$

HQET Sum Rules

Leading Order Eye Contraction

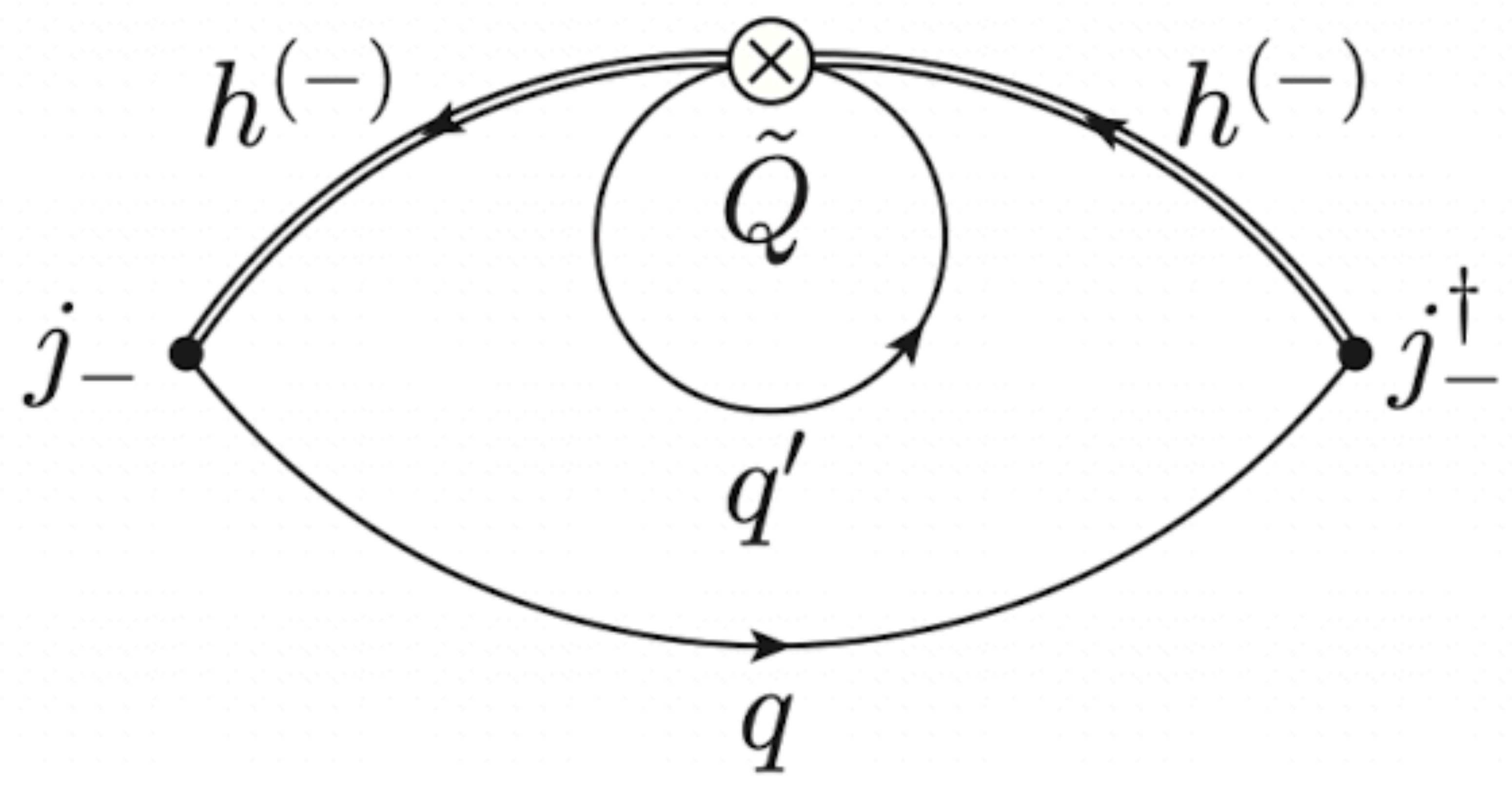
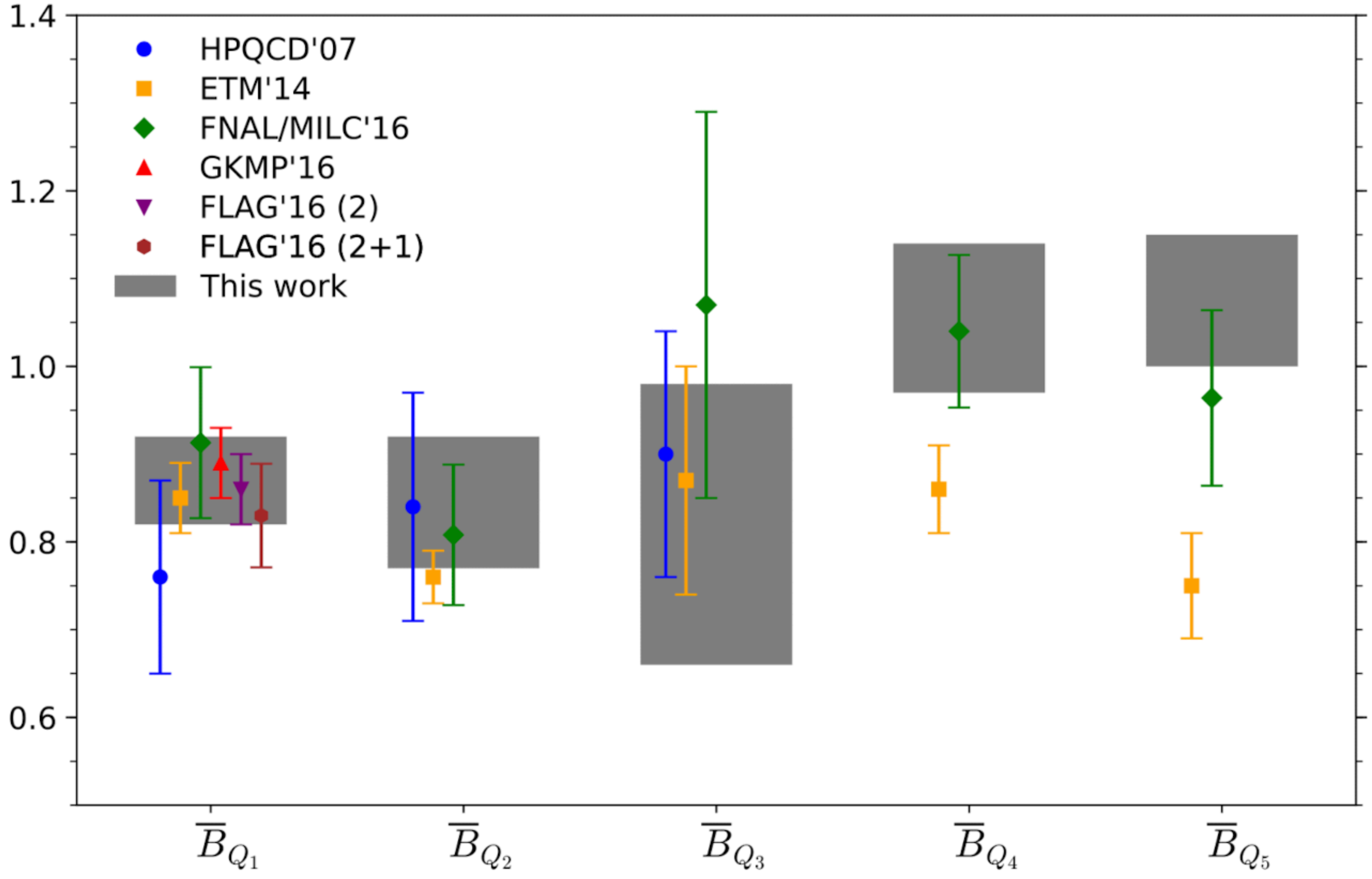


Figure 5: Leading order eye contraction.

Previous Results in Mixing

Full SUSY Basis



M. Kirk, A. Lenz, and T. Rauh, *Dimension-six matrix elements for meson mixing and lifetimes from sum rules*, JHEP **12** (2017) 068, `arXiv:1711.02100`.

Previous Results

Uncertainties

- Variation of $\bar{\Lambda} = M_B - m_b$
- Percent uncertainty for condensate contributions
- Uncertainty in NNLO α_s^2 contributions in the spectral density
- Higher order 1/m_b corrections in the VIA
- Higher order QCD-HQET matching corrections and corrections to the RGE
  - Error estimate from varying renormalization scale

QCD-HQET Matching

Redundant and Excluded Operators

QCD

$$Q_1 = \bar{b}_i \gamma_\mu (1-\gamma^5) q_i \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

$$\widetilde{Q}_1 = \bar{b}_i \gamma_\mu (1+\gamma^5) q_i \bar{b}_j \gamma^\mu (1+\gamma^5) q_j$$

$$Q_2 = \bar{b}_i (1-\gamma^5) q_i \bar{b}_j (1-\gamma^5) q_j$$

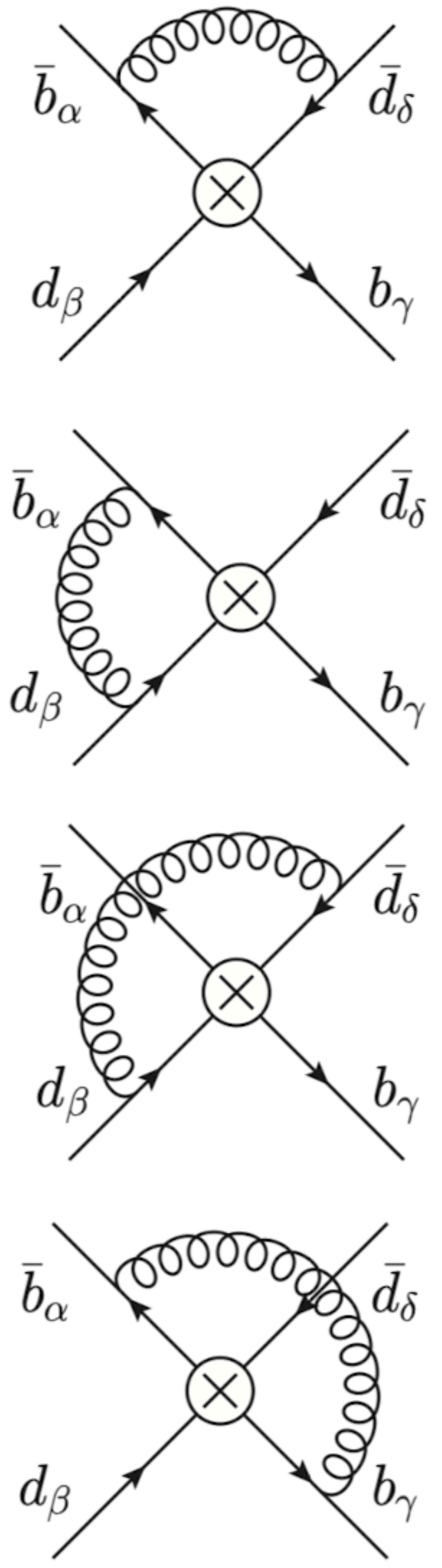
$$\widetilde{Q}_2 = \bar{b}_i (1+\gamma^5) q_i \bar{b}_j (1+\gamma^5) q_j$$

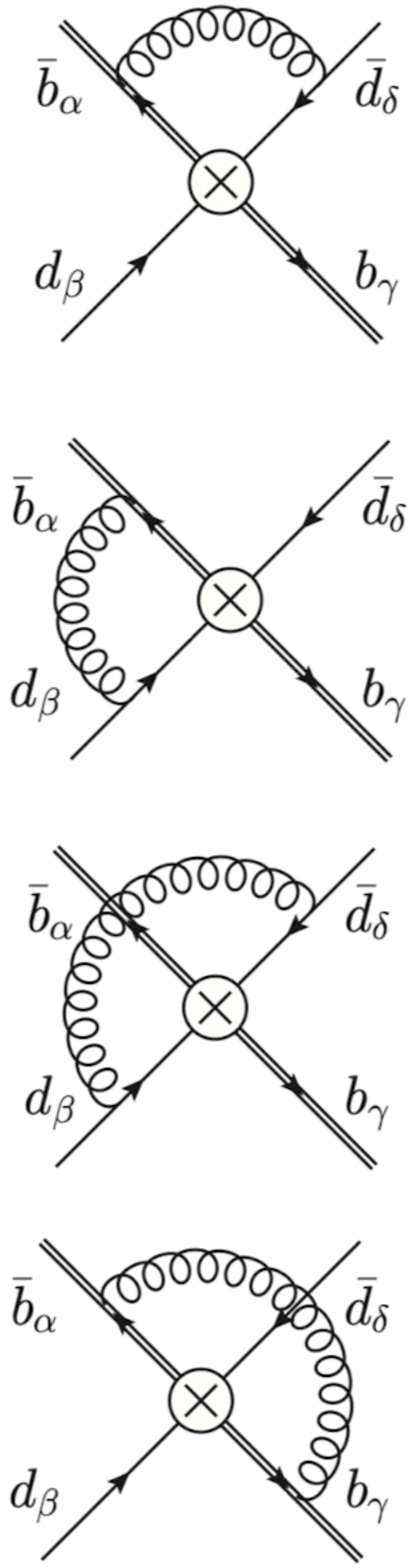
$$Q_3 = \bar{b}_i (1-\gamma^5) q_j \bar{b}_j (1-\gamma^5) q_i$$

$$\widetilde{Q}_3 = \bar{b}_i (1+\gamma^5) q_j \bar{b}_j (1+\gamma^5) q_i$$

$$Q_4 = \bar{b}_i (1-\gamma^5) q_i \bar{b}_j (1+\gamma^5) q_j$$

$$Q_5 = \bar{b}_i (1-\gamma^5) q_j \bar{b}_j (1+\gamma^5) q_i$$





HQET

$$\mathbf{Q}_1 = \bar{h}_i^{\{(+)} \gamma_\mu (1-\gamma^5) q_i \bar{h}_j^{(-)\}} \gamma^\mu (1-\gamma^5) q_j$$

$$\tilde{\mathbf{Q}}_1 = \bar{h}_i^{\{(+)} \gamma_\mu (1+\gamma^5) q_i \bar{h}_j^{(-)\}} \gamma^\mu (1+\gamma^5) q_j$$

$$\mathbf{Q}_2 = \bar{h}_i^{\{(+)} (1-\gamma^5) q_i \bar{h}_j^{(-)\}} (1-\gamma^5) q_j$$

$$\tilde{\mathbf{Q}}_2 = \bar{h}_i^{\{(+)} (1+\gamma^5) q_i \bar{h}_j^{(-)\}} (1+\gamma^5) q_j$$

$$\mathbf{Q}_4 = \bar{h}_i^{\{(+)} (1-\gamma^5) q_i \bar{h}_j^{(-)\}} (1+\gamma^5) q_j$$

$$\mathbf{Q}_5 = \bar{h}_i^{\{(+)} (1-\gamma^5) q_j \bar{h}_j^{(-)\}} (1+\gamma^5) q_i$$