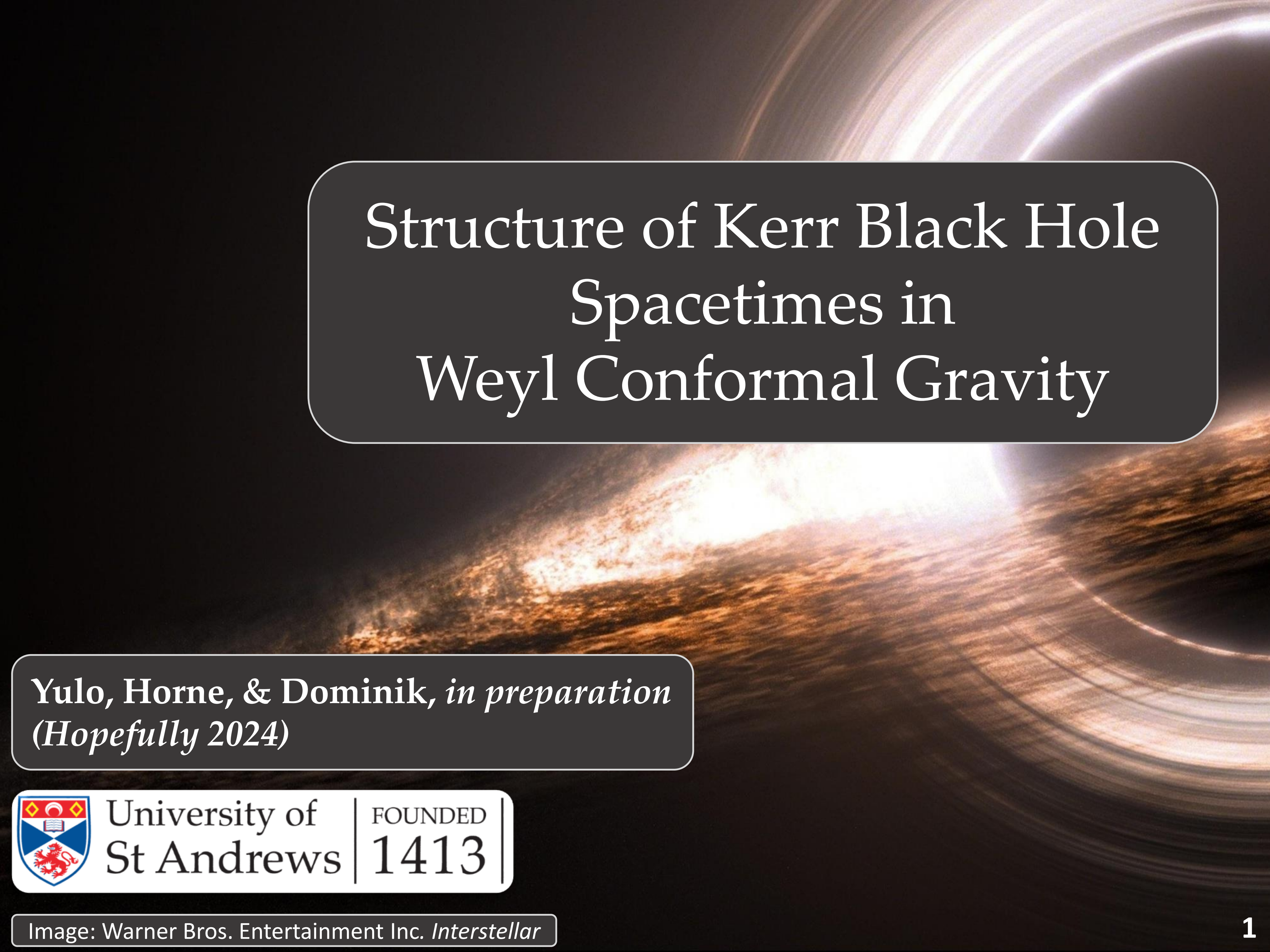


Structure of Kerr Black Hole Spacetimes in Weyl Conformal Gravity

Yulo, Horne, & Dominik, ***in preparation (Hopefully 2024)***

University of St Andrews | FOUNDED 1413

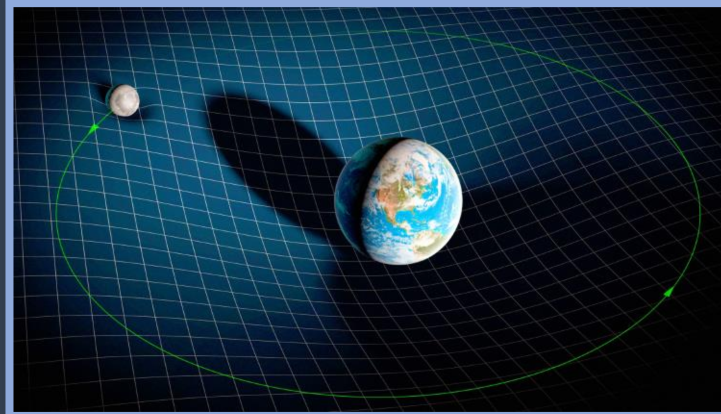
General Relativity

Einstein Field Equations

$$G_{\mu\nu} = 8\pi \, T_{\mu\nu}$$

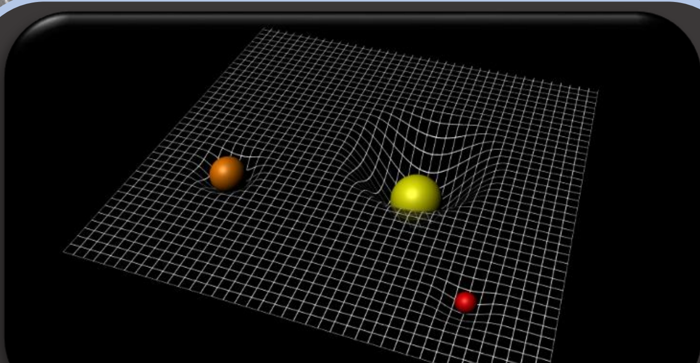
Einstein Curvature Tensor

Stress-Energy Tensor



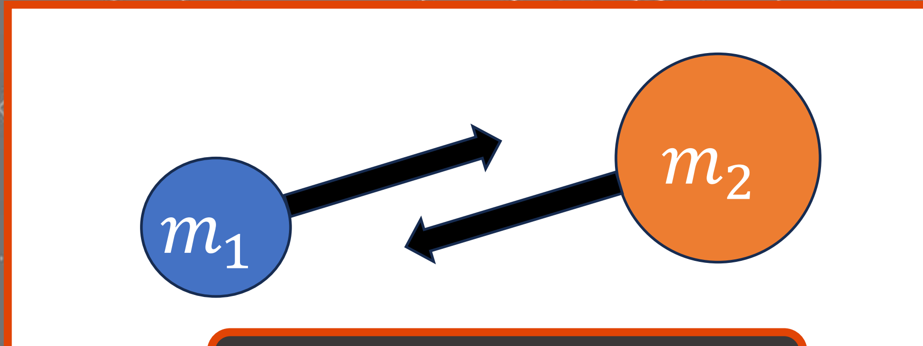
"Matter tells spacetime how to curve; spacetime tells matter how to move." – John Archibald Wheeler

Gravity is spacetime ($g_{\mu\nu}$) itself.



Spacetime Curvature

vs



Newtonian Forces

Image: Mark Garlick/Getty Images, ESA-C. Carreau

Problems with GR

Galactic Scales: Flat Galactic Rotation Curves

Add Dark Matter → $T_{\mu\nu}$

Cosmological Scales: Accelerating Expansion of the Universe

Add Dark Energy → $T_{\mu\nu}$

This is ΛCDM.

Issues

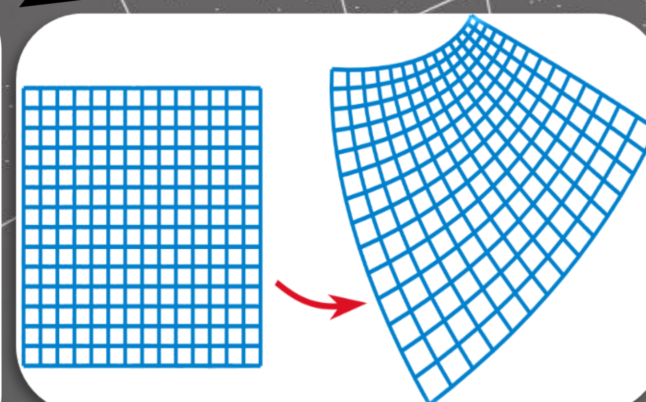
1. *Ad Hoc*
2. No direct evidence
3. Magnitude of Dark Energy is 120 orders of magnitude too low

Weyl Conformal Gravity (1918)

Invariances

	GR	CG
1. Coordinate Transformations: $g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x')$	✓	✓
2. Lorentz Transformations: $x^{\mu} \rightarrow \Lambda^{\mu}_{\nu}\, x^{\nu}$	✓	✓
3. Local Conformal Transformations: $g_{\mu\nu}(x) \rightarrow \widetilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x)$	✗	✓

$\Omega(x)$: Stretching Factor



CG Field Equations

$$W_{\mu\nu} = \frac{1}{4\alpha_g} T_{\mu\nu}$$

Bach Curvature Tensor → $W_{\mu\nu}$

Stress-Energy Tensor → $T_{\mu\nu}$

Gravitational Coupling Constant → α_g

GR Schwarzschild Metric $g_{\mu\nu}$

Static, spherically-symmetric point mass distribution

$$ds^2 = -B(r)\,dt^2 + B(r)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\;d\phi^2)$$

$$B(r) = 1 - \frac{2\beta}{r}$$

G = c = 1

Geometrized Mass

$$\beta = GM/c^2 \text{ (cm)}$$

CG Schwarzschild Metric $g_{\mu\nu}$

$$ds^2 = -\tilde{B}(r)\,dt^2 + \tilde{B}(r)^{-1} \ldots$$

Galactic Scales

Cosmological Scales

$$\tilde{B}(r) = 1 - \frac{\beta(2 - 3\beta\gamma)}{r} - 3\beta\gamma + \gamma r - \kappa r^2$$

Black Holes

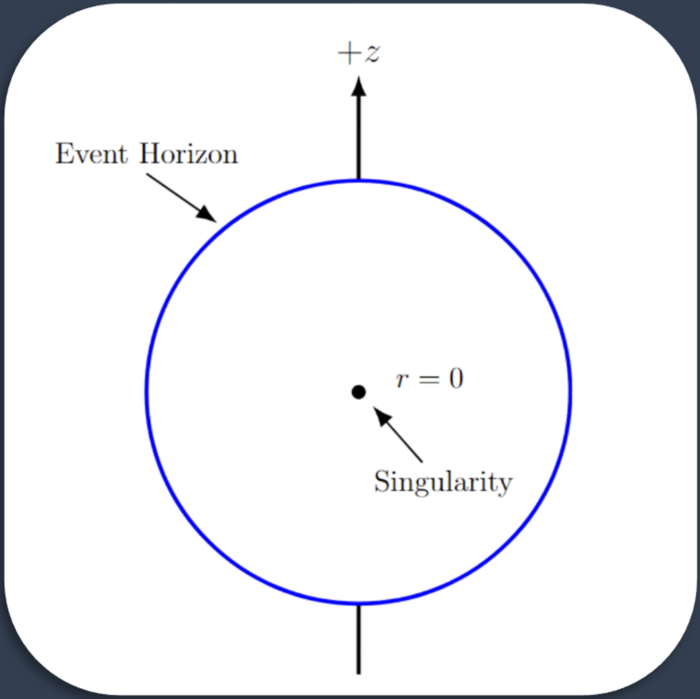
$$ds^2 = g_{tt}\,dt^2 + g_{rr}\,dr^2 + g_{\theta\theta}\,d\theta^2 + g_{\phi\phi}\,d\phi^2$$

Where $g_{rr} \rightarrow \infty$ ⇒ Event Horizon at $r^H = 2\,\beta$

GR Schwarzschild Geometry

One Event Horizon

Point Singularity : Infinite Curvature



GR Kerr Metric $g_{\mu\nu}$

Spin parameter: a (cm)

Stationary, rotating, axially-symmetric mass distribution

$$ds^2 = -\left(1 - \frac{2\beta r}{\rho^2}\right) dt^2 - \left(\frac{4\beta r a \sin^2\theta}{\rho^2}\right) dt\, d\phi + \left(\frac{\rho^2}{\Delta_r}\right) dr^2 + \rho^2\, d\theta^2 + \left(r^2 + a^2 + \frac{2\beta r a^2 \sin^2\theta}{\rho^2}\right) \sin^2\theta\, d\phi^2$$

$\rho^2 \equiv r^2 + a^2 \cos^2\theta$

$\Delta_r \equiv r^2 - 2\beta r + a^2$

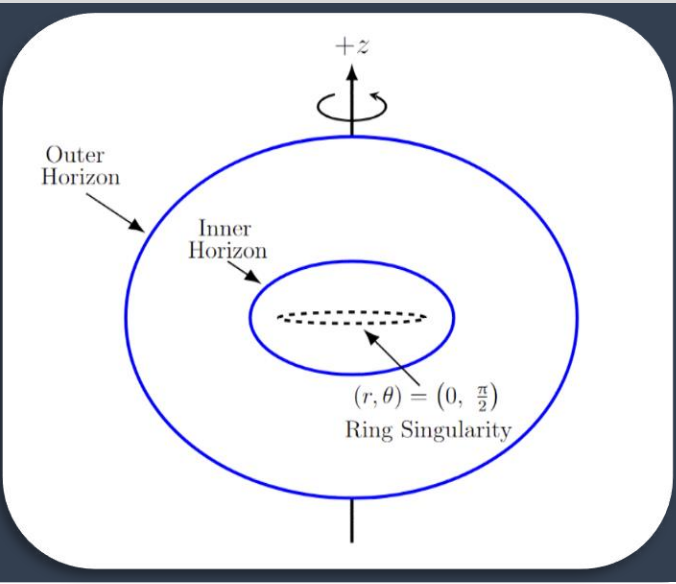
$$ds^2 = g_{tt}\, dt^2 + 2g_{t\phi} dt\, d\phi + g_{rr} dr^2 + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2$$

Two Horizons

Inner Cauchy Horizon

Outer Event Horizon

Ring Singularity



CG Kerr Metric $g_{\mu\nu}$

Stationary, rotating, axially-symmetric mass distribution

$$
\begin{aligned}
\mathrm{d}s^2 = &-\left(1 - \frac{2\widetilde{M}r}{\rho^2} - k\left(r^2 - a^2\cos^2\theta\right)\right)\,\mathrm{d}t^2 \\
&+ 2\left(\frac{-2\widetilde{M}ra\sin^2\theta + ka\left(a^2\left(r^2+a^2\right)\cos^4\theta - r^4\sin^2\theta\right)}{\rho^2}\right)\,\mathrm{d}t\,\mathrm{d}\phi \\
&+ \left(\frac{\rho^2}{\widetilde{\Delta}_r}\right)\mathrm{d}r^2 + \left(\frac{\rho^2}{\widetilde{\Delta}_\theta}\right)\,\mathrm{d}\theta^2 + \left(\frac{\widetilde{\Sigma}^2}{\rho^2}\right)\,\mathrm{d}\phi^2
\end{aligned}
$$

Evaluate on Equatorial Plane $\theta = \frac{\pi}{2}$

$$\widetilde{\Delta}_r \equiv r^2 - 2\widetilde{M}r + a^2 - kr^4$$

$$\widetilde{\Sigma}^2 \equiv \widetilde{\Delta}_\theta\left(r^2 + a^2\right)^2 - a^2\widetilde{\Delta}_r\sin^2\theta$$

$$\widetilde{\Delta}_\theta \equiv 1 - ka^2\cos^2\theta\cot^2\theta$$

$$k = \kappa + \frac{\gamma^2(1-\beta\gamma)}{(2-3\beta\gamma)^2}$$

$$\widetilde{M} \equiv \beta\left(1 - \frac{3}{2}\beta\gamma\right)$$

Recover GR Kerr when $\gamma, \kappa = 0$

Goals: Study Black Hole Solutions

Parametric Study

β (Mass)

a (Spin)

CG Parameters

γ κ

Spacetime Structure

Number and Locations of Horizons

$$g_{rr} \to \infty$$

Causal Structure

- Nature of Horizons (Event, Cauchy, or Cosmological)
- Classify Regions as Timelike (T) or Spacelike (S)

Parameter values

Non-dimensionalization using β (cm)

a (cm) $\rightarrow a/\beta$ $\quad$ γ (cm^{-1}) $\rightarrow \beta\gamma$ $\quad$ κ (cm^{-2}) $\rightarrow \beta^2\kappa$

- Black Hole Masses reach $\beta \sim 10^{16}$ (cm)
- Observational fits (still ongoing) give

$\gamma \sim 10^{-30} - 10^{-28}\ \mathrm{cm}^{-1}$ $\quad$ $\kappa \sim 10^{-54} - 10^{-48}\ \mathrm{cm}^{-2}$

$$-1 \le \beta^2\kappa \le 1 \qquad -1 \le \beta\gamma \le 1$$

$a/\beta = \mathcal{O}(1)$ ⟵ Maximum Spin of GR Kerr is $a/\beta = 1$

CG Kerr Horizons

Equatorial Plane
$\theta = \frac{\pi}{2}$

Horizons $g_{rr} \to \infty$

$$\widetilde{\Delta}^{H} \equiv [(\beta^2\kappa)(2-3\beta\gamma)^2 + (\beta\gamma)^2(1-\beta\gamma)]\left(\frac{r}{\beta}\right)^4 - (2-3\beta\gamma)^2\left(\frac{r}{\beta}\right)^2 + (2-3\beta\gamma)^3\left(\frac{r}{\beta}\right) - (2-3\beta\gamma)^2\left(\frac{a}{\beta}\right)^2 = 0$$

$r/\beta \geq 0$

→ GR Kerr: Two Horizons

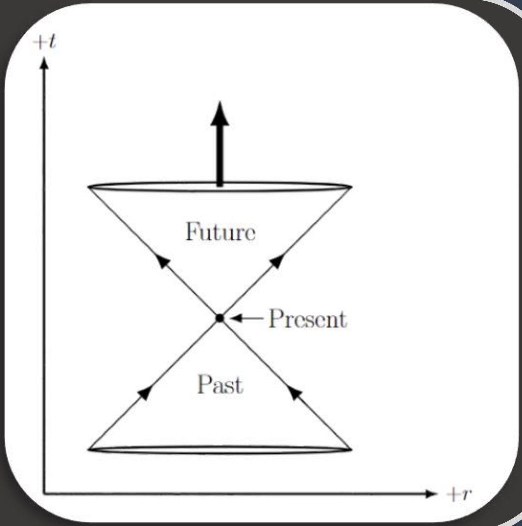
CG Kerr: Maximum of Three Horizons

Causal Structure

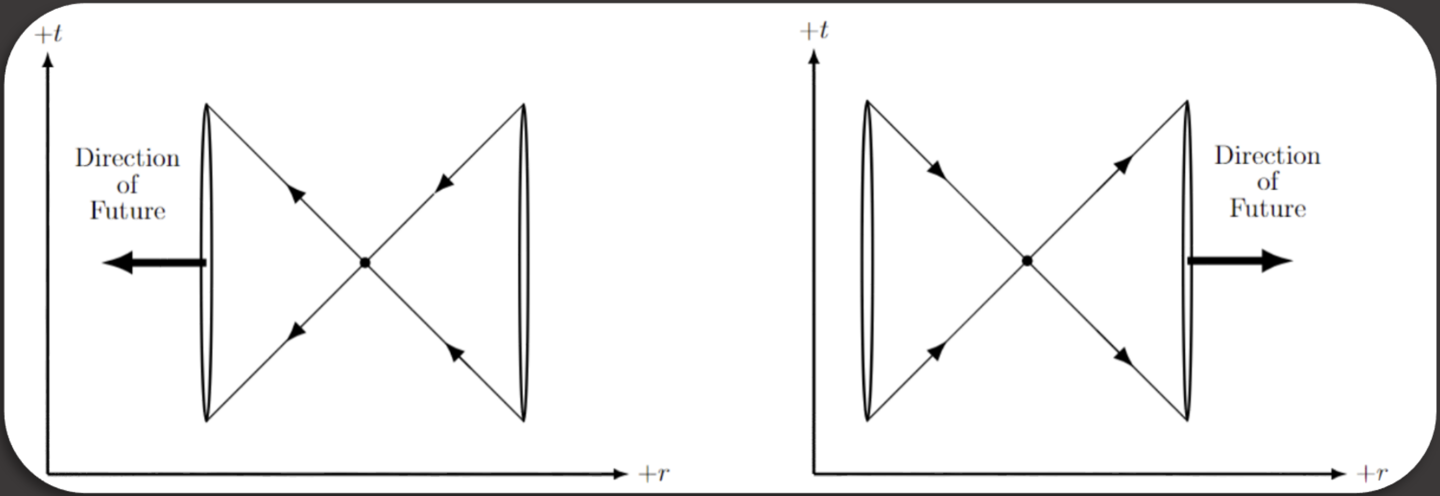
<u>Light Cones:</u> formed by null geodesics

Ingoing (Decreasing r)
Outgoing (Increasing r)

Timelike (T): $g_{rr} > 0$ or $\widetilde{\Delta}^{\mathrm{H}} < 0$



Spacelike (S): $g_{rr} < 0$ or $\widetilde{\Delta}^{\mathrm{H}} > 0$



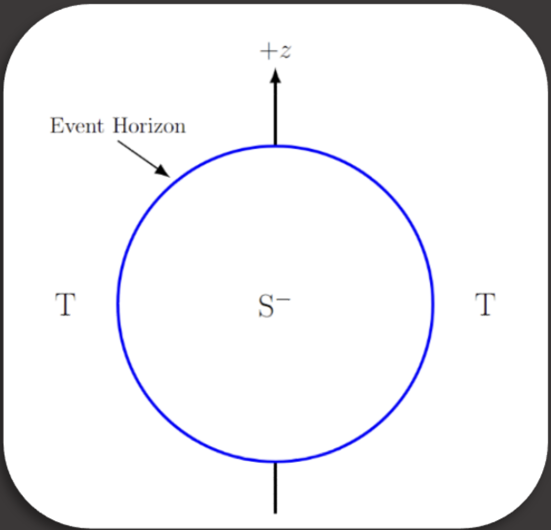
(S^-) (S^+)

Types of Horizons

Event Horizon: $S^{-} \rightarrow T$ (as r increases)

Example:

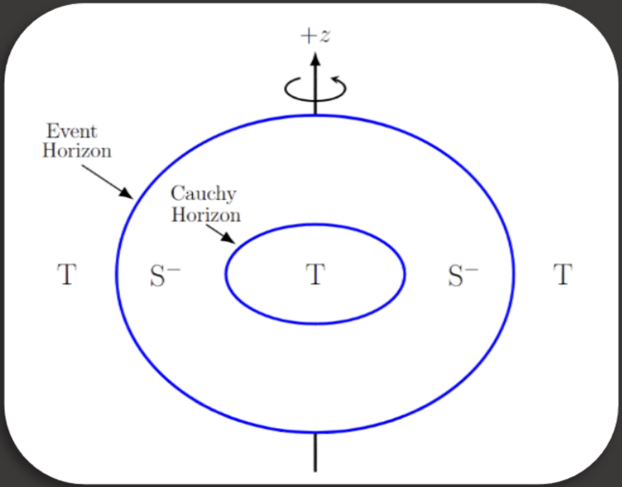
GR Schwarzschild

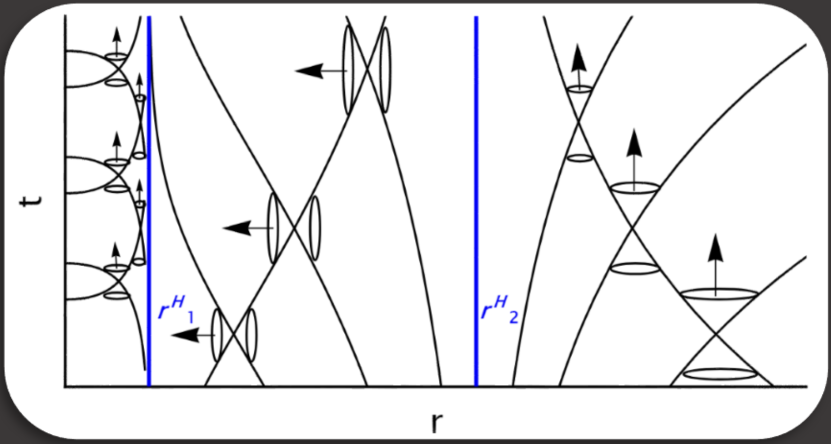


Cauchy Horizon: $T \rightarrow S^{-}$ (as r increases)

Example:

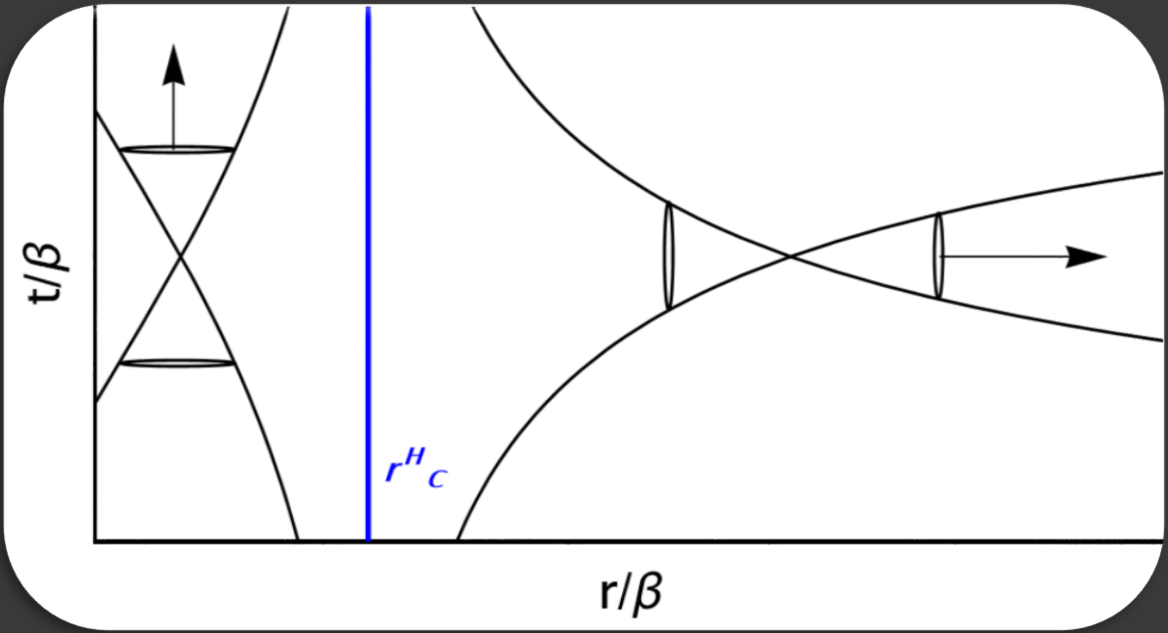
GR Kerr





Types of Horizons

Cosmological Horizon: T → S^+ (as r increases)

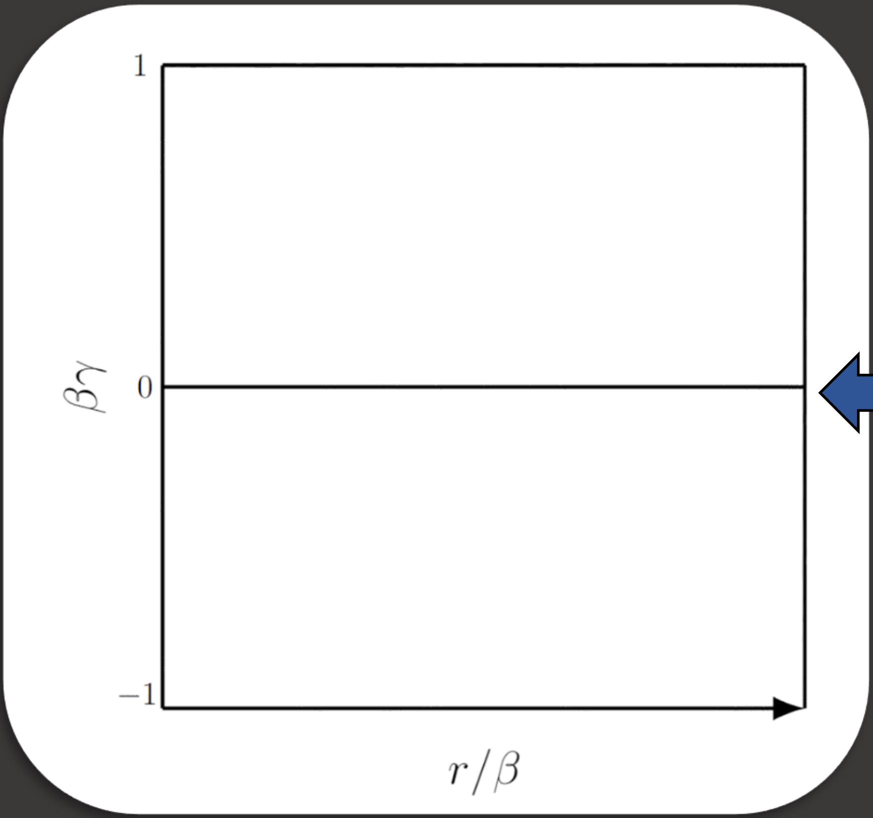


Parameter Maps

$\beta\gamma$ on vertical axis

r/β on horizontal axis

- a/β and $\beta^2\kappa$ set manually and indicated
- Each horizontal slice is a particular spacetime

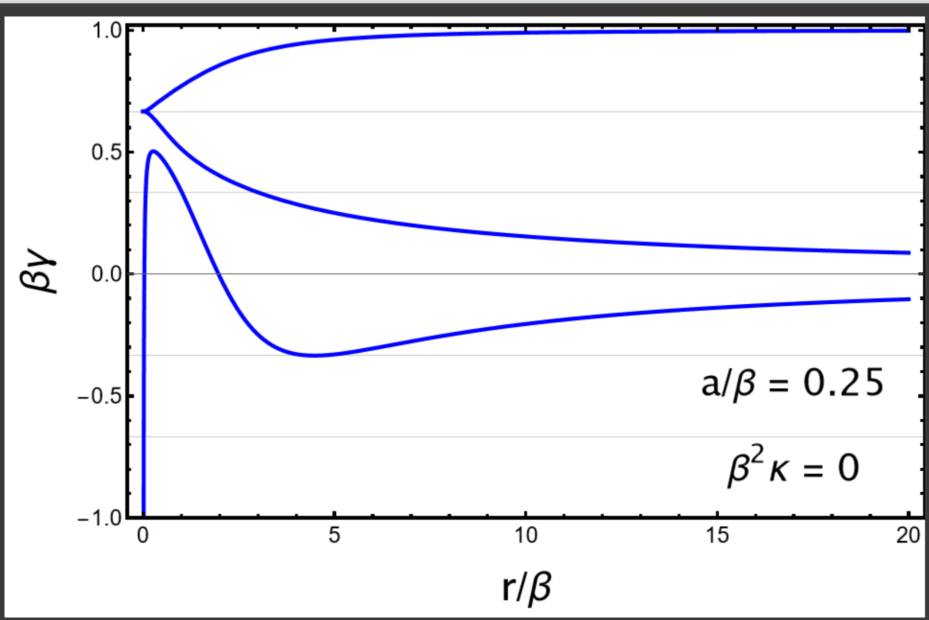


GR solution: $\beta\gamma = 0$

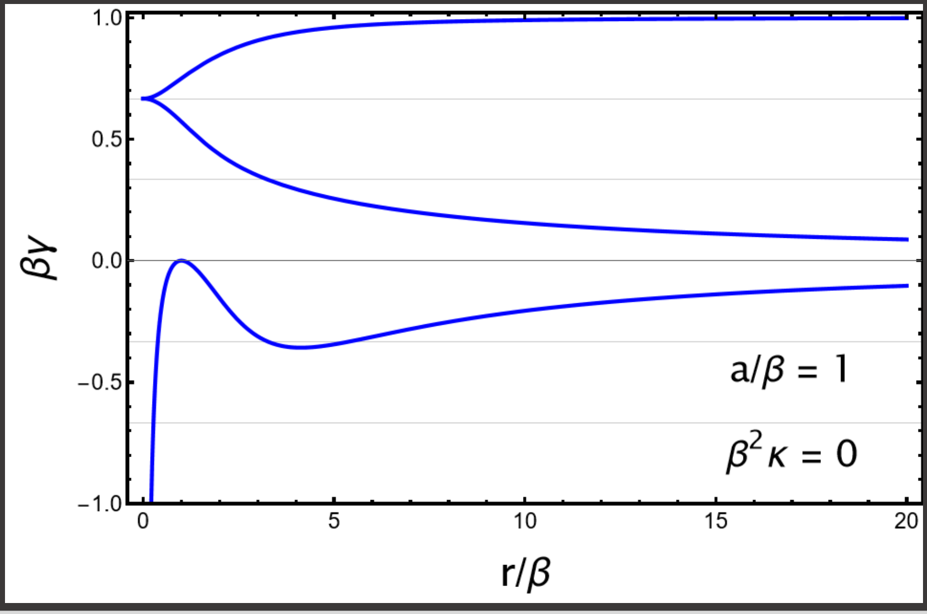
Horizons

Variation of spin: a/β

$\beta^2\kappa = 0$



GR Kerr



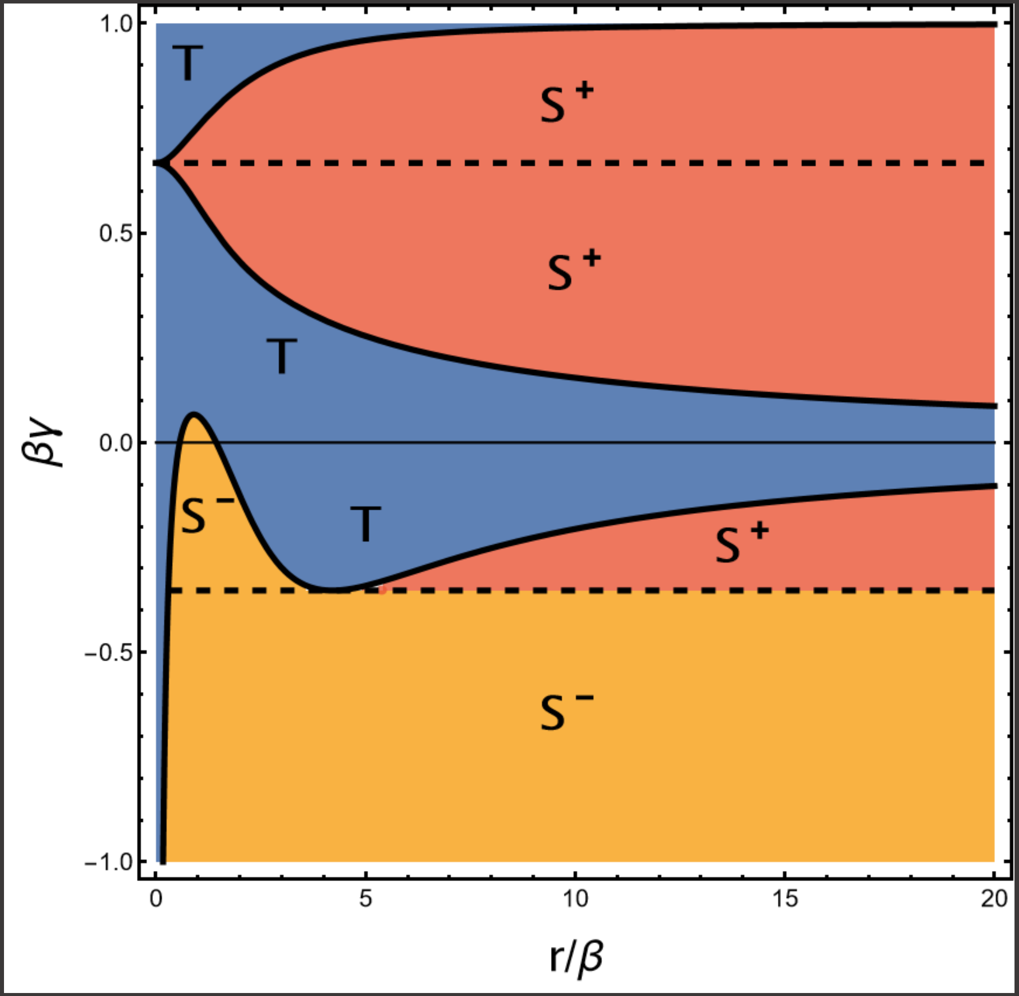
GR Kerr

(Naked Singularity)

Causal Structure

$\beta^2\kappa = 0$

$a/\beta = 0.9$

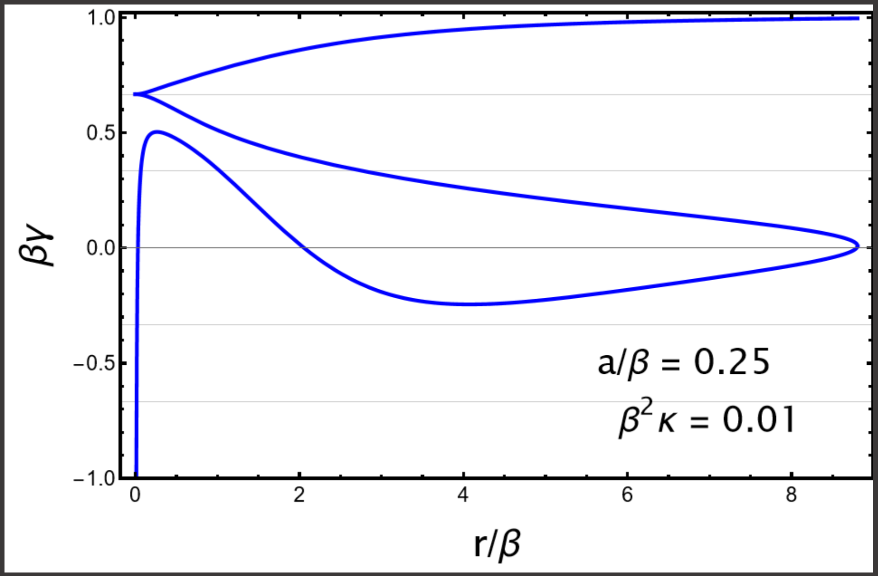


GR Kerr

Horizons

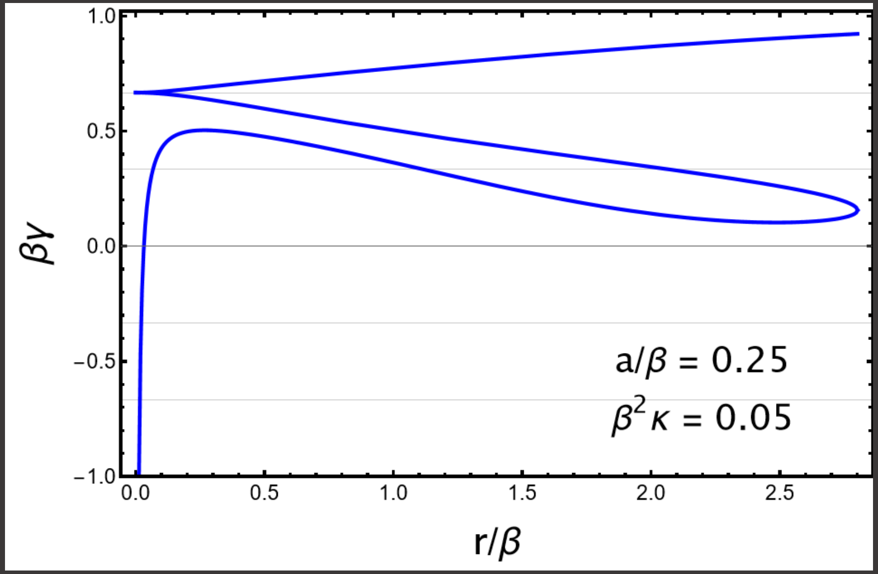
Variation of: $\beta^2\kappa$

$\beta^2\kappa > 0$



GR Kerr-de Sitter (Kerr-dS)

$$\kappa = \frac{\Lambda}{3}$$

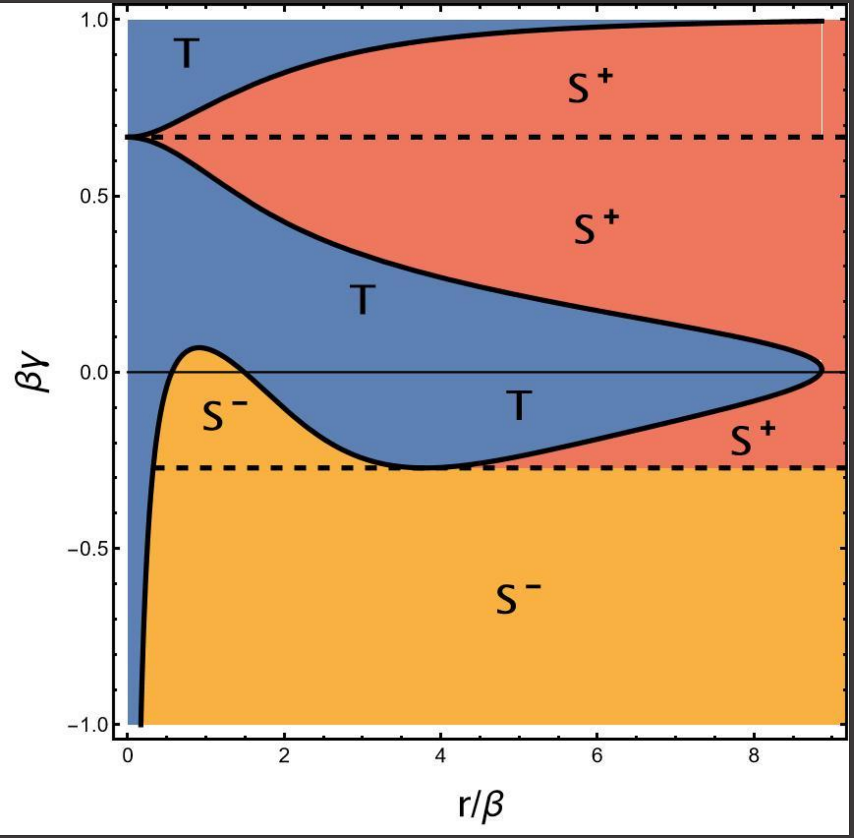


GR Kerr-de Sitter (Kerr-dS)

Causal Structure

$\beta^2\kappa > 0$

$a/\beta = 0.9$
$\beta^2\kappa = 0.01$

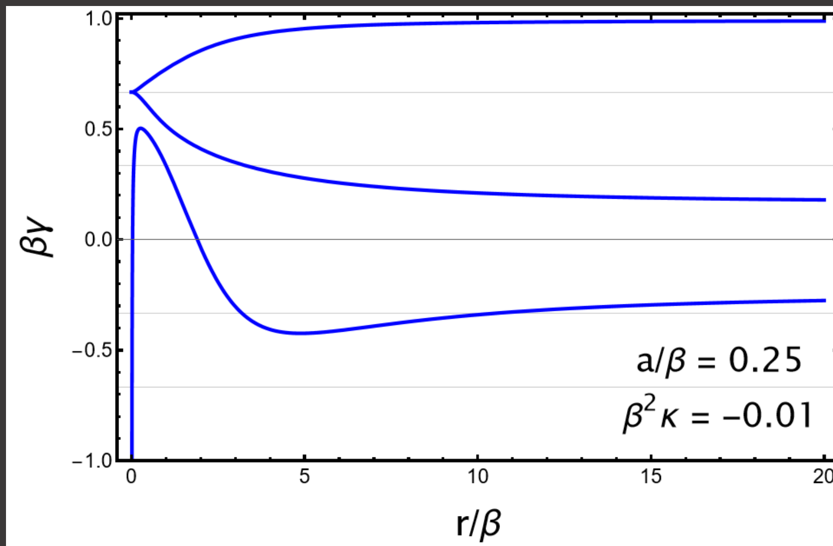


GR Kerr-de Sitter (Kerr-dS)

Horizons

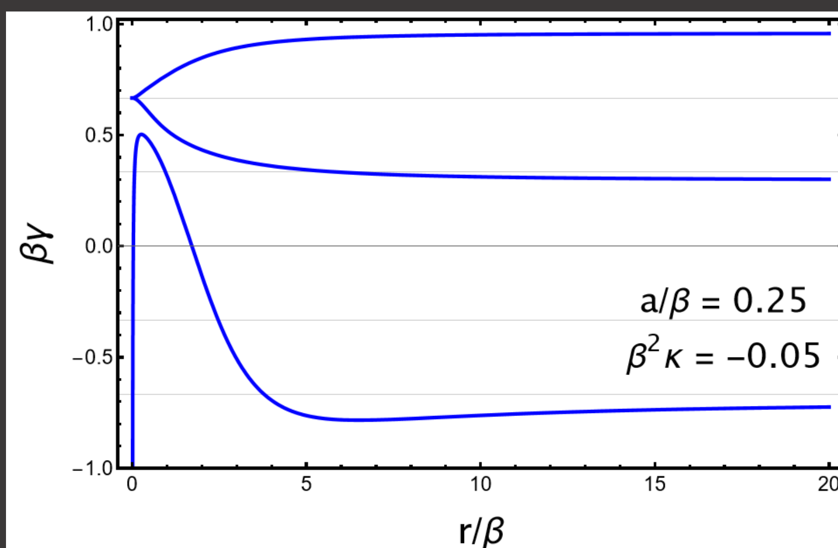
Variation of: $\beta^2\kappa$

$\beta^2\kappa < 0$



GR Kerr-anti-de Sitter (Kerr-AdS)

$$\kappa = \frac{\Lambda}{3}$$

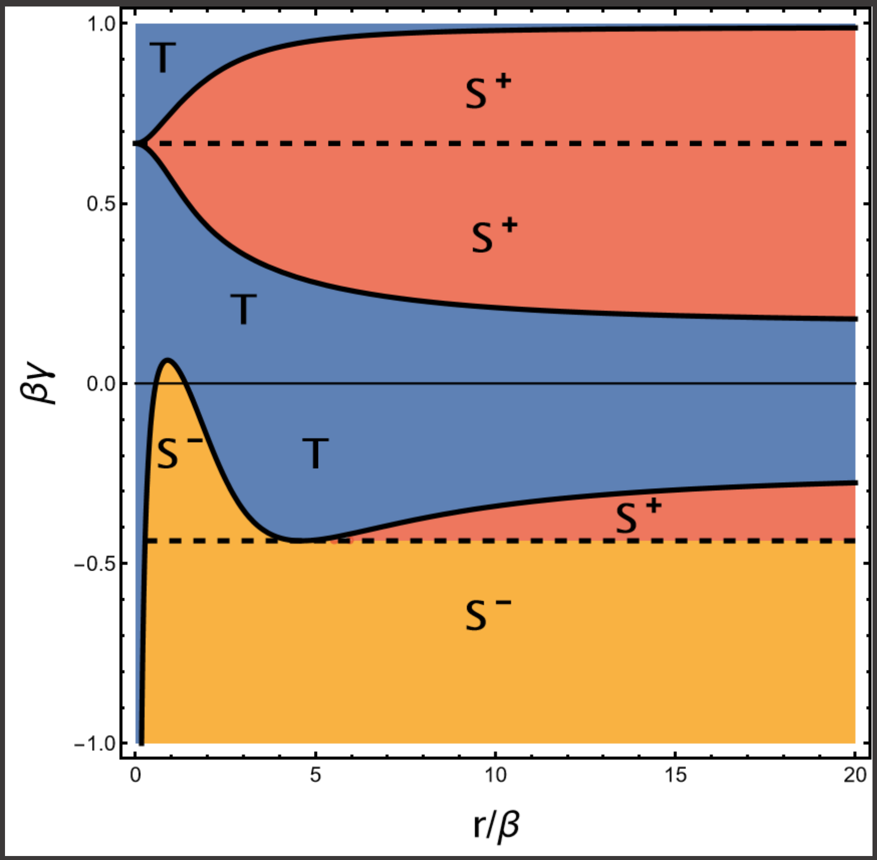


GR Kerr-anti-de Sitter (Kerr-AdS)

Causal Structure

$\beta^2 \kappa < 0$

$a/\beta = 0.25$
$\beta^2 \kappa = -0.01$



GR Kerr-anti-de Sitter (Kerr-AdS)

Conclusions

CG Kerr Black Hole Spacetimes

<u>Two Forms</u>

1. Kerr Black Hole with Cosmological Horizon — Like GR Kerr-dS

2. Kerr Black Hole alone — Like GR Kerr And GR Kerr-AdS

- Observations imply $\gamma > 0$, $\kappa > 0$ and small
  - ⇒ 1st form
  - ⇒ Black Holes are not isolated in reality
- Theoretical presence of cosmological horizon relevant to not needing Dark Energy

Other Results

CG Kerr Spins a/β

- Black Holes can exist above and below GR spin limit
- Important to quantum gravity theories

→ AdS-CFT Correspondence uses Extremal Black Holes

1. Mapped Ergosurfaces ($g_{tt} = 0$) and Ergoregions ($g_{tt} > 0$)
2. Solved Equations of Motion for Principal Null Geodesics
3. Considered structure of Sagittarius A* using CG Kerr metric
4. Explored other non-black hole regions of parameter space

Future Work

1. Elucidate equatorial photon ring structure
2. Deal with nature of singularity in CG Kerr
3. Consider other coordinate systems

Q & A

Extremal Spin: a/β

Naked Singularity Spacetimes

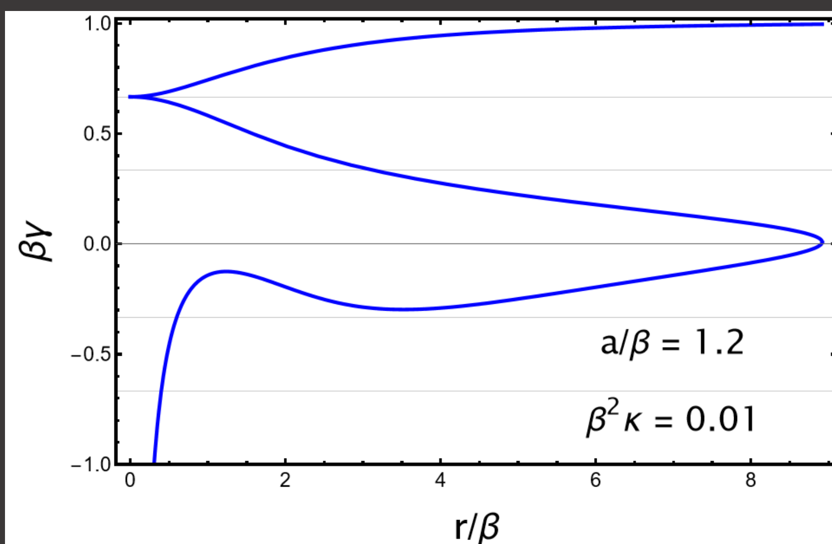
GR Kerr

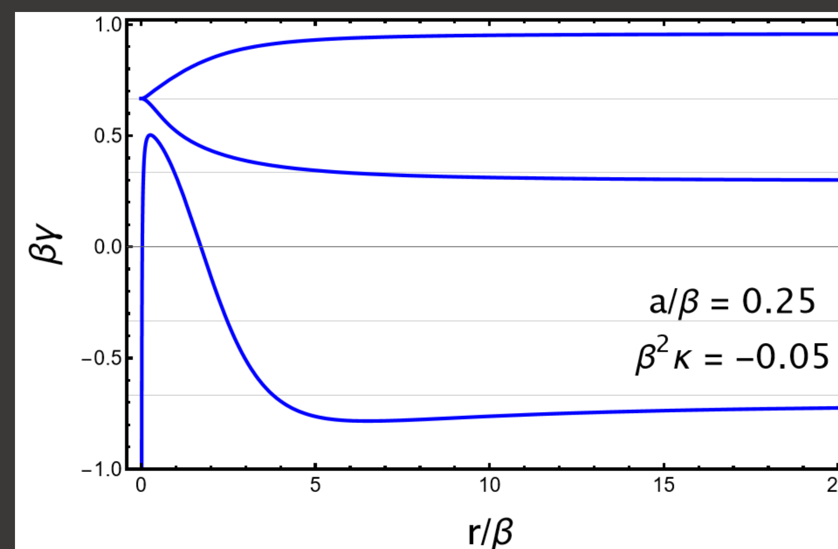
Two Horizons at $r^{\mathrm{H}} = \beta \pm \sqrt{\beta^2 - a^2}$

Horizons merge then become imaginary: $a/\beta \geq 1$

CG Kerr

Black Holes exist past and below $a/\beta = 1$





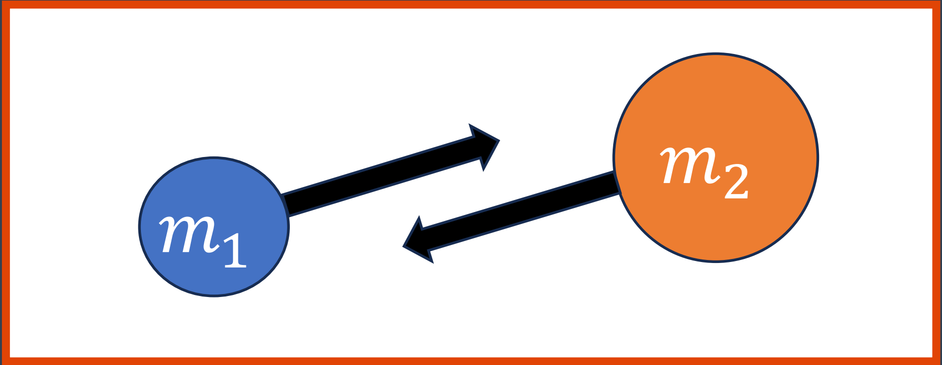
$\beta^2\kappa = 0.01,\ a/\beta = 1$

$\beta^2\kappa = \text{-}0.01,\ a/\beta = 0.25$

Newtonian Gravity

Gravitational Force Equation

$$F_g = -G\frac{m_1 m_2}{r^2}$$



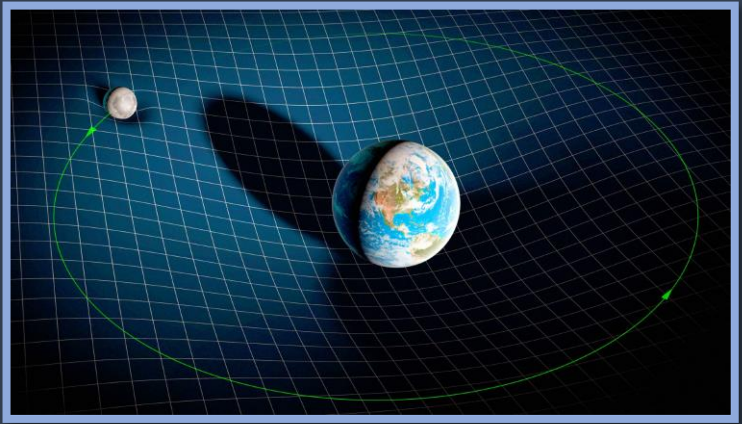
General Relativity

Einstein Field Equations

$$G_{\mu\nu} = 8\pi\, T_{\mu\nu}$$

Einstein Curvature Tensor

Stress-Energy Tensor



"Matter tells spacetime how to curve; spacetime tells matter how to move." – John Archibald Wheeler

Image: Mark Garlick/(Getty Images)

General Relativity

Up to 2nd-order derivatives of metric

Relevant Tensors

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

Ricci Tensor

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$$

Ricci Scalar

$$R \equiv g^{\mu\nu} R_{\mu\nu}$$

Riemann Curvature Tensor

$$R^{d}_{abc} = \partial_b\left(\Gamma^{d}_{ac}\right) - \partial_c\left(\Gamma^{d}_{ab}\right) + \Gamma^{e}_{ac}\Gamma^{d}_{be} - \Gamma^{e}_{ab}\Gamma^{d}_{ce}$$

Image: Mark Garlick/(Getty Images)

Conformal Gravity

$$W_{\mu\nu} = \frac{1}{4\alpha_g} T_{\mu\nu}$$

Up to 4th-order derivatives of metric

Relevant Tensors

Bach Curvature Tensor

$$W_{\mu\nu} = -\frac{1}{6} g_{\mu\nu} R^{;\lambda}{}_{;\lambda} + \frac{2}{3} R_{;\mu;\nu} + R_{\mu\nu}{}^{;\lambda}{}_{;\lambda} - R_{\mu}{}^{\lambda}{}_{;\nu;\lambda} - R_{\nu}{}^{\lambda}{}_{;\mu;\lambda} + \frac{2}{3} R R_{\mu\nu} - 2R_{\mu}{}^{\lambda} R_{\lambda\nu} + \frac{1}{2} g_{\mu\nu} R_{\lambda\rho} R^{\lambda\rho} - \frac{1}{6} g_{\mu\nu} R^2$$

Weyl Curvature Tensor

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu}) + \frac{1}{6} R (g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu})$$

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In GR Schwarzschild:

$g_{rr} = B(r)^{-1} \to \infty$ and $g_{tt} = B(r) \quad = 0$ at $r^H = 2\,\beta$

Not true in GR Kerr:

$g_{tt} = 0$ at Ergosurfaces at r^{E}

GR Kerr Geometry

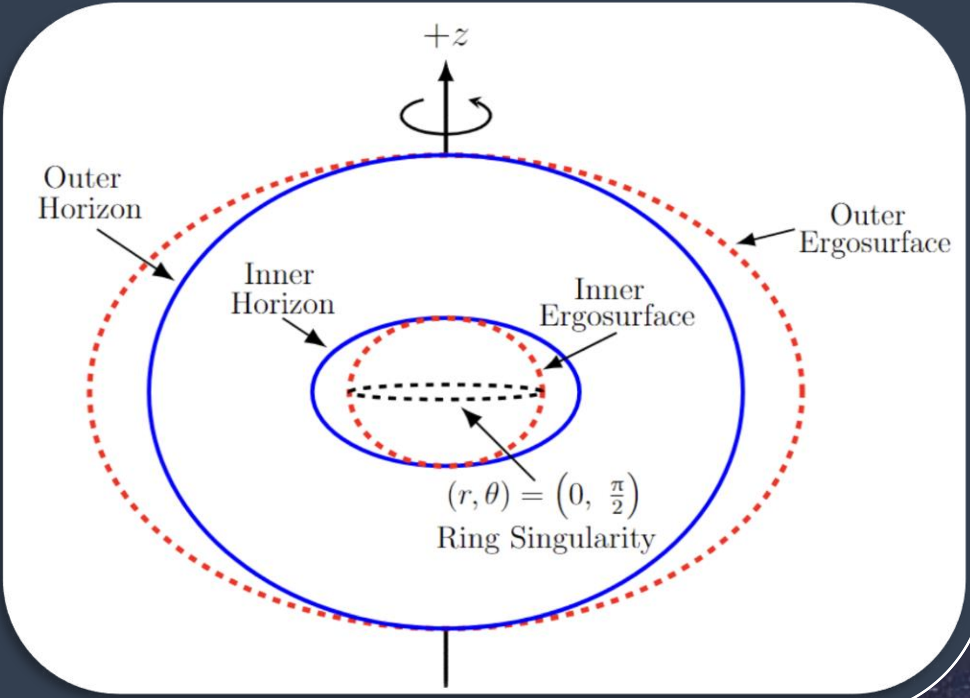
(Oblate-Spheroidal)

Two Horizons: $g_{rr} \to \infty$

Inner Cauchy Horizon
Outer Event Horizon

Two Ergosurfaces: $g_{tt} = 0$

Ring Singularity: Infinite Curvature



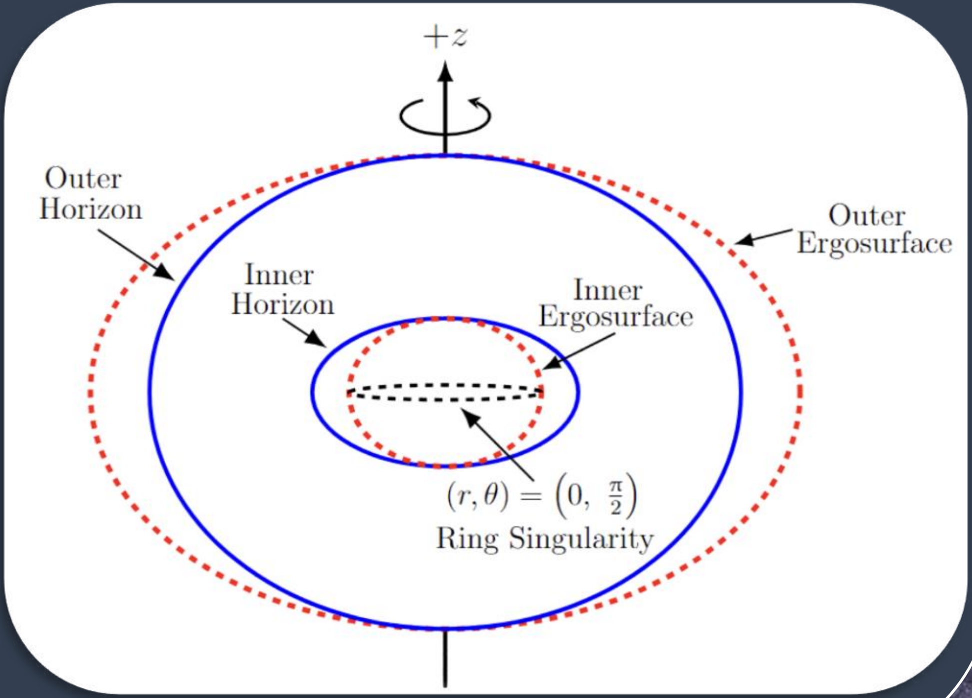
Boyer-Lindquist to “Cartesian” Coordinates

(Oblate-Spheroidal)

$$x = \sqrt{r^2 + a^2}\sin\theta\cos\phi$$

$$y = \sqrt{r^2 + a^2}\sin\theta\sin\phi$$

$$z = r\cos\theta$$



Ring Singularity

$$\rho^2 = r^2 + a^2\cos^2\theta = 0 \quad \Rightarrow \quad (r, \theta) = \left(0, \frac{\pi}{2}\right)$$

Image: NASA/Casey Reed

Goals

Parametric Study

β (Mass)

a (Spin)

CG Parameters

γ κ

Spacetime Structure

Locations of

- Horizons ⇒ $g_{rr} = 0$
- Ergosurfaces ⇒ $g_{tt} = 0$

Causal Structure (Radial r variation)

- Nature of Horizons
- Classify Regions as Timelike (T) or Spacelike (S)

CG Kerr Spacetime Structure

Equatorial Plane
$\theta = \frac{\pi}{2}$

<u>Horizons</u> $g_{rr} = 0$

$$\widetilde{\Delta}^{H} \equiv \left[(\beta^2\kappa)(2-3\beta\gamma)^2 + (\beta\gamma)^2(1-\beta\gamma)\right]\left(\frac{r}{\beta}\right)^4 - (2-3\beta\gamma)^2\left(\frac{r}{\beta}\right)^2 + (2-3\beta\gamma)^3\left(\frac{r}{\beta}\right) - (2-3\beta\gamma)^2\left(\frac{a}{\beta}\right)^2 = 0$$

<u>Ergosurfaces</u> $g_{tt} = 0$

$$\widetilde{\Delta}^{E} \equiv \left[(\beta^2\kappa)(2-3\beta\gamma)^2 + (\beta\gamma)^2(1-\beta\gamma)\right]\left(\frac{r}{\beta}\right)^4 - (2-3\beta\gamma)^2\left(\frac{r}{\beta}\right)^2 + (2-3\beta\gamma)^3\left(\frac{r}{\beta}\right) = 0$$

$r/\beta \geq 0$ → Maximum of Three Horizons/Ergosurfaces

Ergoregions: No Static Observers

Static Observer at fixed spatial coordinates $x^{\mu} = (t, r_0, \theta_0, \phi_0)$
has four-velocity $u^{\mu} = (u^t, 0, 0, 0)$

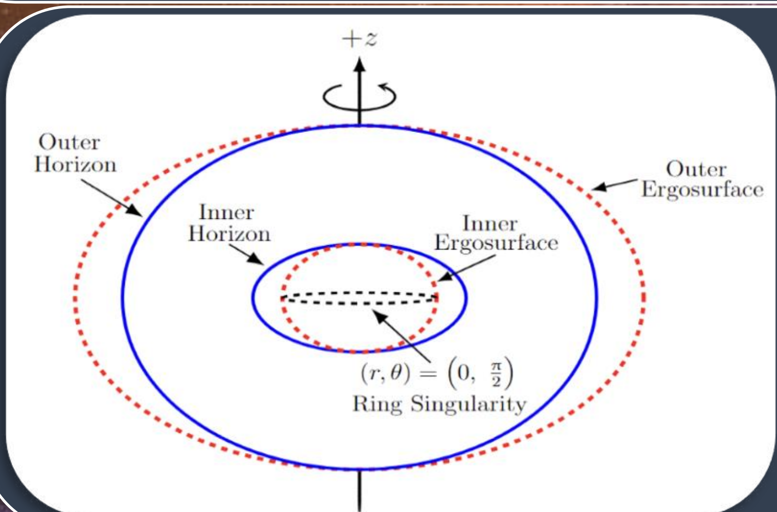
Normalization condition: $g_{tt} u^t \ u^t = -1$

⇓

Static Observers can only exist in non-ergoregions (N) where

$$g_{tt} < 0$$

Ergoregions (E) where $g_{tt} > 0$ or $\widetilde{\Delta}^{E} > 0$



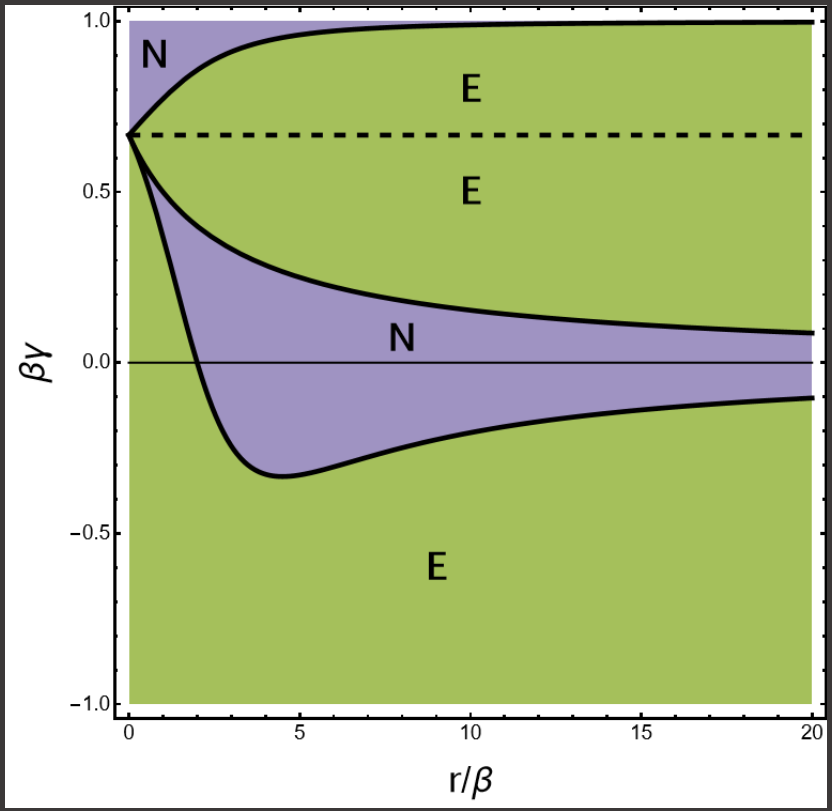
GR Kerr

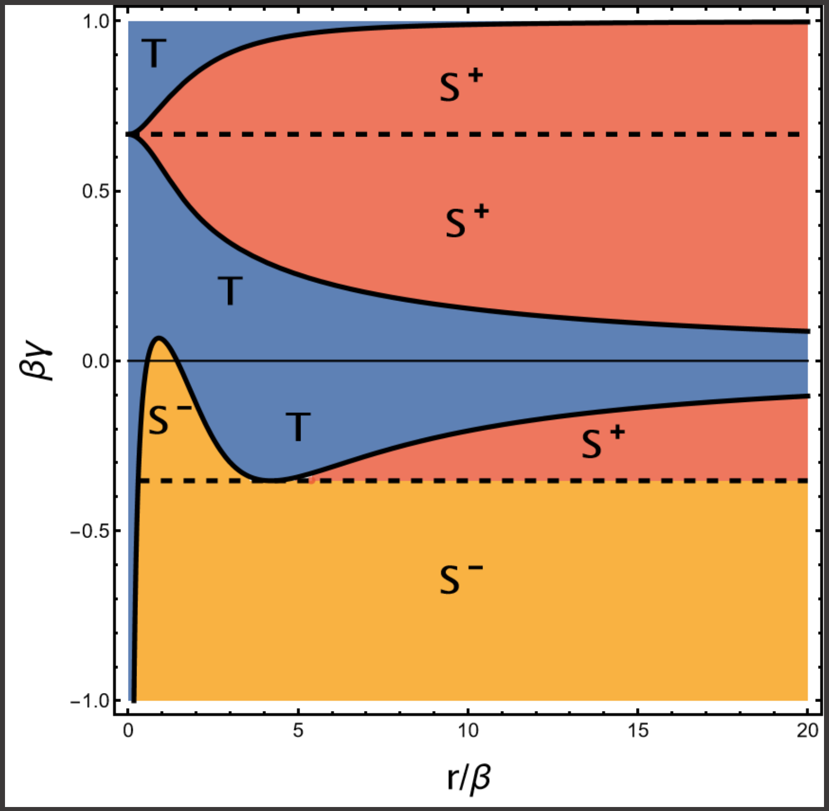
Forced to rotate in region between outer horizon and outer ergosurface

Ergoregion Structure

$\beta^2\kappa = 0$

$a/\beta = 0.25$



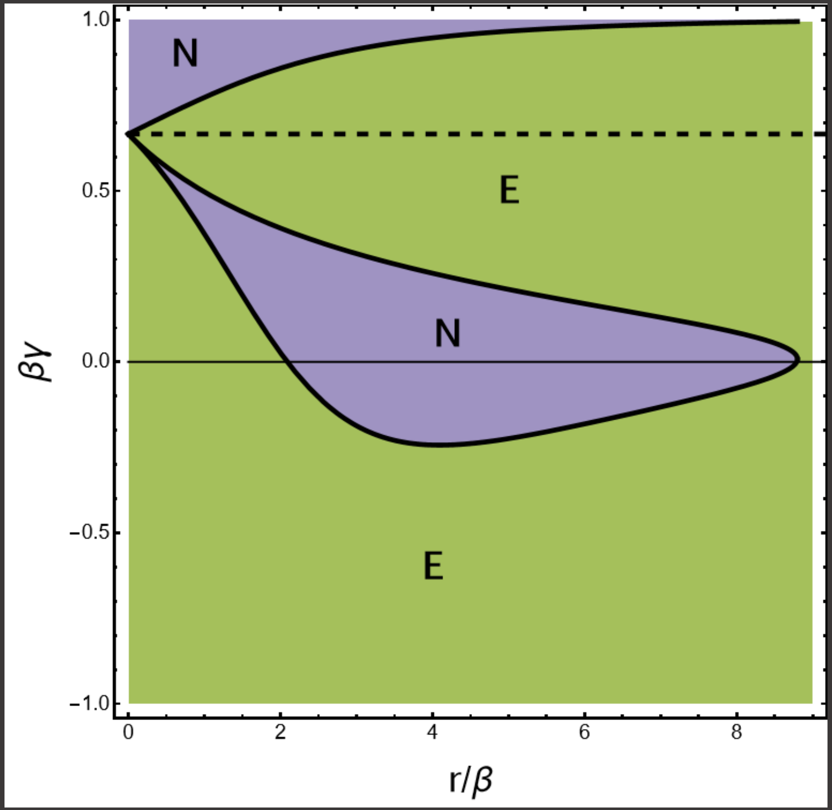


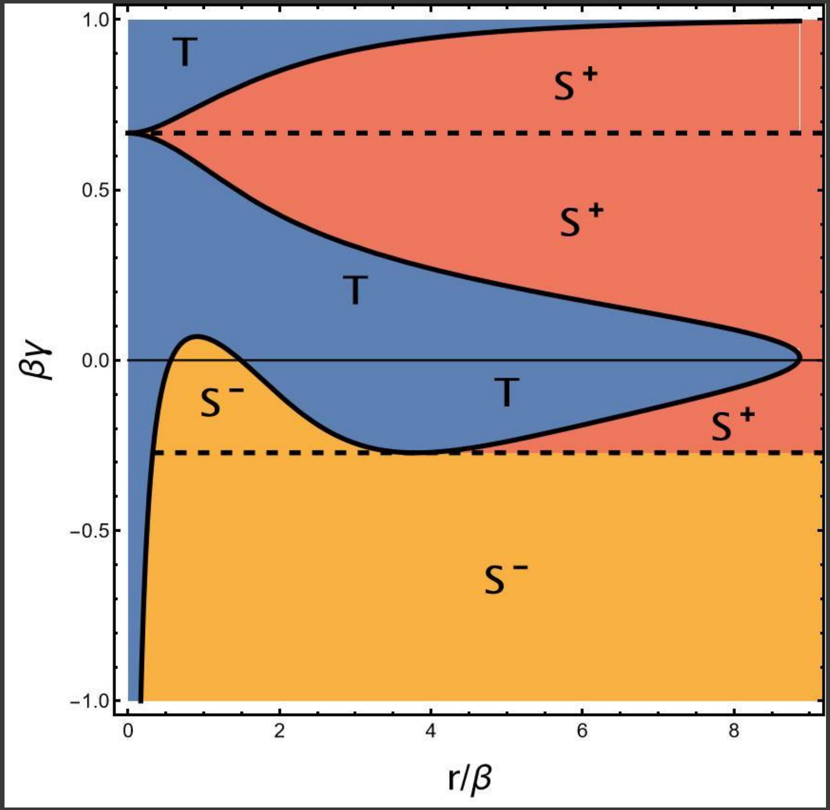
Causal Structure

Ergoregion Structure

$\beta^2\kappa > 0$

$a/\beta = 0.25 \; \beta^2\kappa = 0.01$



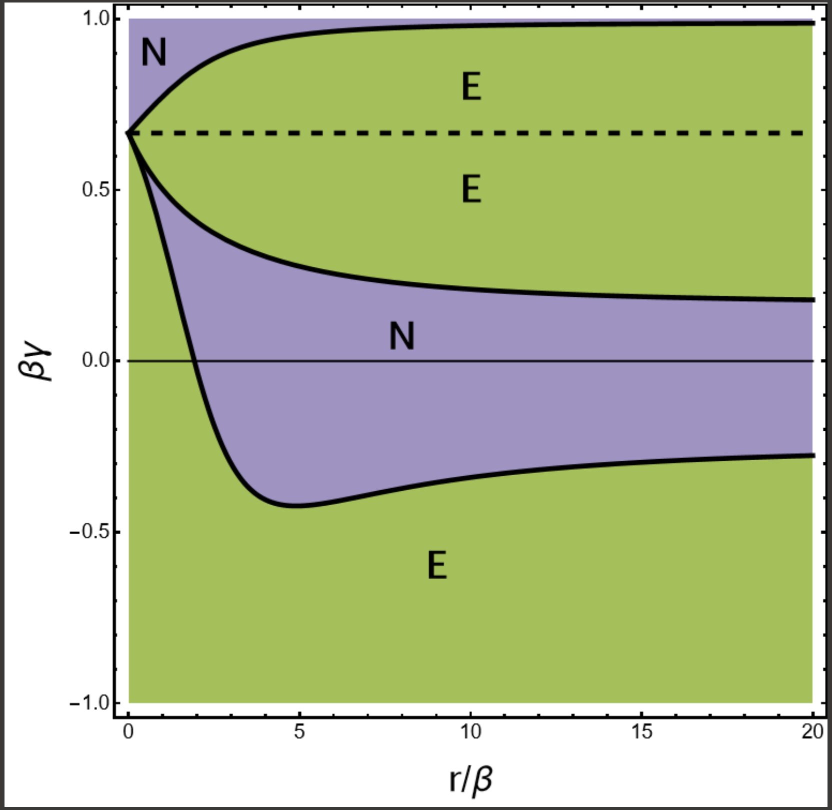


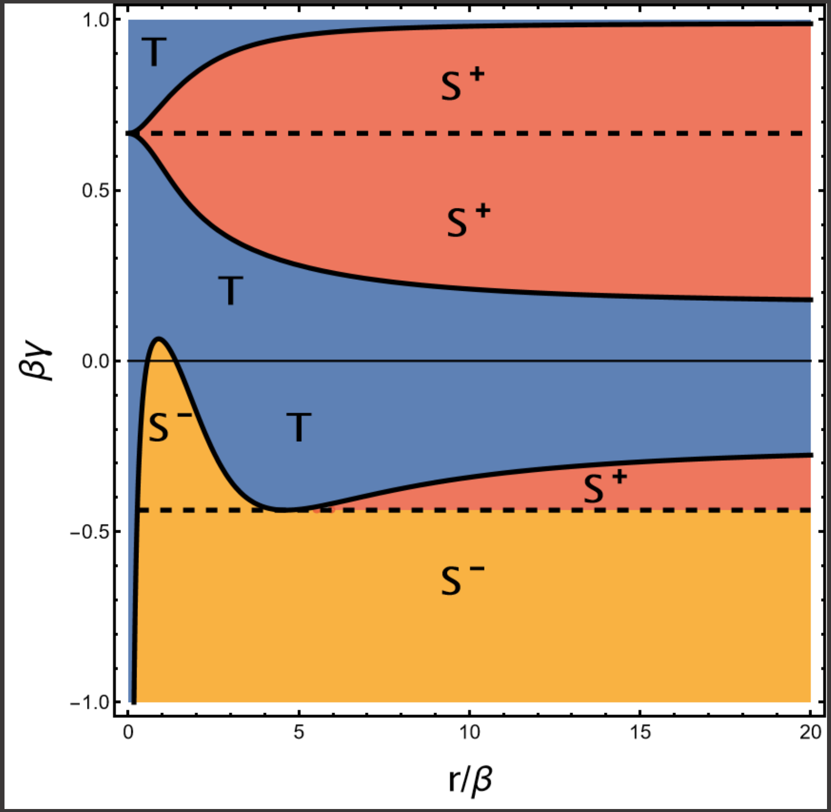
Causal Structure

Ergoregion Structure

$\beta^2\kappa < 0$

$a/\beta = 0.25$ $\beta^2\kappa = -0.01$





Causal Structure

Equations of Motion from CG Kerr Metric

Starting with Lagrangian

$$\mathcal{L} = -\frac{1}{2} g_{\mu\nu} \frac{\partial x^\mu}{\partial \sigma} \frac{\partial x^\nu}{\partial \sigma}$$

Since Null

$$\mathcal{L} = -\frac{1}{2} g_{tt} \dot{t}^2 - g_{t\phi} \dot{t}\dot{\phi} - \frac{1}{2} g_{rr} \dot{r}^2 - \frac{1}{2} g_{\phi\phi} \dot{\phi}^2 - \frac{1}{2} g_{\theta\theta} \dot{\theta}^2 = 0$$

$$\dot{t} = \frac{1}{r^2}\left(\frac{(r^2 + a^2)[(r^2 + a^2)E - aL_z]}{\widetilde{\Delta_r}} + a(L_z - aE) \right)$$

$$\dot{r}^2 = \frac{[(r^2 + a^2)E - aL_z]^2 - \widetilde{\Delta_r}\left(\tilde{Q} + (L_z - aE)^2\right)}{\rho^4}$$

$$\dot{\phi} = \frac{1}{\rho^2}\left(\frac{a[(r^2 + a^2)E - aL_z]}{\widetilde{\Delta_r}} + \frac{(L_z \csc^2\theta - aE)}{\widetilde{\Delta_\theta}} \right)$$

$$\dot{\theta}^2 = \left(\frac{\widetilde{\Delta_\theta}}{\rho^2} \right)^2 p_\theta^2$$

Conserved Quantities

"Energy" $E = p_t = \frac{\partial \mathcal{L}}{\partial \dot{t}}$

"Angular Momentum" $L_z = -p_\phi = -\frac{\partial \mathcal{L}}{\partial \dot{\phi}}$

"Carter's Constant" $\tilde{Q} = \widetilde{\Delta_\theta} p_\theta^2 + \frac{(aE \sin\theta - L_z \csc\theta)^2}{\widetilde{\Delta_\theta}} - (L_z - aE)^2$

Principal Null Geodesics

Analogous to radial trajectories in non-rotating spacetimes

Take $L_z / E = a$

Principal Null Geodesics

Equatorial Plane
$$\theta = \frac{\pi}{2}$$

$$\dot{t} = \frac{E(r^2 + a^2)}{\widetilde{\Delta_r}}$$

$$\dot{r} = \pm E$$

$$\dot{\phi} = \frac{aE}{\widetilde{\Delta_r}}$$

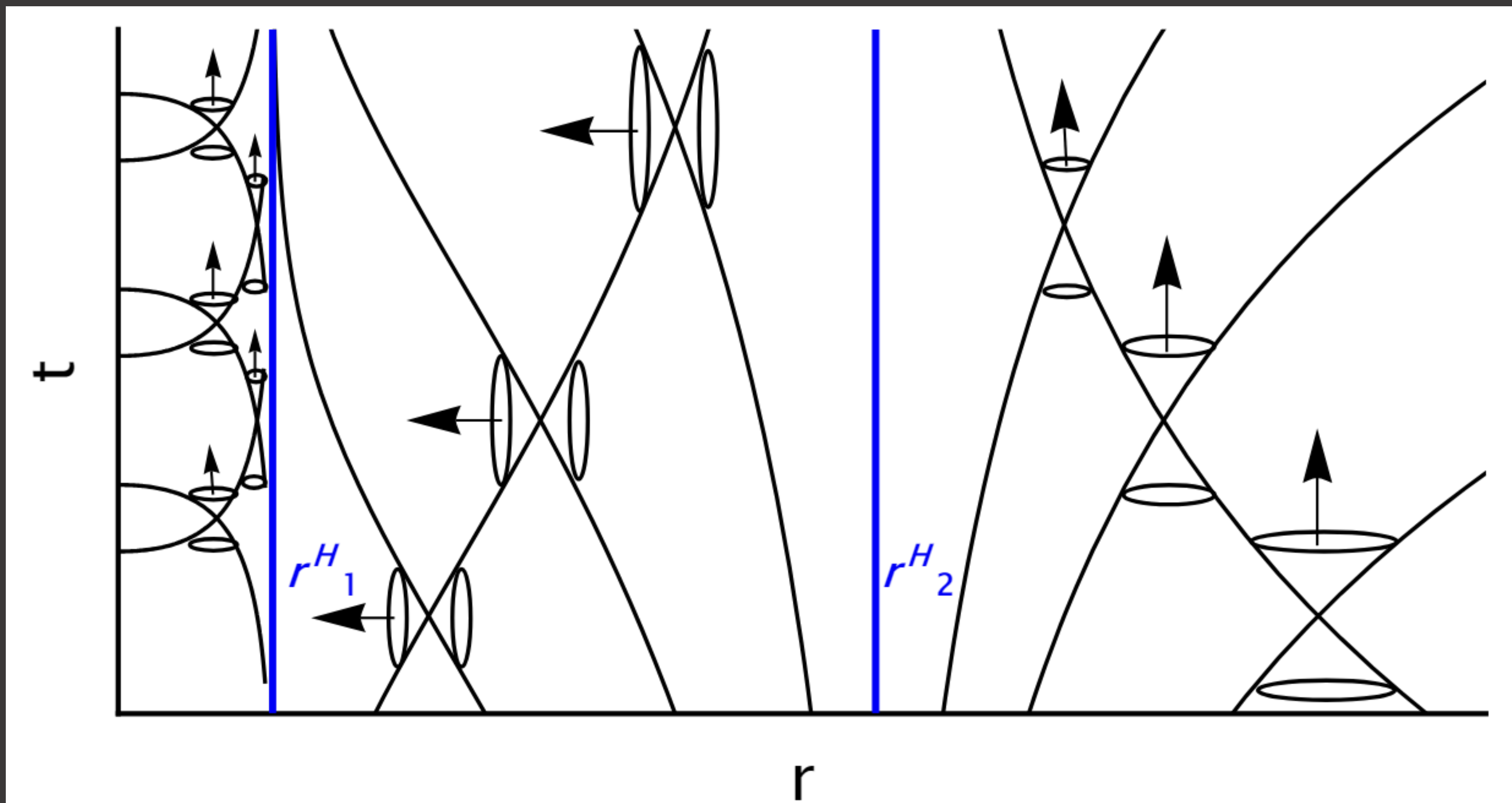
$$\dot{\theta} = 0$$

Get slope of geodesics on spacetime diagram

$$\frac{dt}{dr} = \pm(2 - 3\beta\gamma)^2 \frac{\left(-\left(\frac{r}{\beta}\right)^2 - \left(\frac{a}{\beta}\right)^2\right)}{\widetilde{\Delta^{H}}}$$

Sign of slope and thus timelike or spacelike nature depends entirely on $\widetilde{\Delta^{H}}$

Kerr Black Hole Light Cones



T

S^-

T

Sagittarius A* Structure

Mass: $M \approx 4 \times 10^6 \, M_\odot$ ⟹ $\beta \approx 6 \times 10^{11}$ cm

Spin: $a/\beta = 0.1$

CG Parameters

$\gamma = 1.94 \times 10^{-28}\ \mathrm{cm}^{-1}$ ⟹ $\beta\gamma \approx 1.16 \times 10^{-16}$

$\kappa = 6.42 \times 10^{-48}\ \mathrm{cm}^{-2}$ ⟹ $\beta^2\kappa \approx 2.31 \times 10^{-24}$

Sagittarius A* Structure

Equatorial Plane
$\theta = \frac{\pi}{2}$

GR and CG Kerr

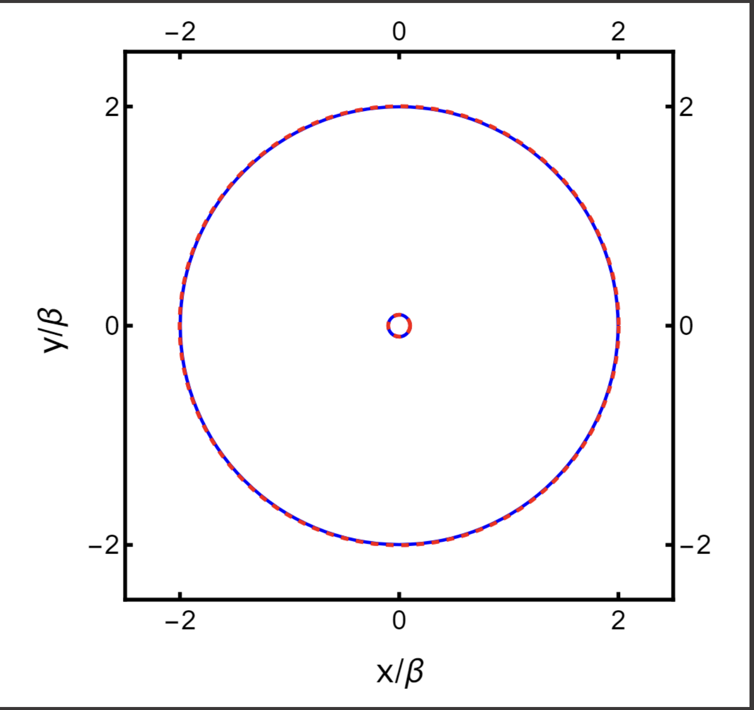
Give same values for black hole region

Inner Ergosurface: $r_1^{\mathrm{E}} = 0$ AU

Inner Horizon: $r_1^{\mathrm{H}} = 2.01 \times 10^{-4}$ AU

Outer Horizon: $r_2^{\mathrm{H}} = 8.00 \times 10^{-2}$ AU

Outer Ergosurface: $r_2^{\mathrm{E}} = 8.01 \times 10^{-2}$ AU



Ergosurfaces (Red) and Horizons (Blue)

CG Kerr has additional

Cosmological Horizon and Ergosurface

$$r_{\mathrm{C}}^{\mathrm{E}} \approx r_{\mathrm{C}}^{\mathrm{H}} \approx 2.64 \times 10^{10} \text{ AU} \approx 128 \text{ pc}$$

Sagittarius A* Structure

Range of CG effects

GR

$$B(r) = 1 - \frac{2\beta}{r}$$

CG

$$\tilde{B}(r) = 1 - \frac{\beta(2 - 3\beta\gamma)}{r} - 3\beta\gamma + \gamma r - \kappa r^2$$

$$\frac{\gamma R}{2} = \frac{\beta}{R}$$

→

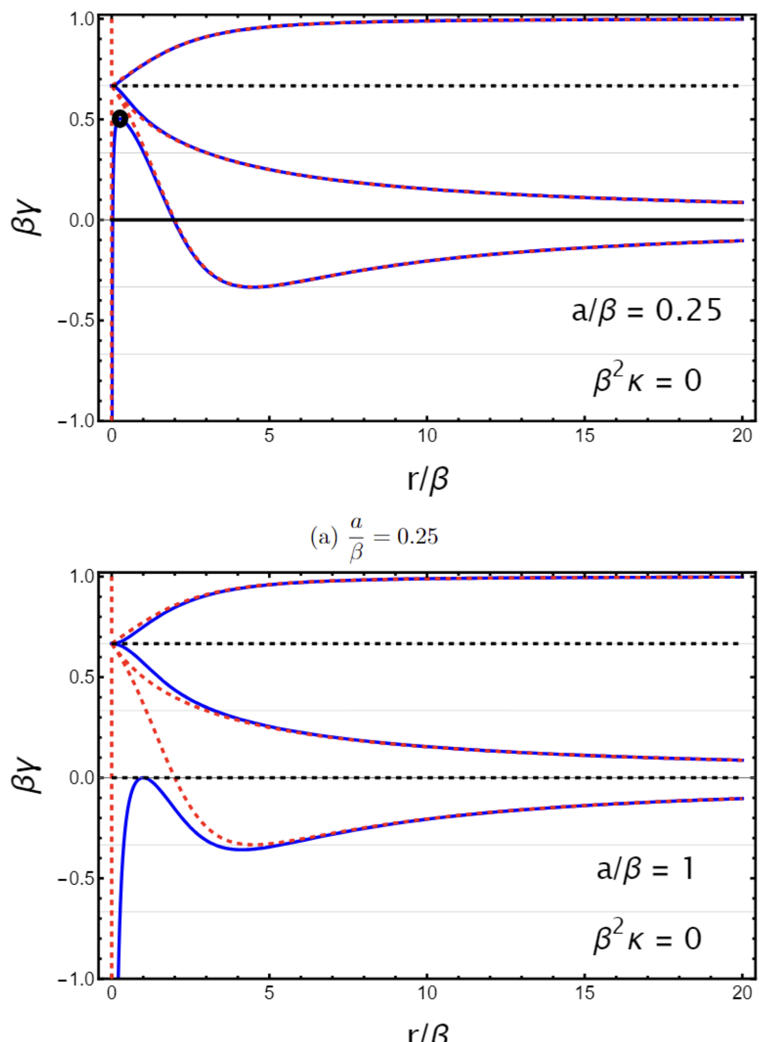
$$R \approx 25 \text{ pc}$$

Mass of material at 5 pc $\approx 10^7 \, M_\odot > 4 \times 10^6 \, M_\odot$ mass of Sgr A*

→ Cosmological horizon not likely observable

Horizons and Ergosurfaces

$\beta^2\kappa = 0$



(a) $\frac{a}{\beta} = 0.25$

(b) $\frac{a}{\beta} = 1$

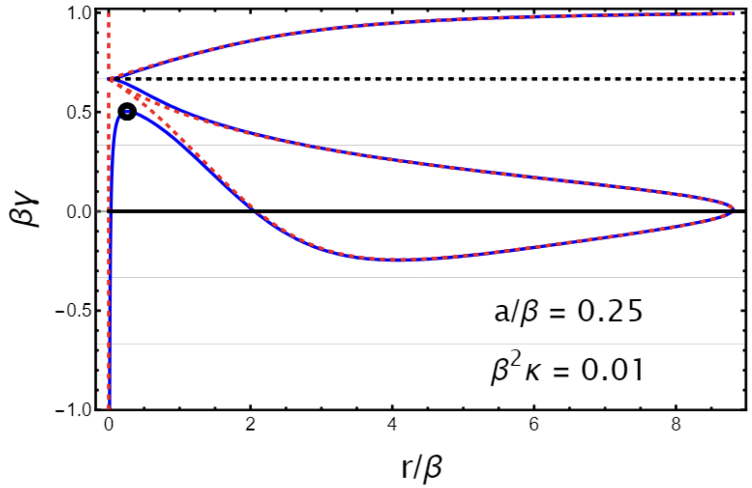
GR Kerr

GR Kerr

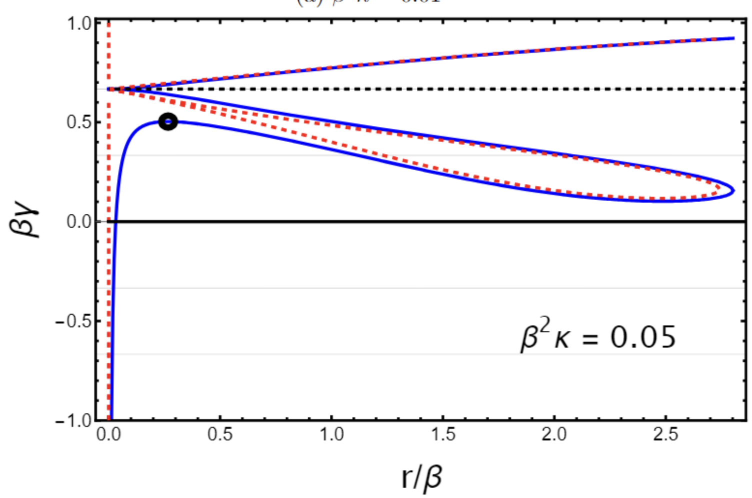
(Naked Singularity)

Horizons and Ergosurfaces

$\beta^2\kappa > 0$



(a) $\beta^2\kappa = 0.01$



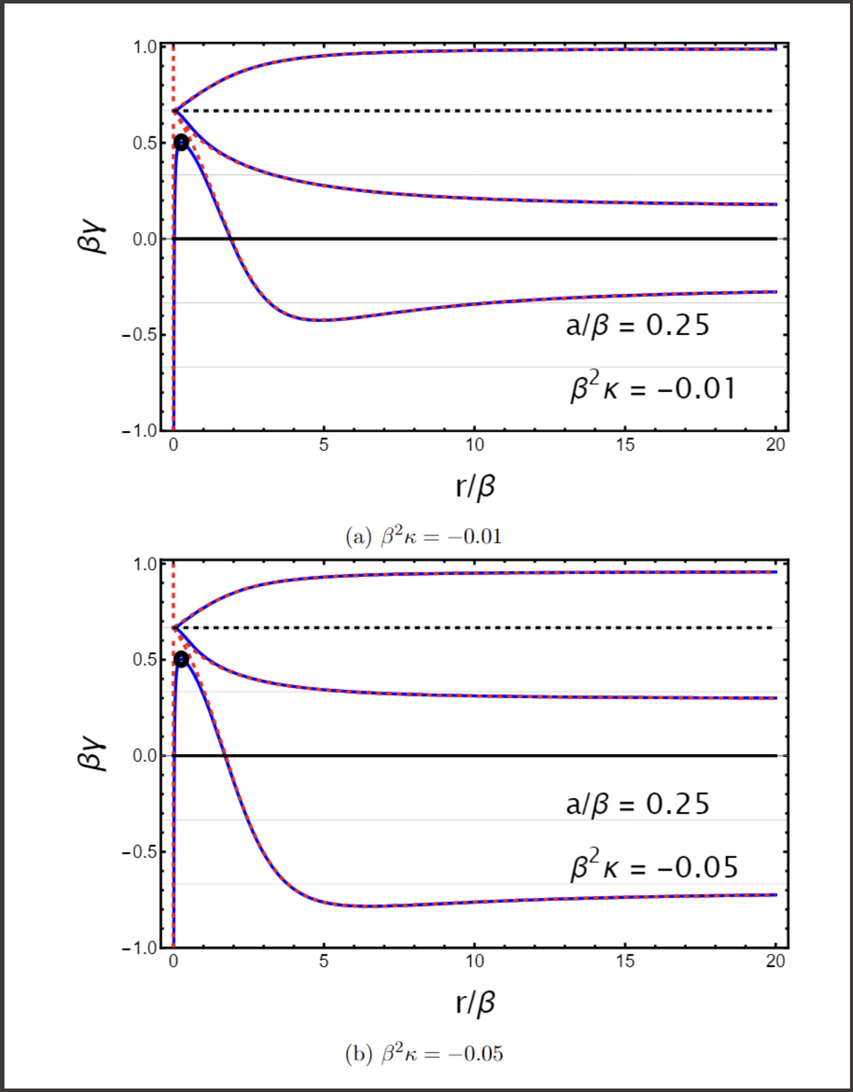
(b) $\beta^2\kappa = 0.05$

GR Kerr-dS

GR Kerr-dS

Horizons and Ergosurfaces

$\beta^2\kappa < 0$



(a) $\beta^2\kappa = -0.01$

(b) $\beta^2\kappa = -0.05$

GR Kerr-AdS

GR Kerr-AdS