technische universität
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Quantum Entanglement in Three-Flavor Neutrino Oscillations

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ISAPP- ”Neutrino and Dark Matter” In The Lab and The Universe

Introduction

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- Applications:

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- Why?
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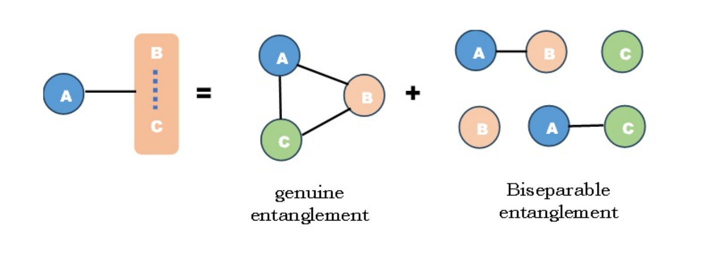
- Applications: Quantum Computing, Quantum Informations, quantum cryptography, High energy Physics, Cosmology

Introduction

- A fundamental principle in quantum entanglement is monogamy entanglement



- The Coffman, Kundu, and Wootters (CKW) inequality, which is quantitatively displayed as

$$T(\rho_{AB}) + T(\rho_{AC}) \leq T(\rho_{A(BC)}) \quad (1)$$

Motivation

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- Investigation quantum entanglement in theree-flavor NOs under wave packet treatment is less explored, especially in high energy level

Objective

- investigating the entanglement distribution (via Tangle) by considering variables:
  - different initial neutrino states (e-channel, μ-channel)
  - different CP violation phases
  - different flavor (mode) entanglement $e(\mu\tau), \mu(e\tau), \tau(e\mu)$
  - different energy levels

  How these variables affect on the Neutrino Oscillations
- study genuine multipartite entanglement by assessing the CKW criteria

Review: Neutrino Oscillations under Wavepacket Treatment

Neutrino Oscillations under Wavepacket Treatment

- The flavor state $|\nu_\alpha(x,t)\rangle$ is depicted as a linear superposition of the mass eigenstates $|\nu_s\rangle$ propagating along the x-direction:

$$|\nu_\alpha(x,t)\rangle = \sum_s V^*_{\alpha s}(\theta)\psi_s(x,t)\,|\nu_s\rangle \tag{2}$$

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$$|\nu_\alpha(x,t)\rangle = \sum_s V^*_{\alpha s}(\theta)\psi_s(x,t)\,|\nu_s\rangle \tag{3}$$

- Each mass eigenstate $|\nu_s\rangle$ is described by a wave packet of the form:

$$\psi_s(x,t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{dp}\psi_s(p)e^{ipx-iE_s(p)t} \tag{4}$$

where:

- the momentum distribution of the wave packet for the s^th mass eigenstate : $\psi_s(p) = (2\pi\sigma_P^2)^{1/4}\,\mathbf{exp} - \frac{(p-p_s)^2}{4\sigma_P^2}$
- $E_s(p) = \sqrt{p^2 + m_s^2}$ is the relativistic energy for mass eigenstate

Neutrino Oscillations under Wavepacket Treatment

After implementing a Gaussian integration over p, the flavor neutrino state in coordinate space becomes:

$$|\nu_\alpha(x,t)\rangle = (2\pi\sigma_x^2)^{-1/4} \sum_s V_{\alpha s}^* \exp\left[ip_s x - iE_s t - \frac{(x - v_s t)^2}{4\sigma_x^2}\right] |\nu_s\rangle \quad (5)$$

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$$E_s \cong E, \qquad p_s \cong E - \frac{m_s^2}{2E}, \qquad v_s \cong 1 - \frac{m_s^2}{2E_s^2} \quad (7)$$

Finally, the density matrix is delivered by :

$$\rho^{(\alpha)}(x) = \sum_{sr} V_{\alpha s} V^{*}_{\alpha r} \varphi_{sr}(x) \left|\nu_s\right\rangle \left\langle\nu_r\right| \tag{8}$$

$$\varphi_{sr} \equiv \exp - \left[i \frac{\Delta m^2_{sr} x}{2E} + \left(\frac{\Delta m^2_{sr} x}{4\sqrt{2} E^2 \sigma_x} \right)^2 \right] \tag{9}$$

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The occupation number of flavors neutrino in the following form:

$$\left|\nu_e\right\rangle = \left|1\right\rangle_e \otimes \left|0\right\rangle_\mu \otimes \left|0\right\rangle_\tau \equiv \left|100\right\rangle_e \tag{10}$$

$$\left|\nu_\mu\right\rangle = \left|0\right\rangle_e \otimes \left|1\right\rangle_\mu \otimes \left|0\right\rangle_\tau \equiv \left|010\right\rangle_\mu \tag{11}$$

$$\left|\nu_\tau\right\rangle = \left|0\right\rangle_e \otimes \left|0\right\rangle_\mu \otimes \left|1\right\rangle_\tau \equiv \left|001\right\rangle_\tau . \tag{12}$$

Neutrino Oscillations under Wavepacket Treatment

Adopting $|\nu_s\rangle = \sum_\alpha V_{\alpha r} |\nu_g\rangle$ with $g = e, \mu, \tau$, the density matrix is displayed as

$$\rho^{(\alpha)}(x) = \sum_{gj} F^{\alpha}_{gj}(x) |\nu_g\rangle \langle \nu_j| \tag{13}$$

and

$$F^{\alpha}_{gj}(x) = \sum_{sr} V^*_{\alpha s} V_{\alpha r} \varphi_{sr} V_{gs} V^*_{jr} \tag{14}$$

In a matrix form

$$\rho^{(\alpha)}_{e\mu\tau}(x) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F^{\alpha}_{ee}(x) & 0 & F^{\alpha}_{e\mu}(x) & F^{\alpha}_{e\tau}(x) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F^{\alpha}_{\mu e}(x) & 0 & F^{\alpha}_{\mu\mu}(x) & F^{\alpha}_{\mu\tau}(x) & 0 \\ 0 & 0 & 0 & F^{\alpha}_{\tau e}(x) & 0 & F^{\alpha}_{\tau\mu}(x) & F^{\alpha}_{\tau\tau}(x) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{15}$$

Entanglement Measure in Neutrino Oscillations

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- $T_{e\mu} = \mathrm{Tr}\left(\rho^{(\alpha)}_{e\mu}(x)\tilde{\rho}^{(\alpha)}_{e\mu}(x)\right) = 4|F^{\alpha}_{ee}(x)||F^{\alpha}_{\mu\mu}(x)|$

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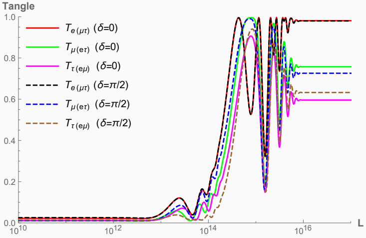
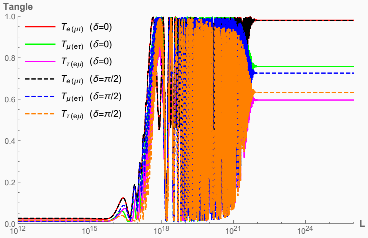
Entanglement Measure in Neutrino Oscillations

Tangle distribution for e-channel



E = 10GeV

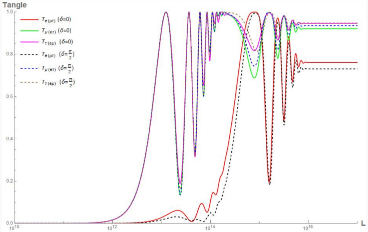


E = 10 TeV

- At short distances, minimal entanglement between the flavor occurred and the initial system was almost pure state.
- The oscillatory behavior portrays the quantum interference effects.
- The plateauing of the tangle at a very large L indicates that even after extensive propagation and decoherence, some entanglement remains in the system.
- this entanglement level depends strongly on the CP phase

Entanglement Measure in Neutrino Oscillations

Tangle distribution for μ-channel



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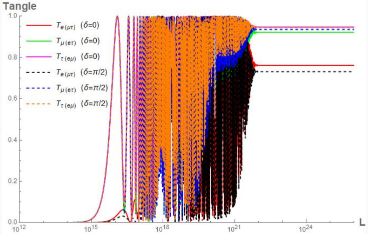
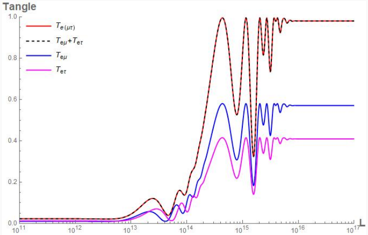


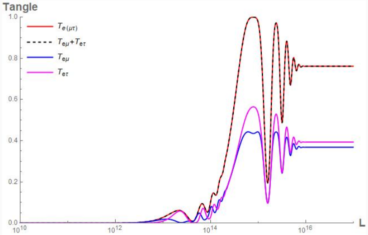
E = 10 TeV

- At short distances, No entanglement occurred. The initial system was almost pure state.
- the oscillatory and the decoherence parts have the same behavior as in the e-channel
- in both channels, combinations $e(\mu\tau)$ produce the same entanglement distributions when the different cp-phases applied

The CKW Inequality Check



e-channel : $T_{e\mu} > T_{e\tau}$



$\mu - channel : T_{e\mu} < T_{e\tau}$

- The CKW equality relation of the tangles holds true
- Entanglement is not overly concentrated in just one pair of neutrino states but is more evenly distributed.
- the residual entanglement vanishes, i.e., $\mathcal{T}_e = T_{e(\mu\tau)} - T_{e\mu} - T_{e\tau} = 0$ then the genuine tripartite entanglement is not well-defined but entanglement is still there

Summary

- Different flavor mode combinations and the CP-phases produce different the entanglement distributions
- The difference between the entanglement levels reflects how the CP-violating phase affects the long-distance behavior of entanglement
- All entanglement distributions show monogamy properties which implying that entanglement is shared among the neutrino flavors to preserve the structure of multipartite quantum states.
- Genuine multipartite entanglement is not well-defined in term of tangle reflecting that the entanglement is ”purely bipartite”, with correlations existing between pairs of qubits or subsystems.

Thank You