Feedback System for Hybrid Filling with Large Bunch Current Contrast

T. NAKAMURA*

KEK / J-PARC

This work was carried out while I was previously affiliated with JASRI/SPring-8

Based on collaboration of

T. NAKAMURA, K. KOBAYASHI#, T. FUJITA, M. MASAKI, H. DEWA

JASRI/ SPring-8

R. SREEDHARAN, R. NAGAOKA

Synchrotron SOLEIL

LEE Jaeyu, KIM Dotae, SHIN Seunghwan+

POHANG ACCELERATOR LABORATORY / PLS-II

K. KOBAYASHI, et al., https://www.pasj.jp/web_publish/pasj2015/proceedings/PDF/WEOL/WEOL03.pdf
Jaeyu LEE, et al., J Synchrotron Radiat . 2021 Sep 1;28(Pt 5):1417-1422.
A. Gamelin, R. Sreedharan, et.al , “SOLEIL transverse bunch-by-bunch feedback system”, This workshop

Programmed and manufactured by Tokyo Electron Device (TED) , based on our conceptual design

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*nkmr@post.kek.jp , https://research.kek.jp/people/nkmr/ , # kkoba@spring8.or.jp

+ Current Instittution : Korea University

I.FAST workshop on bunch-by-bunch feedback systems, KIT, 04/Mar/2024

Most Contents of this talk is overlapping of the talk at

IPAC 18 http://ipac18.org/

https://accelconf.web.cern.ch/ipac2018/papers/tuzgbd2.pdf

https://accelconf.web.cern.ch/ipac2018/talks/tuzgbd2_talk.pdf

and

The Joint ARIES Workshop on Electron and Hadron Synchrotrons:

Next Generation Beam Position Acquisition and Feedback Systems (2018)

https://indico.cern.ch/event/743699/contributions/3112134/attachments/1747270/2840473/SP8_BBF_ARES_WS_181114.pdf

SPring-8 standard filling modes (2024)

Bunch mode*1
203 bunches
4 bunch train x 84
11 bunch train x 29
1/7-filling + 5 bunches
2/29-filling + 26 bunches
406 x 11/29-bunches + 1 bunch

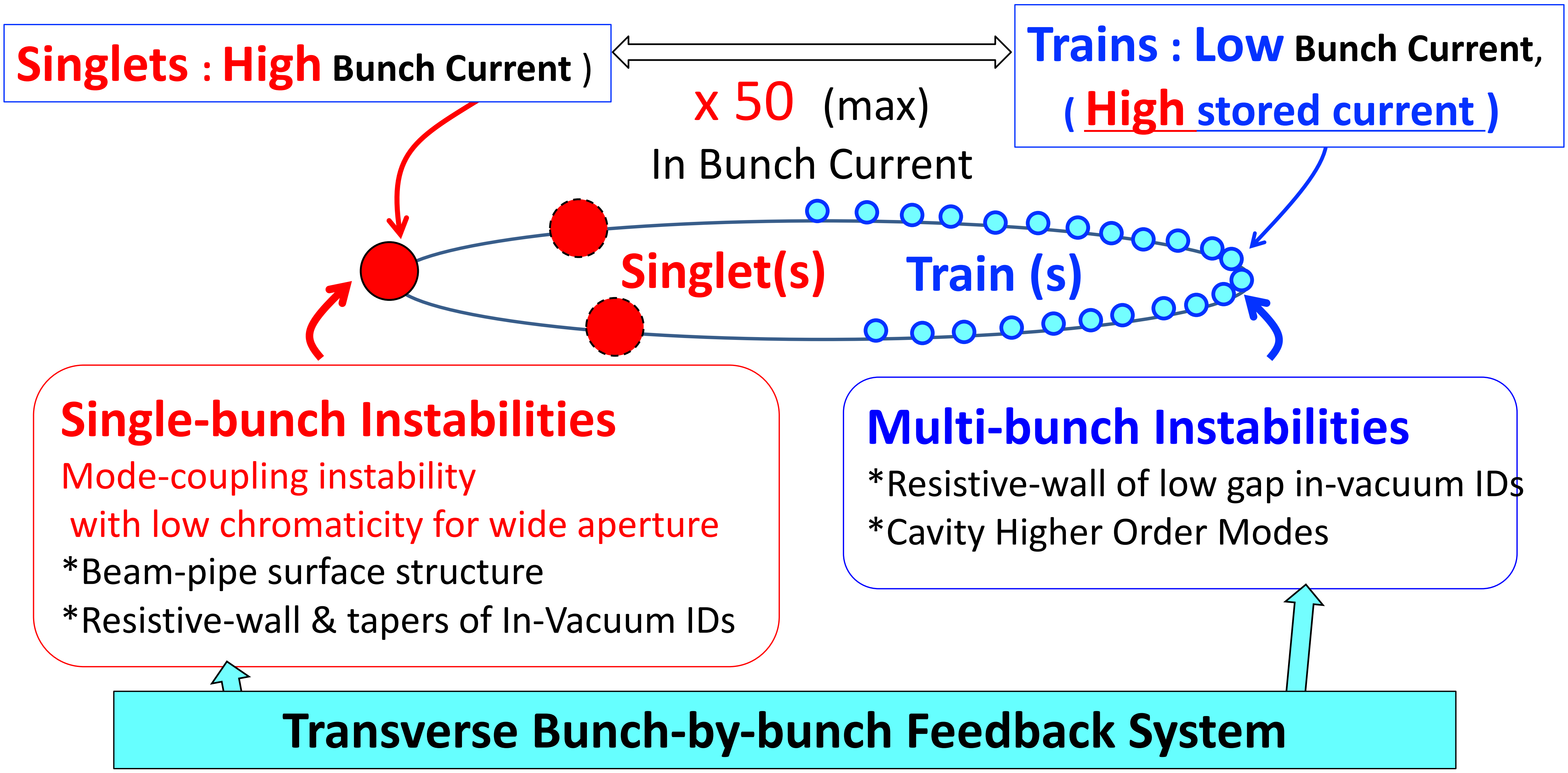
Trains

High Current Singlet(s)

Used be ~ 1000 bunches x **0.01 mA/bunch** and 1 x **5 mA/bunch** x 1

X 50

Hybrid Filling with Large Contrast Bunch Current



WHY and HOW

1) Single Analog Front-end   Conventional System

X : Saturation at High Current Bunch / Too **Low Gain** for Low Current Bunch

2) Single Analog Front-end + **Digital Gain Control**

X : Lost of ADC Resolution for Low Current Bunch (<- analog signal is so small)

3) One Analog Front-end + Signal Level Control

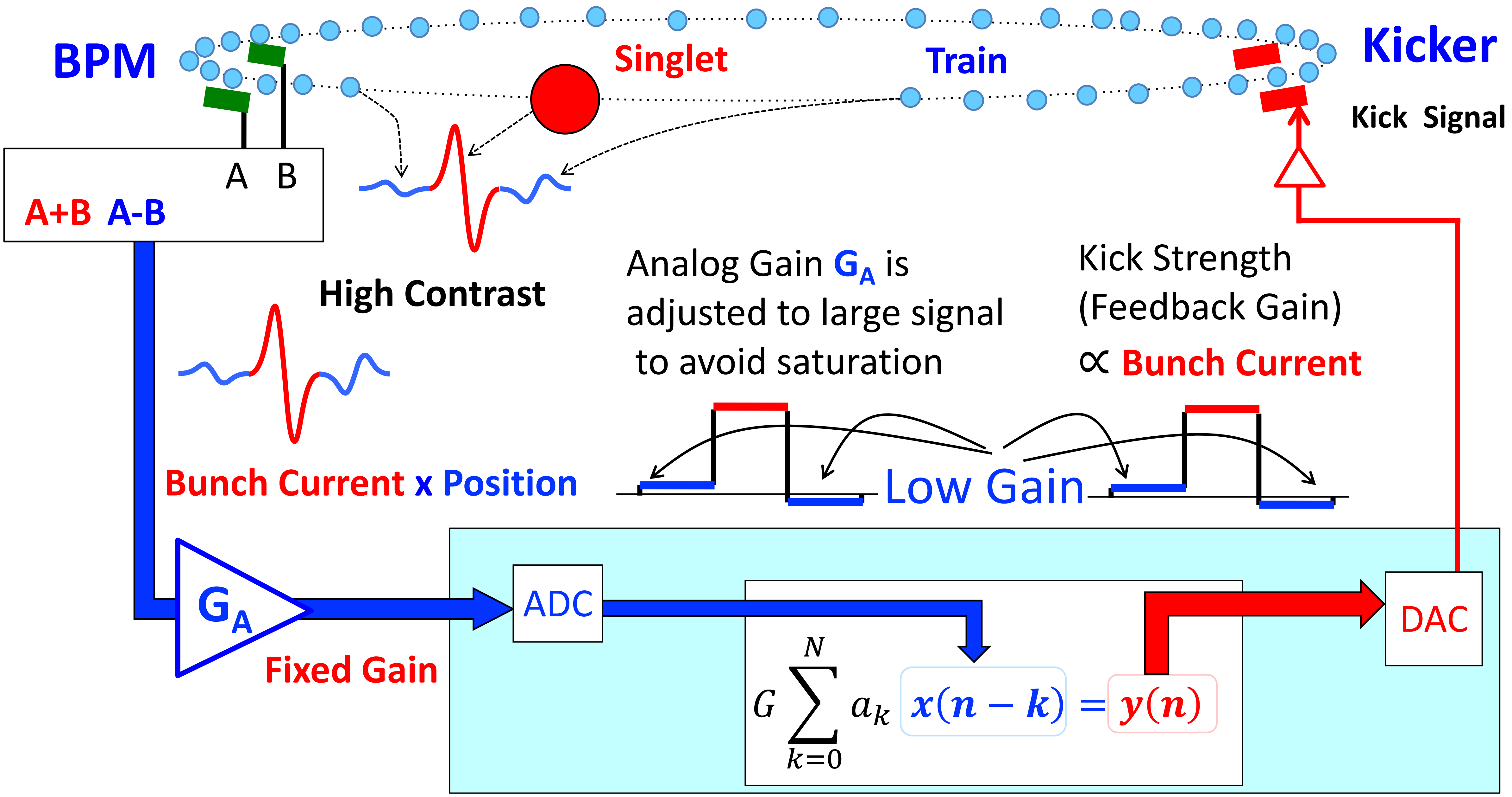
by **Fast Variable Attenuator**

X : Loss of SN ratio for High current bunch (High Gain is required => High SNR)

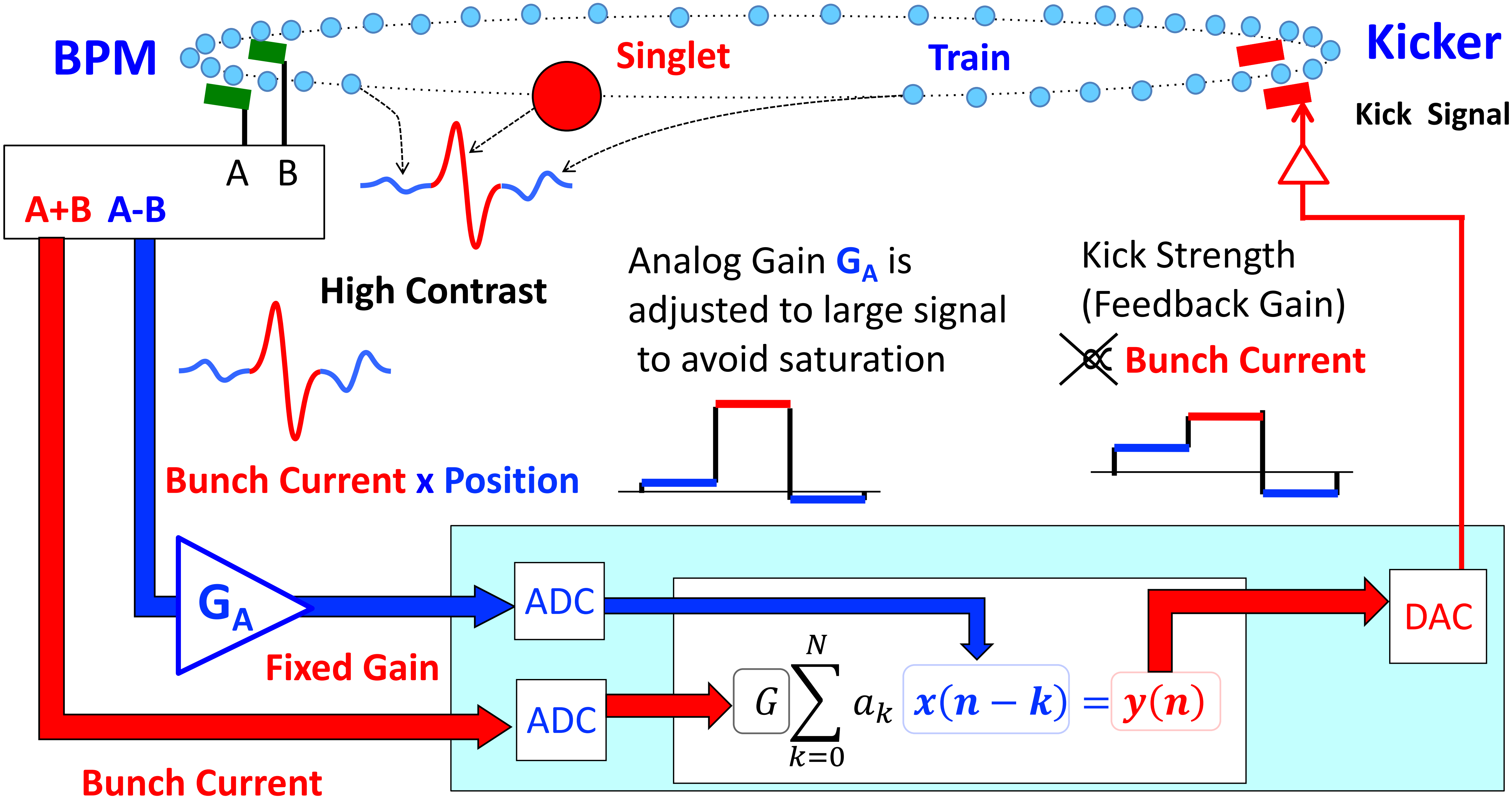
X : Complex System (two processor and attenuators)

4) "Multiple (Analog Front-end + ADC)" Switching with Bunch Current

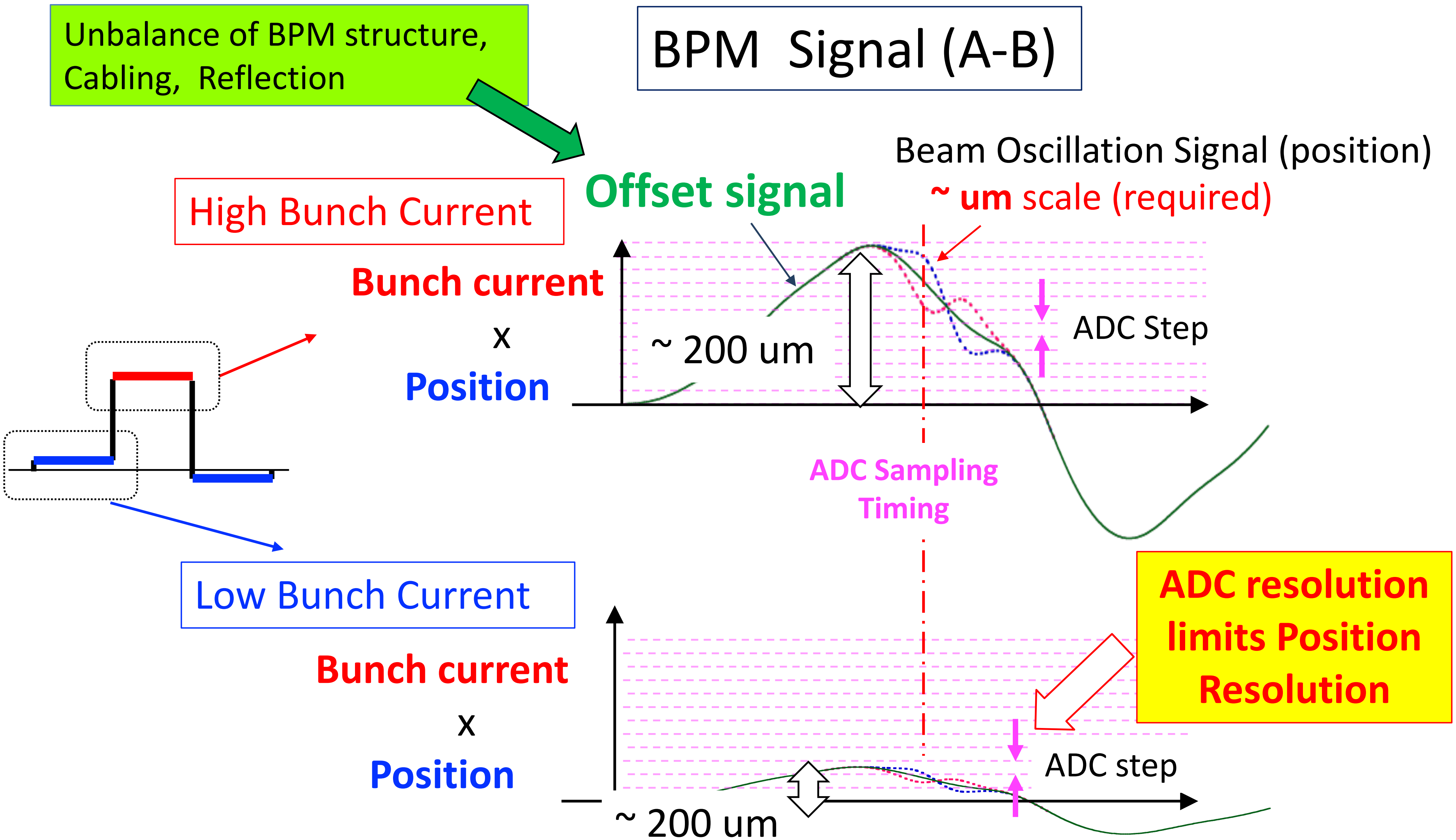
1) Single Front-end



2) Single Front-end + Digital Gain Control



However, ADC resolution is lost



Fixed Analog Gain + **Digital Gain** Switching

⇩

Analog Gain Switching

WHY and HOW

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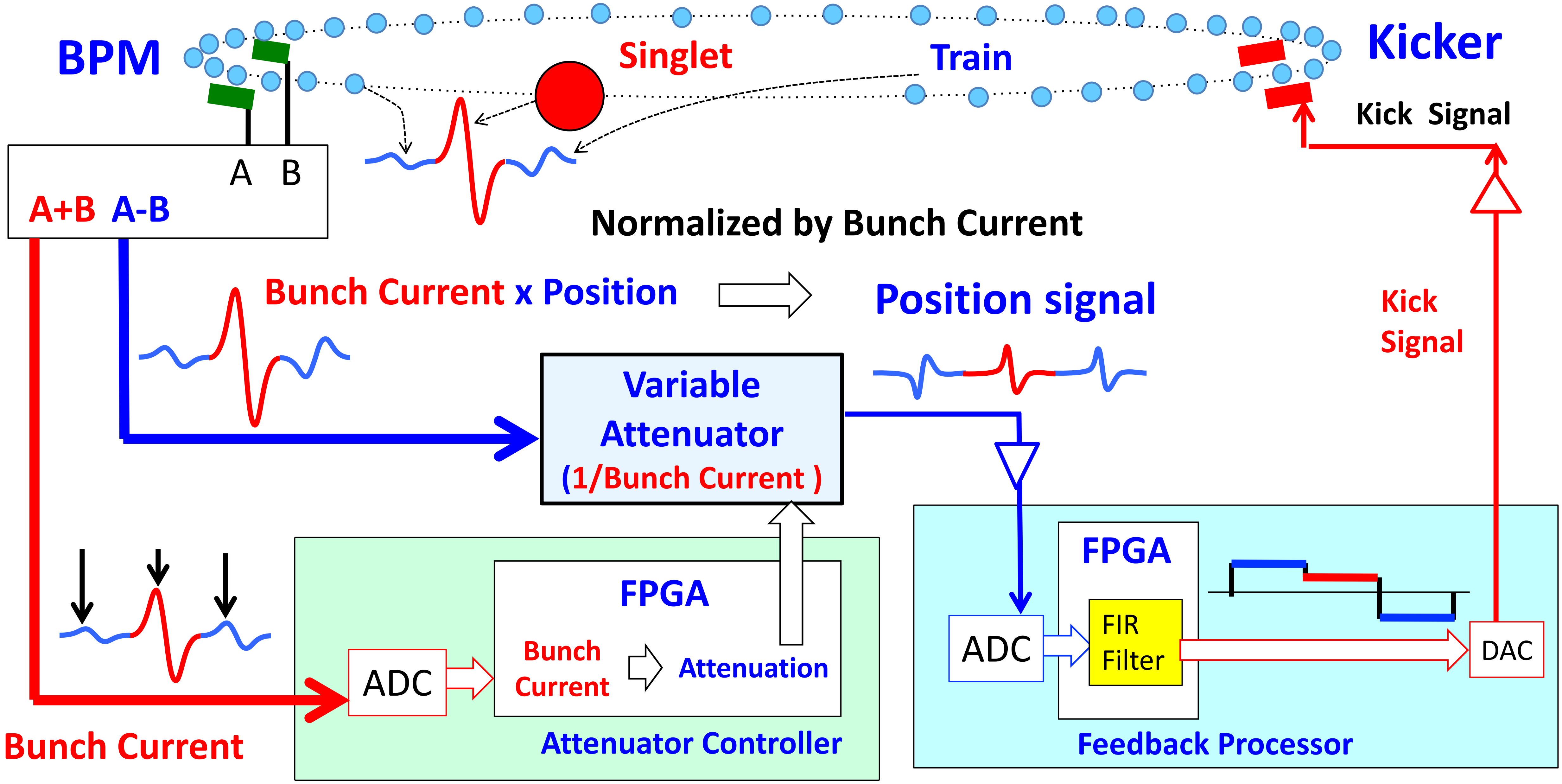
X : Complex System (two processor and attenuators)

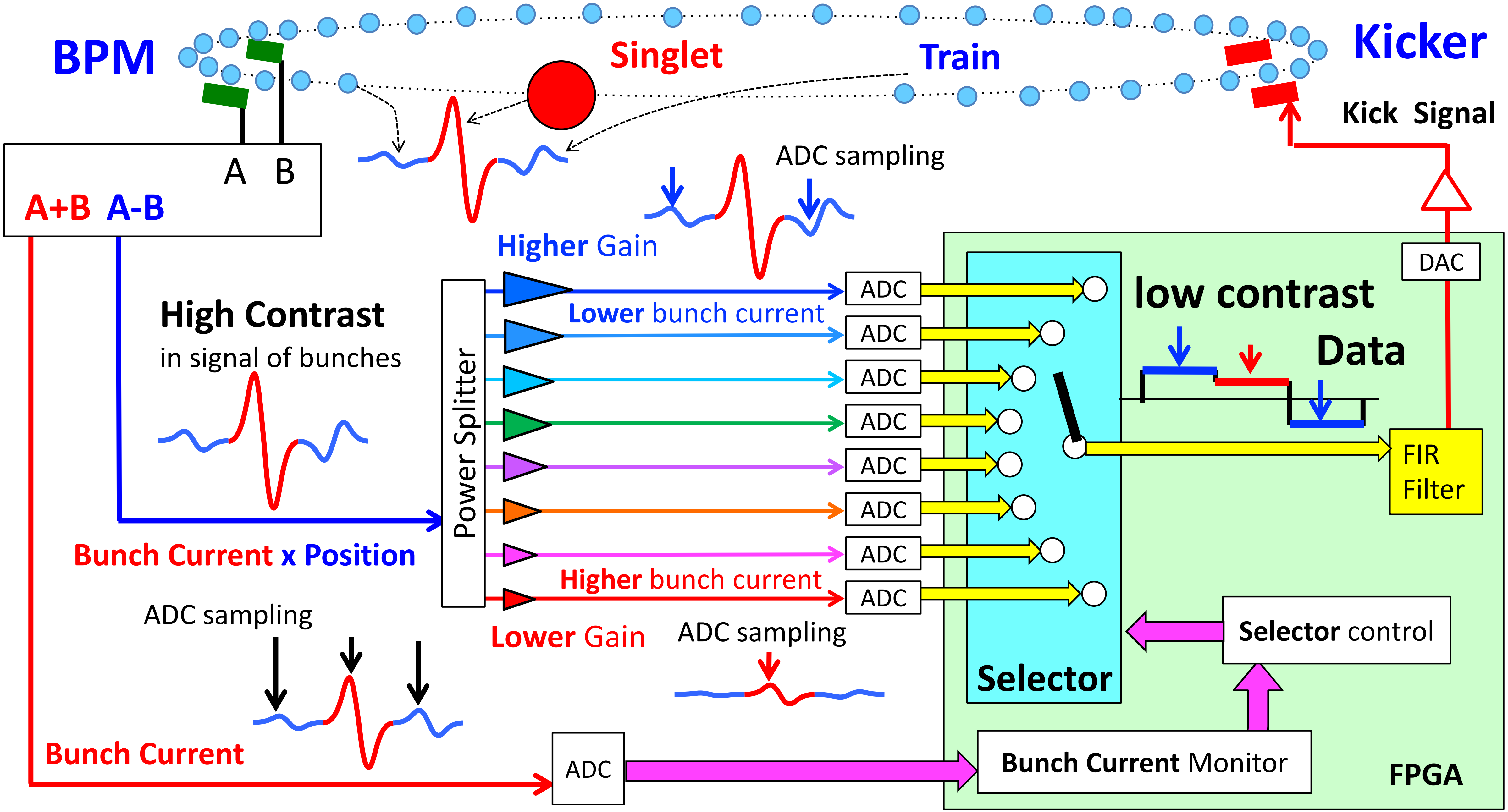
4) "Multiple (Analog Front-end + ADC)" Switching with Bunch Current

3) Analog Signal Level Control by Fast Variable Attenuator



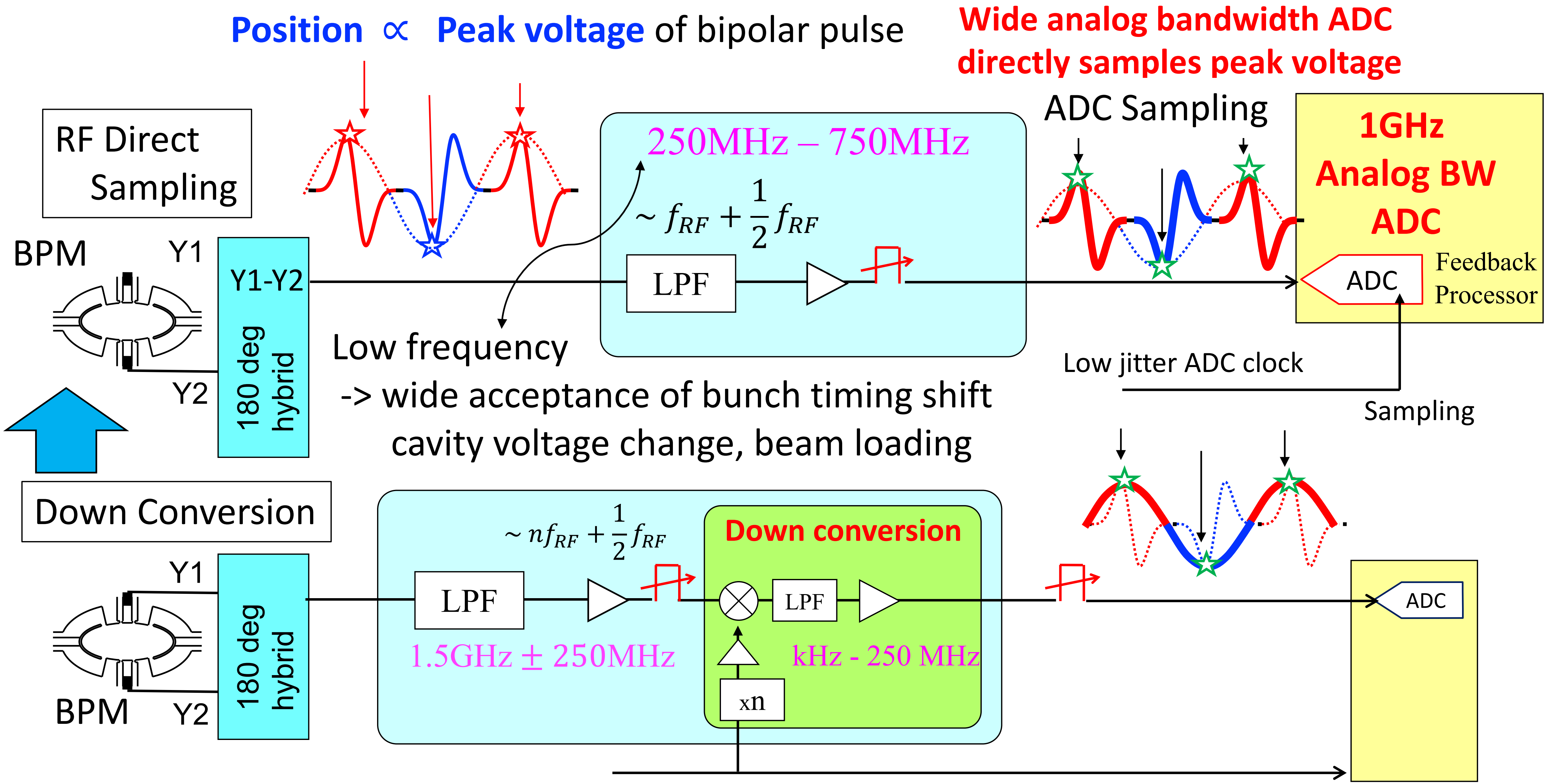
K. Kobayashi, T. Nakamura, THB006, ICALEPCS’09. (very primitive scheme :T. Nakamura, K. Kobayashi, et al., THPC127, EPAC08)

4) Analog Gain Switching with Multiple "Analog Front-end + ADC"



Front-End with RF Direct Sampling for Transverse Feedback

Front-End with RF Direct Sampling for Transverse Feedback



T. Nakamura, K. Kobayashi, Z. Zhou, https://accelconf.web.cern.ch/e08/papers/thpc128.pdf

Bunch Timing Spread at Hybrid Filling (Localized Filling)

Localized Filling (1/3 fill, 1/14 fill, …)
at Large Ring (5us revolution)

Beam time structure ~ a few us

Normal RF Acceleration Cavities

Filling time ~ a few us

Beam Loading by Localized Filling Modulates
Voltage and Phase of Cavity Voltage

Timing Spread of Bunches
~ 100ps (SPring-8 case)

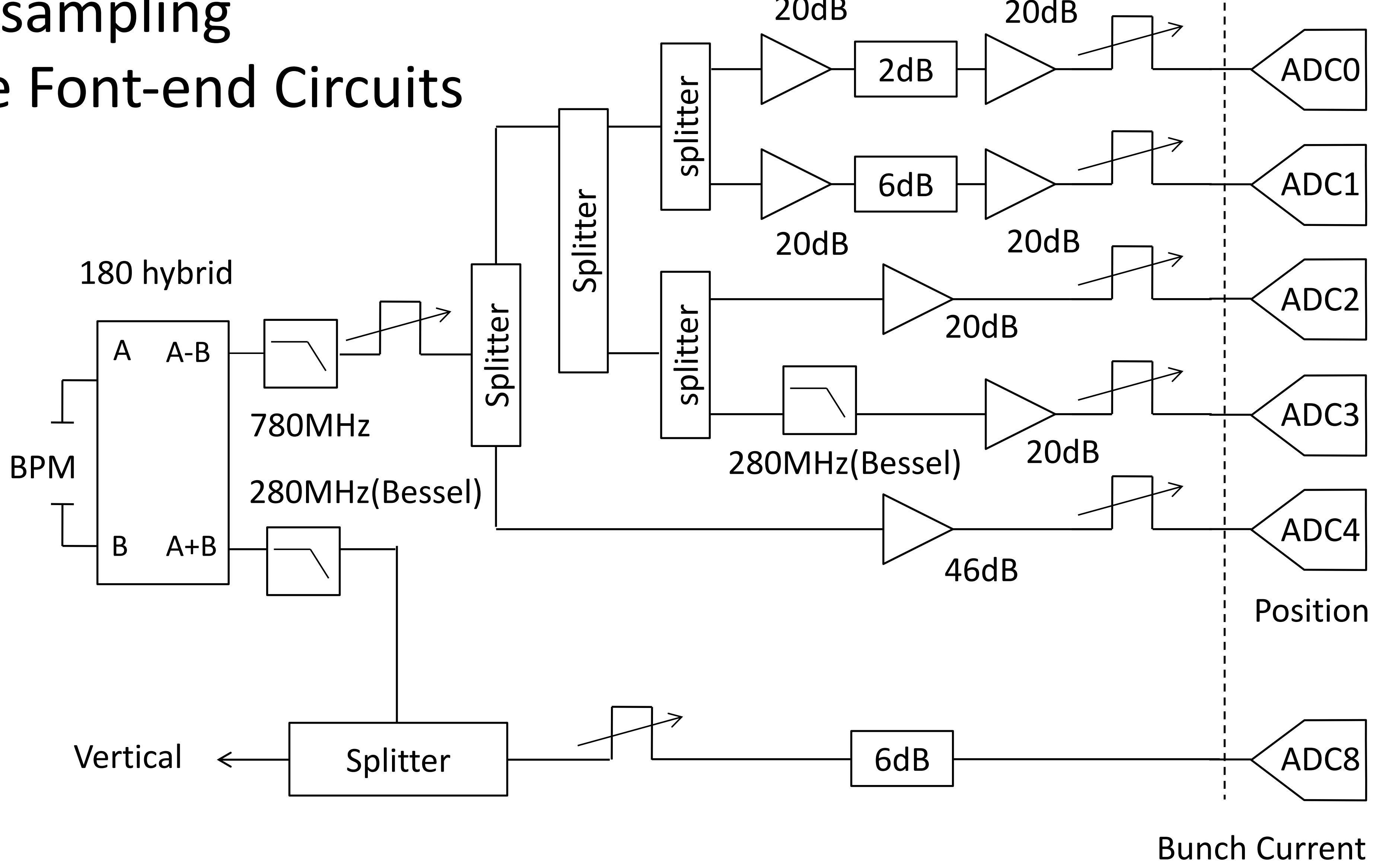
We choose Lowest Carrier frequency for BPM

$n \times f_{RF} = f_{RF}$ (n=1) ~ 500MHz

for wider acceptance for timing

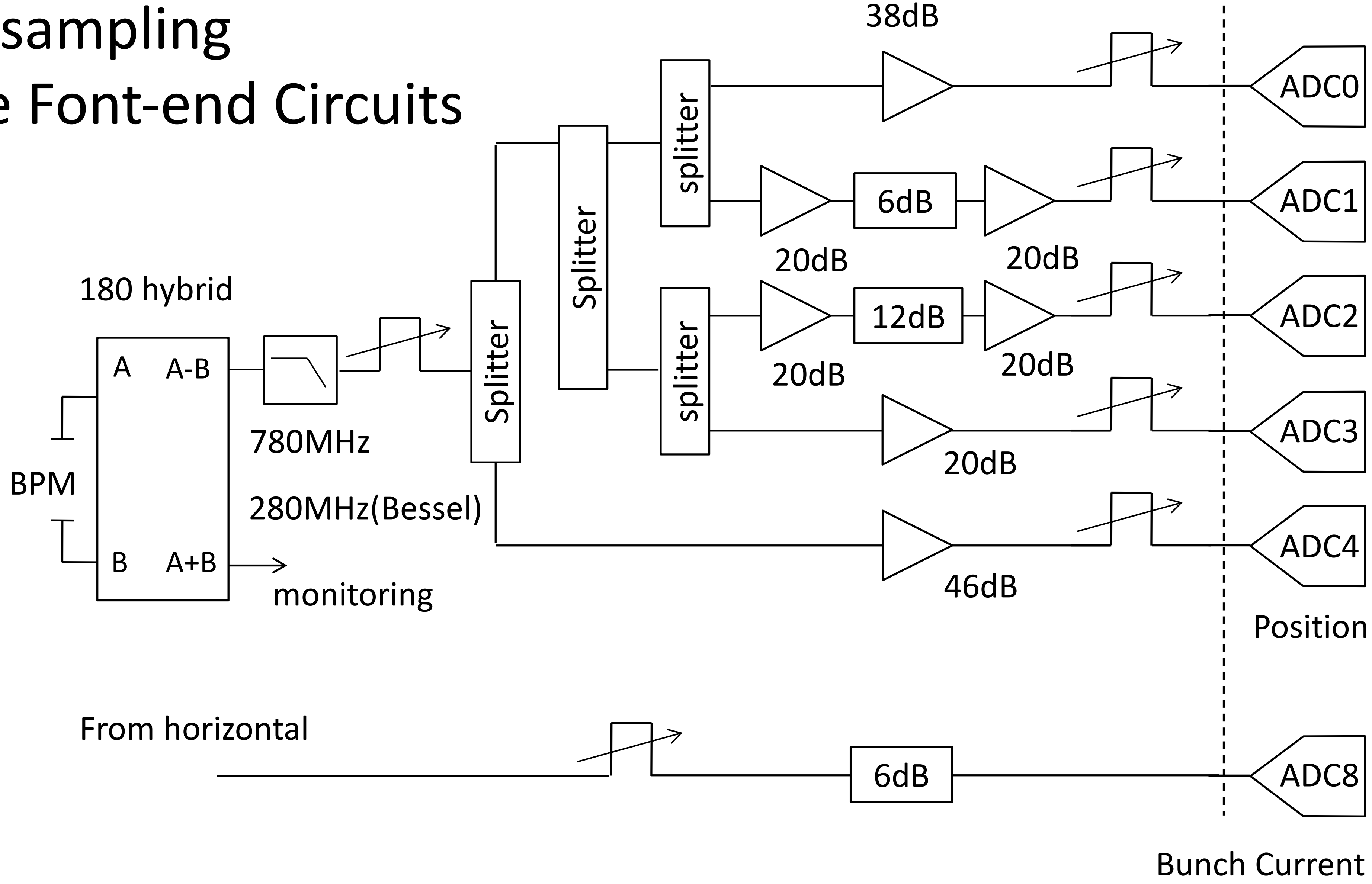
Front-end Horizontal for SPring^8

RF direct sampling
-> Simple Font-end Circuits



Front-end Vertical

RF direct sampling
-> Simple Font-end Circuits



Suppression of Single-bunch instability by Feedback

mode-coupling (fast head-tail) for V (and H : weak)

Chromaticity = 1 (< 3) for wide dynamic aperture

In-vacuum IDs **Open**

3.5 mA/bunch => 14 mA/bunch

Feedback **OFF** **ON**

~ simulation result

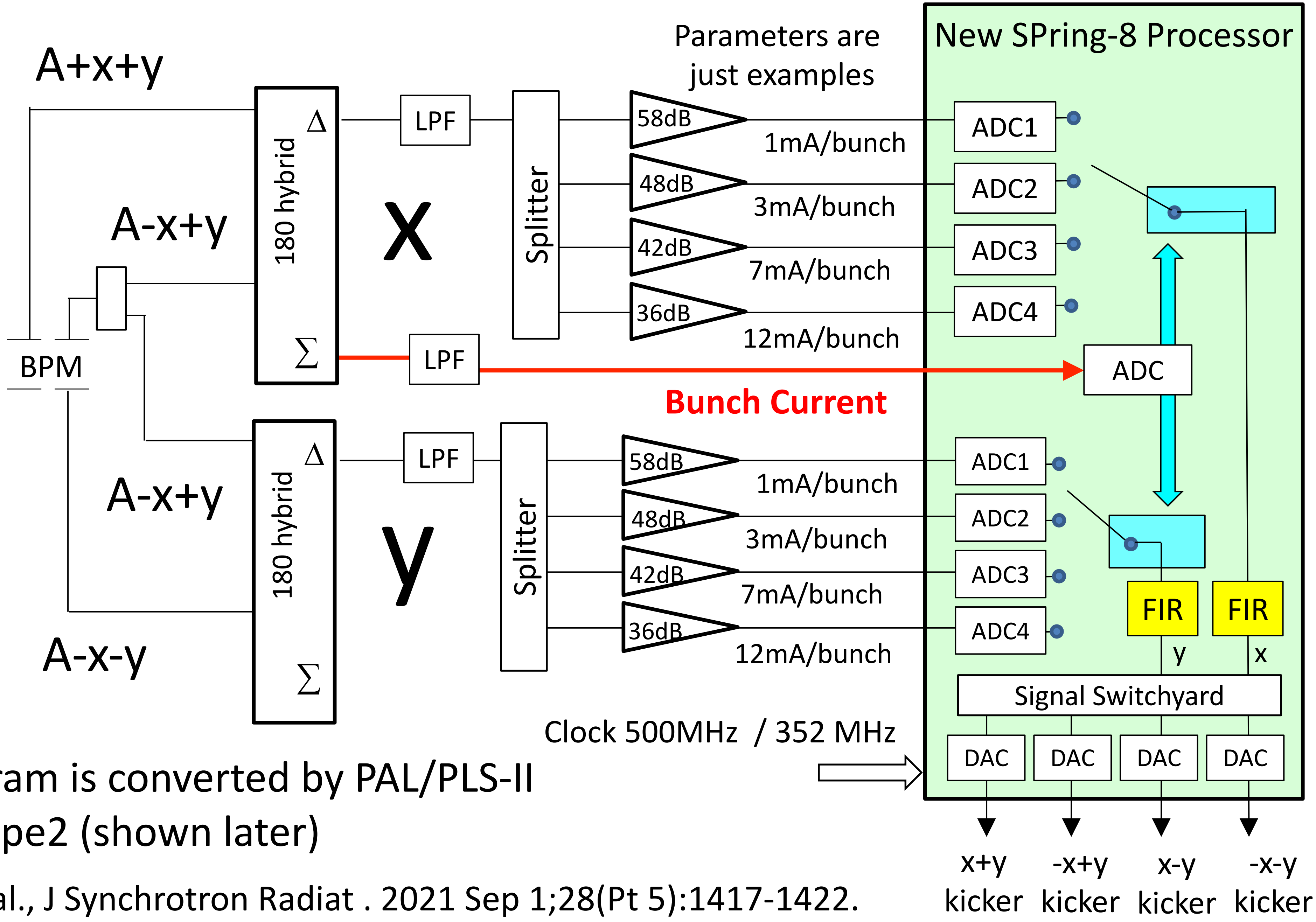
In-vacuum IDs **Close** **(**Partly ~ user operation**)**

2.5 mA/bunch => **6 mA/bunch**

Feedback **OFF** **ON**

5 mA/bunch for User operation

For Hybrid Filling for PLS-II : H and V in one processor

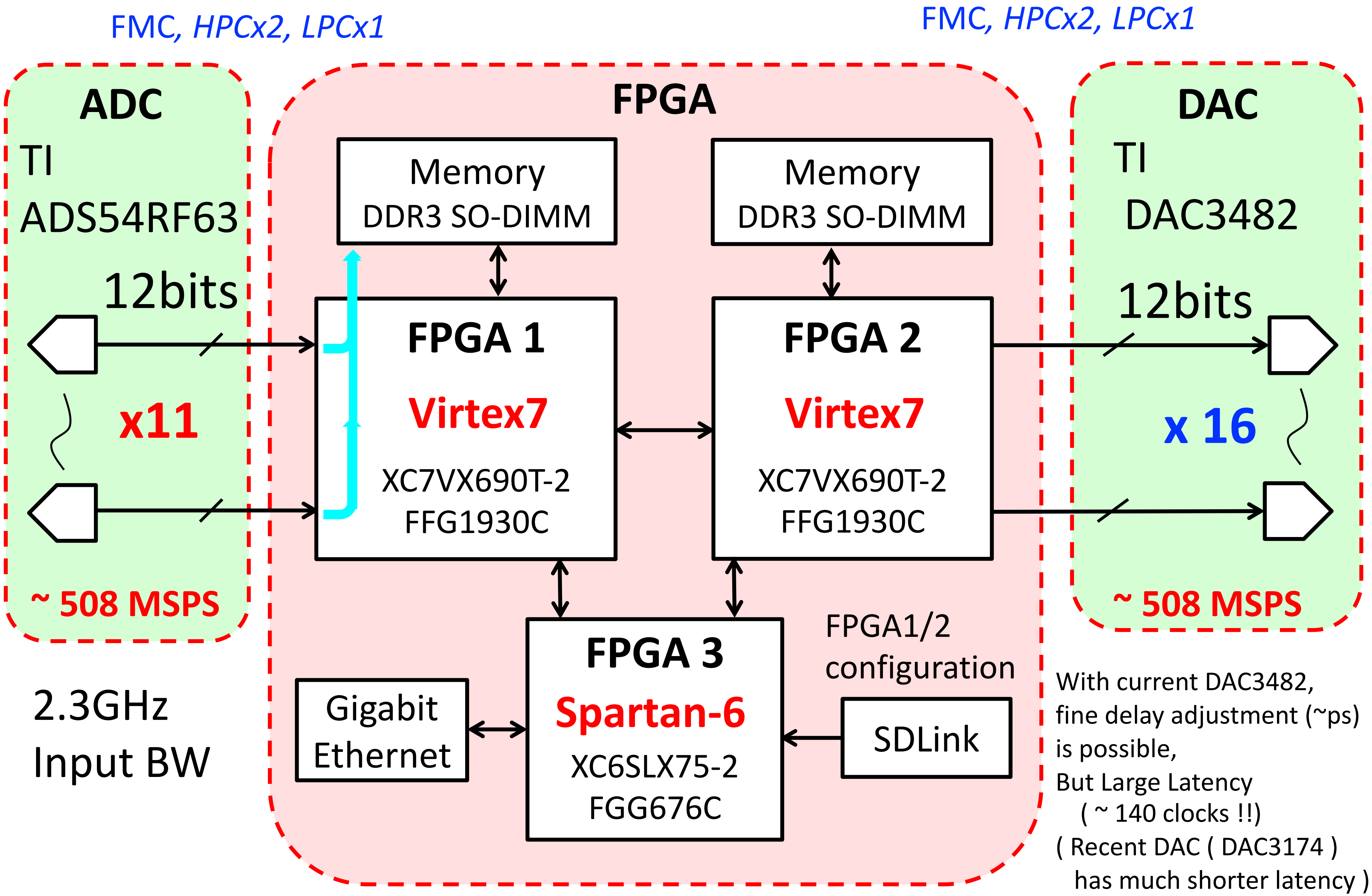


FPGA Program is converted by PAL/PLS-II
from type2 (shown later)

Jaeyu Lee, et al., J Synchrotron Radiat . 2021 Sep 1;28(Pt 5):1417-1422.

Hardware and Brock Diagram

Hardware Block Diagram

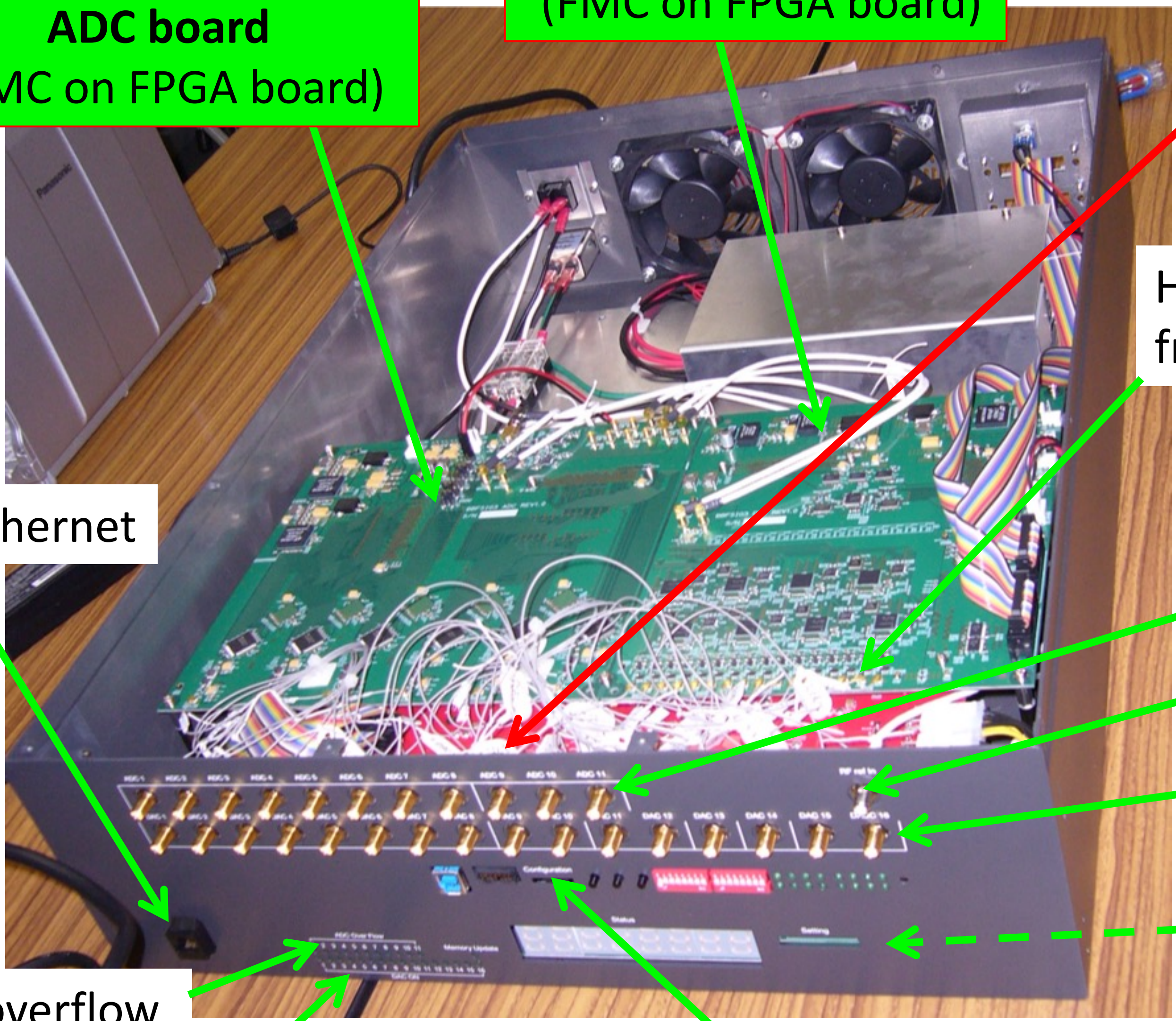


With current DAC3482,
fine delay adjustment (~ps)
is possible,
But Large Latency
(~ 140 clocks !!)
(Recent DAC (DAC3174)
has much shorter latency)

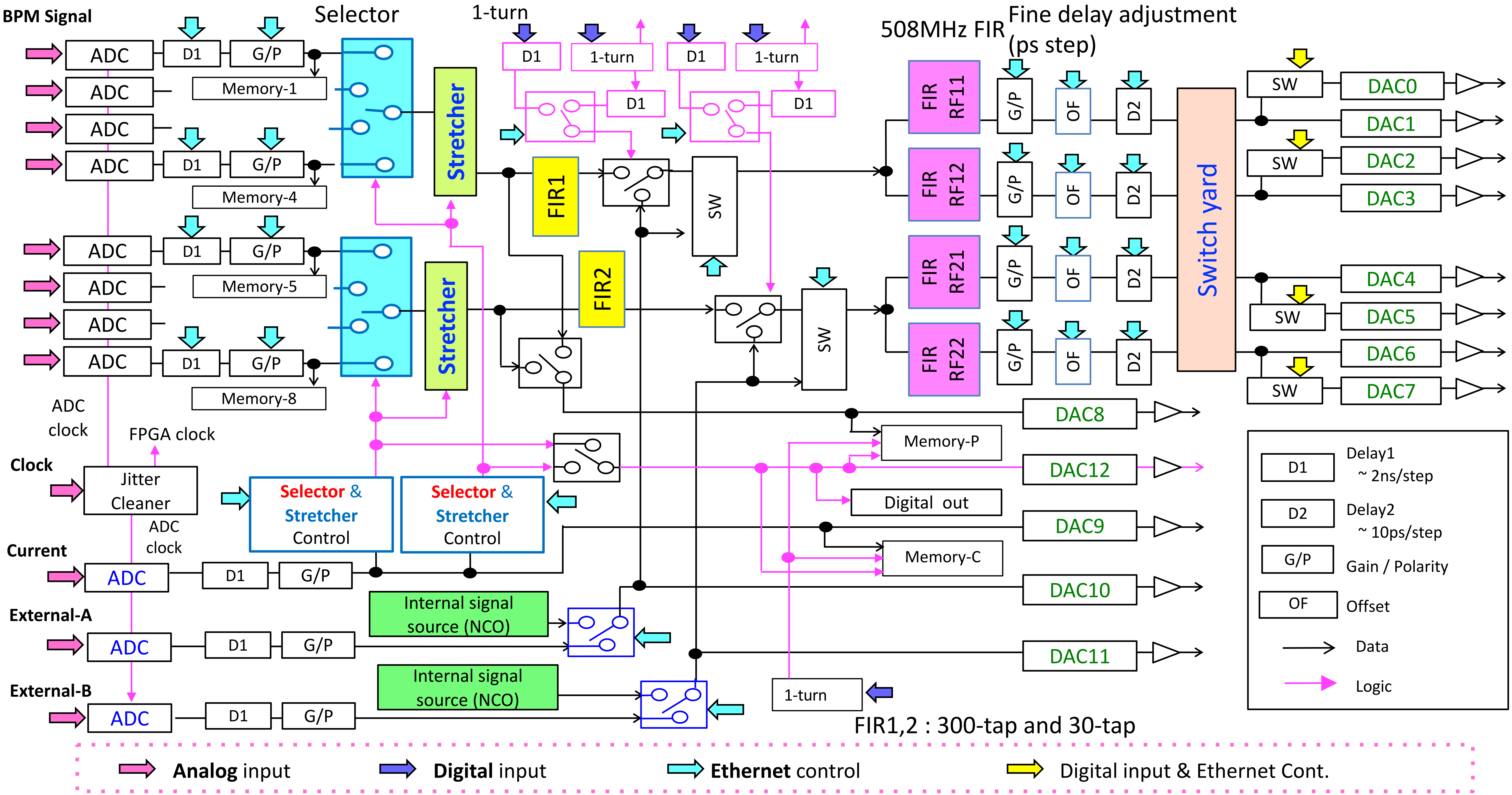
http://www.design-gateway.com/SDLink.html

Kazuo KOBAYASHI / SPring-8

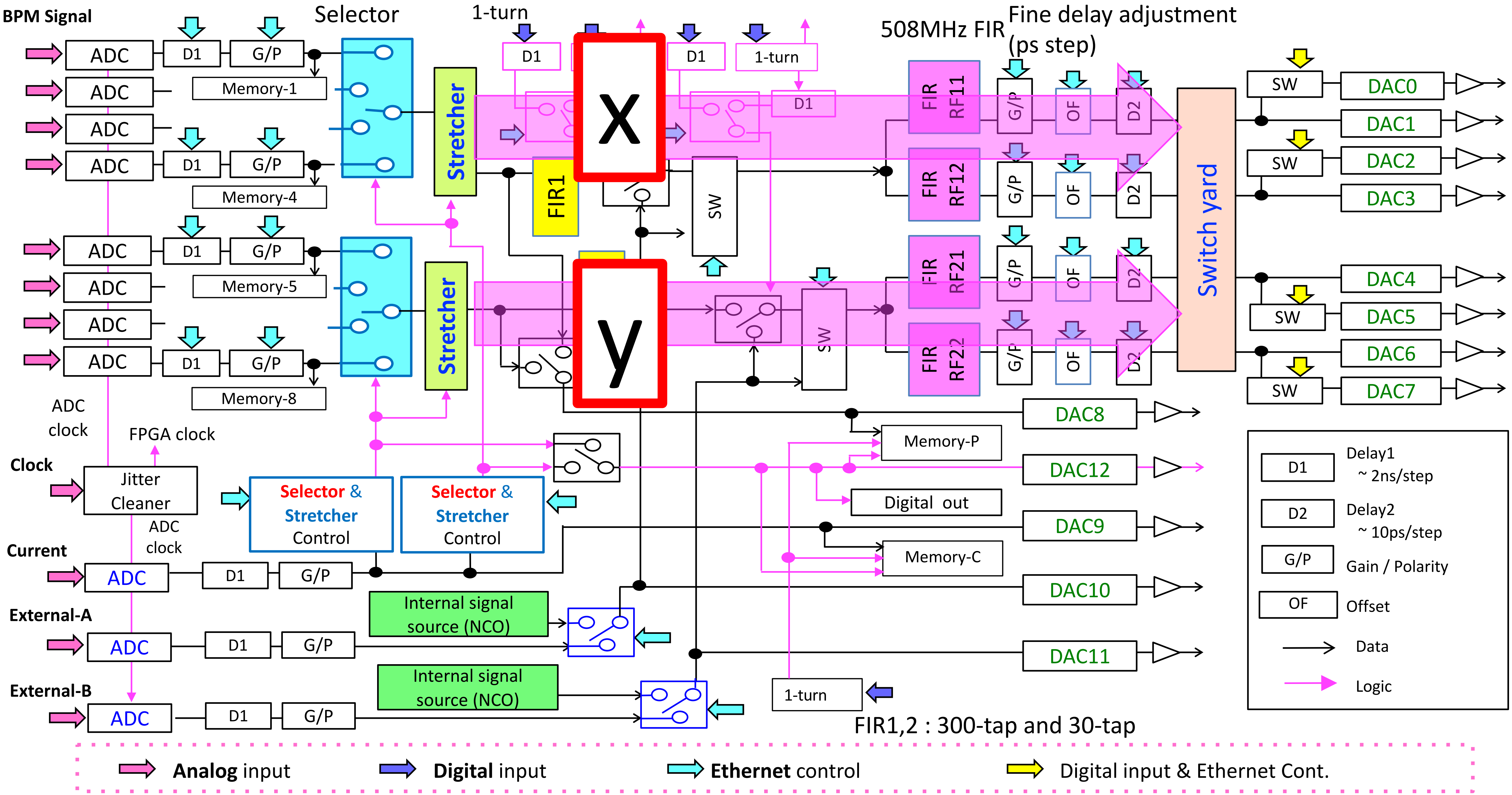
New SPring-8 Signal Processor (upside down)



Function Block Diagram (4 ADC x 2 direction version)



Function Block Diagram



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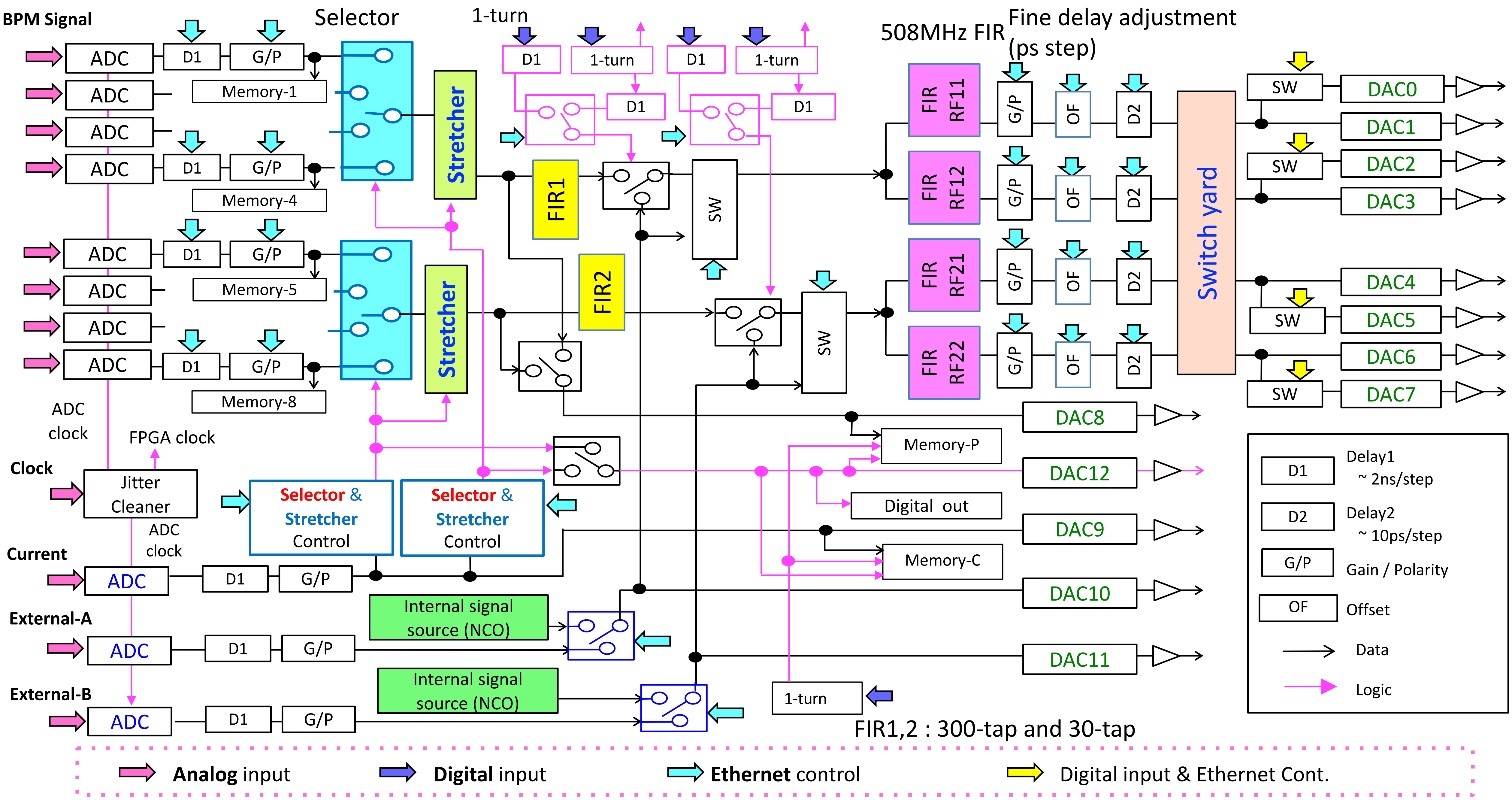
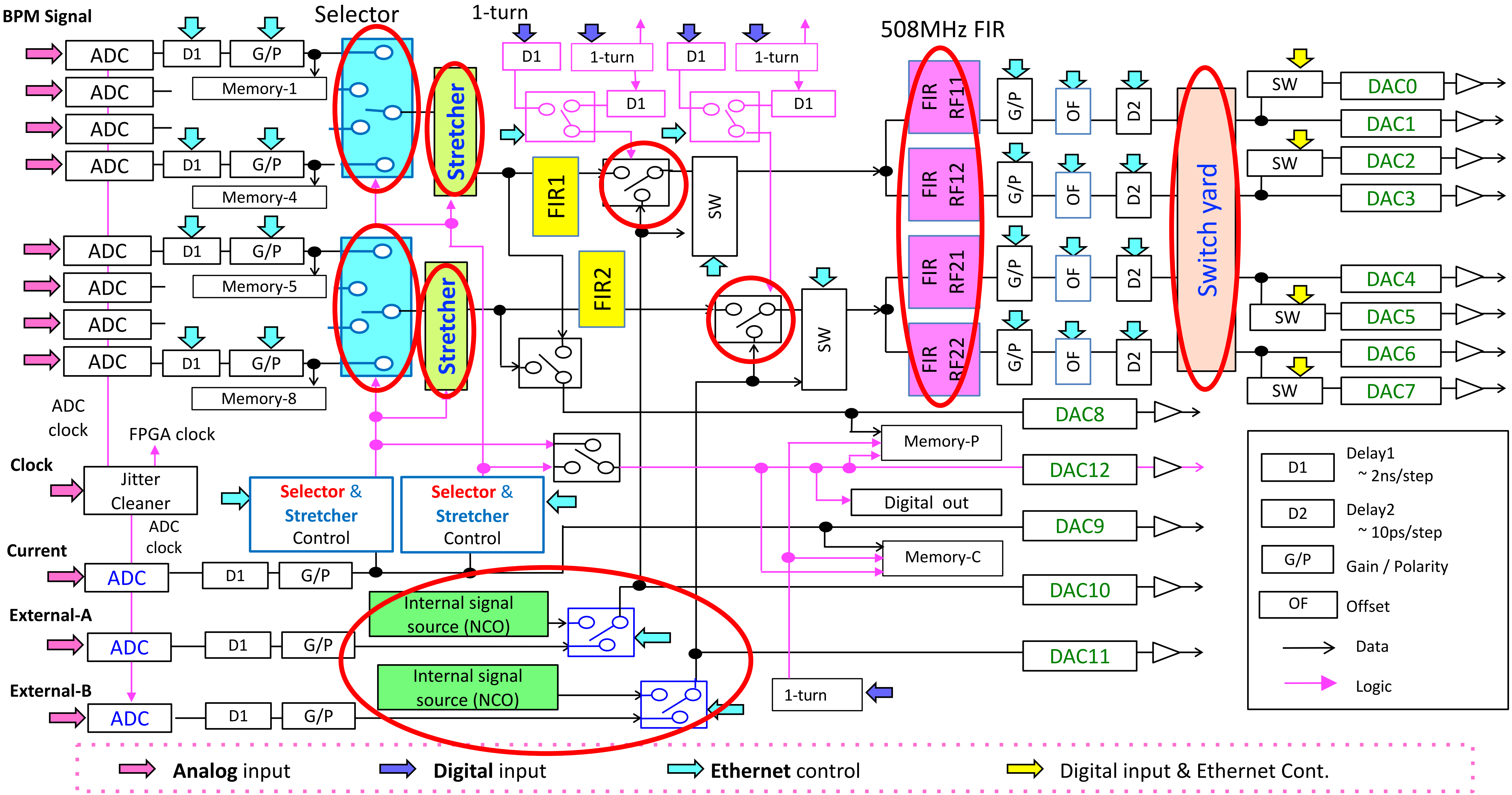
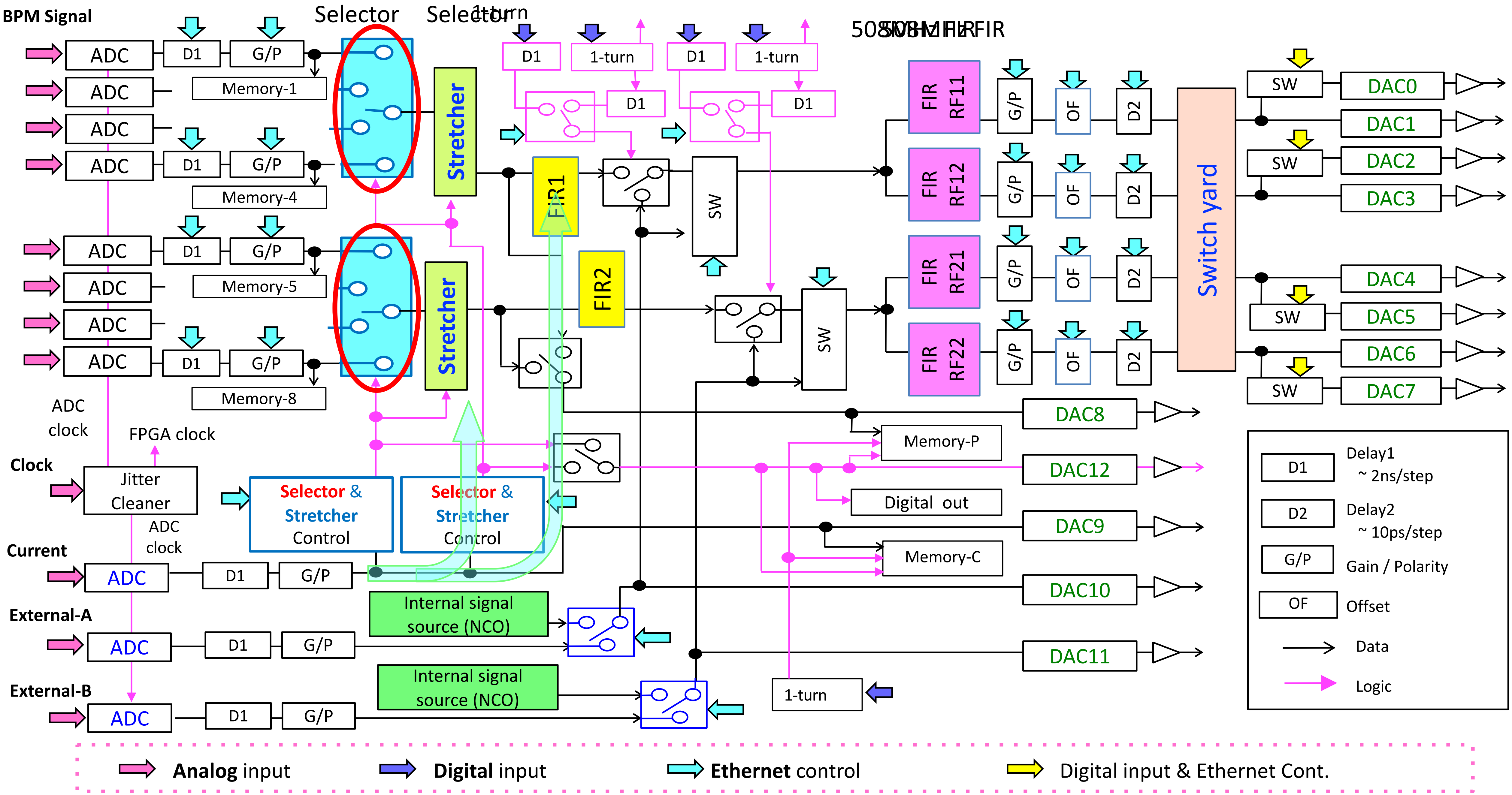


Gadgets



BPM Signal
ADC
D1
G/P
Memory-1
Memory-4
Memory-5
Memory-8
Selector
Stretcher
1-turn
FIR1
FIR2
SW
508MHz FIR
FIR RF11
FIR RF12
FIR RF21
FIR RF22
OF
D2
Switch yard
DAC0
DAC1
DAC2
DAC3
DAC4
DAC5
DAC6
DAC7
DAC8
DAC9
DAC10
DAC11
DAC12
Memory-P
Memory-C
Digital out
ADC clock
FPGA clock
Clock
Jitter Cleaner
Selector & Stretcher Control
Current
External-A
External-B
Internal signal source (NCO)
D1 Delay1 ~ 2ns/step
D2 Delay2 ~ 10ps/step
G/P Gain / Polarity
OF Offset
Data
Logic
Analog input
Digital input
Ethernet control
Digital input & Ethernet Cont.

Selection of ADC with Bunch Current Controlled Selector



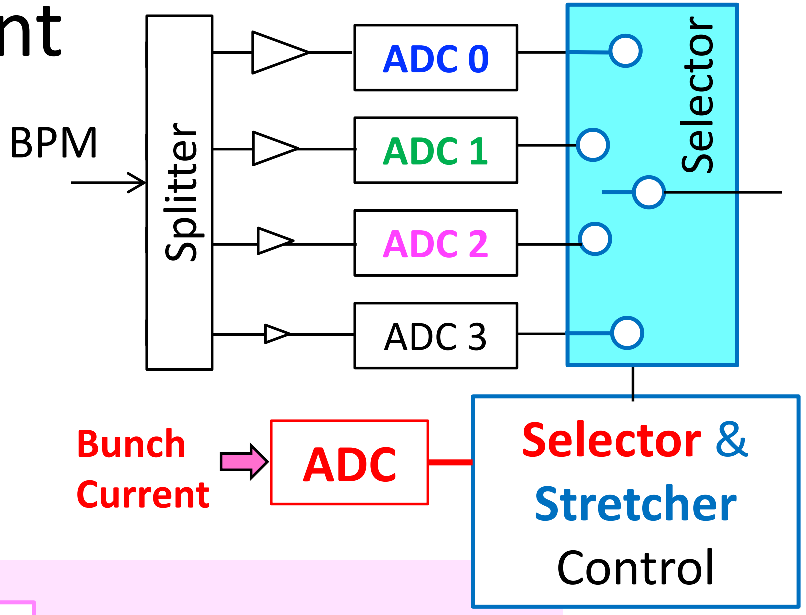
Selector Control with anti-chattering

ADC Selection for bunch current

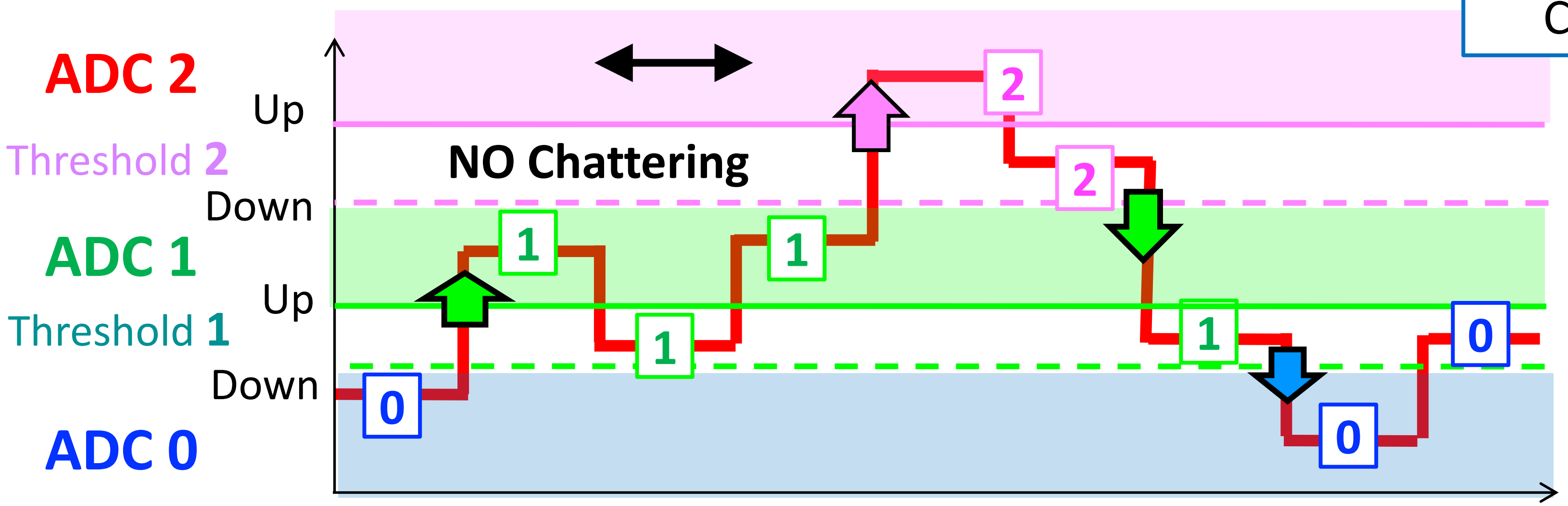
Anti-chattering by

Up-ward threshold and

Down-ward threshold

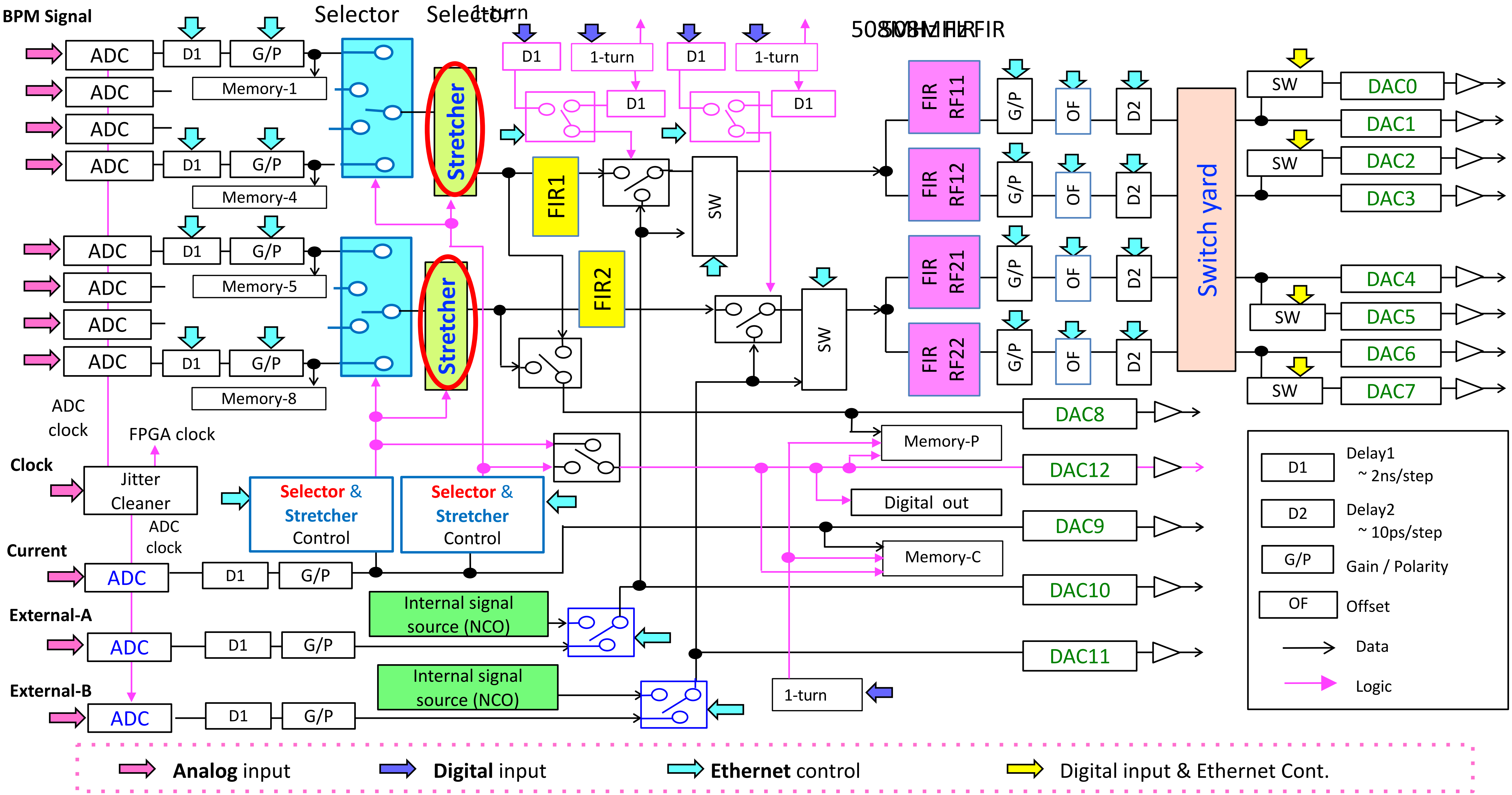


Turn-by-turn Bunch current history



Selected ADC #	ADC 0	ADC 1	ADC 1	ADC 1	ADC 2	ADC 2	ADC 1	ADC 0	ADC 0

Stretcher



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Selector
1-turn
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508MHz FIR
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Switch yard
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Analog input
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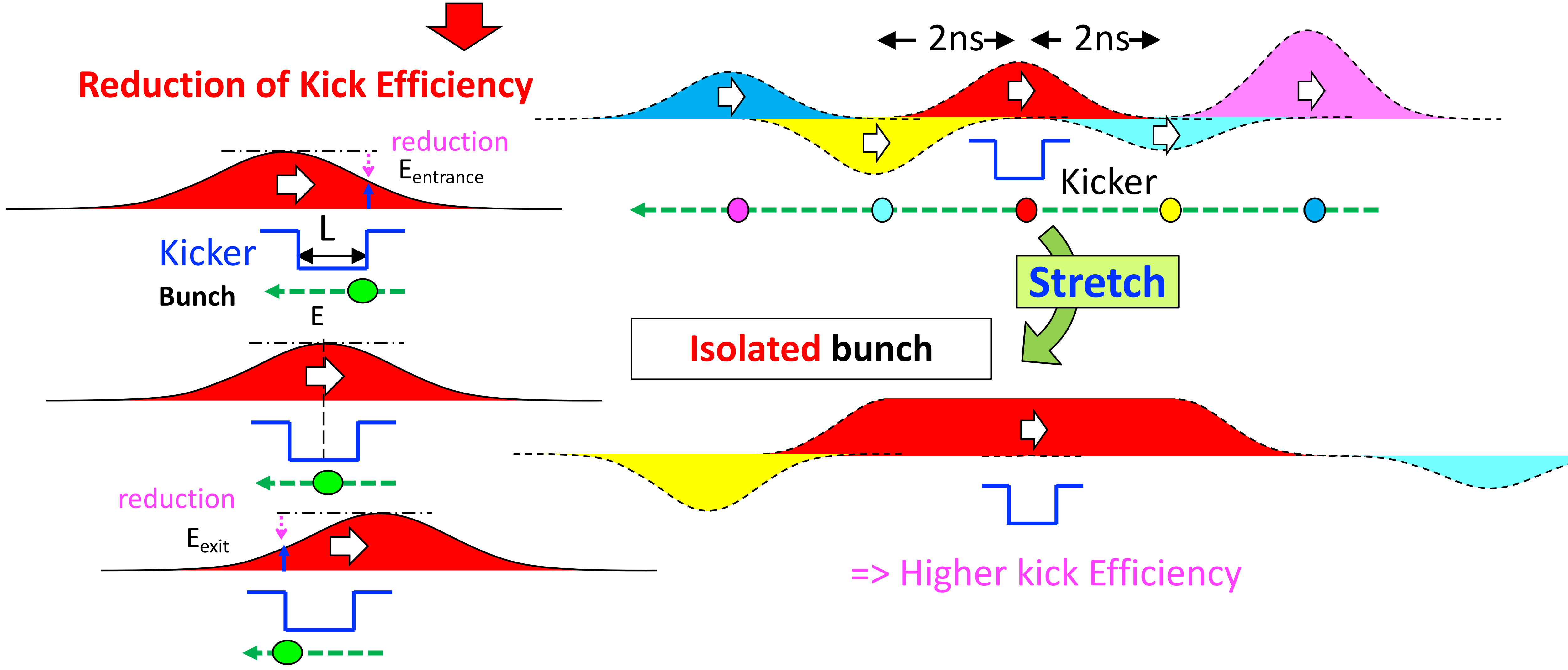
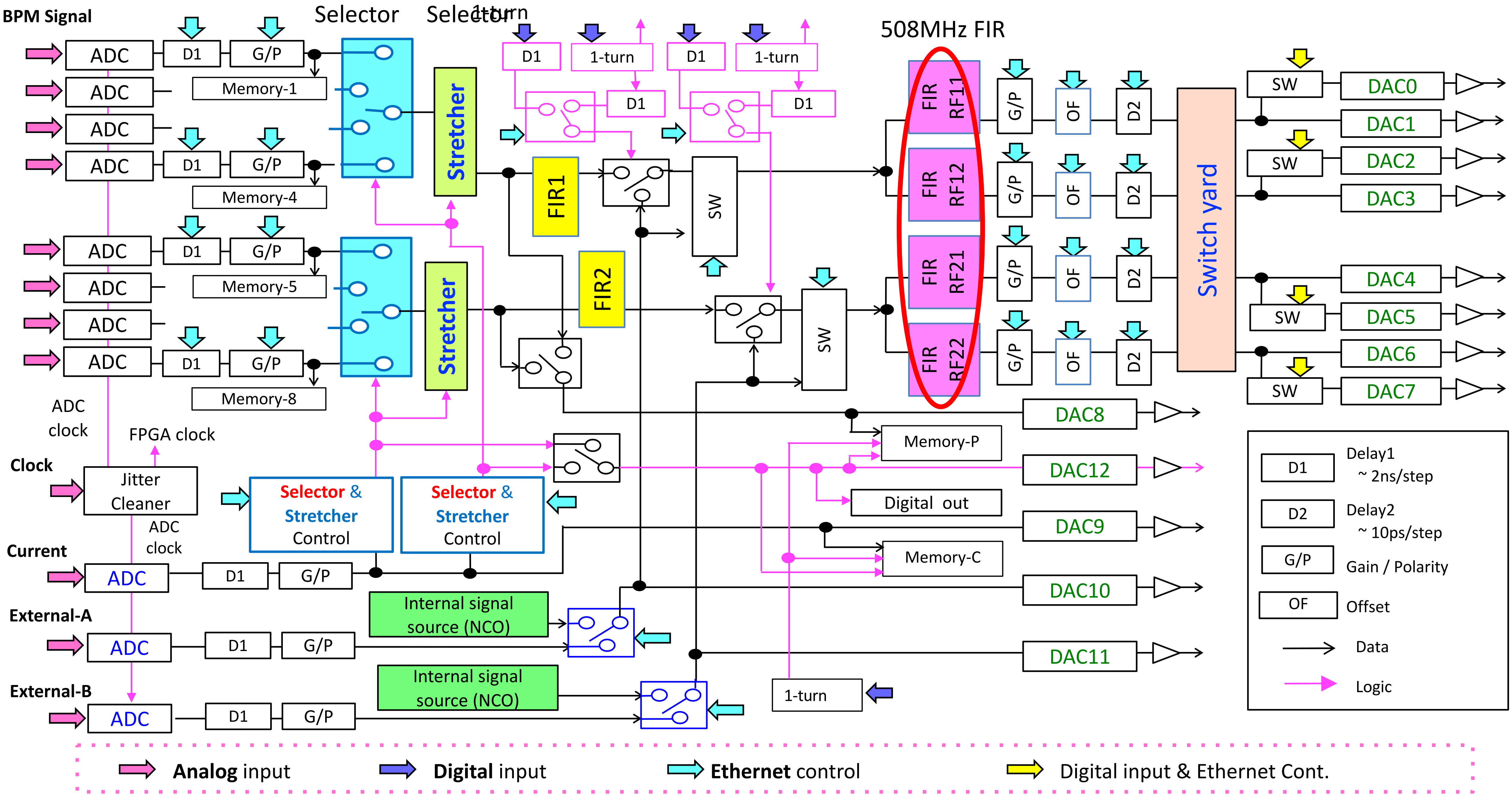
Stretcher

Bunch-by-bunch Kick signal

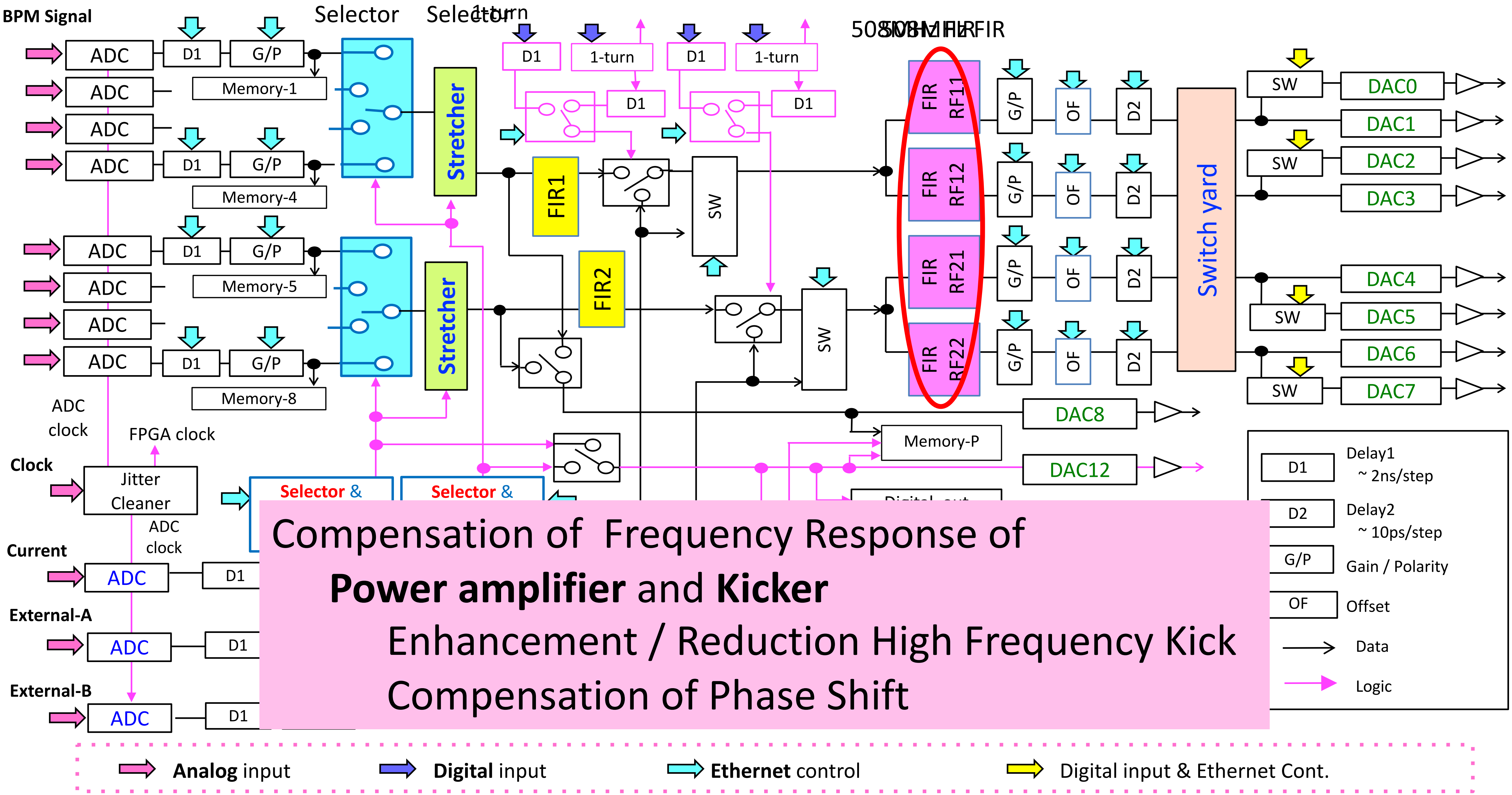
Kick Pulse Switches in 2ns



500MHz FIR filter



500MHz FIR filter



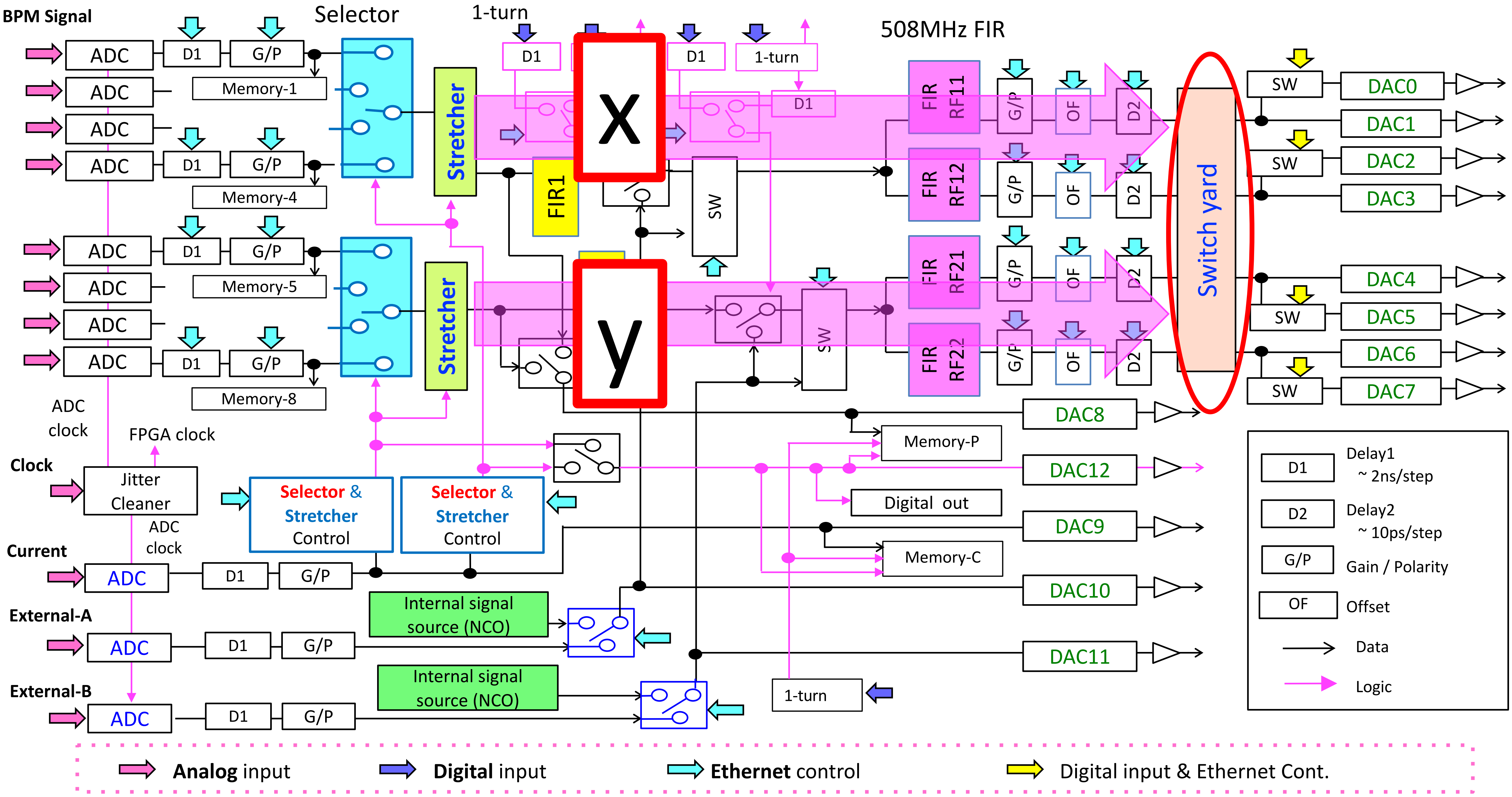
Compensation of Frequency Response of

Power amplifier and **Kicker**

Enhancement / Reduction High Frequency Kick

Compensation of Phase Shift

Switch yard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Memory-8
Selector
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FIR1
X
y
SW
508MHz FIR
FIR RF11
FIR RF12
FIR RF21
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Switch yard
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Logic
Analog input
Digital input
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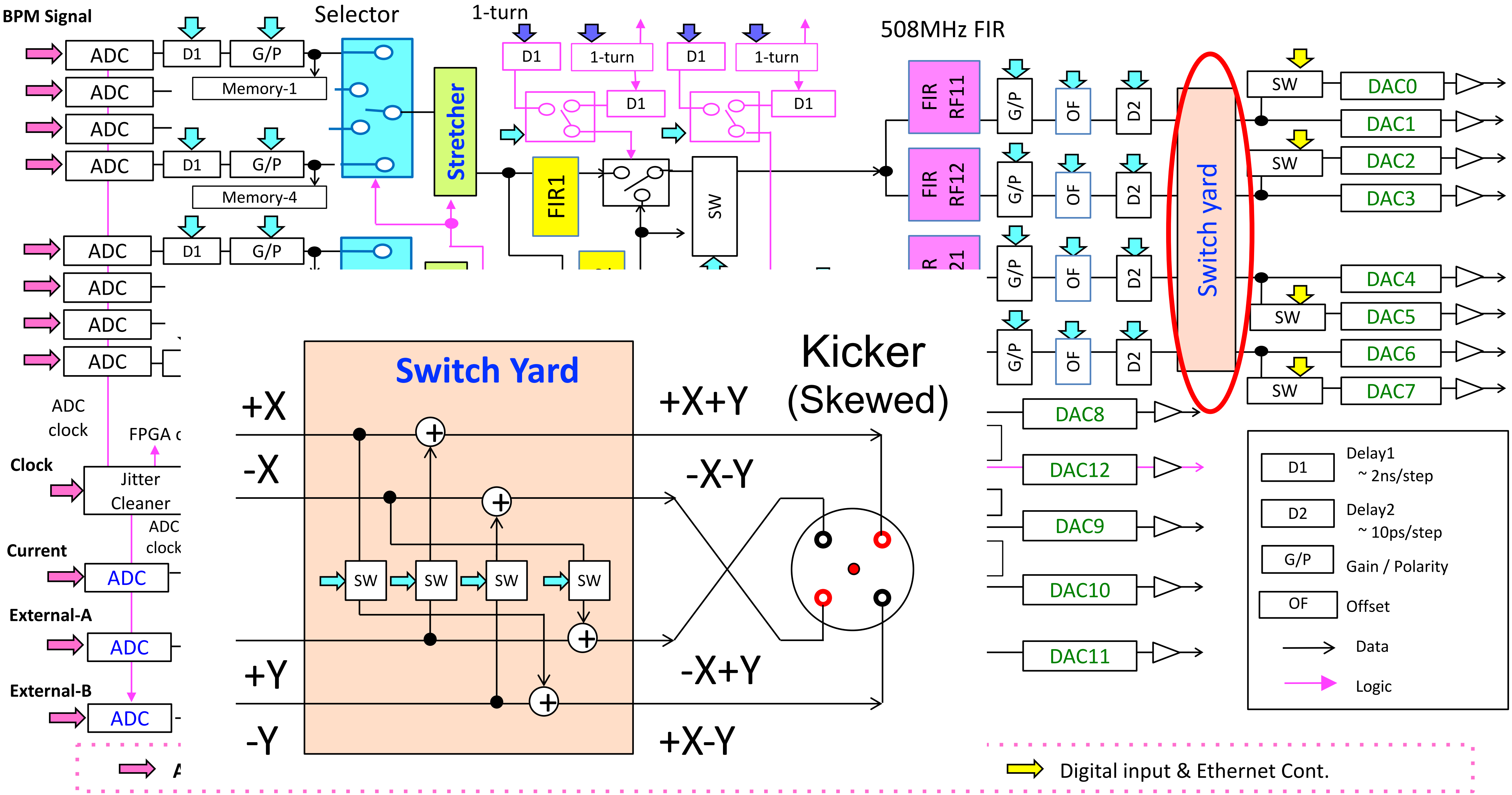
Digital input & Ethernet Cont.

Switch yard



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G/P
Memory-1
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Selector
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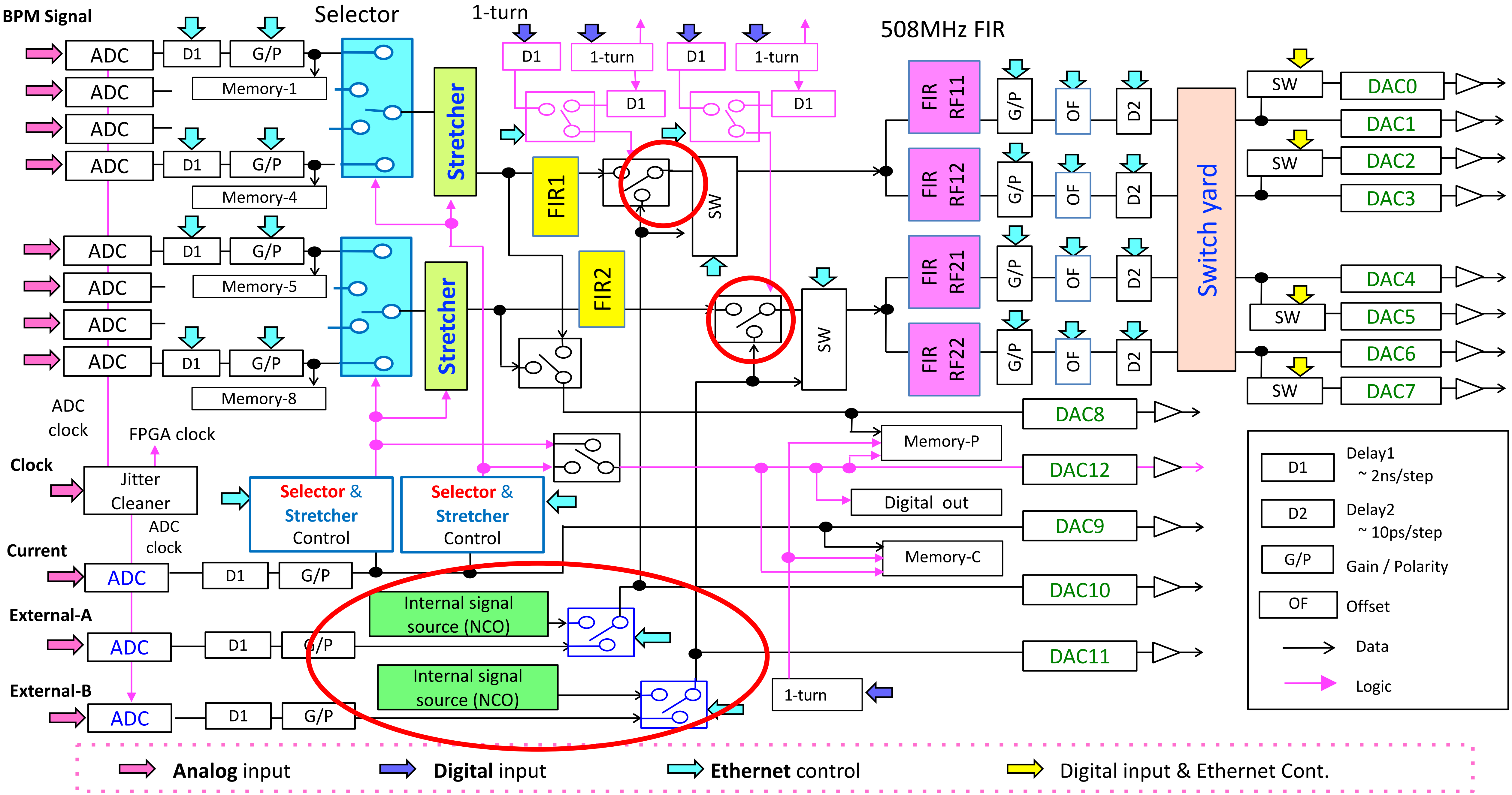
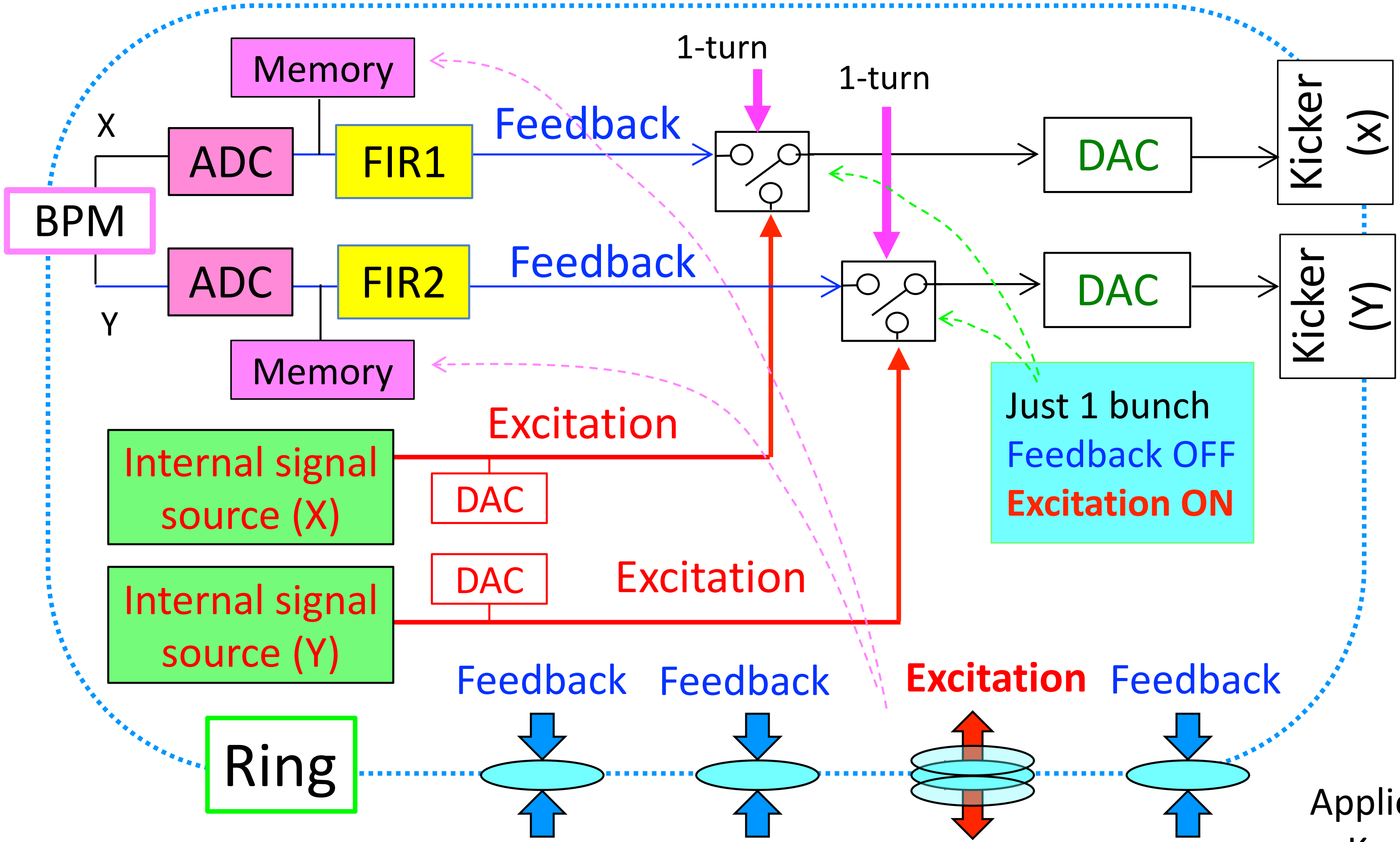
DAC12
DAC9
DAC10
DAC11
ADC clock
FPGA c
Clock
Jitter Cleaner
Current
External-A
External-B
Switch Yard
+X
-X
+Y
-Y
+X+Y
-X-Y
-X+Y
+X-Y
Kicker (Skewed)
D1 Delay1 ~ 2ns/step
D2 Delay2 ~ 10ps/step
G/P Gain / Polarity
OF Offset
Data
Logic
Digital input & Ethernet Cont.

Tune Measurement with One Bunch Excitation



Tune Measurement with One Bunch Excitation

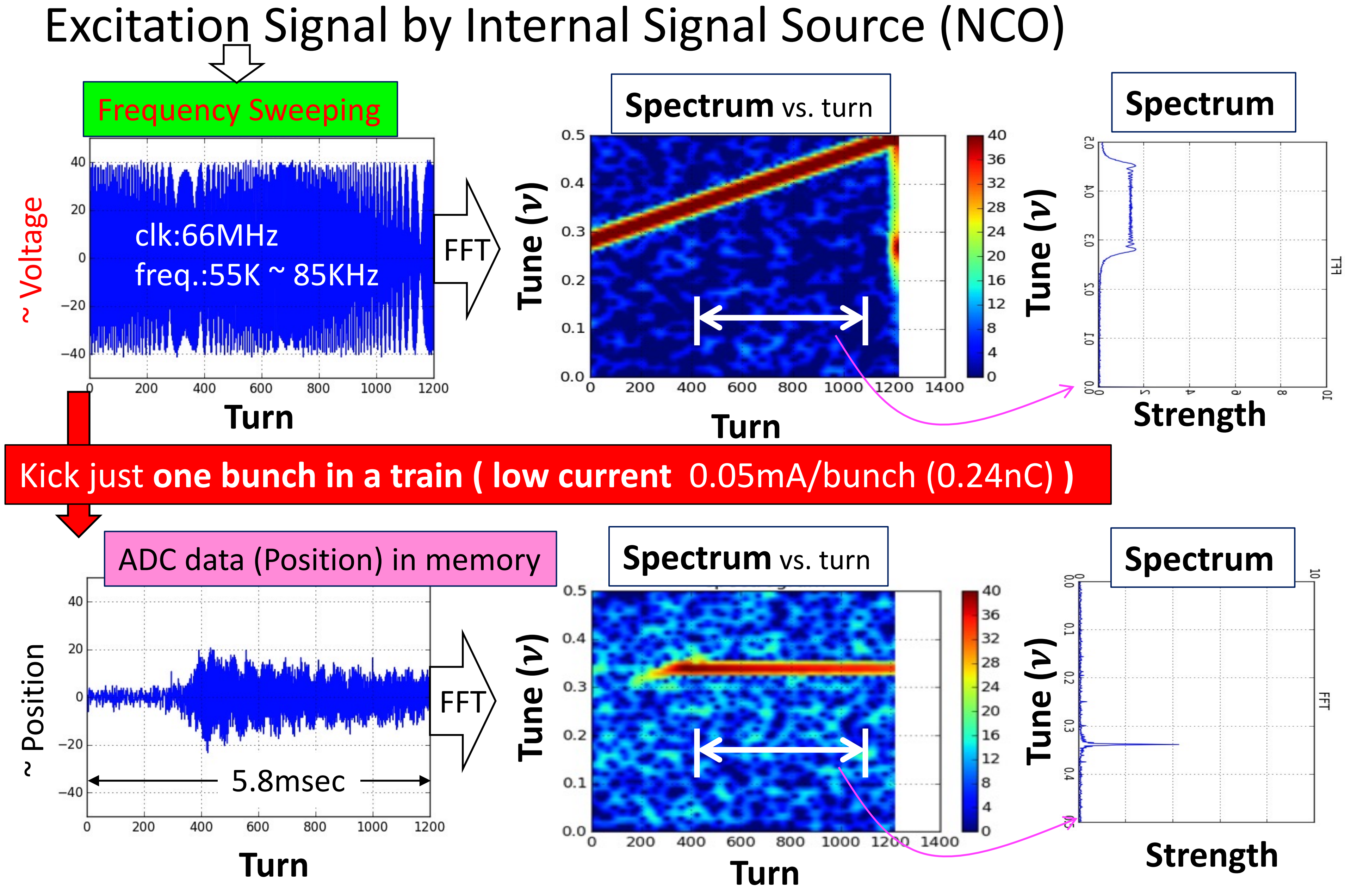
Just one bunch is excited, others are feedbacked => small effect to users



Based on the tune measurement system for the previous version of the processor developed by **R. Sreedharan and R. Nagaoka (SOLEIL)**

Applied to SPring-8 by
Kazuo KOBAYASHI
JASRI/SPring-8

Tune Measurement with One Bunch Excitation

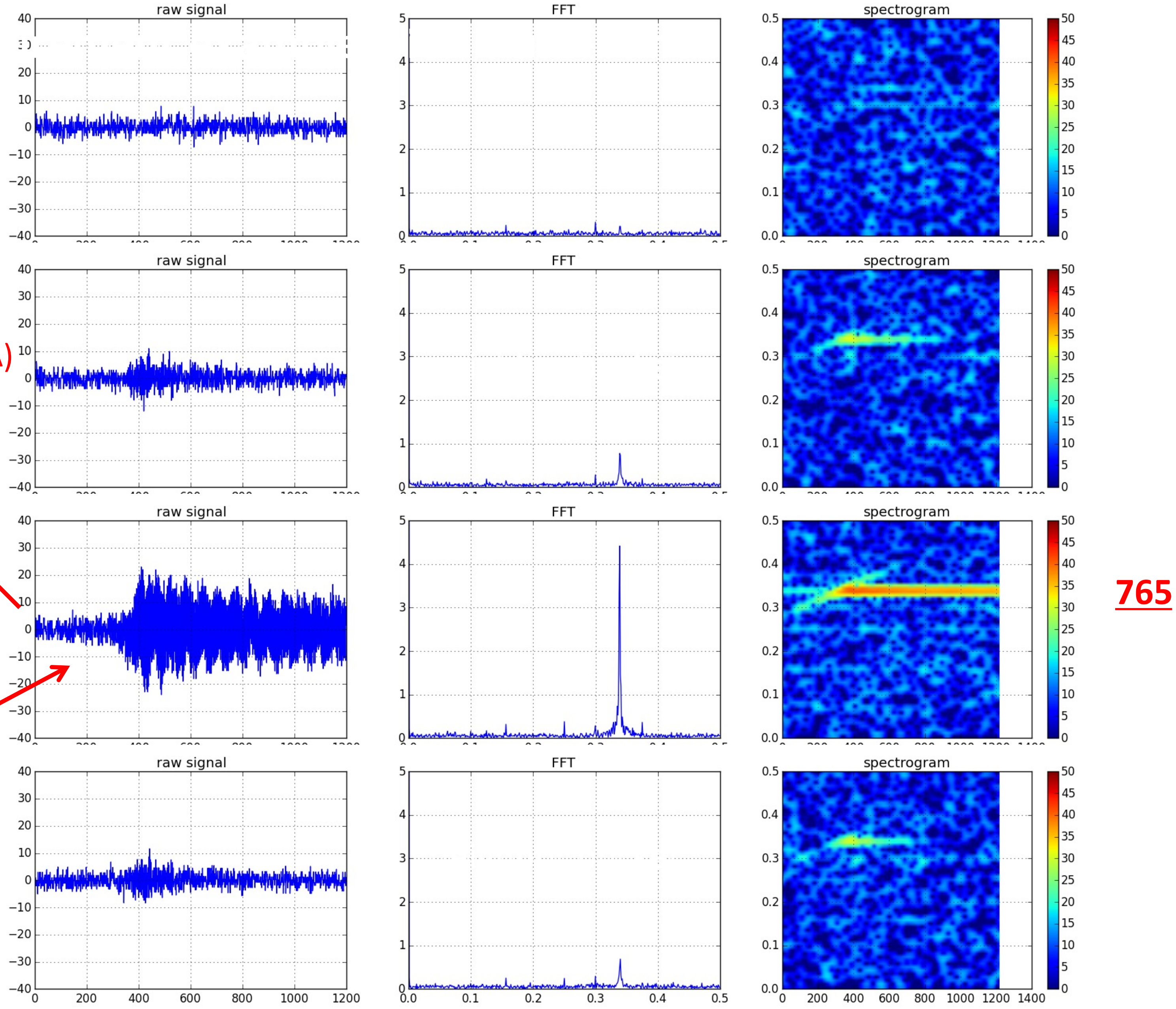


By Kazuo KOBAYASHI / SPring-8

❑ Tune observation system with New Signal Processor

bucket address	current(mA)
762	0.05103
763	0.05486
764	0.05298
765	**0.05696**
766	0.05655

target bunch



By Kazuo KOBAYASHI / SPring-8

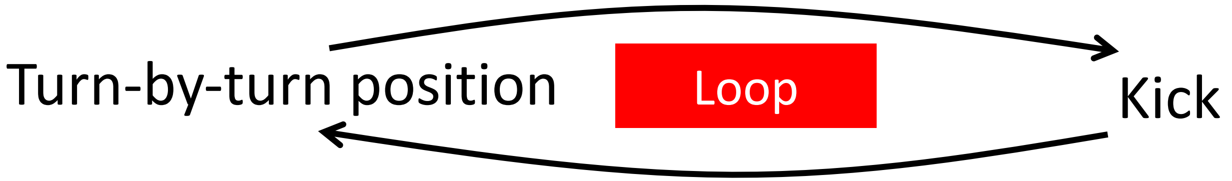
Stability limit of Feedback
and
Multiple-BPM scheme
to remove it

Simplify saying, it’s Digitalized Analog feedback scheme with Two BPMs

Its Extreme Case for Damping time ~ several turns

Feedback with Multiple BPMs for Stability at High Gain

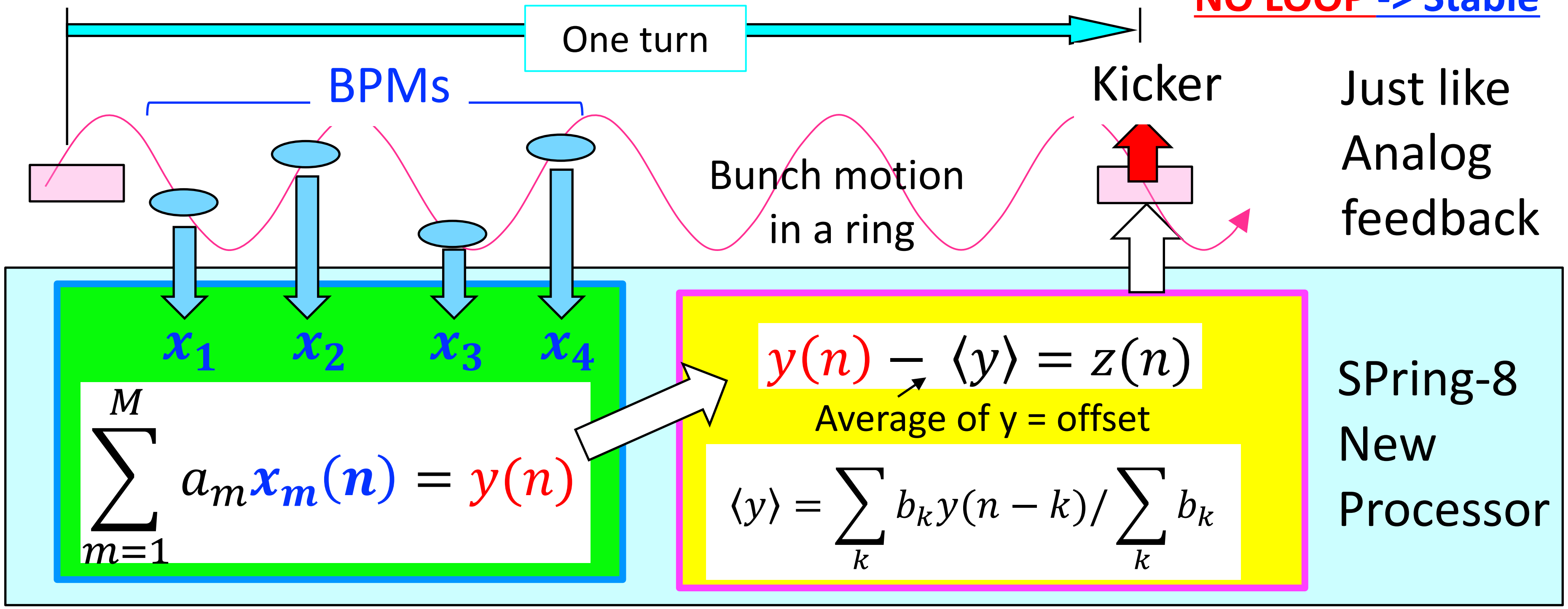
Kick <= **Turn-by-turn Position** with a single BPM



Loop ->
Unstable At High Gain

Kick <= **Multiple Positions (BPMs) in a SINGLE TURN**

NO LOOP -> Stable



Just like Analog feedback

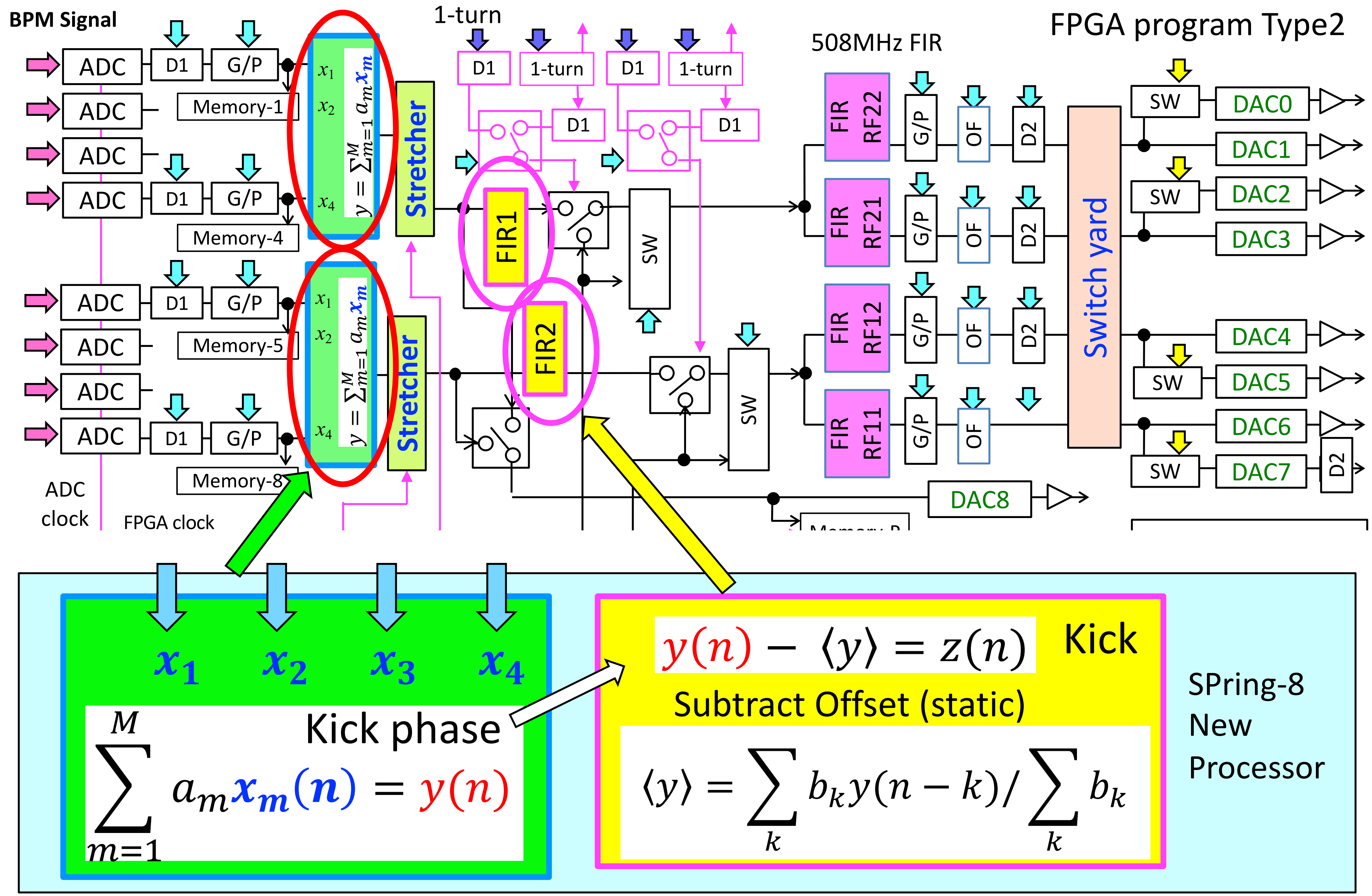
SPring-8 New Processor

1 or 2 BPMs : enough if those have good phase relations each other and kicker

T. Nakamura, Proc. of *14th Ann. Meet. Part. Accel. Soc. Japan,* paper TUP090, Aug. 1-3, 2017

Proc. of IPAC18, . tuzgbd2

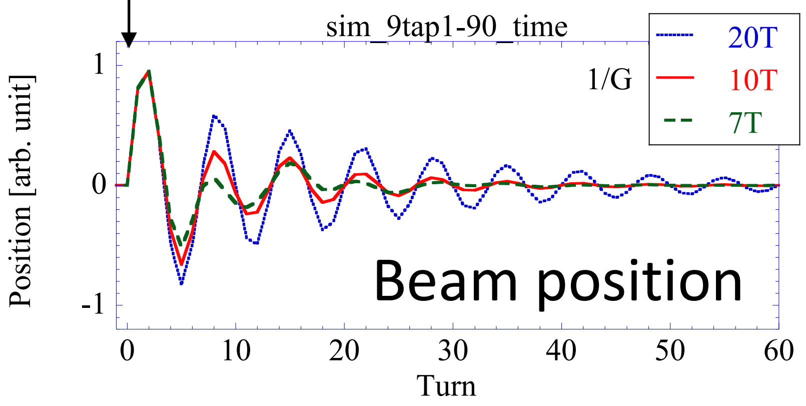
Feedback with Multiple BPMs for Stability at High Gain



Instability of Feedback at High Gain

At High Gain, feedback system drives beam oscillation

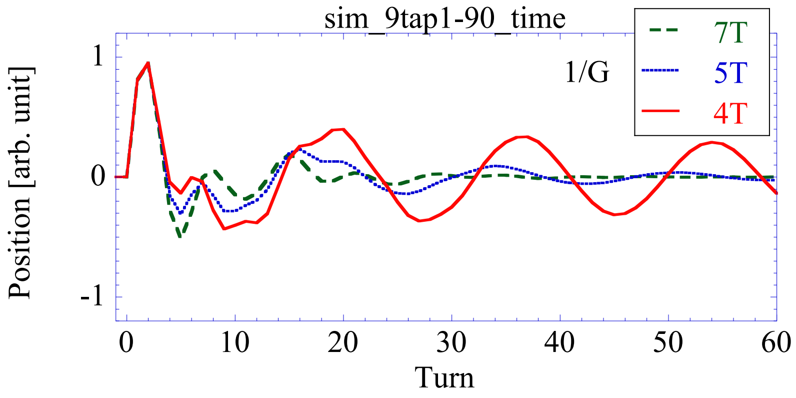
Kick



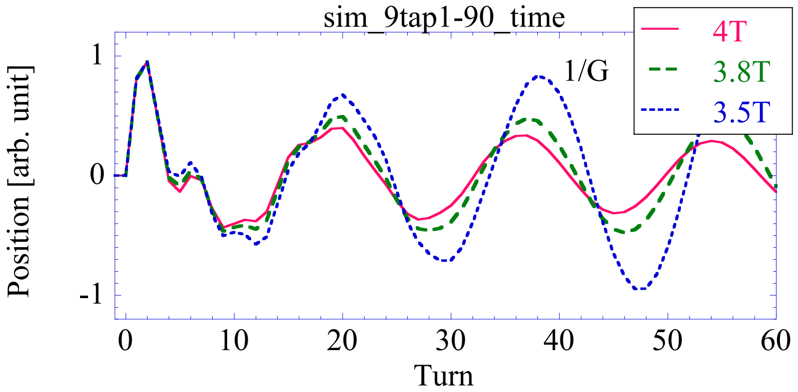
nT <= Gain corresponding to Damping time n turns

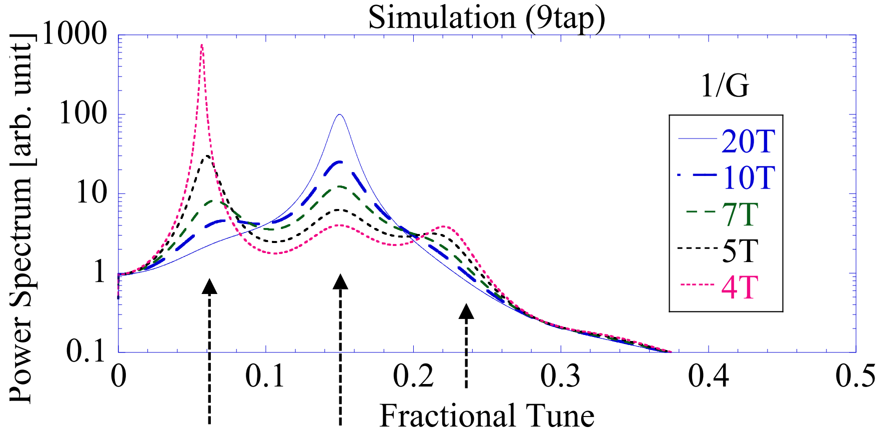
Higher Gain

Beam is unstable at High gain with Different tune from original



Fourier Trans.





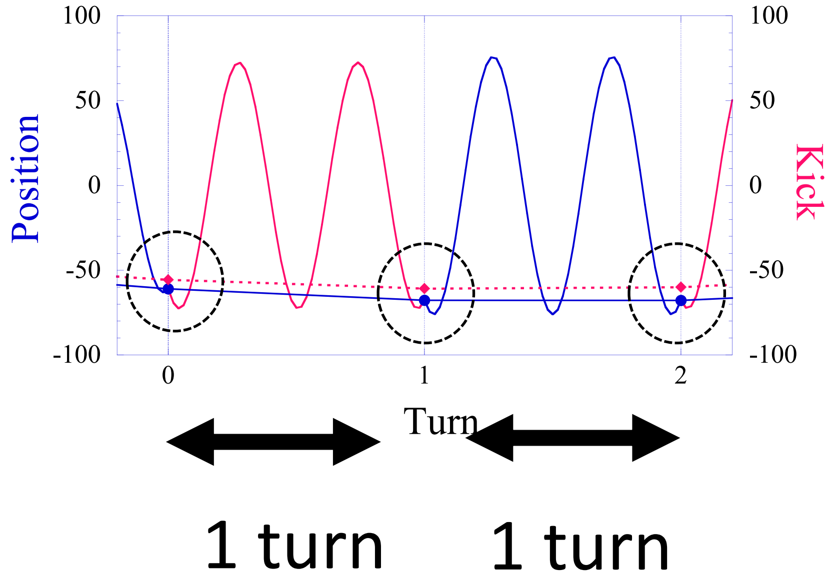
Excited 0.05 | Original 0.15 | Excited 0.23

1/Threshold Gain = 3.8 turns

Tune Shift by Feedback at High Gain

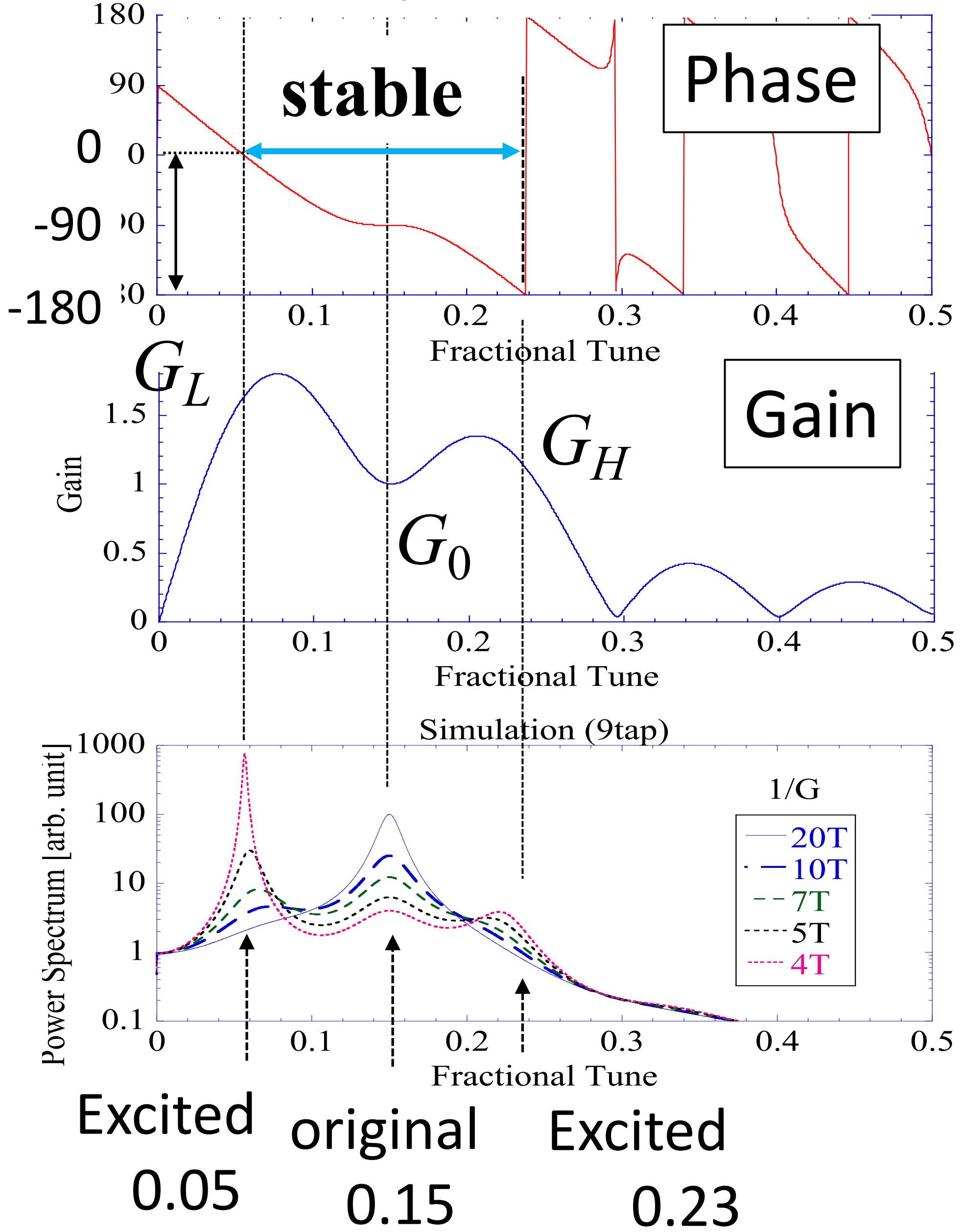
High Gain Feedback Drives Tune
to Unstable Region

1/G = 2.8 T < 3.8 T (threshold)

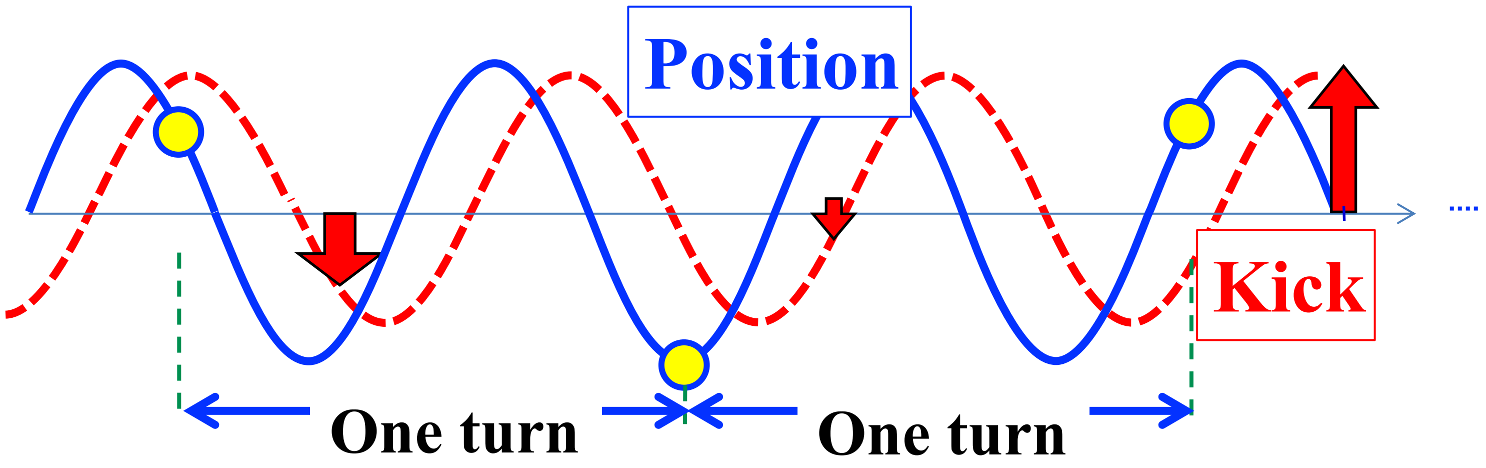


Feedback is Defocusing kick
gain GL is strong enough
to shift tune from
0.15 -> 0.05

Tune response of FIR filter

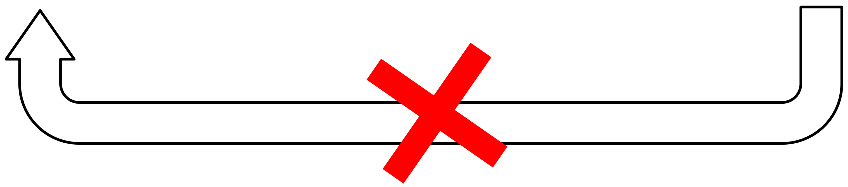


Instability of Feedback Driven by Closed Loop : Position - Kick



Turn-by-turn position data ⇨ **Kick**

Position data at multiple Locations at one previous turn ⇨ **Kick**



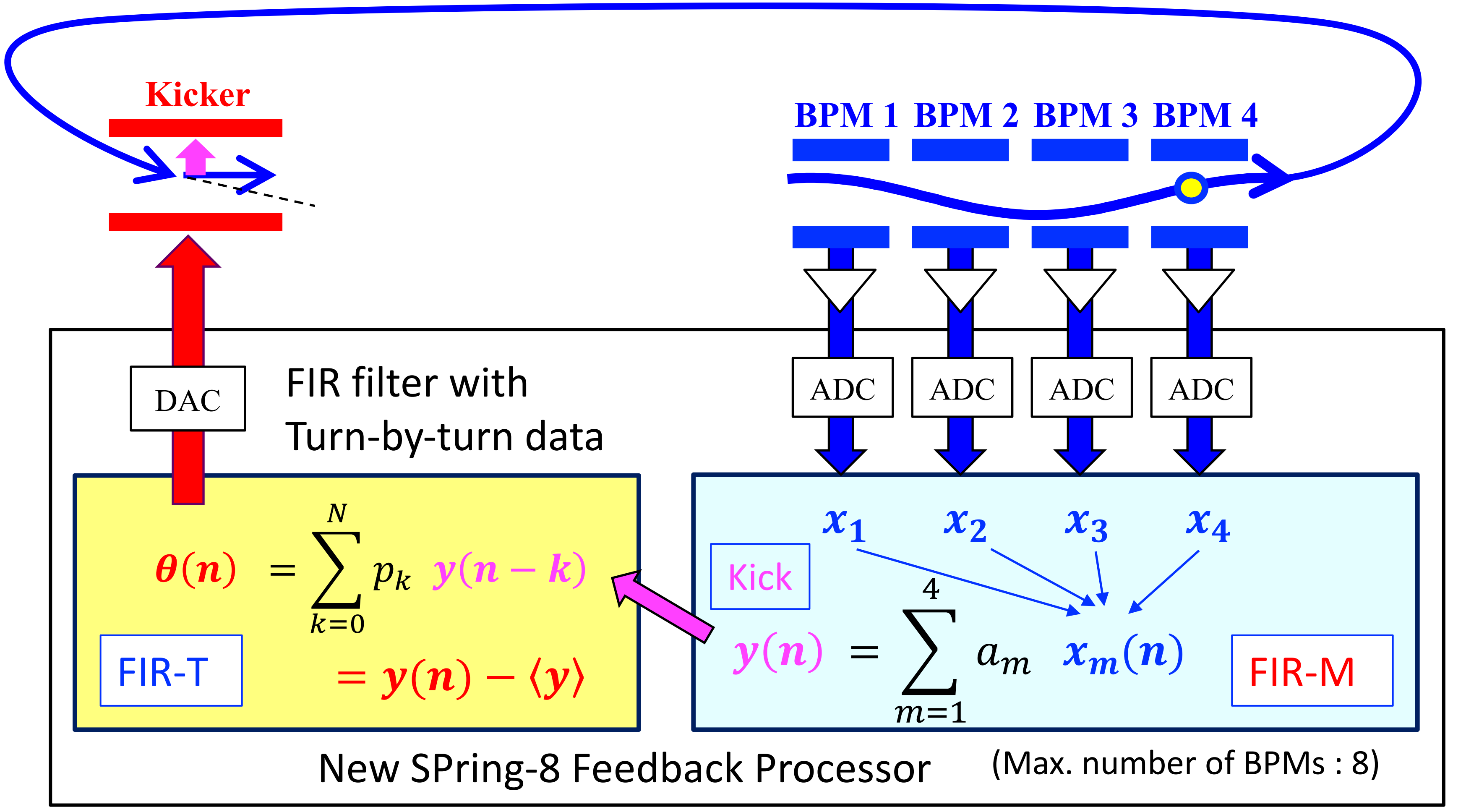
Digital Feedback with Position Data at Multiple Locations (BPMs)

FIR-M for **Kick from beam positions at multiple locations**

4 BPM -> **DC offset subtraction** is possible for **Ideal case (static)**

FIR-T for **Subtraction of DC offset**

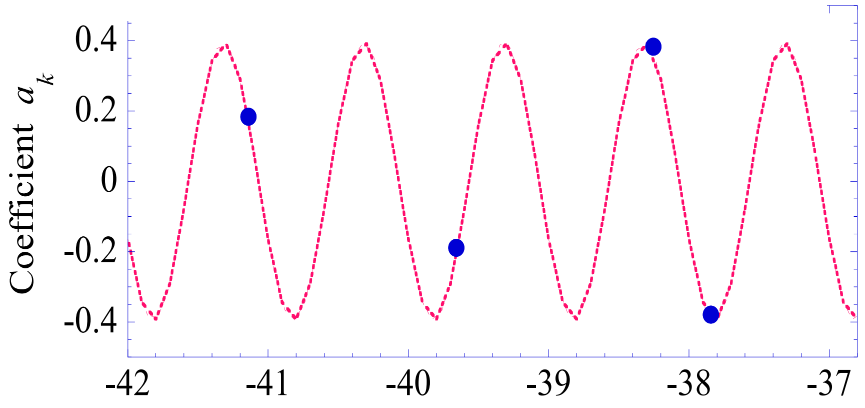
Produced by drift of **Closed orbit**, **Amplifier gain**, **ADC gain and timing**



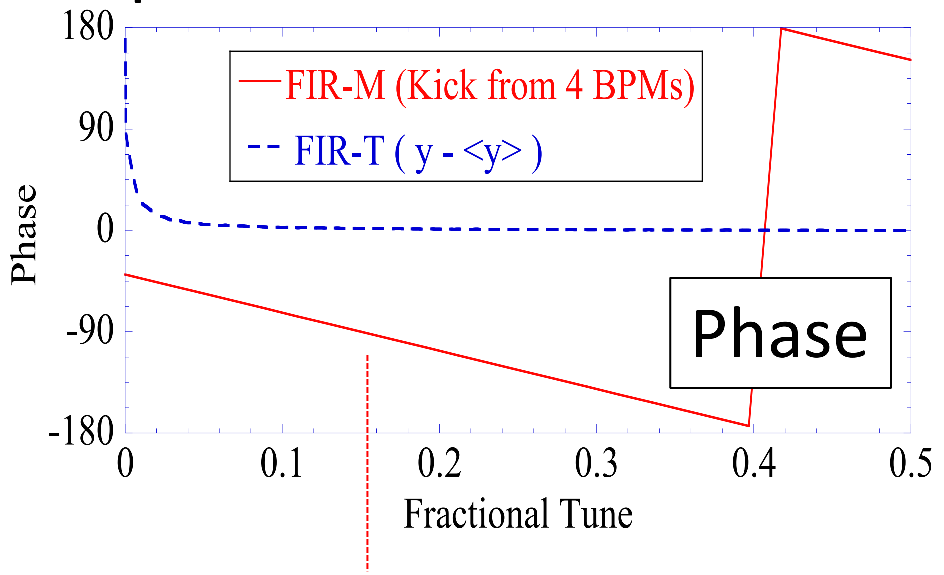
Digital Feedback with Position Data at Multiple Locations (BPMs)

FIR-M $y(n) = \sum_{m=1}^{4} a_m \, x_m(n)$ **FIR filter response**

Coefficients of and Phase of **4 BPMs**

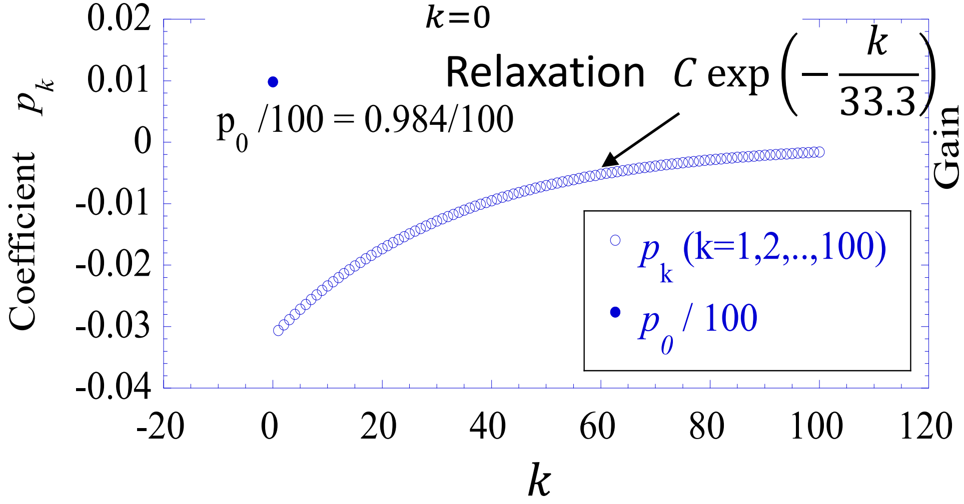


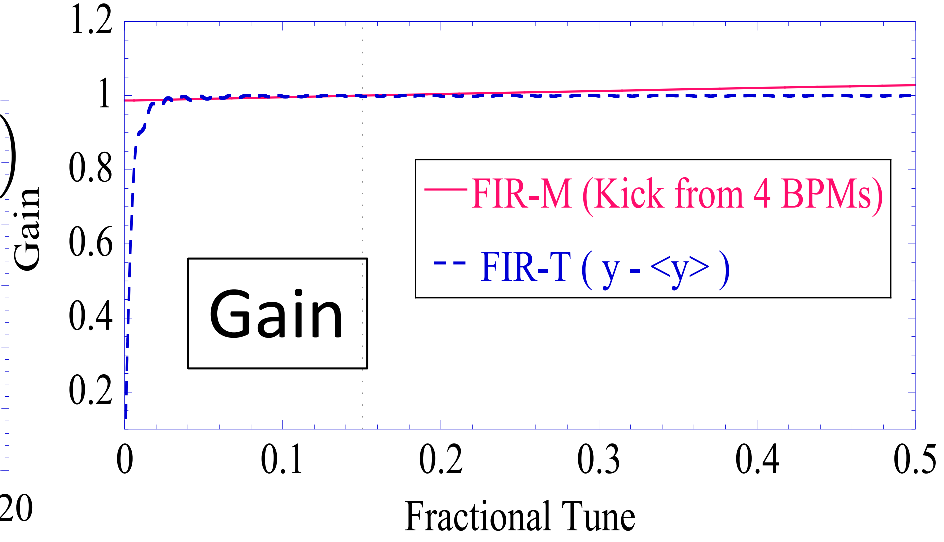
Phase Shift / (2π) at BPM from Kicker



Tune should be FIXED even at High Gain

FIR-T $\theta(n) = \sum_{k=0}^{100} p_k \, y(n-k)$

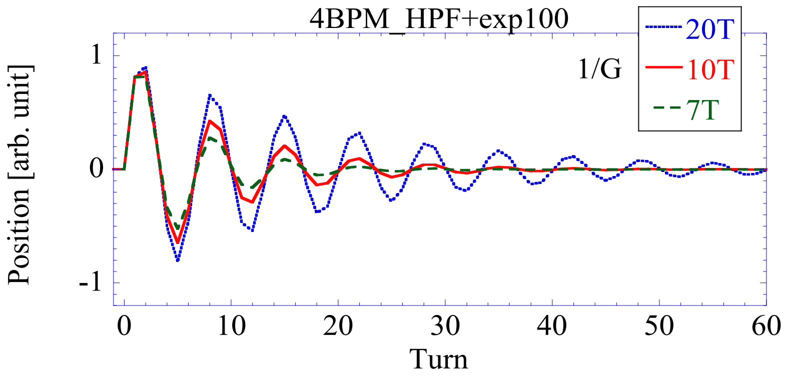


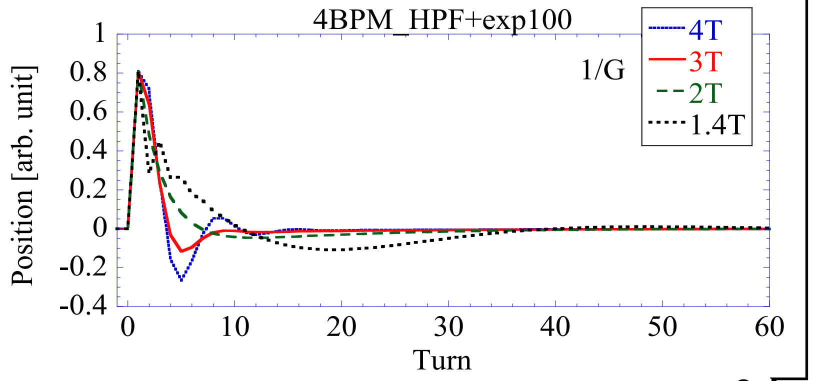


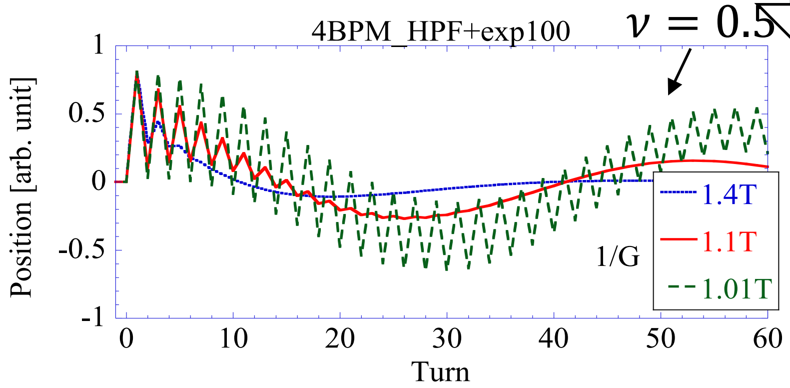
Simulation Results : Feedback with Position Data at Multiple Locations

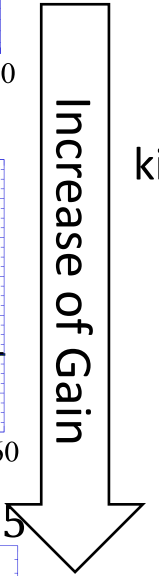
Damping time up to ~ 1.1 turns

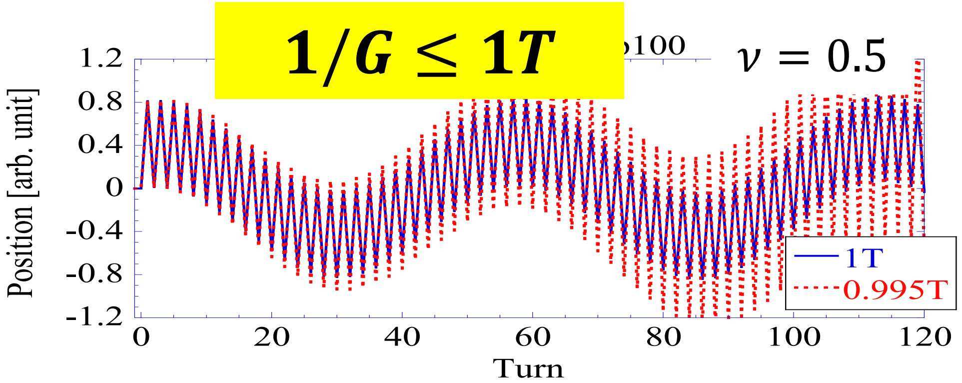
No growth







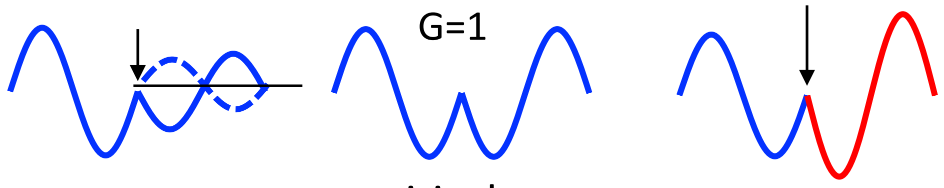




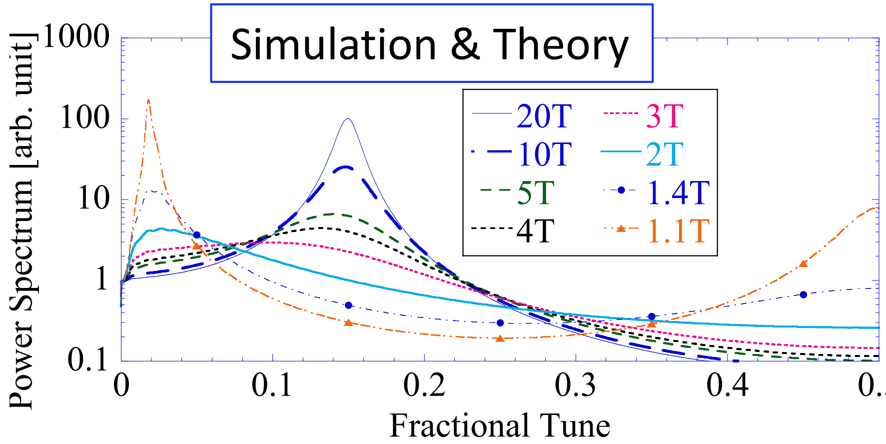
Beam is unstable for > G=1T

even with ideal feedback : **kick = - 2 G x angle**

kick < -2 x angle   kick = -2 x angle   kick > -2 x angle



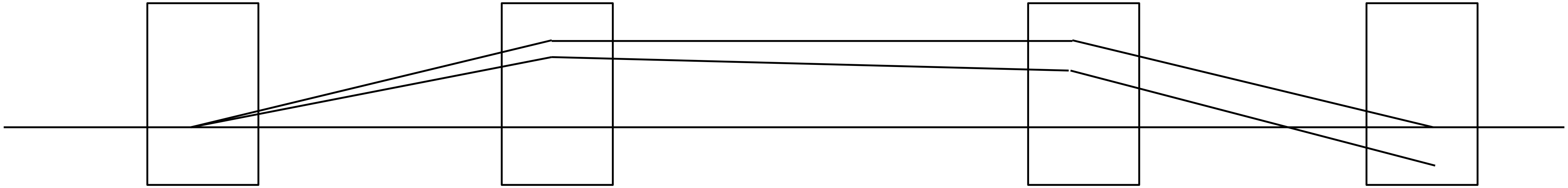
stable   critical   unstable



Fast Correction Kicker for Reduction of Transient Beam Oscillation Excitation by **Injection Bump Orbit Formation**

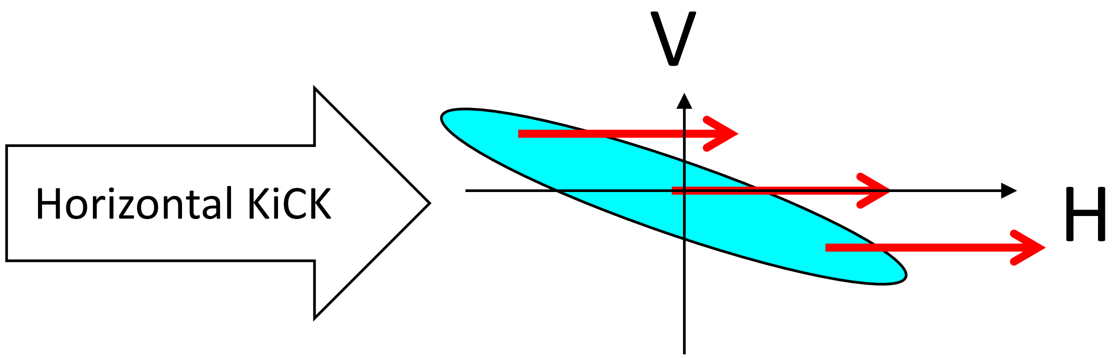
Fast Correction Kicker for Reduction of Unwanted **Main Kicker** Effect

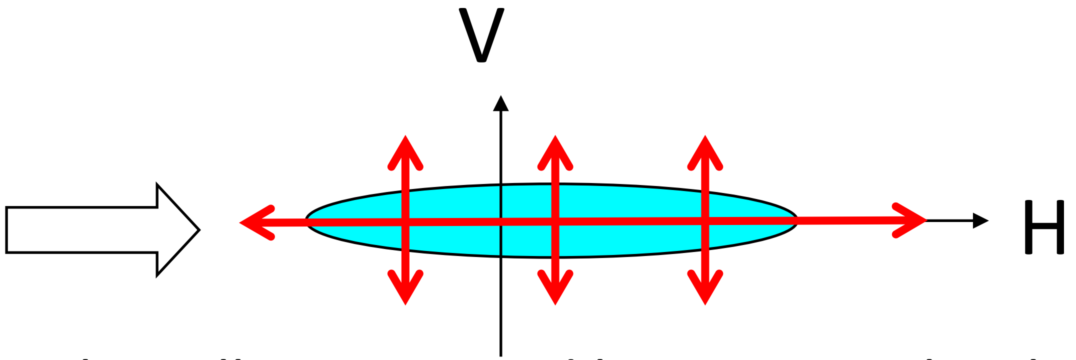
Horizontal : Mismatching at Fast Rising/Falling Edge



Vertical : X-Y coupling at Injection Kickers

X-Y coupling at Injection Kickers



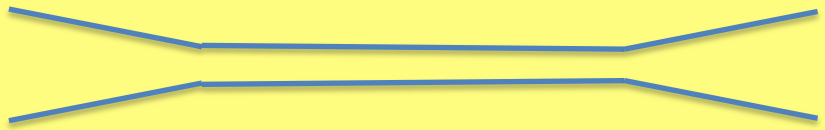


Vertical Oscillation excited by Horizontal Kick

(Rotation of Kicker Magnets are optimized with Remotely Controlled Magnet Base)

SPring-8 (and some of light sources)

Low Chromaticity for Large Dynamic Aperture
for high Injection Efficiency, long lifetime
Many low gap IN-VACUUM Insertion Devices



Large Value of sum of [Resistive-wall Impedance + Taper Impedance]/ gap^3

VERY STRONG Single-bunch Instabilities (Mode-coupling)

High Damping Rate is required in Vertical and Horizontal

Damping Rate is proportional to
Kick / **Position Amplitude**
x Revolution Frequency (0.2MHz) / Energy (8GeV)

limited kicker performance (two 30cm kickers + 500Wpeak amp)

Small Position Amplitude is required

HOWEVER

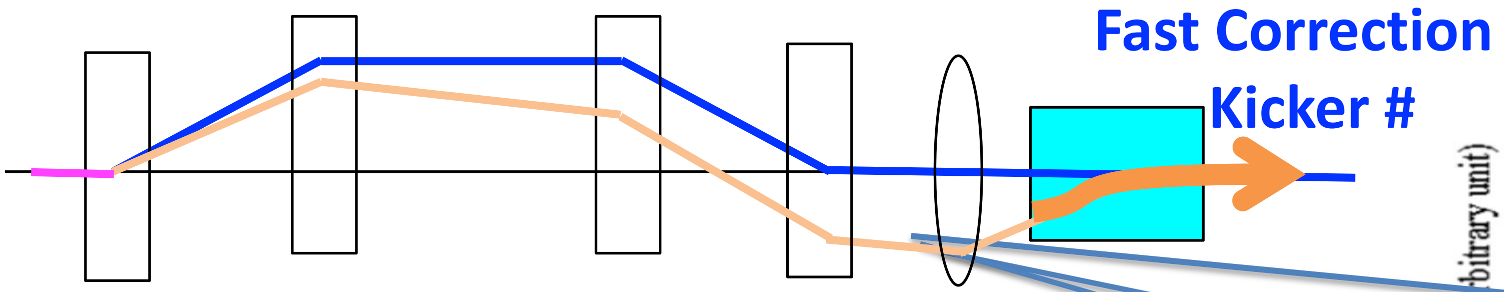
Large Horizontal and Vertical Motions are EXCITED at INJECTION

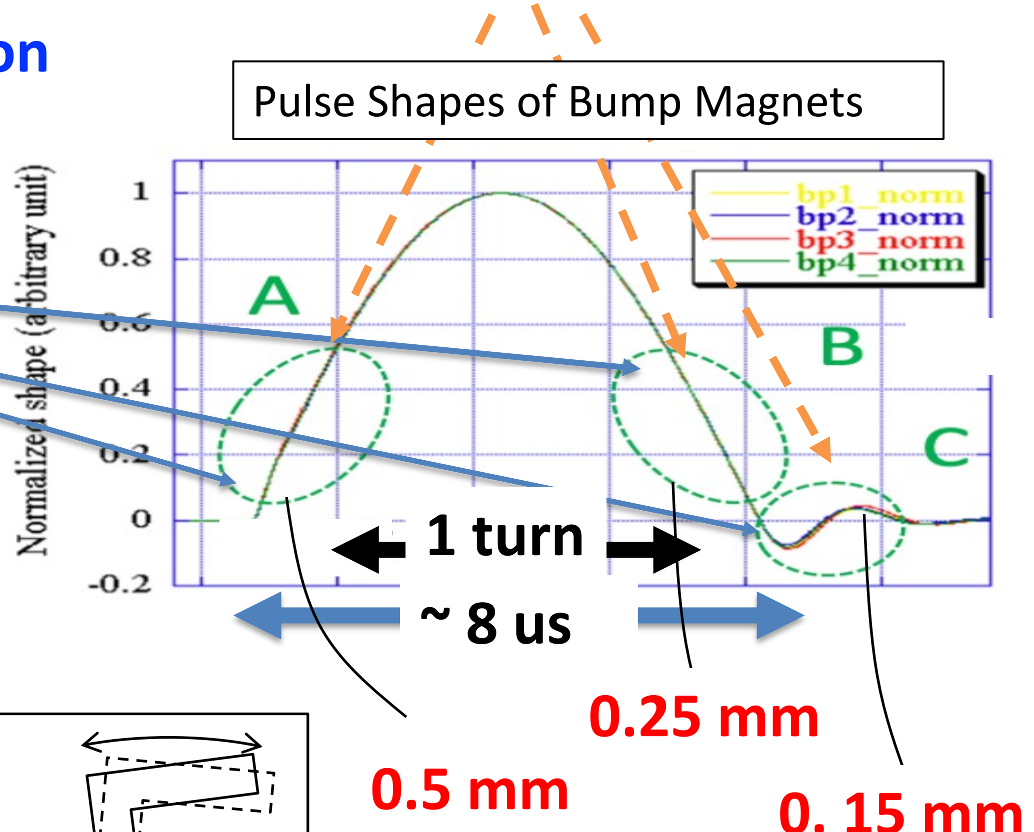
FAST CORRECTION KICKERS
for Reduction of Horizontal/Vertical Oscillation of STORED BEAM at Injection

* Horizontal : Pulse Shape Difference of **Bump Magnet**: mostly at **Rising/Falling Edges**

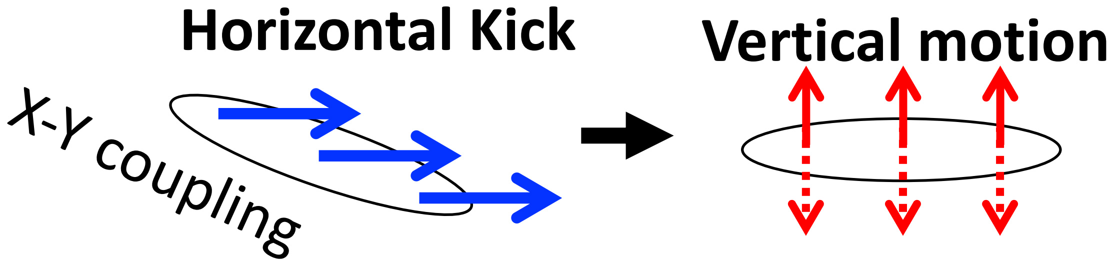
Several Parts of Stored Beam

Bump orbit is not closed => Large Horizontal Oscillation

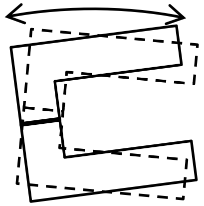




* Vertical : X-Y coupling at Injection Kickers



Rotation of Bump Magnets are optimized with Remotely Controlled Magnet Base*; **Coupling Changes run to run**



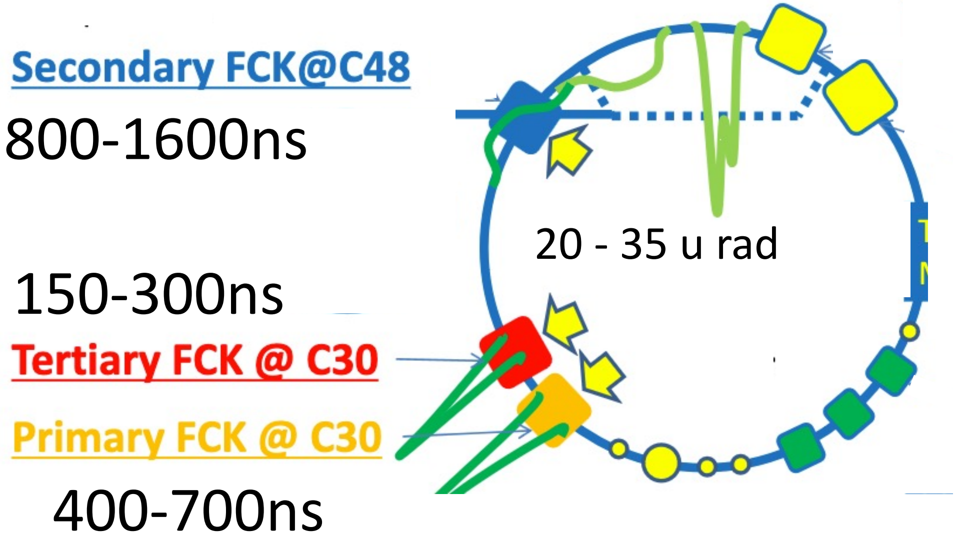
*K. Fukami, et al., http://accelconf.web.cern.ch/e08/papers/wepc076.pdf

C. Mitsuda, K. Fukami, K. Kobayashi, et al., https://accelconf.web.cern.ch/IPAC2014/papers/mopro082.pdf

C. Mitsuda, https://indico.cern.ch/event/635514/contributions/2660454/attachments/1513848/2370449/twiss_2017_v6_pub.pdf

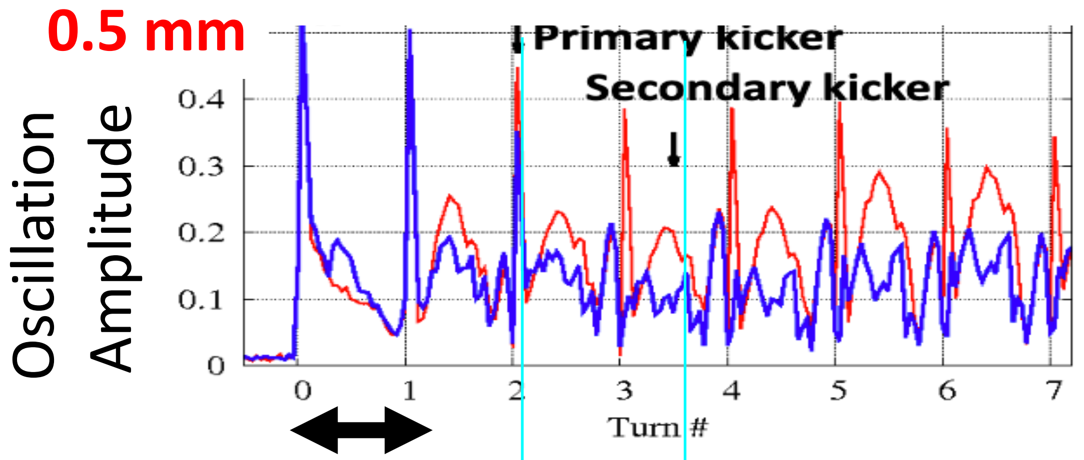
FAST CORRECTION KICKERS
for correction of Horizontal/Vertical Oscillation of STORED BEAM at Injection

Three Fast Correction Kickers (FCK) for different kick timings



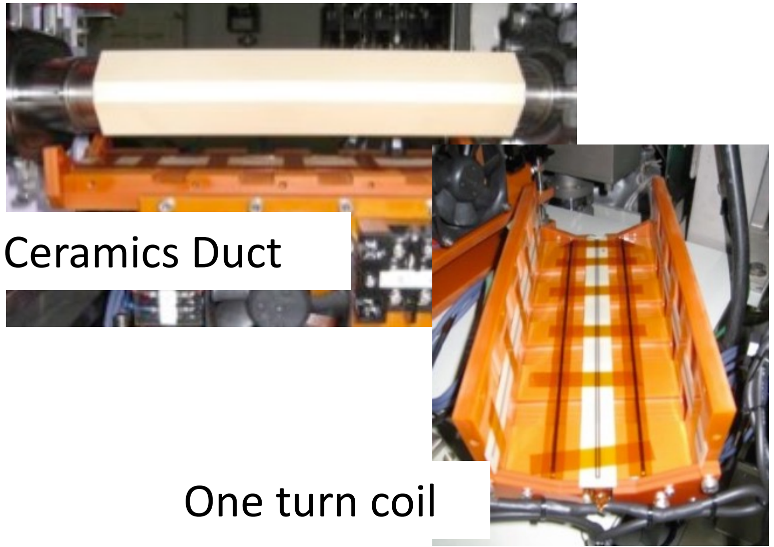
Blue : two FCKs ON
Red : OFF

Horizontal

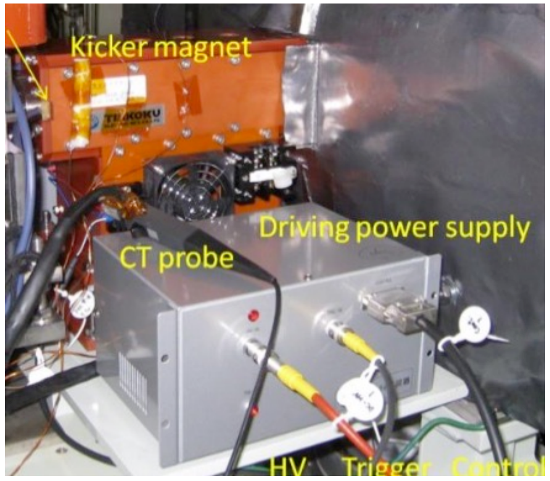


1 turn

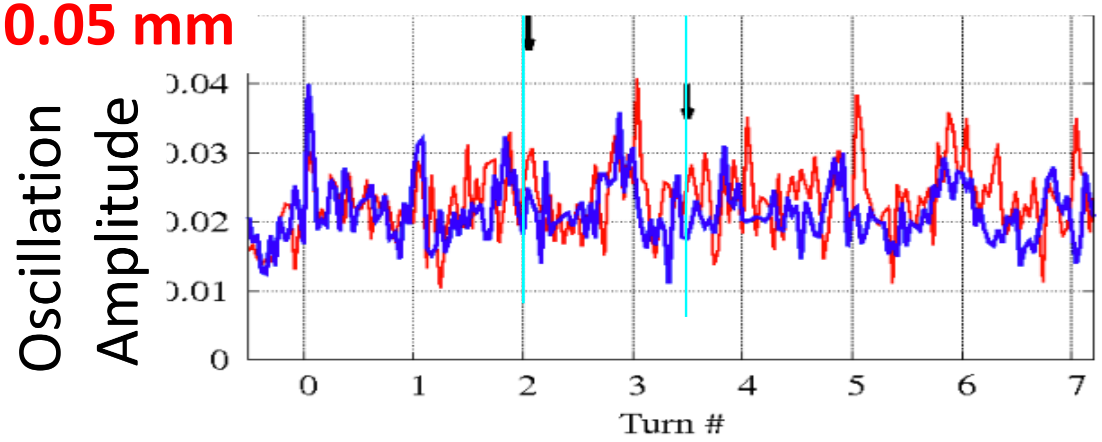
Vertical FCK also installed but enough with FCKs (horizontal)



Installed with Power supply



Vertical (also reduced with FCK)



Instability Strength Monitoring for In-Vacuum Insertion Devices (ID)

Instability Strength Monitoring for In-Vacuum Insertion Devices (ID)

Resistive-wall, taper, step, ..

$$\sum_{i=1}^{ALL\ In-Vacuum\ IDs} \frac{(ID\ Length)_i \langle \beta_y \rangle_i}{(Gap)_i^3}$$

14 mA/bunch

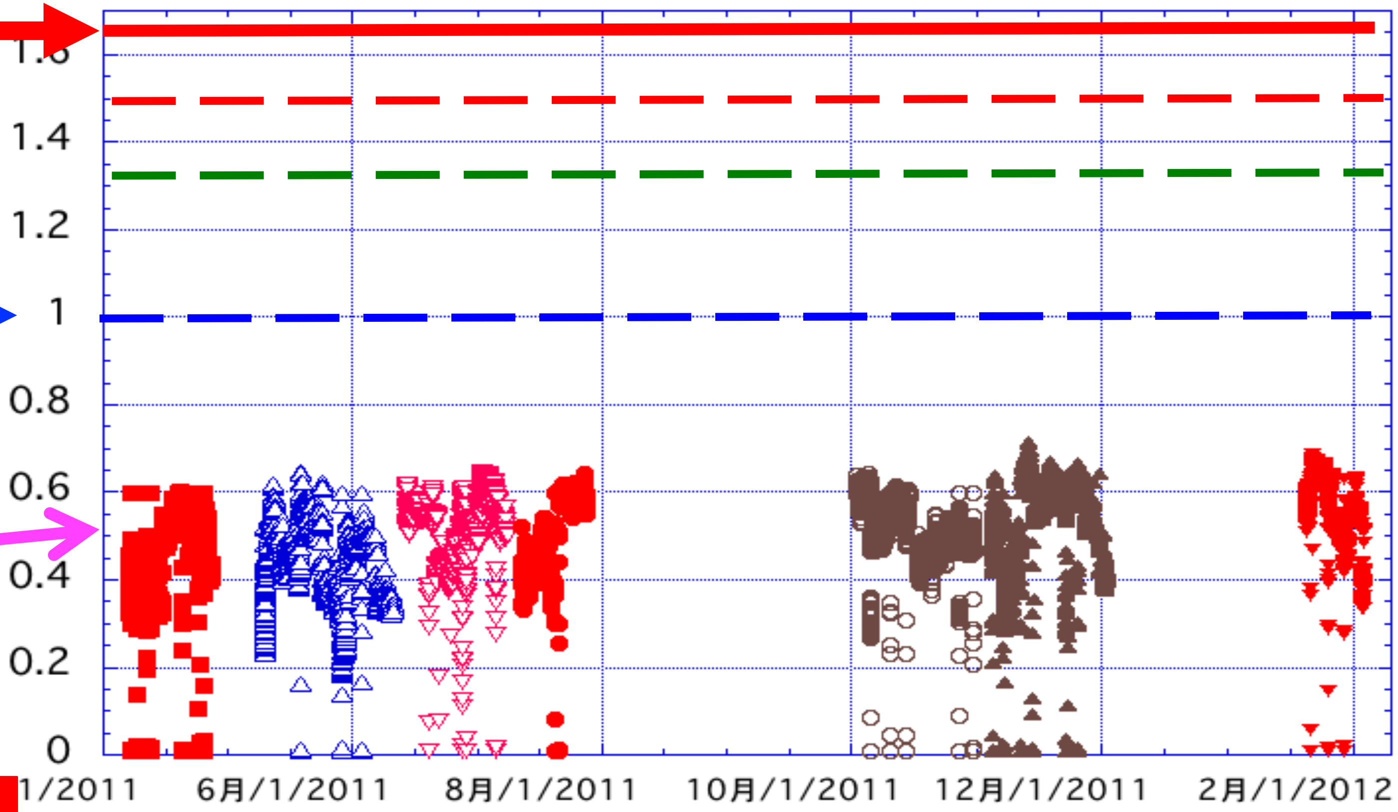
$$+\ Fixed\ Value\ Impedances \times \beta$$

HISTORY of the value

All In-vacuum IDs with their **minimum gap** (**No HOPE** for suppression)

Routine Stability Test **(OK : up to** 6mA/Bunch user operation **5mA/bunch)**

At User Operation (a piece of cake!)

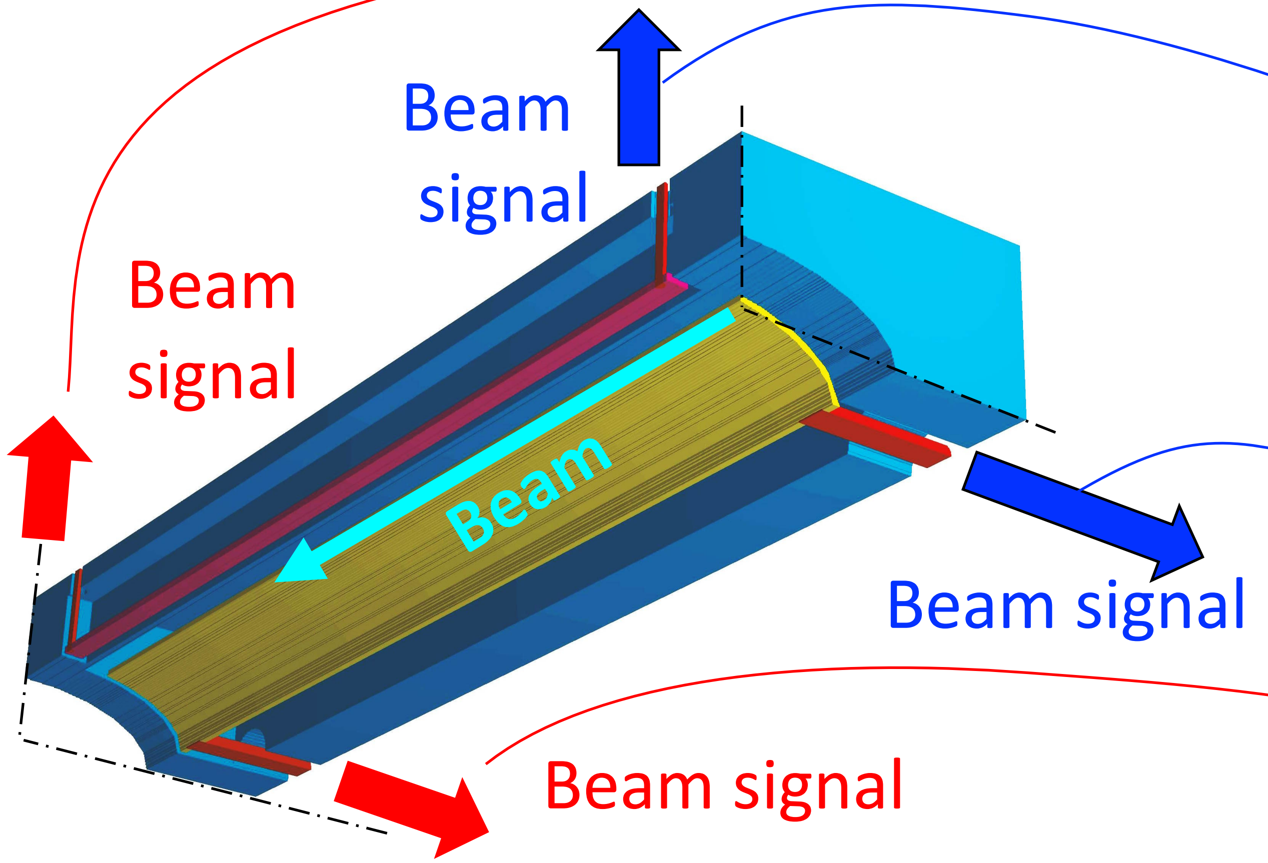


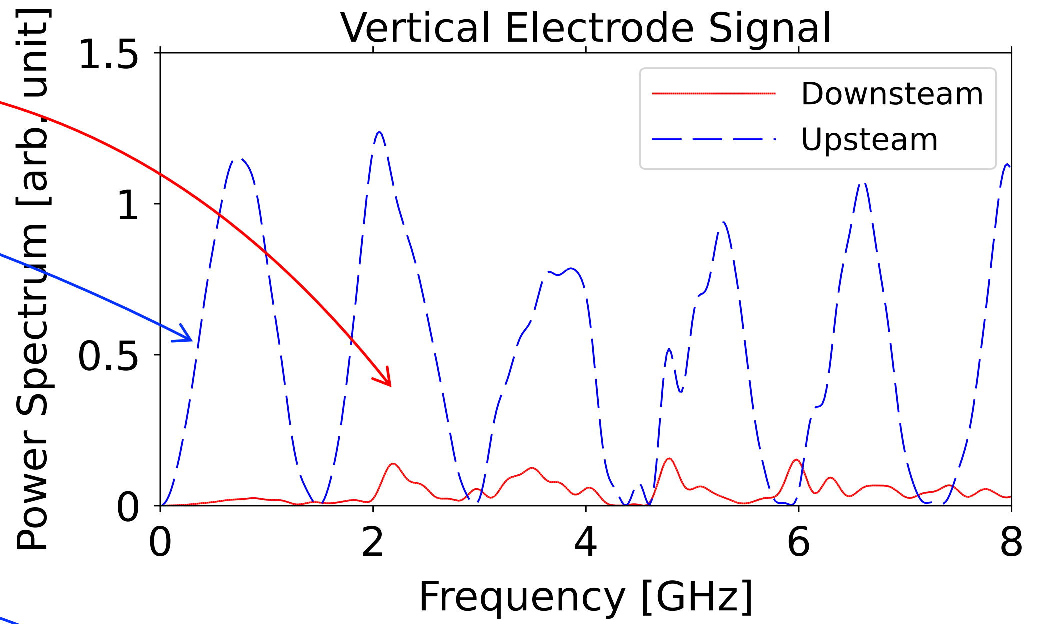
1 year

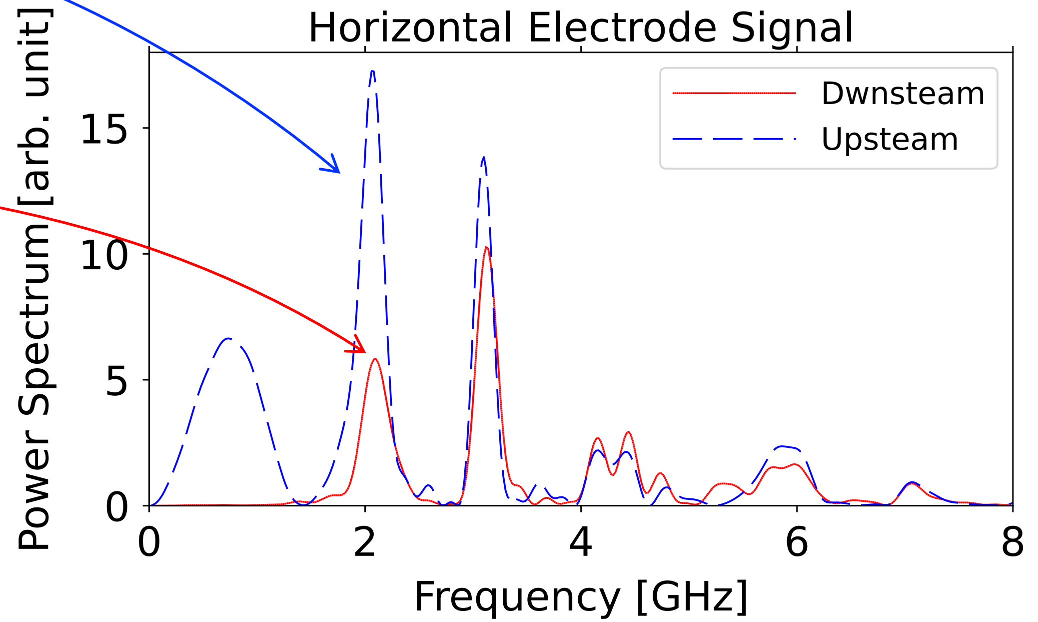
BUT Users requests > 10mA/bunch …
NOT YET

Trip of **Power Amplifier** with “Reverse Power” by High Current Bunch

Trip of Power Amplifier with “Reverse Power” by High Current Bunch



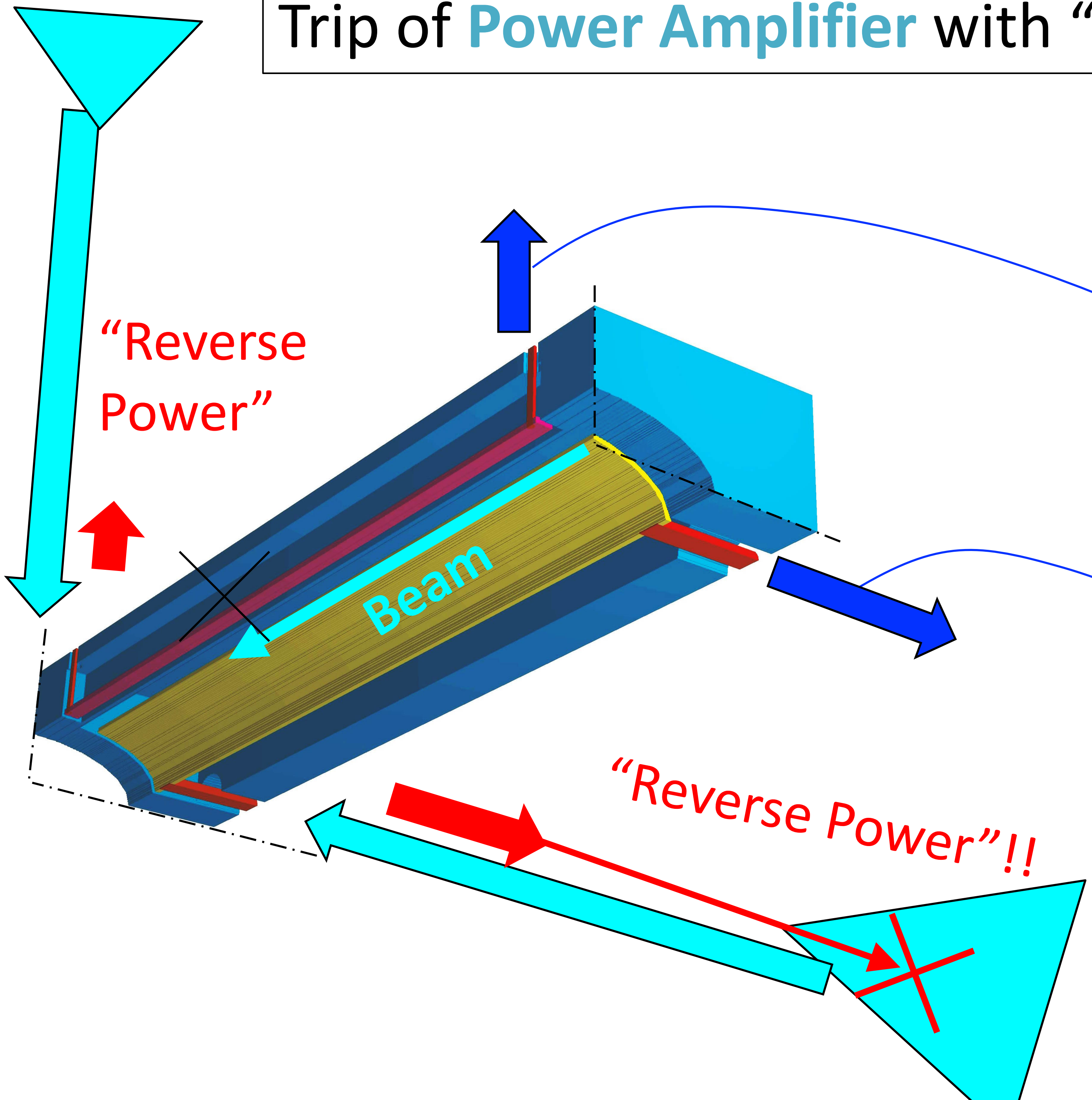


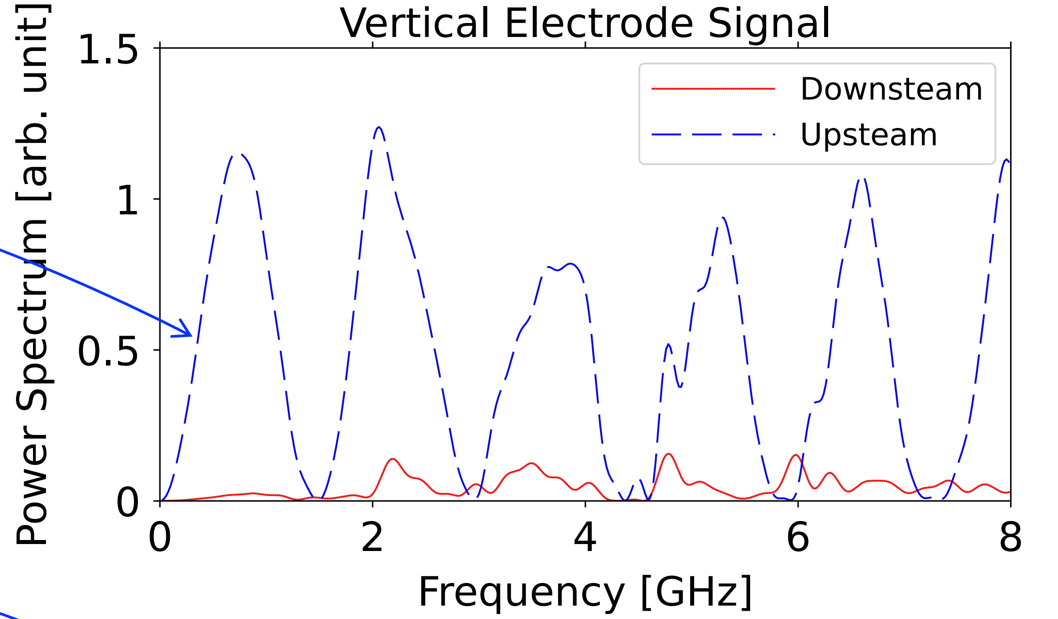


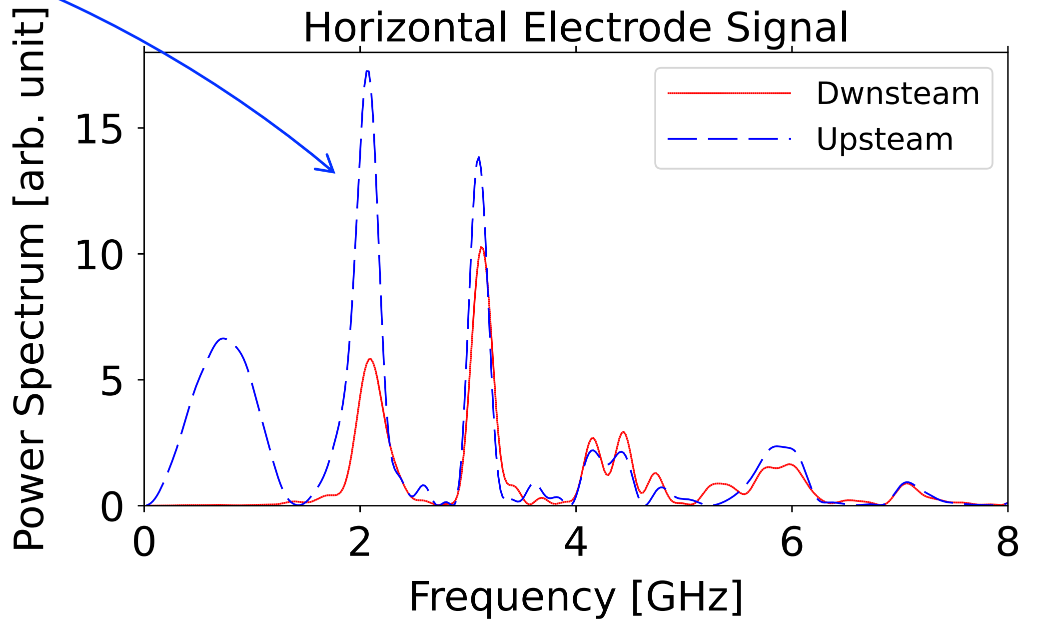
Signal Produced by Beam

mismatching at **kicker end structure**

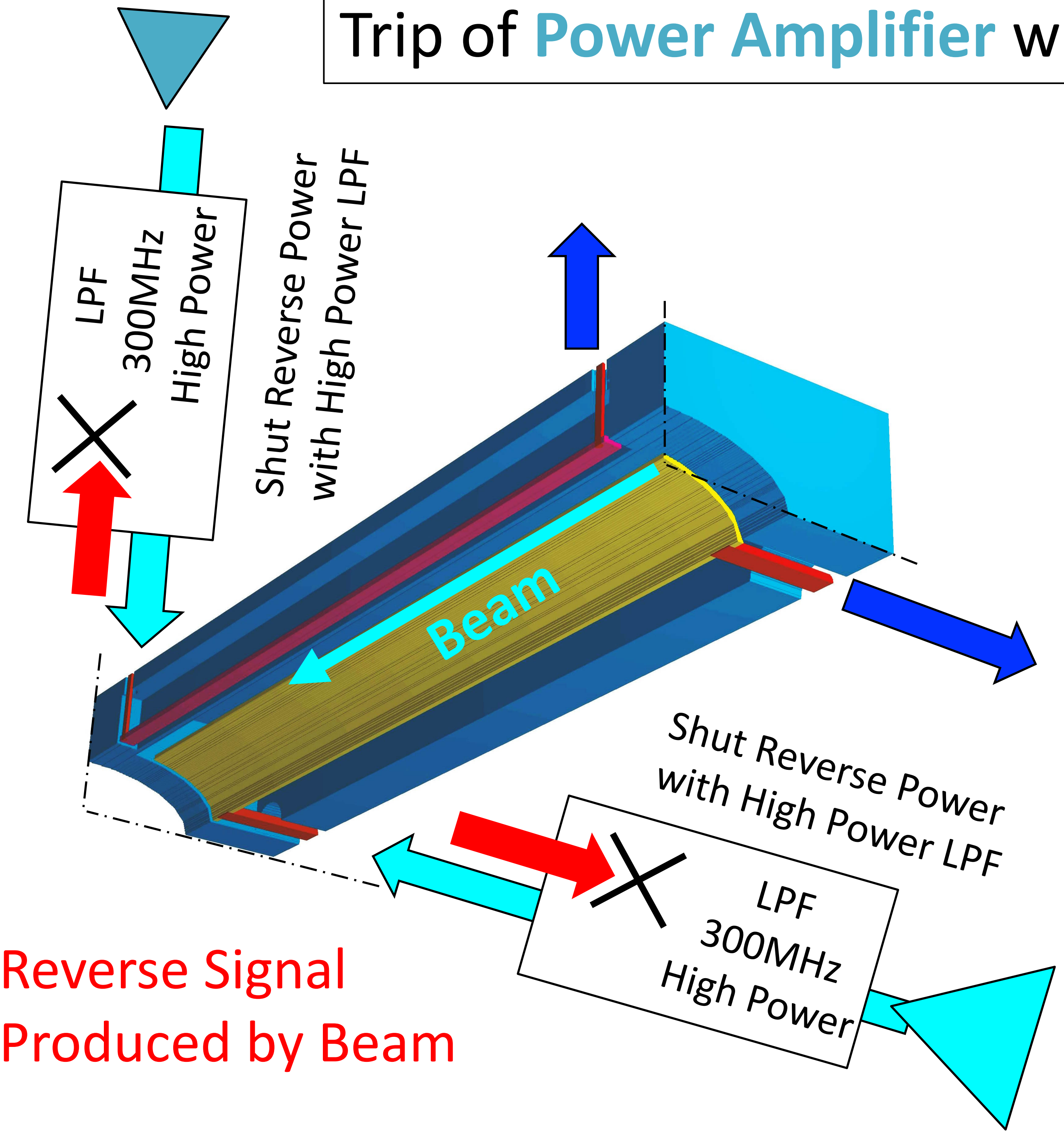
Trip of Power Amplifier with "Reverse Power"



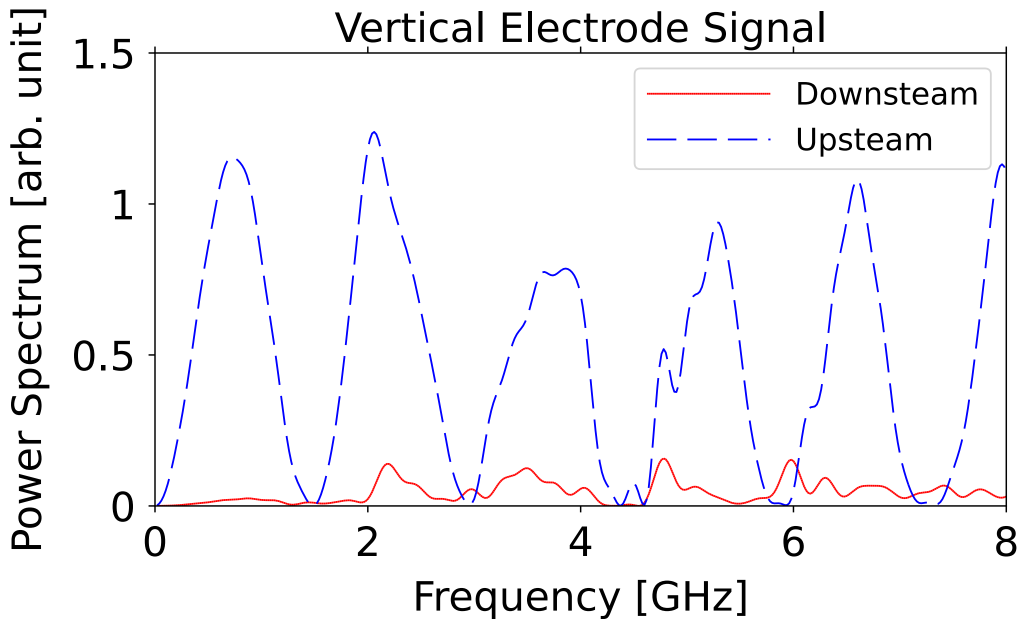


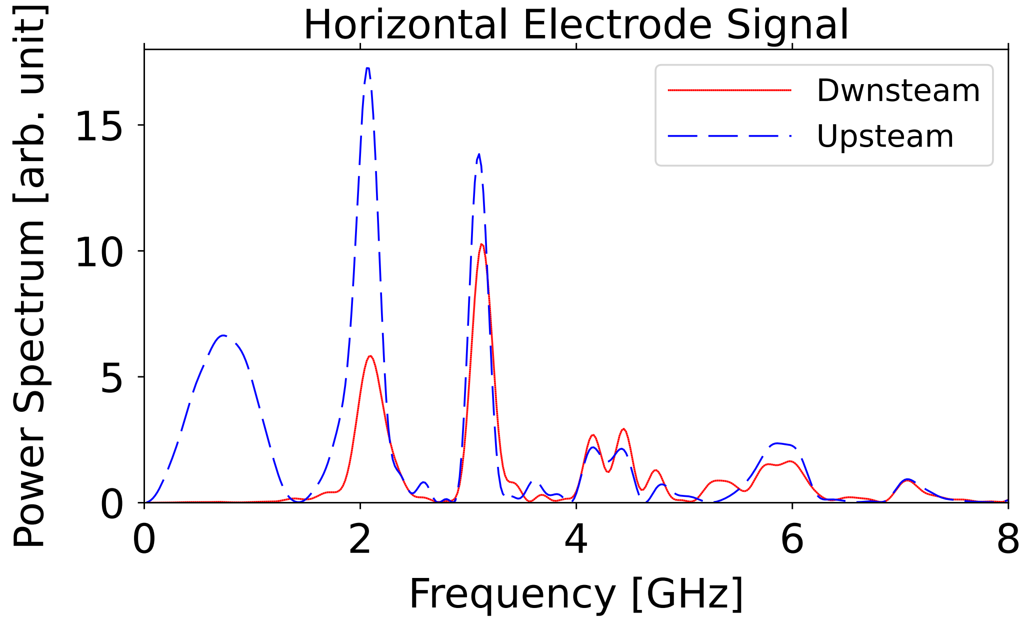


Trip of Power Amplifier with "Reverse Power"



Reverse Signal
Produced by Beam



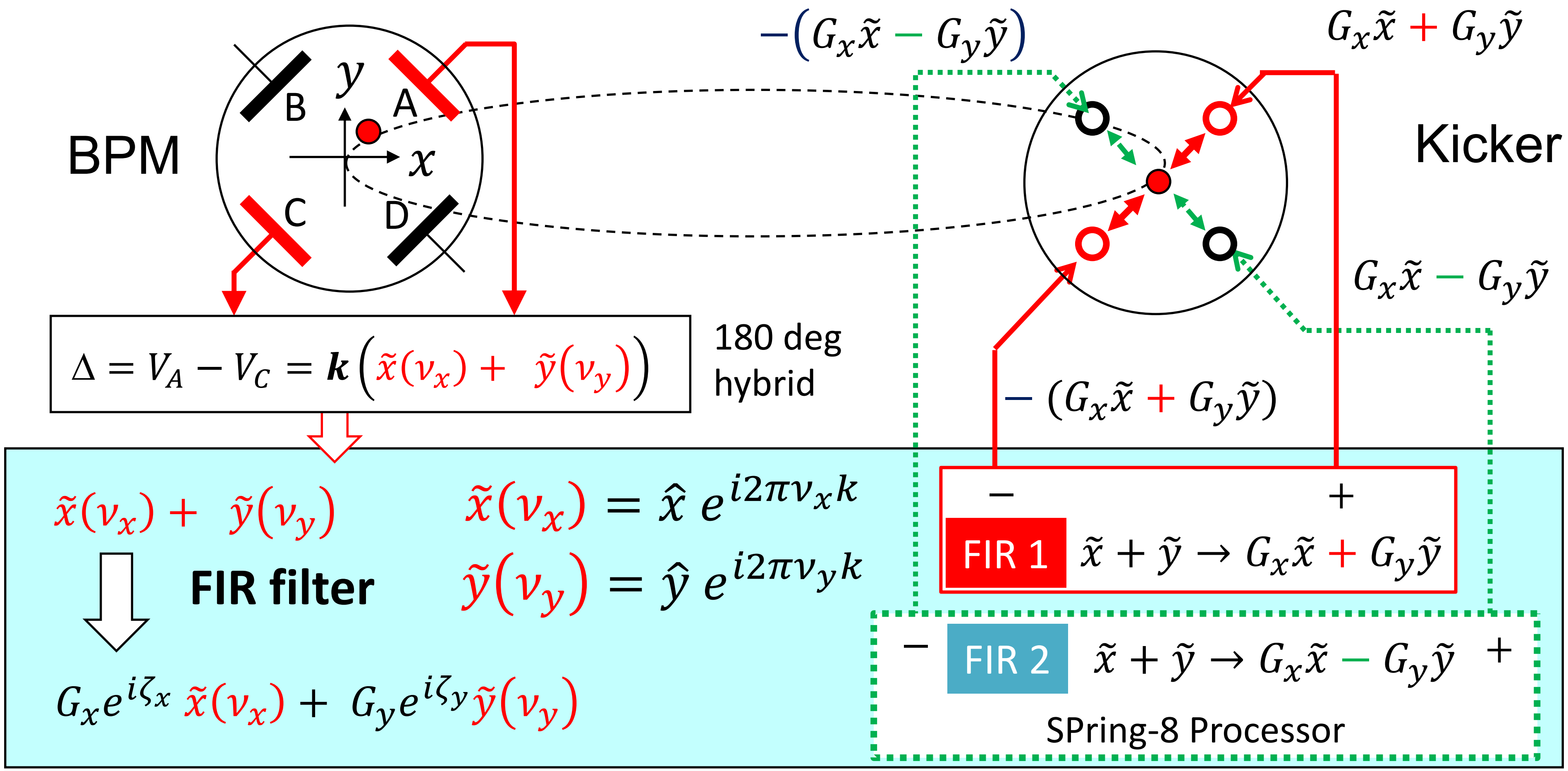


Single-Loop Two-Dimensional Transverse Feedback

Single-Loop Two-Dimensional Transverse Feedback with Previous version Processor

One Position Signal, **One** Processor, for **Horizontal and Vertical feedback**

* Less components, cost and tuning points
* No special devices are needed (but SPring-8 Processor for all kick electrodes)



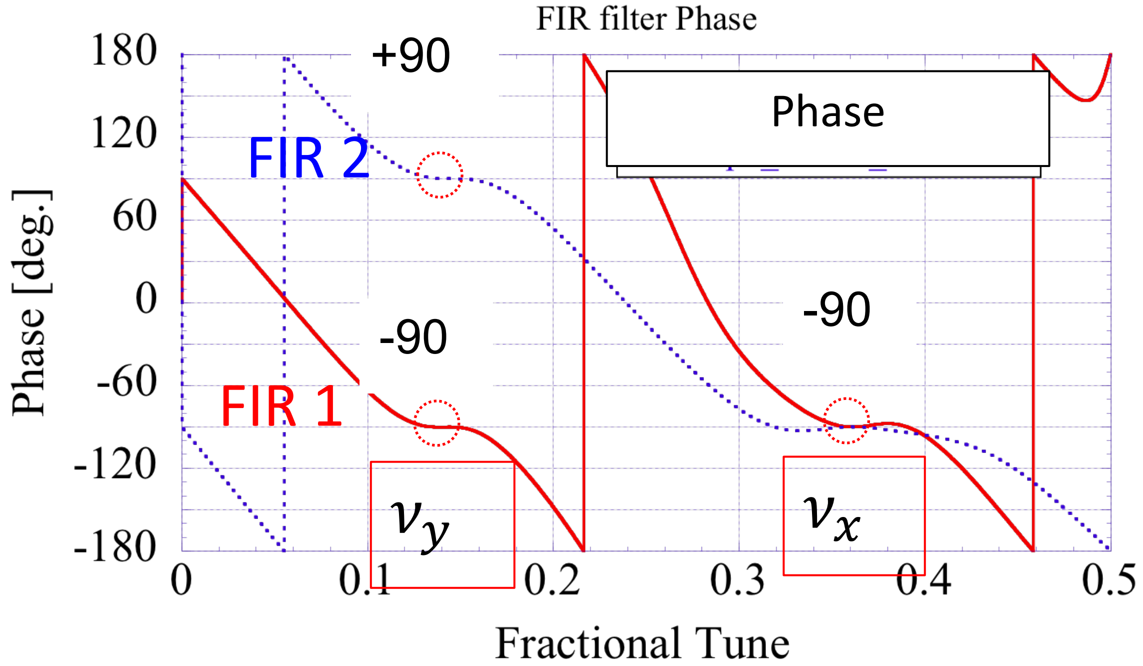
T. Nakamura, et al, EPAC06, “Single-loop Two-Dimensional Transverse Feedback for Photon Factory”
E.-H. Lee, et al., Review of Scientific Instruments **85**, 125102 (2014)

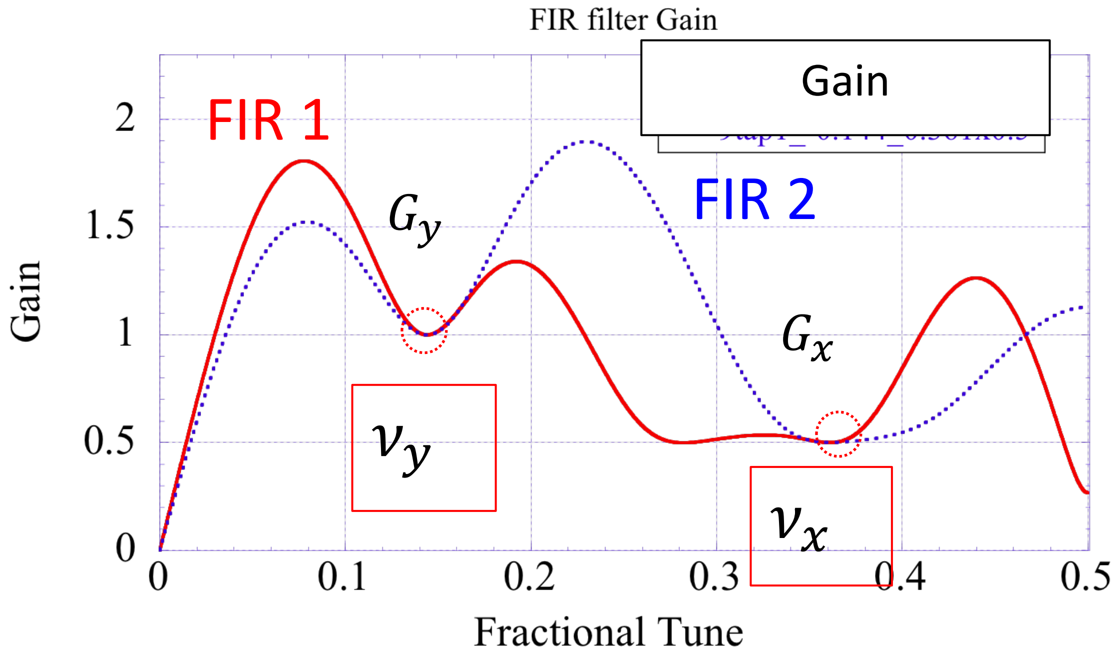
PF, TLS, SOLEIL, PLS-II
in their early stage

PLS-II Two-Dimensional Feedback

FIR 1 $\tilde{x} + \tilde{y} \rightarrow G_x\tilde{x} + G_y\tilde{y}$

FIR 2 $\tilde{x} + \tilde{y} \rightarrow G_x\tilde{x} - G_y\tilde{y}$





Adjust gain for required damping / acceptance
with beta function at BPM and Kicker

E.-H. Lee, et al., Review of Scientific Instruments **85**, 125102 (2014)

Increase of beam size by BPM noise
and
High Resolution BPM

T. Nakamura, NANOBEAM 2005, p. 401 in https://lib-extopc.kek.jp/preprints/PDF/2005/0525/0525020.pdf

Increase of beam size by BPM noise

BPM random noise

Resolution σ_{BPM}

→ Transverse Feedback →

Effective beam size at BPM by noise kick

$$\sigma_x = \frac{\sqrt{\tau_{total} T_0}}{\tau_{FB}} \sigma_{BPM} \sim \sqrt{\frac{T_0}{\tau_{FB}}} \sigma_{BPM} \qquad \tau_{total} \sim \tau_{FB}$$

$$\tau_{FB} = 0.5\text{ms} = 100 T_0$$

$$\sigma_x = \frac{1}{10} \sigma_{BPM} \ll$$ Vertical Beam size $\sim 5\ \mu m$

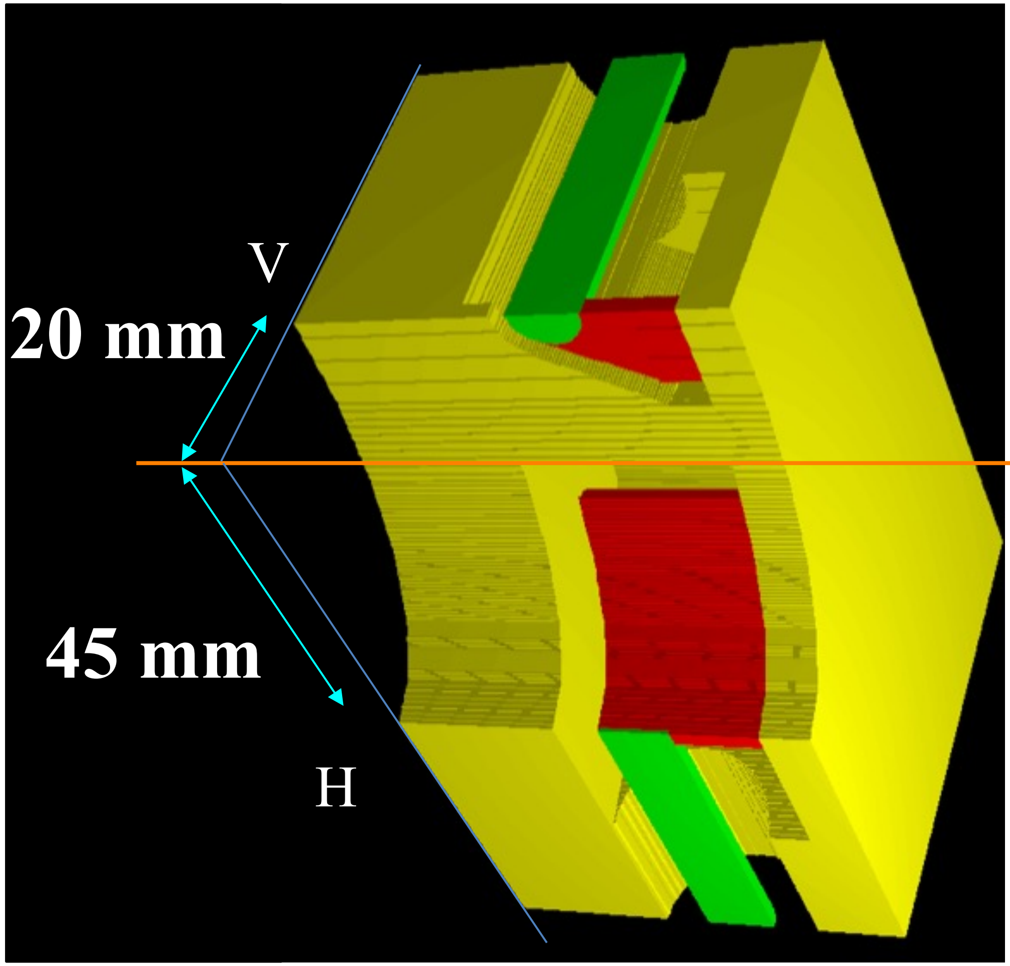
$$\sigma_{BPM} \sim 5\ \mu m$$

for one passage of bunch

T. Nakamura, NANOBEAM 2005, p. 401 in https://lib-extopc.kek.jp/preprints/PDF/2005/0525/0525020.pdf

High Resolution BPM by Shorted Stripline Structure

1/4 of structure



Beam axis

$\sigma_V = 5\mu m$

for 0.2nC bunch
(= 100mA x 2ns)

T. Nakamura, DIPAC05, https://accelconf.web.cern.ch/d05/PAPERS/POW027.pdf

SPring-8 Longitudinal Kicker

SPring-8 Longitudinal Kicker

SPring-8 Storage ring (at 6 GeV operation)

* Large Revolution Period (5us) and High Energy (6GeV) => Large kick / revolution
* Limited space for kickers => Short kicker

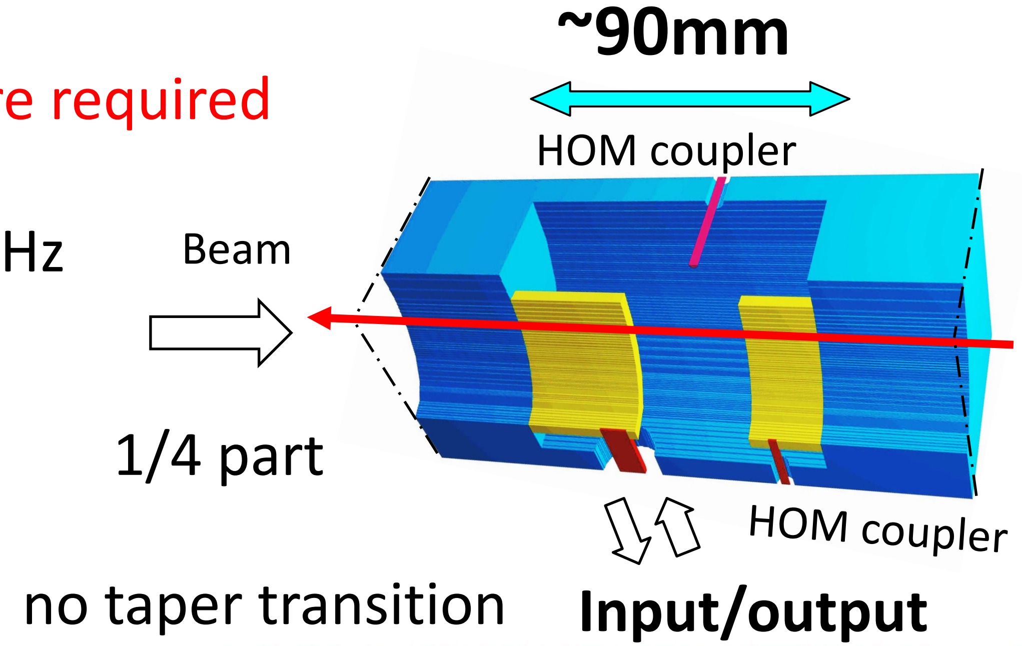
High Shunt Impedance / Length Kickers are required

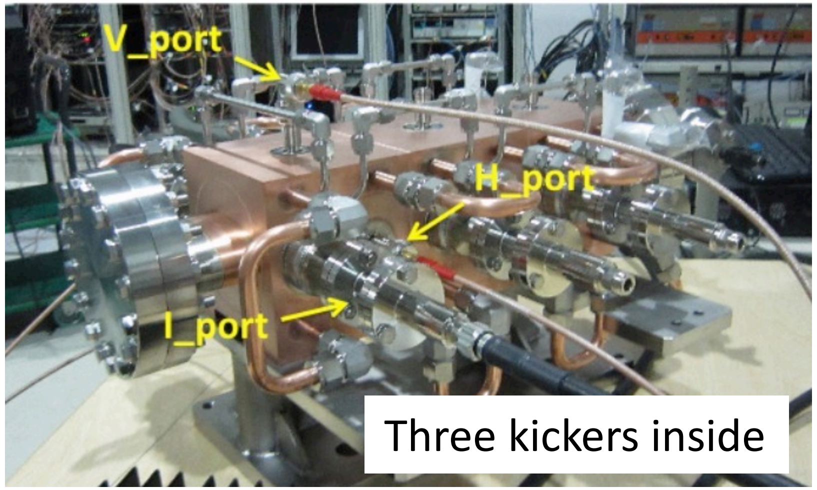
Higher frequency is chosen :

3 + 1/4 period / bunch spacing (2ns) @500MHz

* **Small Kicker**
* **Simple Drive Circuit**

without QPSK modulator



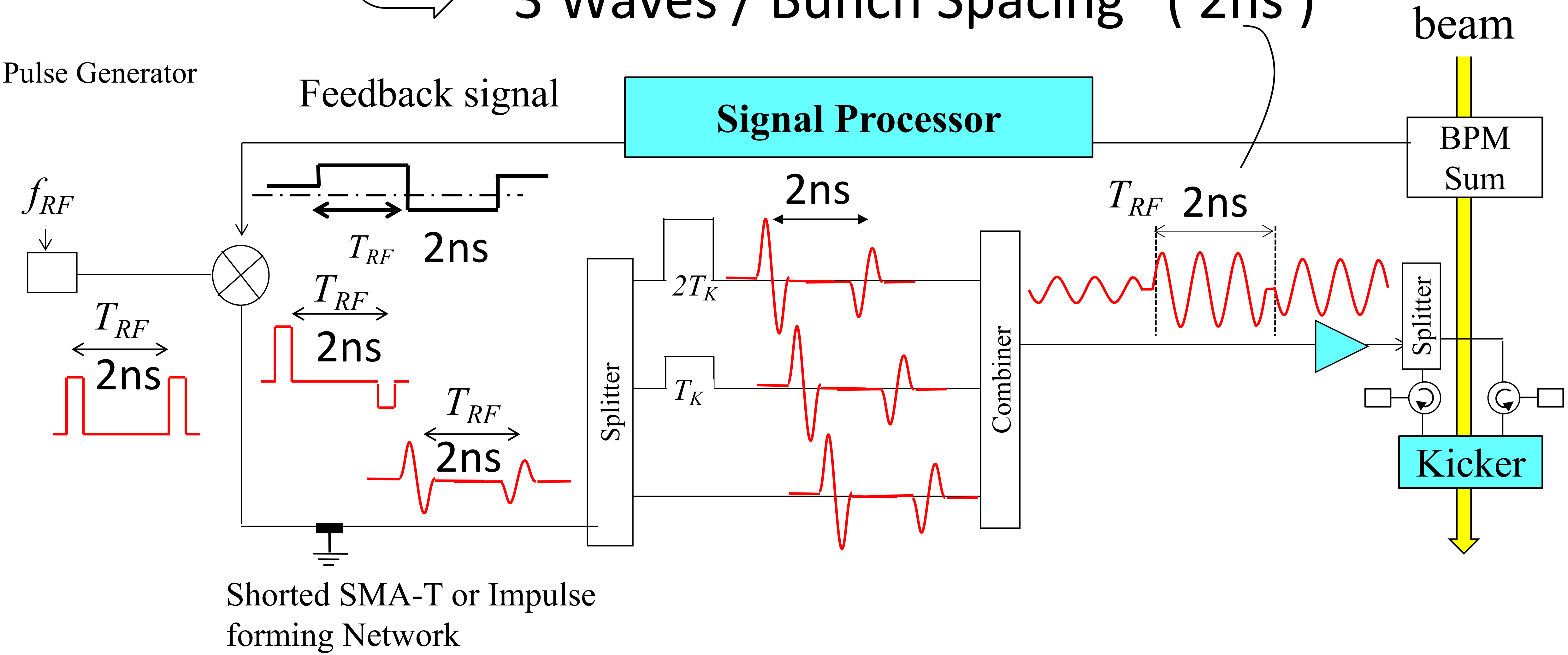


T. Nakamura, proposed and test with prototype,
https://accelconf.web.cern.ch/IPAC2011/papers/mopo007.pdf
M. Masaki, et al, (actual kicker and feedback test with beam)
https://www.pasj.jp/web_publish/pasj2015/proceedings/PDF/WEP0/WEP088.pdf
https://accelconf.web.cern.ch/ibic2013/papers/tupc18.pdf

Simple drive circuit without QPSK modulator

1.6 GHz = (3 + 1/4) x 500MHz => (3 + 1/4) period / bunch

3 Waves / Bunch Spacing (2ns)



Longitudinal Kicker (Comparison)

SPring-8 Type Kicker

1.6 GHz : 3 + 1/4 period / bunch

3 period drive : 3% loss

No QPSK modulator

Standard Kicker widely used

DAΦNE type Overdamped Cavity

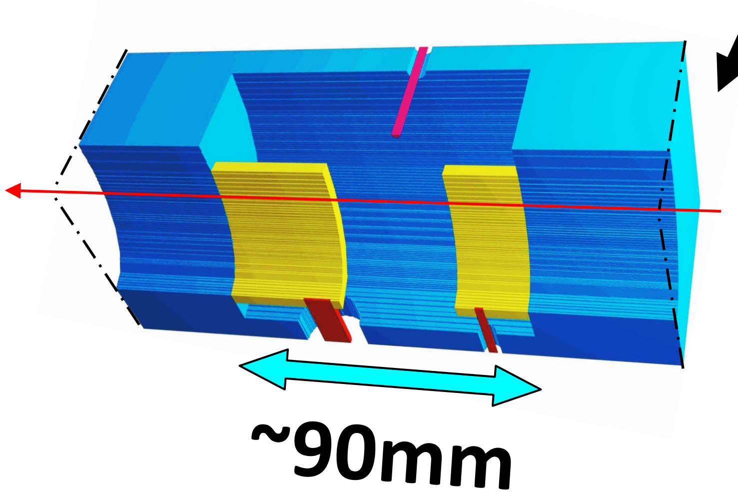
1.4 GHz :

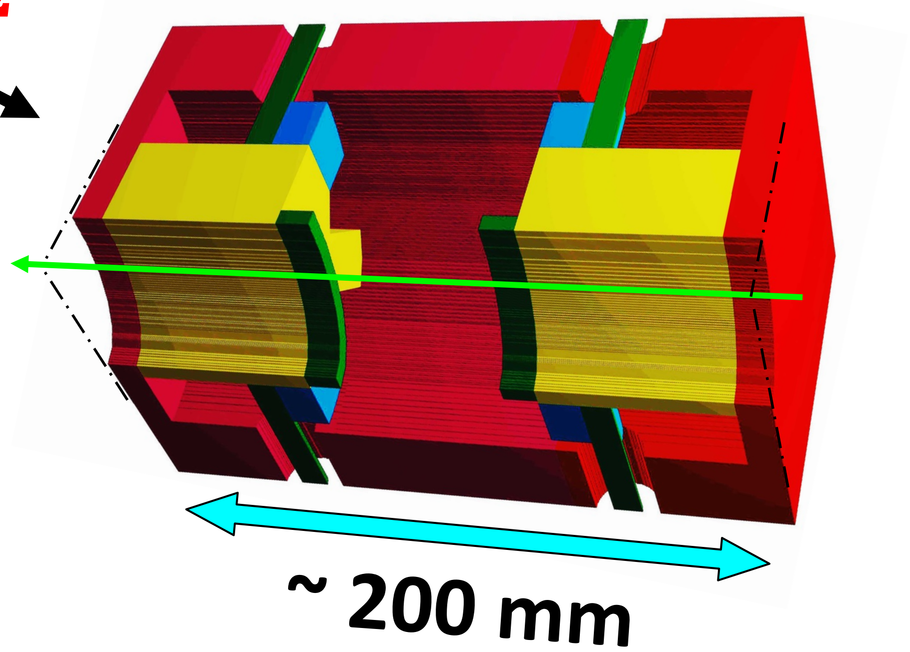
2 + 3/4 period / bunch

needs **QPSK modulator**

(2 period drive : 20 % loss)

Shunt Impedance ~ 1.3 – 1.4 kΩ





f_{RF} ~ 500MHz region

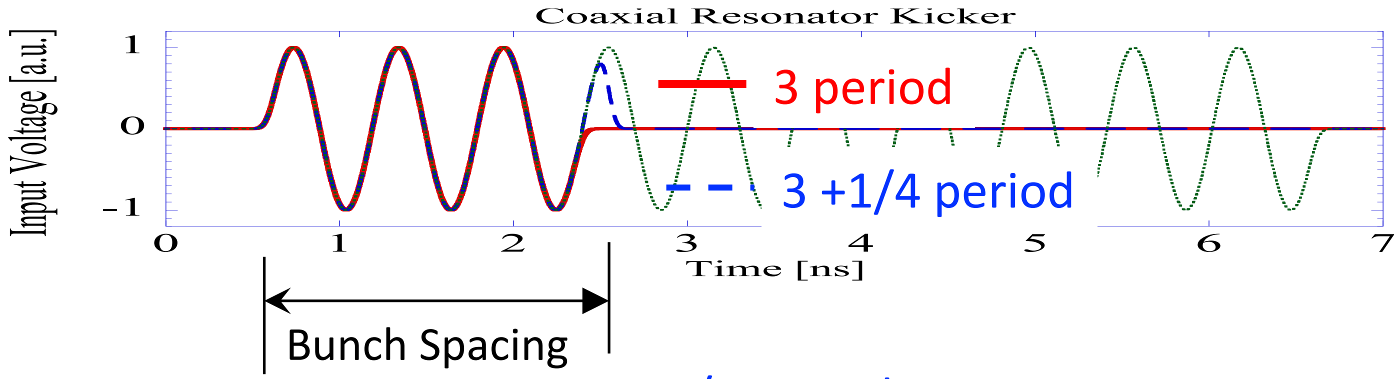
BESSYII/SLS/Elettra/TLS kicker shape

SPring-8 Longitudinal Kicker

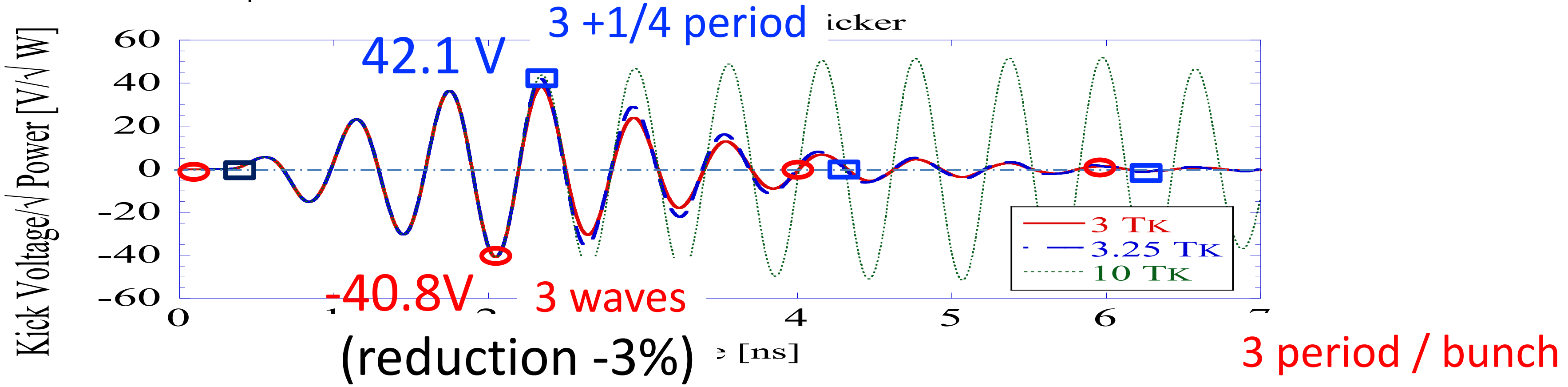
3 + 1/4 period / bunch spacing @500MHz is preferable

=>

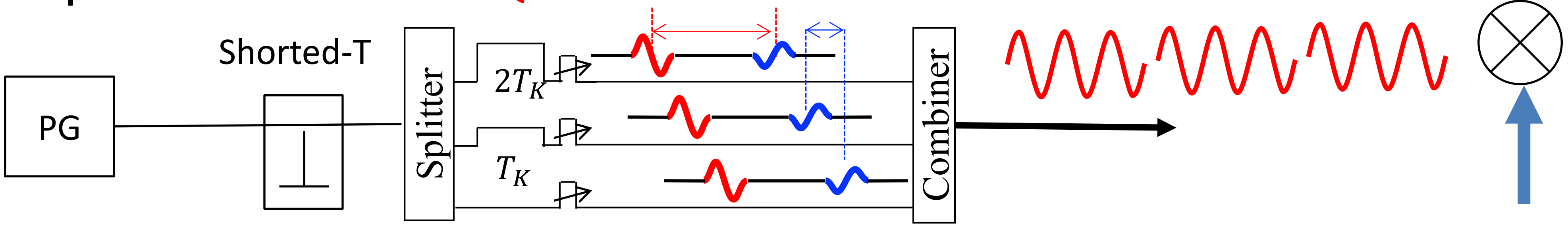
Kicker Drive Signal



Kick Voltage



Simple drive circuit without QPSK modulator



FIR filter Coefficients

Least Square Fitting (TDLSF method) for Coefficients

First, ~2002, we developed this method and used in
APS(US), SOLEIL(France), HLS (China), TLS(Taiwan), PLS-II(Korea),
KEK-PF, and was contacted from IHEP (China)

Finally, I realized that
this method and
the frequency domain method
 that I also developed (or might be re-invented),
 and I show in previous discussion
are equivalent (at least, some cases).

We made the code with Python, C/C++, Fortran version and soon Julia

And the conversion code with Python from
FIR Phase and gain to Dimtel’s definition of Phase and Gain

Least Square Fitting (TDLSF method) for Coefficients

$$x[n] = A\cos(n\phi + \psi) + \mathrm{B} = \boldsymbol{p_0} + \boldsymbol{p_1} \cos n\phi + \boldsymbol{p_2} \sin n\phi$$

$\boldsymbol{p_1} = B$
$\boldsymbol{p_1} = A\cos\psi$
$\boldsymbol{p_2} = -A\sin\psi$

Least Square Fitting

$\boldsymbol{p_m}$ to "measured" data $\boldsymbol{x_k}$

$$\boldsymbol{p_m} = \sum_{k=0}^{N} C_{m,k}\boldsymbol{x_{-k}}$$

$$\left(\begin{array}{c}\boldsymbol{definition\ of\ a_k} \\ \boldsymbol{y}[0] = \sum_{k=0}^{N} a_k \boldsymbol{x_{-k}}\end{array}\right)$$

With this, we can construct kick data at n = 0 turn

$$y[0] = \boldsymbol{G} A\cos(\psi + \boldsymbol{\zeta}) = \boldsymbol{p}_1 \boldsymbol{G}\cos\boldsymbol{\zeta} + \boldsymbol{p}_2 \boldsymbol{G}\sin\boldsymbol{\zeta} = \sum_{k=0}^{N} \underline{\left(C_{1,k}\boldsymbol{G}\cos\boldsymbol{\zeta} + C_{2,k}\boldsymbol{G}\sin\boldsymbol{\zeta}\right)}\boldsymbol{x_{-k}}$$

$$a_k = C_{1,k}\boldsymbol{G}\cos\boldsymbol{\zeta} + C_{2,k}\boldsymbol{G}\sin\boldsymbol{\zeta}$$

We have a_k , k = 0, 1, 2, ... , N

Extension to multiple tunes is easy

T. Nakamura, et al. https://accelconf.web.cern.ch/e04/PAPERS/THPLT068.PDF

Constraints on Coefficients of FIR filter

Setting Phase and Gain

$$G(\phi_j)e^{i\zeta(\phi_j)} = \sum_{k=0}^{N} a_k \exp(-ik\phi_j)$$

$$\left.\frac{\partial}{\partial\phi}\left(G(\phi)e^{i\zeta(\phi)}\right)\right|_{\phi=\phi_j} = 0 \rightarrow \sum_{k=0}^{N} a_k k \exp(-ik\phi_j) = 0$$

For Flat response at $\phi \sim \phi_j$

Minimization

$$P = \sum_{k=0}^{N} |a_k|^2$$

Frequency Domain Condition for FIR filter

TDLSF is Equivalent to Following Frequency Domain condition

Gain, Phase and "Flat response" : Equivalent to

$$\boldsymbol{G}(\boldsymbol{\phi_j})e^{i\boldsymbol{\zeta}(\boldsymbol{\phi_j})} = \boldsymbol{G}(\boldsymbol{\phi_i} \pm \Delta)e^{i\boldsymbol{\zeta}(\boldsymbol{\phi_i} \pm \Delta)}$$

$$= \sum_{k=0}^{N} a_k \exp(-ik(\boldsymbol{\phi_i} \pm \Delta))$$

$$\boldsymbol{G}(\boldsymbol{\Delta})e^{i\zeta(\boldsymbol{\Delta})} = \mathbf{0}$$

with setting $\Delta \ll 1$

Minimization

$$\boldsymbol{P} = \sum_{k=0}^{N} |a_k|^2$$

Least Square Fitting (TDLSF method) for Coefficients : 1st order

$$x[n] = A\cos((1+\Delta)\phi^{(n)} + \psi) + (1 + n\Delta_0)\mathrm{B} \quad \text{for } |\Delta| \ll 1$$

$$\rightarrow \boldsymbol{p_{0,1}} + \boldsymbol{p_{0,2}n} + \boldsymbol{p_{1,1}}\cos\phi^{(n)} + \boldsymbol{p_{1,2}}\sin\phi^{(n)} + \boldsymbol{p_{2,1}}\phi^{(n)}\cos\phi^{(n)} + \boldsymbol{p_{2,2}}\phi^{(n)}\sin\phi^{(n)}$$

$$p_{0,1} = B$$
$$p_{0,2} = \Delta_0 B$$
$$p_{1,1} = A\cos\psi$$
$$p_{1,2} = -A\sin\psi$$
$$p_{2,1} = -A\Delta\cos\psi$$
$$p_{2,2} = -A\Delta\sin\psi$$

Least Square Fitting

$\boldsymbol{p_{i,j}}$ to $\boldsymbol{x_k}$ turn-by-turn Positions

$$\boldsymbol{p_{i,j}} = \sum_{k=0}^{N} C_{i,j,k}\boldsymbol{x_{-k}}$$

$$y[0] = \boldsymbol{G}A\cos((1+\Delta)\phi_0 + \psi + \boldsymbol{\zeta}) = \boldsymbol{G}A\cos(\psi + \boldsymbol{\zeta})$$

$$= \boldsymbol{p_{1,1}}\boldsymbol{G}\cos\boldsymbol{\zeta} + \boldsymbol{p_{1,2}}\boldsymbol{G}\sin\boldsymbol{\zeta} = \sum_{k=0}^{N}\left(C_{1,k}\boldsymbol{G}\cos\boldsymbol{\zeta} + C_{2,k}\boldsymbol{G}\sin\boldsymbol{\zeta}\right)\boldsymbol{x_{-k}}$$

$$a_k = C_{1,k}\boldsymbol{G}\cos\boldsymbol{\zeta} + C_{2,k}\boldsymbol{G}\sin\boldsymbol{\zeta}$$

We have a_k , k = 0, 1, 2, … , N

Example

5 Constraints : Target tune **0.15** with **flat response**

$G(0) = 0$

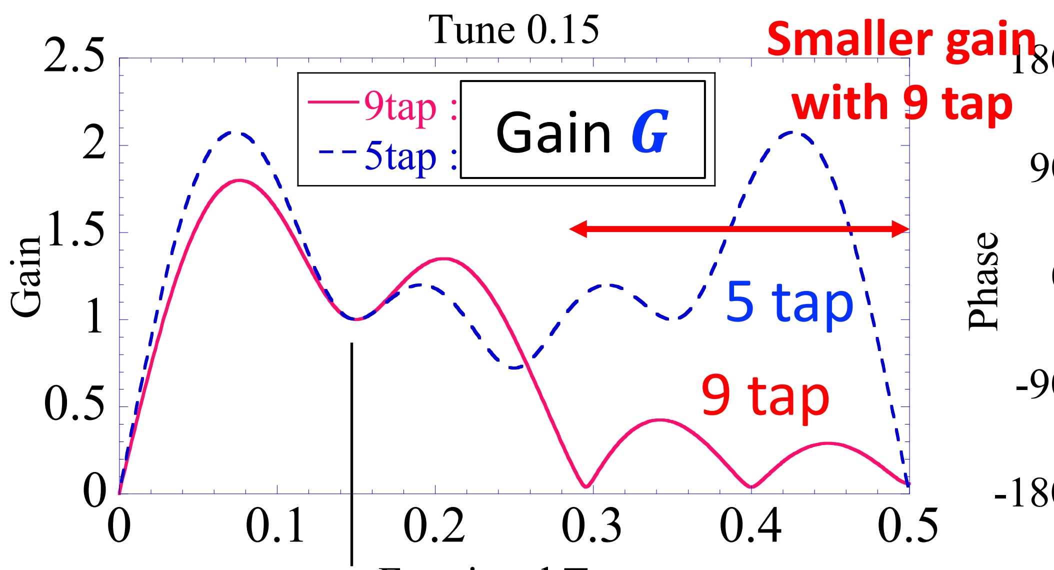
$G(0.15 - 0.01) = G(0.15 + 0.01) = 1$

$\zeta(0.15 - 0.01) = \zeta(0.15 + 0.01) = -90$ deg

position data

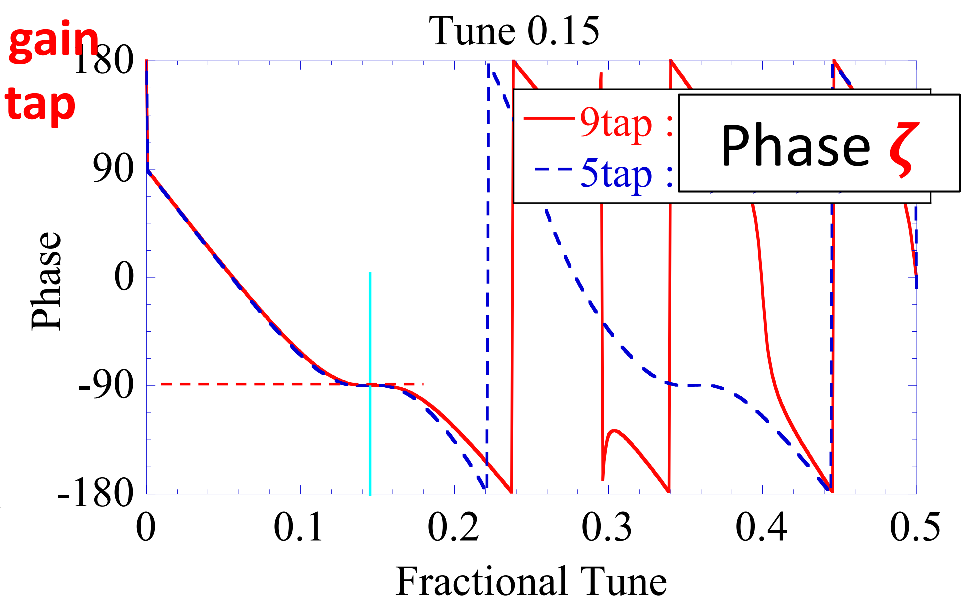
5 tap : **-1, -3, -5, -7, -9** turns

9 tap : **-1, -2, -3, -4, -5, -6, -7, -8, -9** turns



$G(0) = 0$

$G(0.15 - 0.01) = G(0.15 + 0.01) = 1$



$\zeta(0.15 - 0.01) = \zeta(0.15 + 0.01) = -90$