Longitudinal Feedback Using a Low-pass Differentiator

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I.FAST Workshop 2024 on Bunch-by-Bunch Feedback Systems

Outline

Longitudinal Stabilization and Tune Variation

- Traditionally, longitudinal feedback controllers are designed to satisfy three requirements:
  - 90° phase shift at the synchrotron frequency;
  - DC rejection;
  - Bandpass response.
- This approach is limited in its tune acceptance range;
- Many modern light sources use harmonic cavities to lengthen the bunches for the improved lifetime;
- This leads to wide range of synchrotron frequency variation versus beam current, as well as wide spread of frequencies at nominal operating current.

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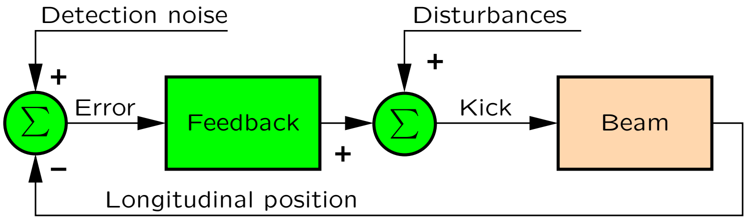
Differentiator Feedback in the Longitudinal Plane



- Beam response $B(s) = \frac{C}{s^2 - 2\lambda s + \omega^2}$;
- C absorbs front-end and back-end gains, as well as optics parameters;
- Differentiator $F(s) = Ks$ is mathematically ideal to stabilize $B(s)$:
- Closed loop response $H_{\text{cl}}(s) = \frac{B(s)}{1+F(s)B(s)} = \frac{C}{s^2-(2\lambda-KC)s+\omega^2}$;
- Differentiator only affects the growth rate and provides constant damping independent of the synchrotron frequency.

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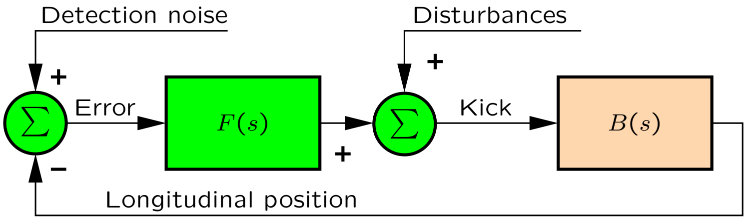

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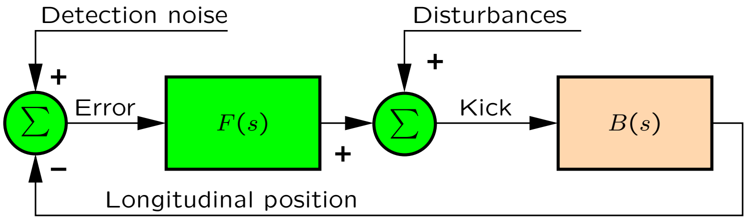

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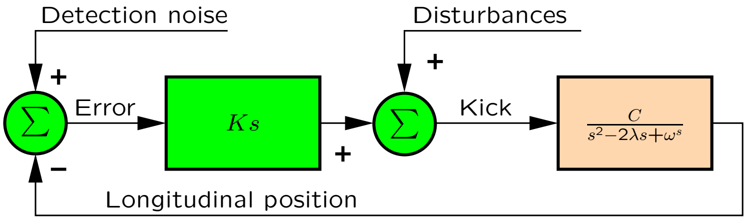

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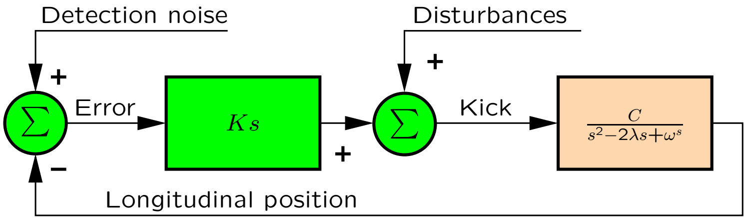

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It Could Never Work...

- When I started working on longitudinal bunch-by-bunch feedback as a first year graduate student I almost immediately learned that "differentiator could never work".
- Approximate differentiator by one turn difference $u_n = z_n - z_{n-1}$
- Problems:
  - Very little gain at synchrotron frequencies — small phase advance per turn;
  - Controller gain increases linearly with frequency — output will be swamped with the amplified wideband detection and quantization noise.
- For me the matter rested there until summer 2022 (NAPAC 2022), when I learned that Ryan Lindberg of APS-U arrived at the differentiator idea;
- I set out to demonstrate why this was a bad idea and was surprised to discover that, at least in the Simulink model, it worked as well as the conventional approach;
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Test in an Accelerator

- A month later I got a call from the ALS — they had trouble reaching 500 mA operating current;
- What is ALS:
  - 1.5 GeV storage ring with normal conducting main and harmonic RF cavities;
  - Multiple unstable modes in the longitudinal plane;
  - Zero current synchrotron frequency of 8.5 kHz decreases to 5.5 kHz at full current due to the action of the harmonic cavities;
  - Mode 0 is shifted to 4 kHz by the interaction with the main RF impedance, normally stable.
- Feedback filter for the ALS is designed to damp instabilities from 5 to 9 kHz, rolls off gently below 5 kHz to avoid destabilizing mode 0.
- In September 2022, mode 0 was going unstable at the ALS and the existing feedback filter could not damp the instability;
- Tried a low-pass differentiator — worked like a charm.

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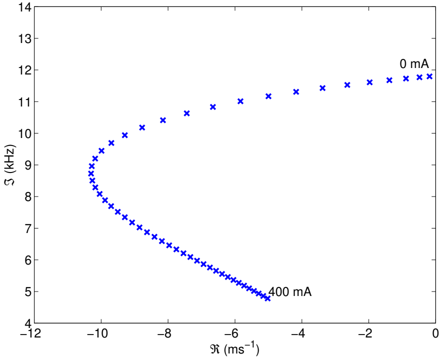
Mode 0 vs. Beam Current



- Root locus of the modal eigenvalue versus beam current;
- Damping on the real axis, frequency on the imaginary;
- Old ALS configuration (higher RF voltage).

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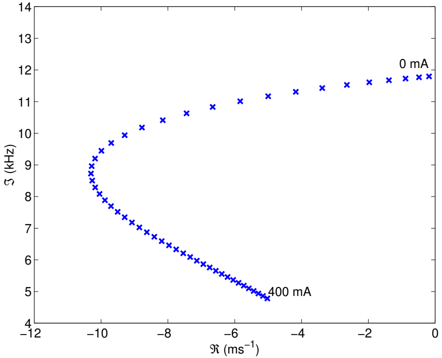

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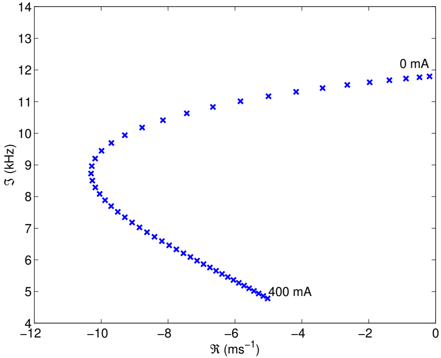

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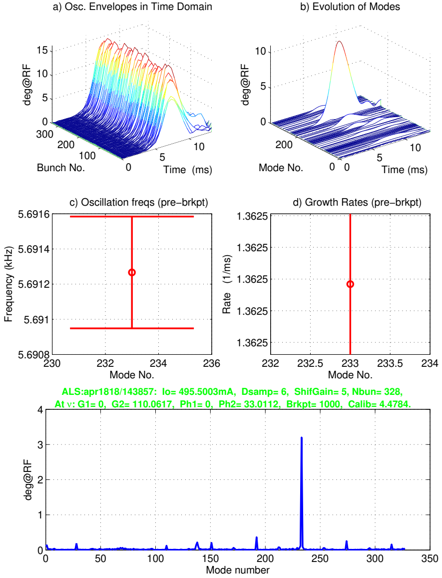

- A grow/damp measurement from 2018;
- 12 ms growth time;
- Well-known HOMs in main RF cavities drive mode 233;
- A measurement on September 8, 2022;
- 3 ms growth time;
- Modes 0, 1, and -1 grow.

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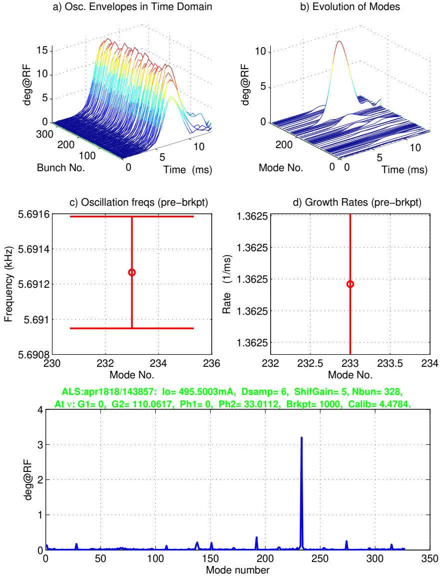

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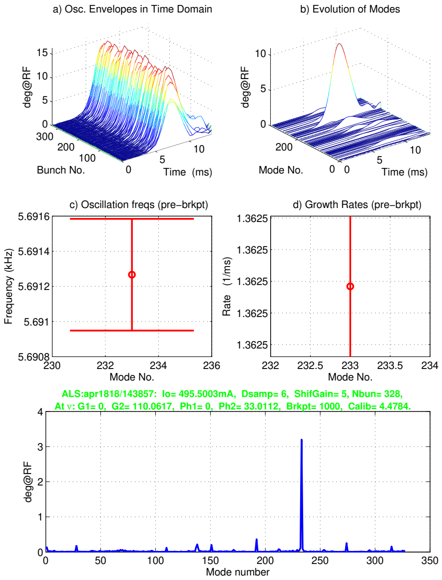

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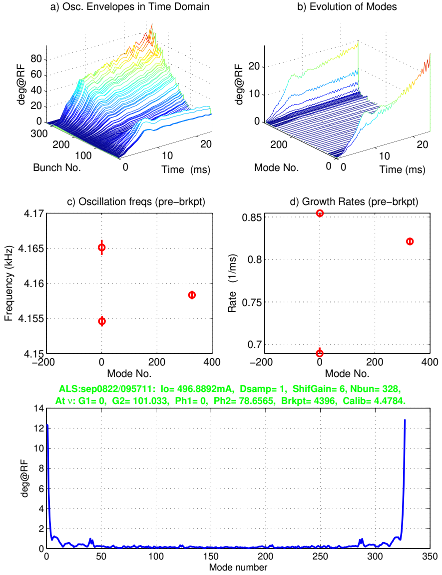

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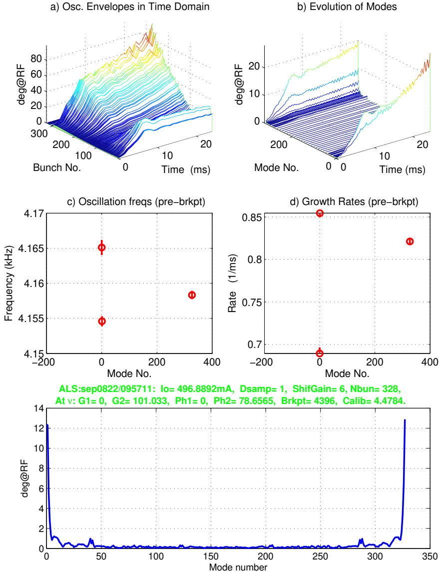

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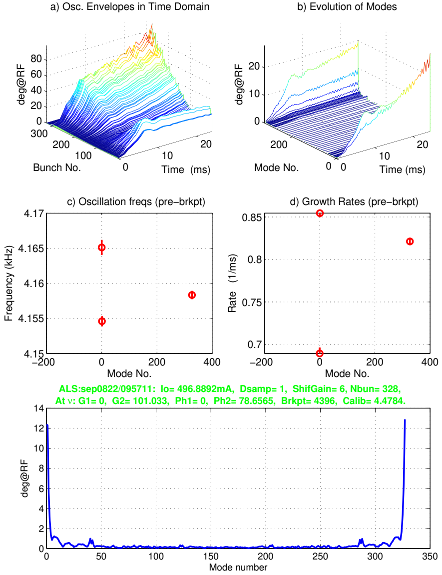

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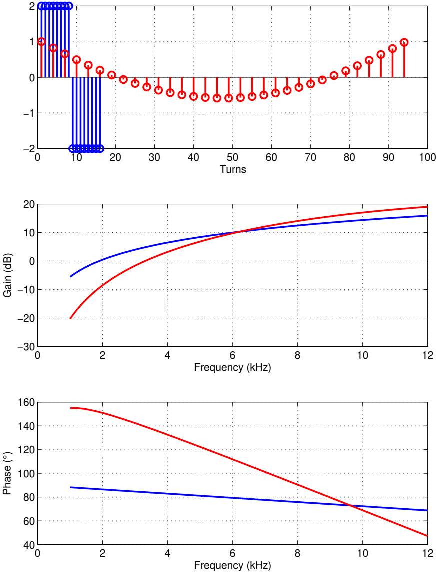

- Comparison of the normal ALS filter and a 16-tap low-pass differentiator;
- Similar gains in the working range;
- Much steeper slope for the normal filter (32 taps, downsampling of 3);
- Log scale plot over a wider frequency range.

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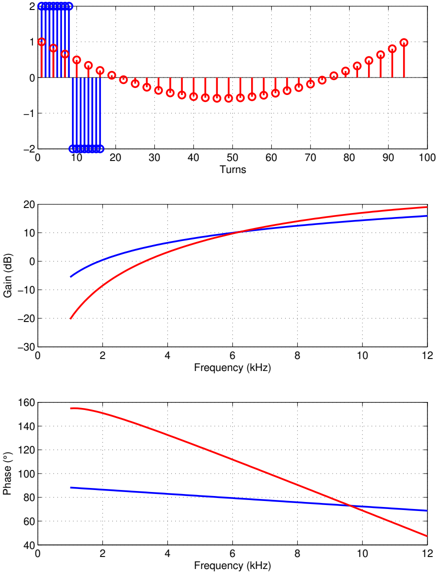

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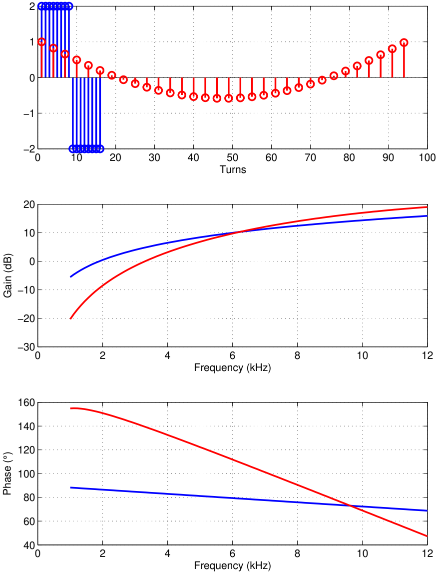

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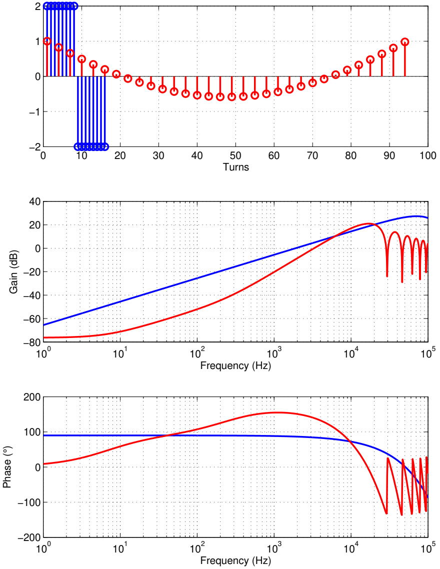

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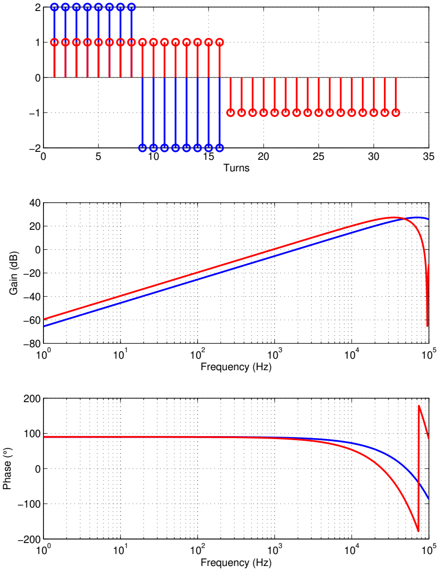

- Since the original “emergency intervention” we have updated the ALS differentiator from 16 to 32 taps;
- Factor of 2 in the shorter filter’s coefficients reflects higher shift gain;
- 6 dB gain difference in the linear range; 3);
- As expected, a longer filter has faster phase roll due to higher group delay;
- Identical peak gains.

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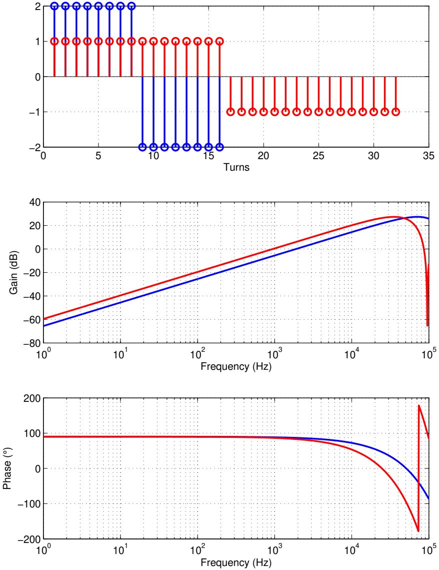

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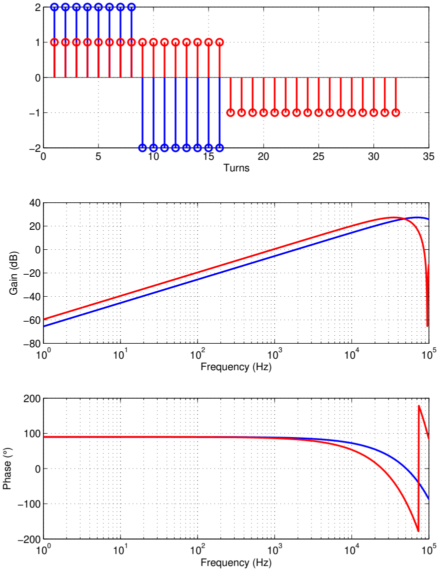

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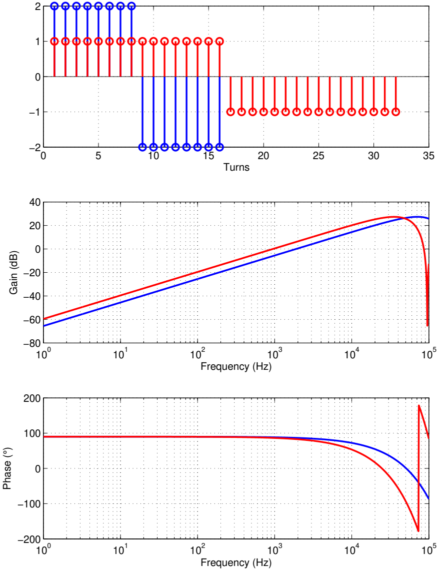

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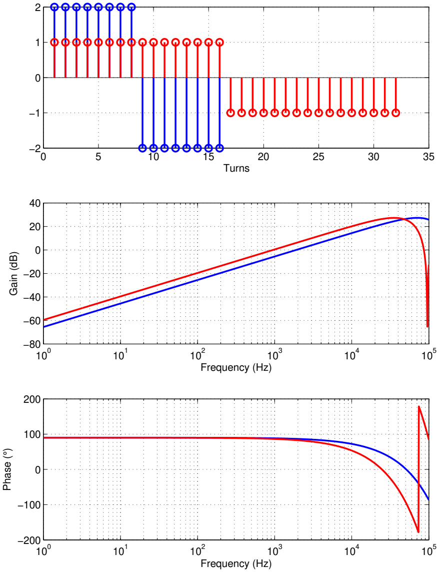

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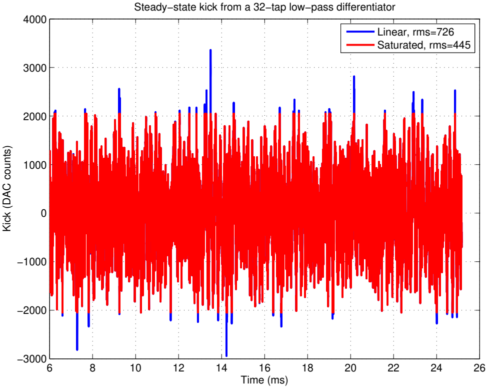

- Standard worry — differentiator will saturate the output with amplified noise, leaving no power for feedback correction;
- Steady-state kick at 500 mA with a 32-tap low-pass differentiator;
- Consistently hits full scale, reflected in the difference in the RMS kick;
- When longitudinal motion develops, power is directed exactly where it's needed.

Motivation
Concept
Surprise Discovery
Some Observations
Future Directions
Summary

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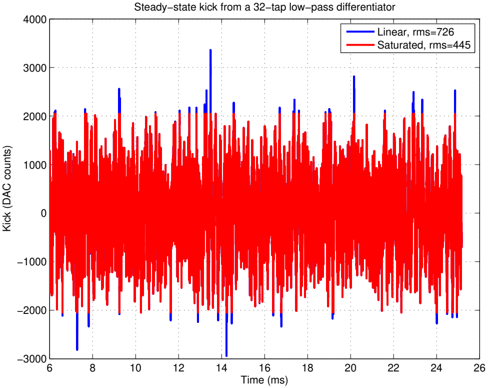

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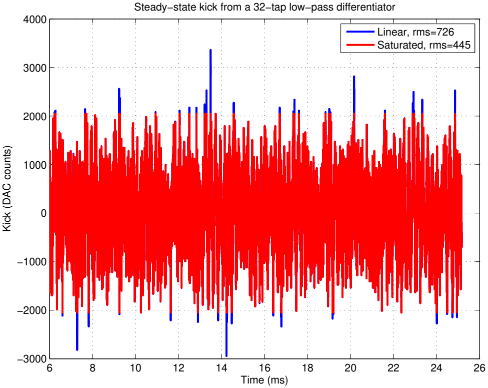

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- Consistently hits full scale, reflected in the difference in the RMS kick;
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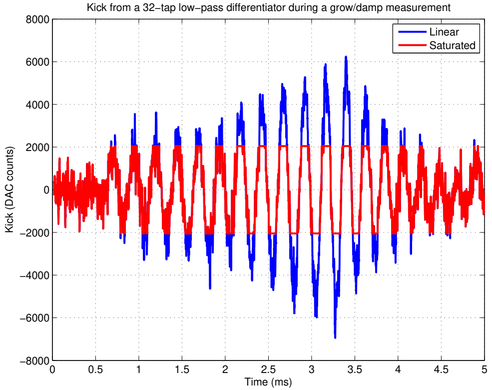
Kick Signals



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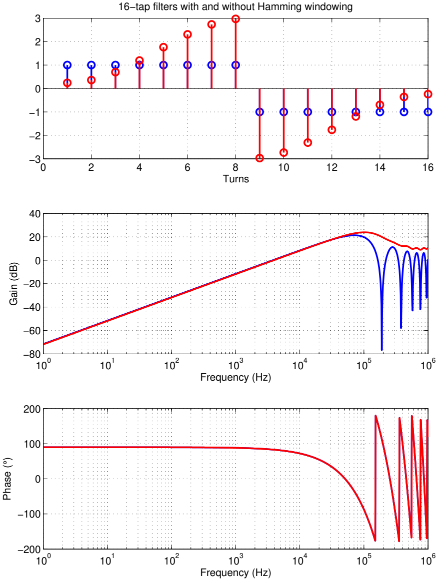
Low-pass Differentiator Optimization



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- Worse than the boxcar — more high-frequency gain;
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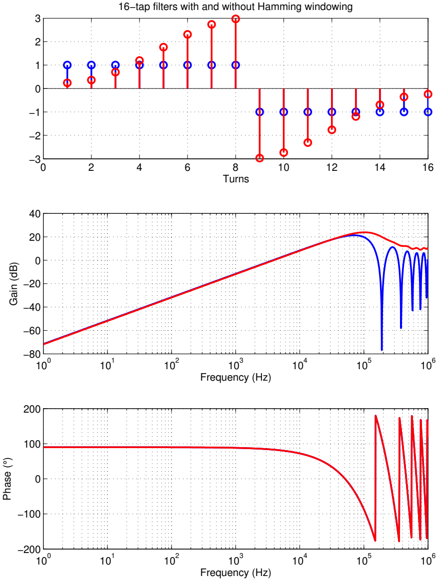

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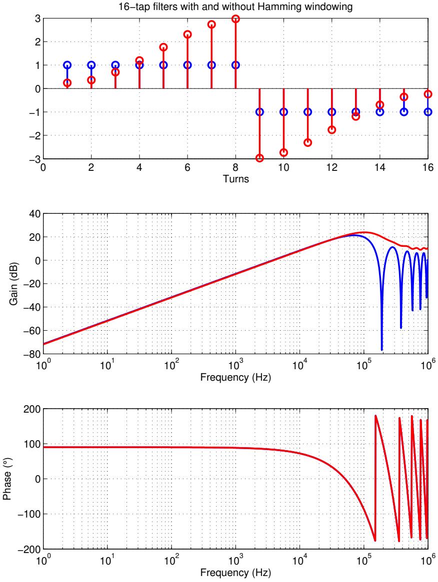

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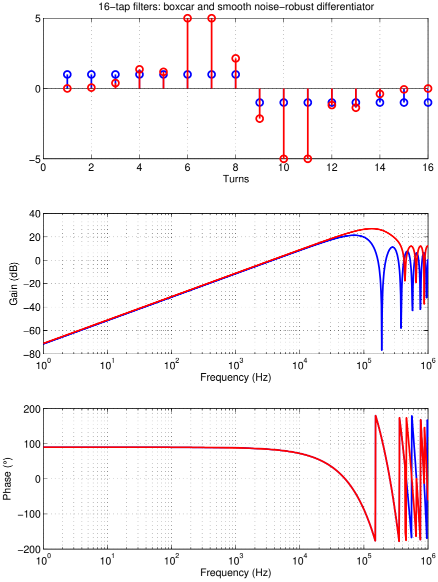

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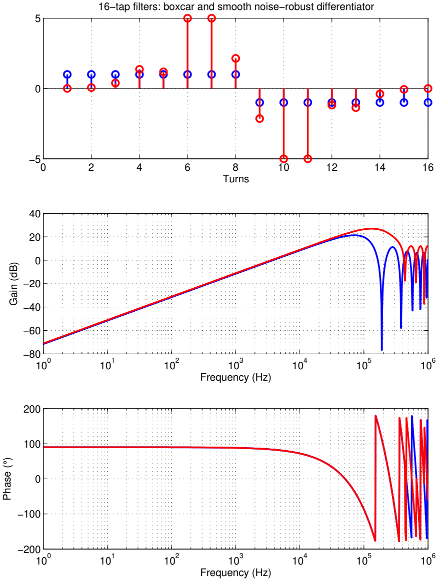

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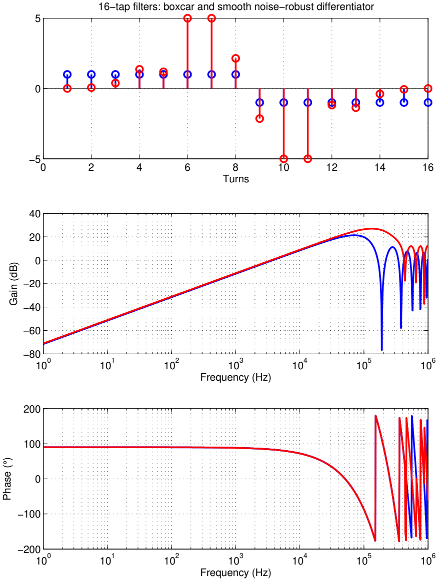

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