



The First and Second-Order Fermi Acceleration Processes in BL Lacertae Objects

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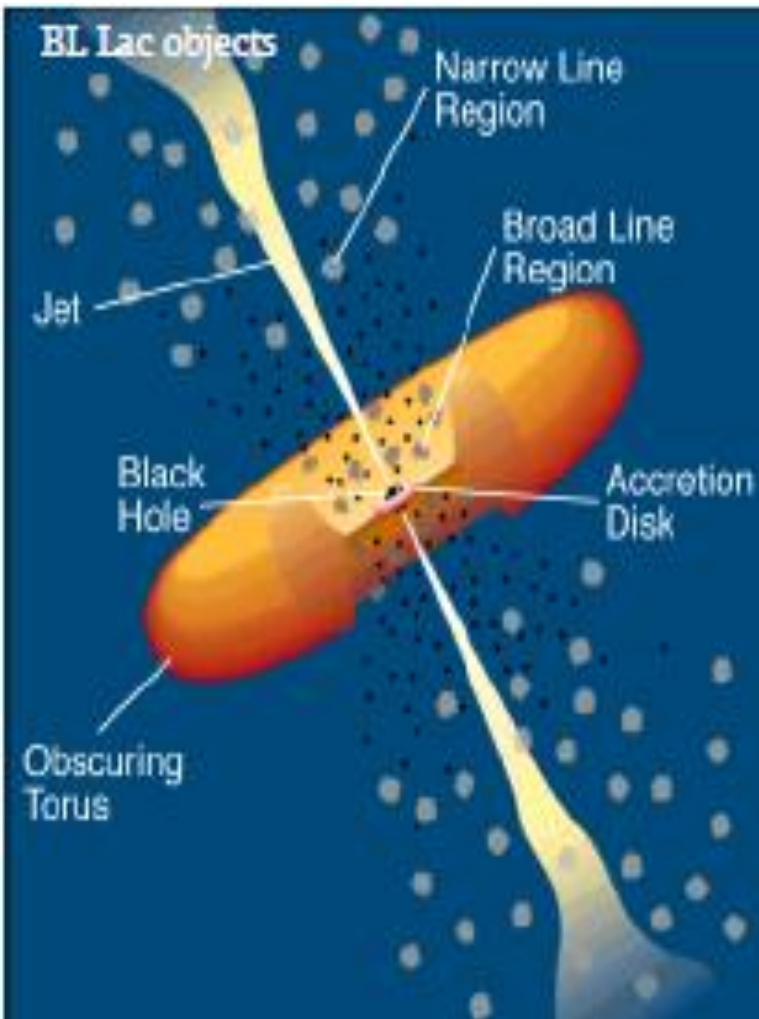
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Outline

- **BL Lacertae Objects**
- **BLL Jets: Some Open Problems**
- **First-Order Fermi Acceleration**
- **Second-order (stochastic) Acceleration**
- **Summary and Conclusions**

BL Lacertae Objects



- BL Lacertae objects (BLLs, a blazar subclass) – AGNs of elliptical galaxies :
 - Non-thermal continuum emission stretching from radio to TeV band (17-19 orders of the frequencies)
 - Absence of emission lines.
 - Strong flux variability in all spectral bands
 - Compact and flat-spectrum radio emission
 - apparent superluminal motion of some components
 - high and variable radio/optical polarization
 - strong γ -ray emission: **BLLs form a majority of extragalactic TeV sources and one of the most important constituent of Fermi-LAT catalogues**
- Hypothetic structure: central supermassive BH (SMBH; 10^8 - $10^9 M_{\odot}$) + accretion disc (AD) + two opposite jets (closely aligned to the observer)

❖ **Open problem: How are BLL jets launched and what is the source of their tremendous energy?**

- The current paradigm describing launching and acceleration processes of relativistic jets of BL Lac jets: key role to magnetic fields → the energy stored in a rapidly spinning Kerr SMBH can be extracted and channeled into a Poynting flux (Blandford & Znajek 1977, Tchekhovskoy+ 2011).
- The jet power, originally carried by a magnetically-dominated beam (with magnetization parameter $\sigma \equiv P_B/P_{\text{kin}} \gg 1$):
 - ✓ Progressively used to accelerate matter (\equiv conversion from magnetic to kinetic energy), until a substantial equipartition between the magnetic and the kinetic energy fluxes ($\sigma \approx 1$) is established (Tchekhovskoy +2009) →
 - ✓ Strong support from the relativistic MHD simulations (e.g. Tchekhovskoy +2009, 2011): the power actually extracted from the BH–AD system is larger than the sole accreting power $\dot{M}c^2$, if the BZ-mechanism is at work (Tavecchio & Ghisellini 2016) →
Observationally, supported by studies comparing the jet and the accretion power: the jet indeed carries a power larger than that associated to the accreted matter, estimated through the disk radiation (Ghisellini +2010, 2014)

❖ **Open problem: particle acceleration processes in BL Lac jets**

- Electrons (and, possibly, positrons and protons) should be accelerated to **(ultra)relativistic energies of TeV-order to produce X-ray - VHE photons (via synchrotron and IC mechanisms)**
- In the bulk frame of the electrons moving down the jet, for frequencies $\nu \sim 10^{17}$ Hz and $B \sim 0.1$ G:
 - Radiative lifetimes of the order of one hour, corresponding to minutes in the observer's frame (assuming $\delta \sim 10$) →
 - The electron accelerated to ultra-relativistic energies by **BZ-mechanism** **lose their energy, emitting X-ray photons (+ IC-scattering), very fastly**
 - The higher keV-MeV-GeV-TeV states observed on daily-weekly timescales and X-ray emission detected at sub-pc, pc and sometimes even at the pc distances (Chandra observations; e.g. Marscher & Jorstad 2011): **some local acceleration mechanisms in BLL jets to be continuously at work**
- Significantly higher X-ray luminosity during the flares than the maximal one expected from the initial acceleration

- **Rapid TeV variability observed in BL Lacs (e.g. in PKS 2155–304 (Aharonian+2007) and Markarian 501 (Albert+2007)):** the time-scales of a few minutes → shorter, by at least an order of magnitude, than the light-crossing time of the central SMBH with a typical mass → Variability is associated with small regions of the highly relativistic jet rather than the central region
- **With the observed t_{var} and jet Lorentz factor Γ , the flare should occur at a distance greater than $c t_{var} \Gamma^2$ (Begelman+2008) → Flaring region at a distance in excess of $100r_s$ from the central SMBH →**
 - Relativistic particles that are ultimately responsible for the emission are then required to have been ejected from the central region along with the jet, and subsequently survived out to r_s where they then radiate away their energy quickly (barely possible!)
 - Alternatively, the particles are accelerated within the jet itself, close to the emission region

- The most plausible “additional” acceleration mechanisms for the particles responsible for the nonthermal emission:
 - **Diffusive shock acceleration (DSA, or first-order Fermi-acceleration; Kirk+1998)** at the front of relativistic shocks:
 - *intermittent increase in the collimated matter or other instable mechanism in the vicinity of SMBH* → *propagation of relativistic shock moving down the jet* (Marscher 2008)
 - *collision high-energy plasma “blobs” entering the jet base with different velocities* (Bottcher & Dermer 2010) → *internal shocks*
 - *Jet instabilities (e.g. Kelvin-Helmholtz instability): stationary or internal shocks*
 - **Stochastic (second-order Fermi) acceleration by magnetic turbulence** in shocked jet area (Tramacere+2009)
 - The presence of the first and second-order Fermi mechanisms: **strengthened by the observed X-ray spectral curvature**, i.e., by the presence of log-parabolic (LP) spectra emitted by the **LP particle energy distribution (PED; Massaro+11)** → yielding LP photon spectra more naturally than the original interpretation of the spectral curvature is in terms of radiation cooling of high energy electron population, injected with a power law spectrum (Massaro+2004)

- X-ray spectra of HBLs (BL Lacs with synchrotron peaks at UV—X-ray frequencies): generally curved, fitted with the logparabolic model (LP, Massaro+2004)

$$F(E) = K(E/E_1)^{-(a+b \log(E/E_1))} \quad \text{ph/cm}^2/\text{s}$$

with **K**: normalization factor

E₁: reference energy, fixed to 1 keV

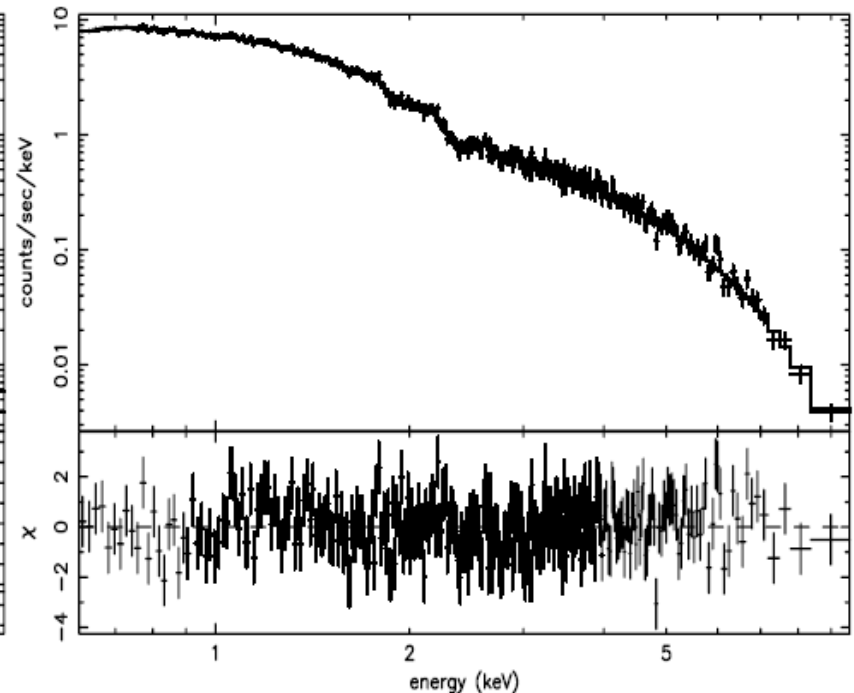
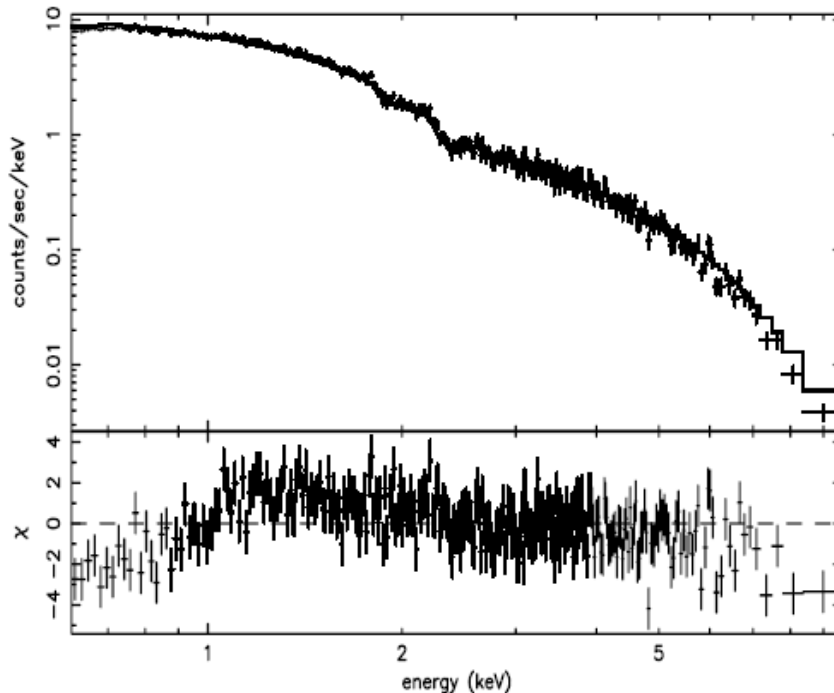
a: photon index at 1 keV

b: curvature parameter

The position of the synchrotron

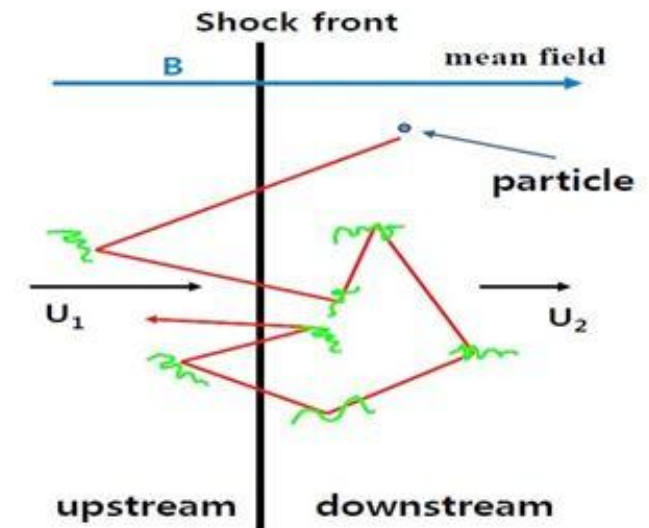
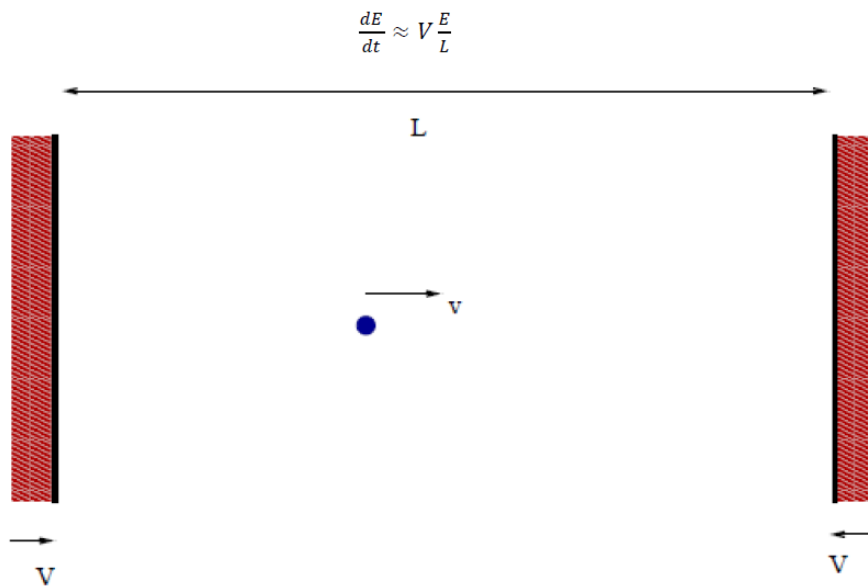
SED peak (Massaro+2004):

$$E_p = E_1 10^{(2-a)/2b} \quad \text{keV}$$

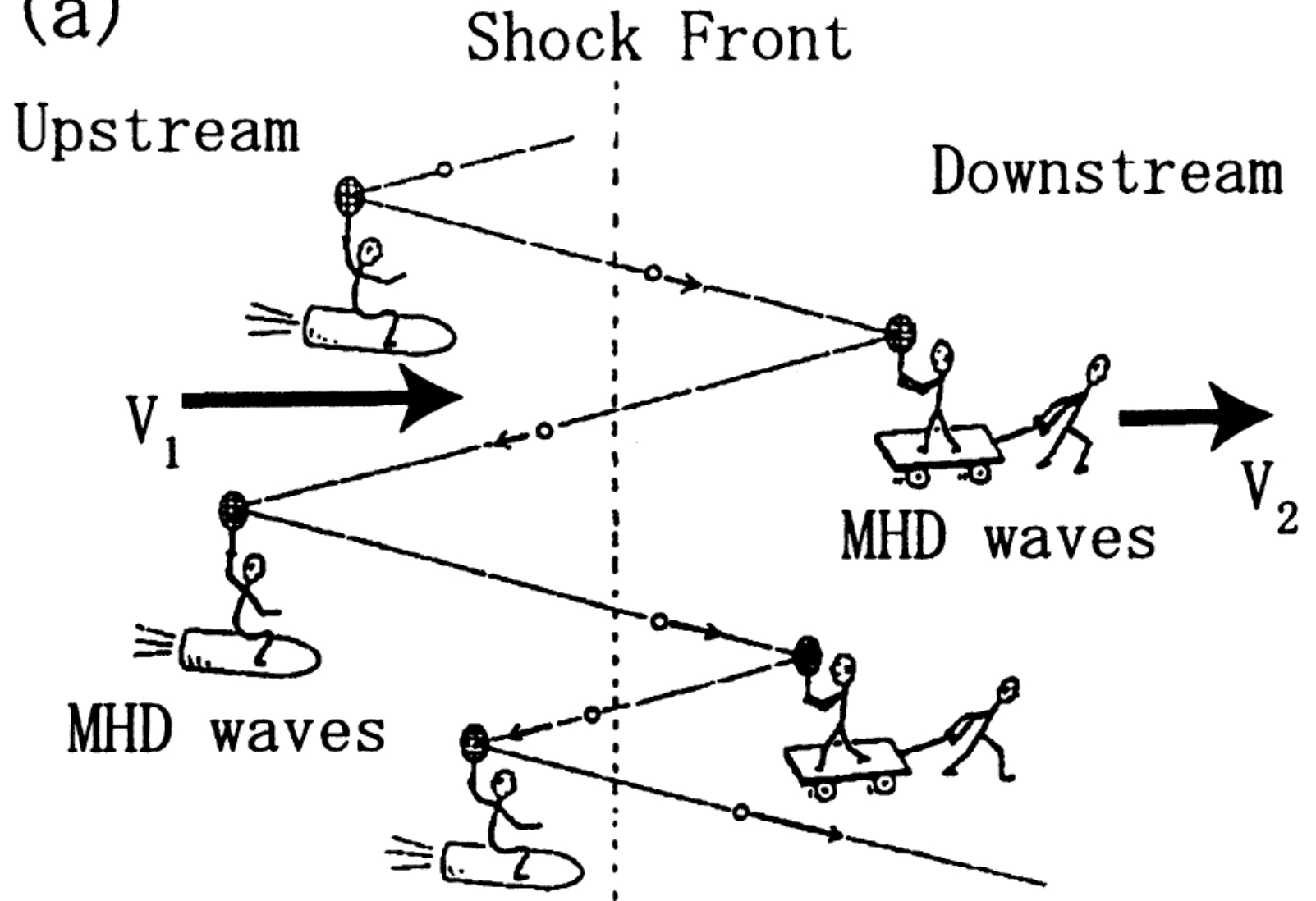


❖ First-Order Fermi Acceleration at Shock Front

- When particles are scattered by magnetic fluctuations, they gain energy whenever two subsequent scattering centres are moving towards each other, leading to a 'head-on' collision
- Suitable conditions are provided around a shock wave, where a relativistic particle crossing the shock always sees the plasma and the scattering centres on the other side of the shock approaching (Tammi & Duffy 2009)
- DSA relies on **repeated scattering of charged particles by magnetic irregularities (Alfven waves)** to confine the particles for some time near the shocks



(a)



- **Particles gain energy by multiple crossing the shock front** (Tammi & Duffy 2009):
 - **An average cycle increases the particle energy by a factor of Γ^2 (with plasma flow having Lorentz factor Γ) for the first cycle**
 - **By a factor of ~ 2 thereafter**
- **The duration of each cycle, as well as the probability for a particle to be injected into the acceleration zone, or re-cross the shock front, depends heavily on the details of the scattering of the particles in the turbulent plasma and the geometry of the shock**
- **A source of radius R can not confine particles with gyroradius $r_g > R$**
- **Eventually, a particle escapes from “acceleration zone” and cool by synchrotron radiation and IC scattering in the magnetic field behind it (“emission zone”;** Kirk+1998)

Generally, first-order Fermi mechanism yield a powerlaw spectrum (Massaro+2004):

$$N(> \gamma) = N_0(\gamma/\gamma_0)^{-s+1}, \quad (10)$$

where $N(> \gamma)$ is the number of particles having a Lorentz factor greater than γ and s is the spectral index given by:

$$s = -\frac{\text{Log } p}{\text{Log } \varepsilon} + 1, \quad (11)$$

here p is the probability that a particle undergoes an acceleration step i in which it has an energy gain equal to ε , generally assumed both independent of energy:

$$\gamma_i = \varepsilon\gamma_{i-1} \quad (12)$$

and

$$N_i = pN_{i-1} = N_0 p^i. \quad (13)$$

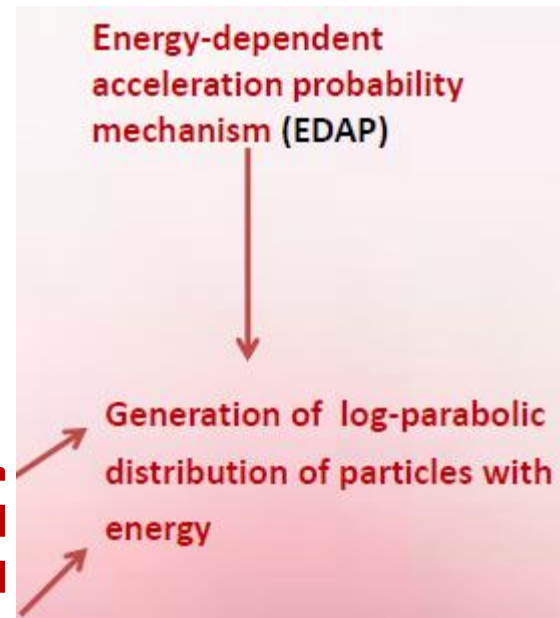
A log-parabolic energy spectrum follows when the condition that p is independent of energy is released and one assumes that it can be described by a power relation as:

$$p_i = g/\gamma_i^q, \quad (14)$$

where g and q are positive constants; in particular, for $q > 0$ the probability for a particle to be accelerated is lower when its energy increases. Such a situation can occur, for instance, when particles are confined by a magnetic field with a confinement efficiency decreasing for an increasing gyration radius. After

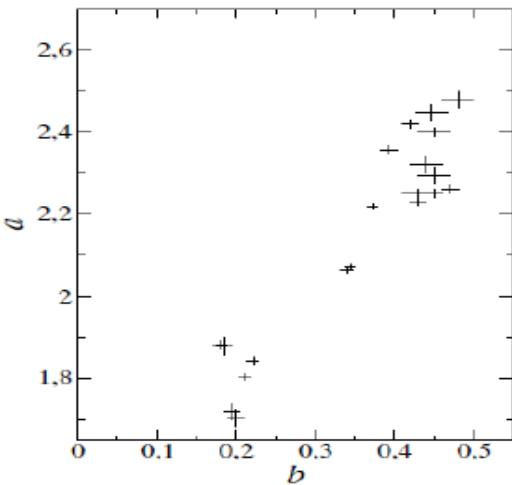
Energy-dependent
acceleration probability
mechanism (EDAP)

Generation of log-parabolic
distribution of particles with
energy

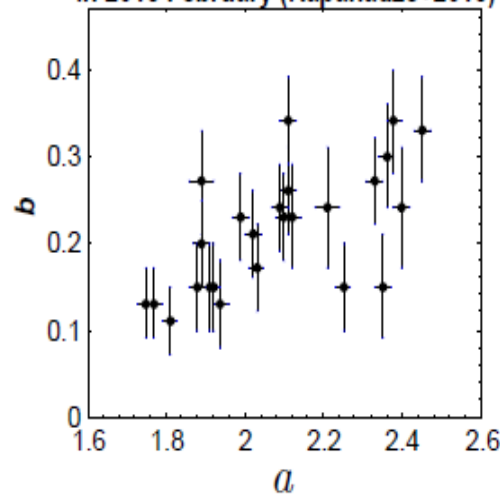


- **EDAP predicts a positive correlation between the parameters a and b**
- **Very weak positive a - b correlation from the Swift-XRT observations of Mrk 421 during 2005—2015** (Kapanadze 2016a, 2017a, 2018a,b) **and in 1ES 1959+650 in 2016 August – 2017 November** (Kapanadze+2018c): **“contamination” of the EDAP by other types of the acceleration and cooling processes (not yielding such correlation)?** →
- **Katarzynski et al. (2006): charged particle can be accelerated at the shock front by the first-order Fermi process and then continue gaining an additional energy via the stochastic mechanism in the shock downstream region. Eventually, the particle will be able to re-enter the shock acceleration region and repeat the combined acceleration cycle** → **Positive a - b correlation will be weak and may not even be observed**

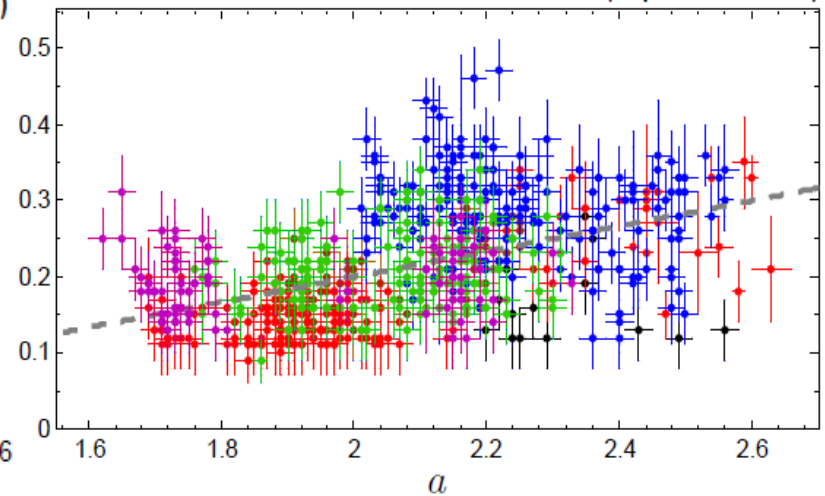
BeppoSAX observations of Mrk 421 in 1997–1998 (Massaro+2004)



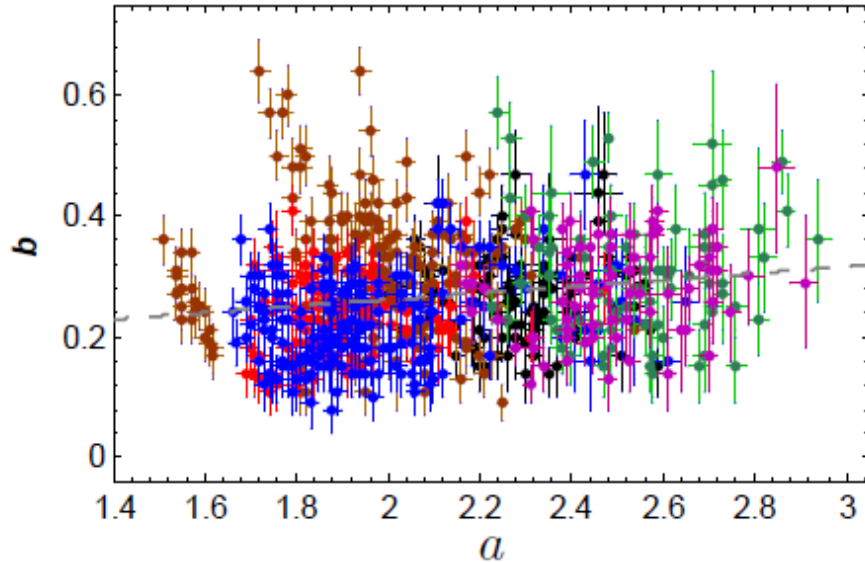
Swift-XRT observations of Mrk 421 in 2015 February (Kapanadze+2015)



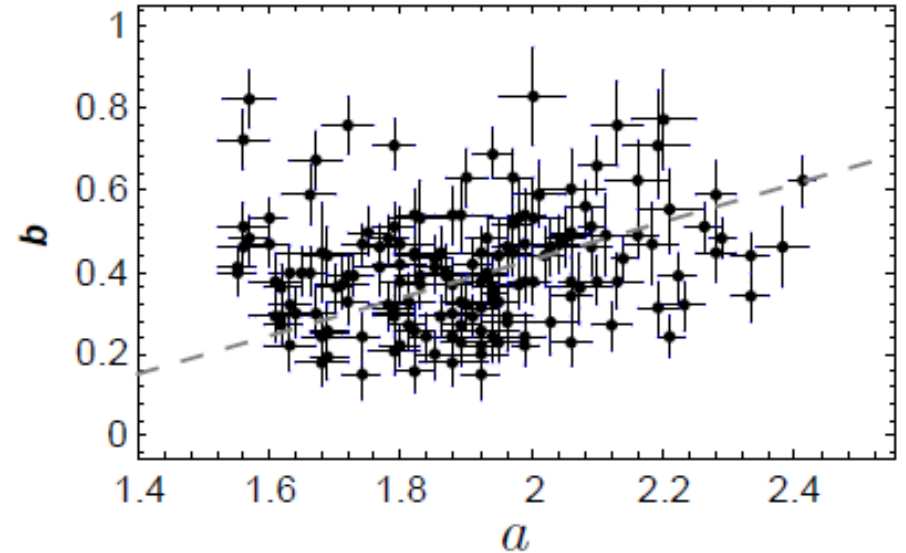
Swift-XRT observations of Mrk 421 in 2005--2008 (Kapanadze+2018a)



Swift-XRT observations of Mrk 421 in 2009–2012
(Kapanadze+2018b)



Swift-XRT observations of 1ES 1959+650 in
2016 Aug – 2017 Oct (Kapanadze et al. 2018c)



- **No positive positive a - b correlation from the Swift-XRT observations of 1ES 1959+650 during 2005 April—2016 July** (Kapanadze 2016b, 2016c, 2018d), **Mrk 501 in 2014 March-October** (Kapanadze+2017b), **PKS 2155-304 during 2005--2013** (Kapanadze+2014)

shock. In the Bohm limit, where the particle's mean free path is equal to its gyroradius, $\delta B \sim B$, in which case

$$r_g = \frac{\gamma mc^2}{eB} \approx 1700 \sqrt{\gamma^2 - 1} \left(\frac{B}{1\text{G}} \right)^{-1} \text{ cm.} \quad (1)$$

In this case, the acceleration time-scale can be simplified to

$$\tau_{\text{FI}} \gtrsim 6 \left(\frac{c}{v_s} \right)^2 \frac{\lambda}{c} \approx 6 \frac{r_g c}{v_s^2}, \quad (2)$$

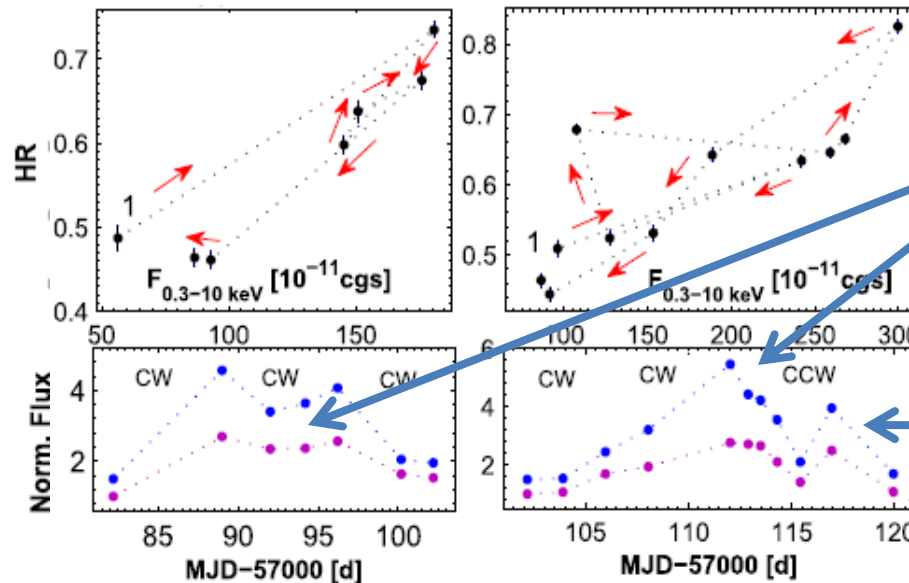
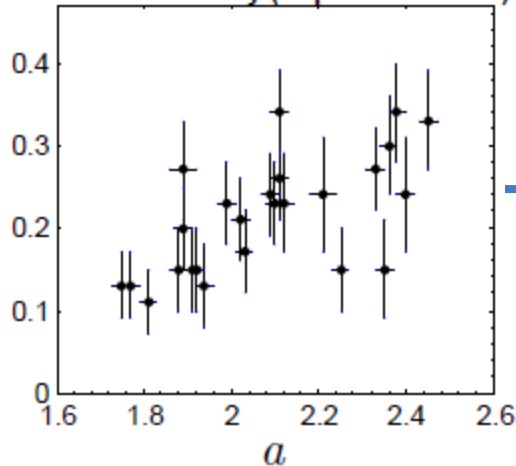
where v_s is the speed of the shock. For a 1-G magnetic field and a relativistic shock ($v_s \rightarrow c$), this gives, for an electron with $\gamma = 10^4$, an acceleration time-scale of a few milliseconds. For lower energy particles, the acceleration is even faster. With time resolution of the observations of the order of minutes, this is 'instantaneous'

Tammi & Duffy (2009):

• EDAP: rapid injection of very energetic particles in the emission zone rather than a gradual acceleration (Cui 2004)

• Clockwise (CW) spectral evolution in the hardness ratio – flux plane (Mastichiadis & Moraitis 2008)

Swift-XRT observations of Mrk 421 in 2015 February (Kapanadze+2015)



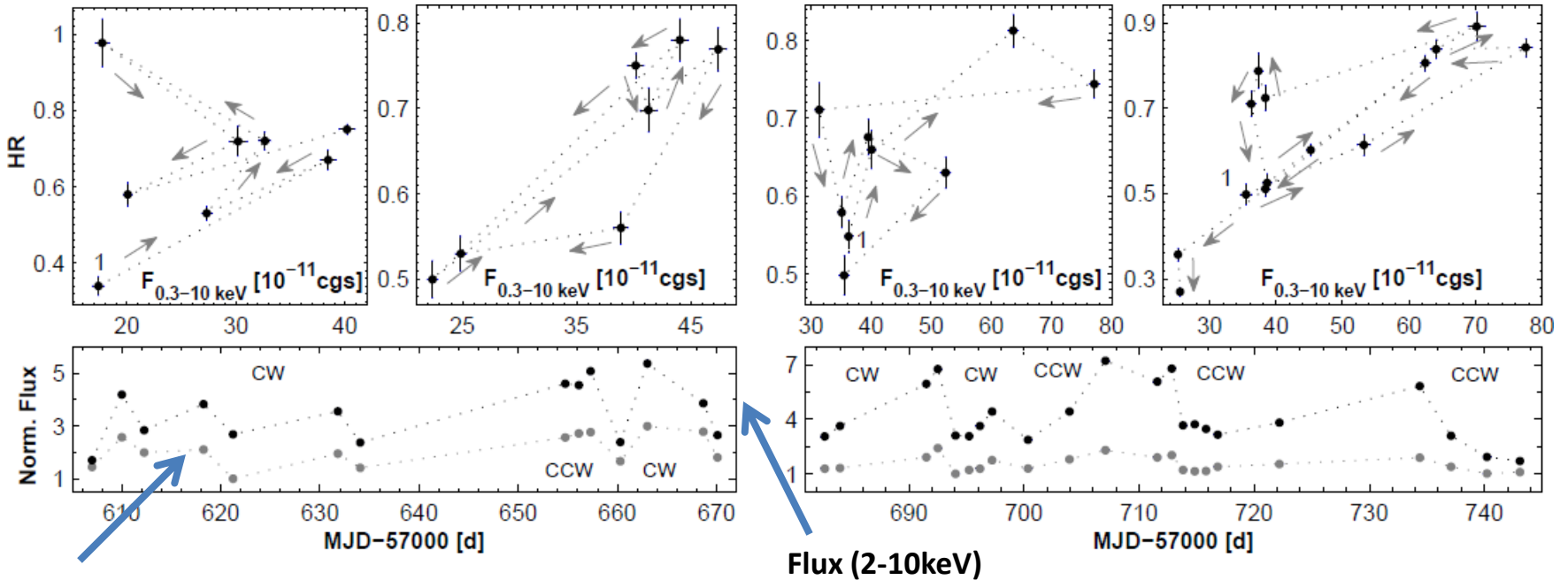
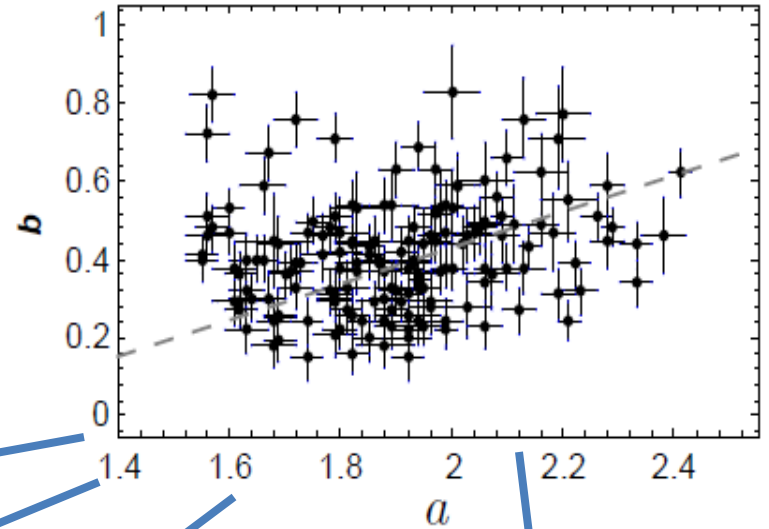
• Observation of the soft lag expected

Flux (0.3-2keV)

Flux (0.3-2keV)

- For protons, however, the mass and acceleration time-scale 1000 times larger than for electrons → **No instantaneous injection for the emission zone with significant hadron contribution**
- Similar situation for the electron-positron jet with the **magnetic field strength significantly lower than 1 G** (e.g. $B \sim 0.05$ G, often inferred from one-zone SSC modelling) → **CCW-type spectral evolution (gradual acceleration)** often observed along with the CW-loops in the epochs with the positive **a-b correlation**

Swift-XRT observations of 1ES 1959+650 in 2016 Aug - 2017 Oct (Kapanadze et al. 2018c)

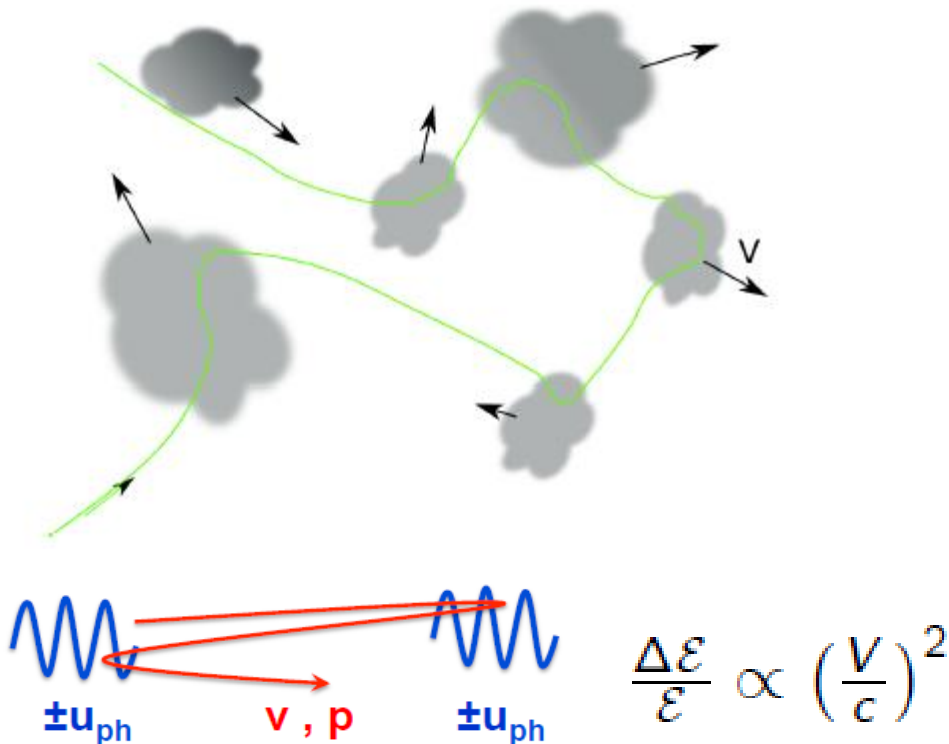


Flux (0.3-2keV)

Flux (2-10keV)

Stochastic Acceleration

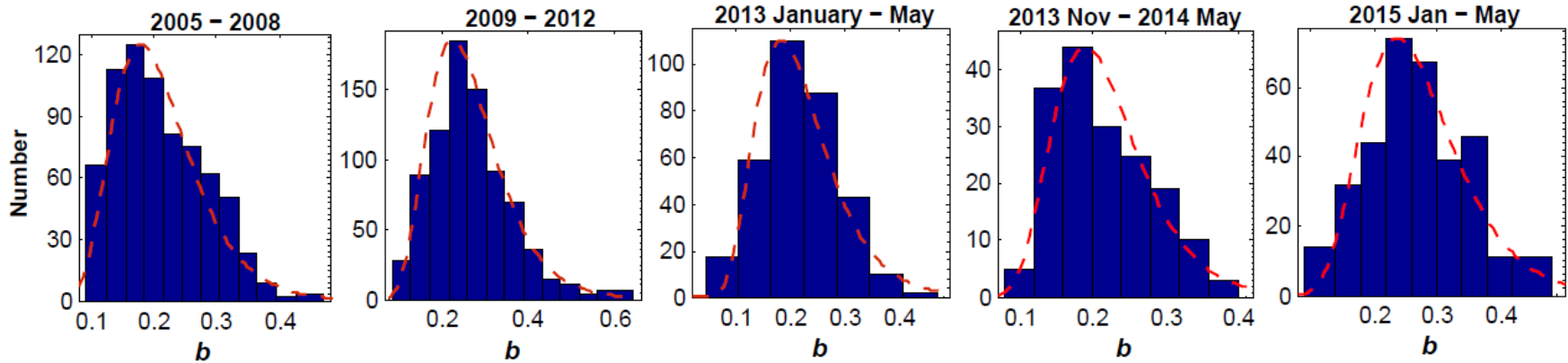
- LP particle energy distributions (PEDs) can be also established via the **stochastic (second-order Fermi) mechanism** operating in the **turbulent jet area** which accelerates particles using scattering centres moving relative to each other even without differences in the actual flow speed
- **Relativistic shocks In BL Lac jets: turbulent structures can be strongly amplified in shocked material** (Marcher 2014, Mizuno+2014)



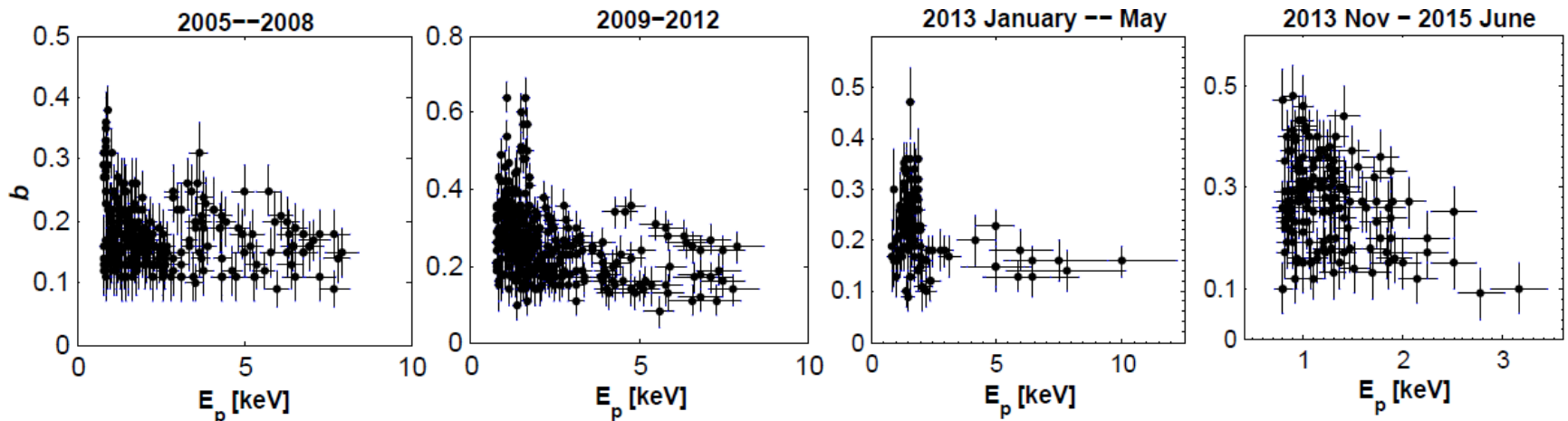
- **Alfven waves in the turbulent downstream of a relativistic shock: can provide promising conditions for efficient stochastic acceleration** (Virtanen & Vainio 2005)
- **Stochastic process: not tied to the plasma speed → continue to accelerate particles far away from the shock and for much longer than the first-order process – provided there is sufficient turbulence present** (Tammi & Duffy 2009)

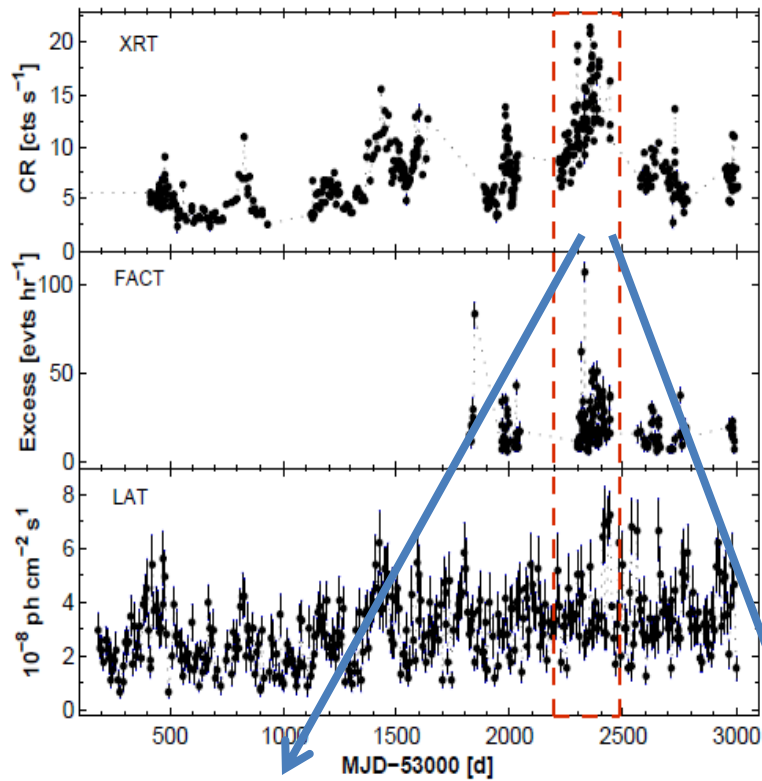
- Paggi +2009 and Tramacere +2011: PEDs represent the general solution of the **energy- and time-dependent Fokker–Planck equation** that includes **systematic** (e.g. BZ-mechanism) and **stochastic** (momentum diffusion due to resonant interactions with turbulent MHD modes) accelerations together with **radiative/adiabatic cooling** as well as **particle escape and injection terms**
- **Synchrotron SED expected to be** (Massaro+ 2011)
 - **relatively broader ($b \sim 0.3$)** when the **stochastic acceleration is efficient**
 - **narrower ($b \sim 0.7$): less efficient stochastic acceleration**

- **0.3-10 keV spectra of Mrk 421 during 2005-2015 (Swift-XRT observations): more than 90% of b values with $b \sim 0.3$ or smaller** (Kapanadze+2016a, 2017a, 2017d, 2017e)



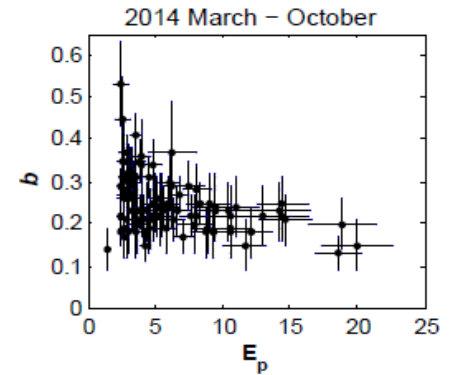
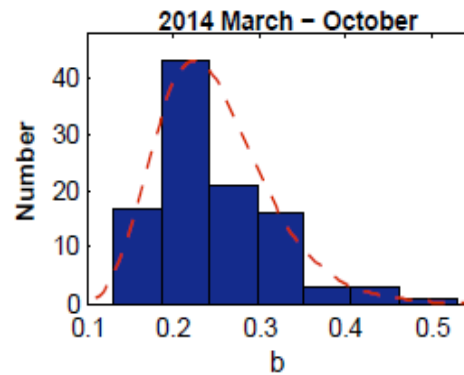
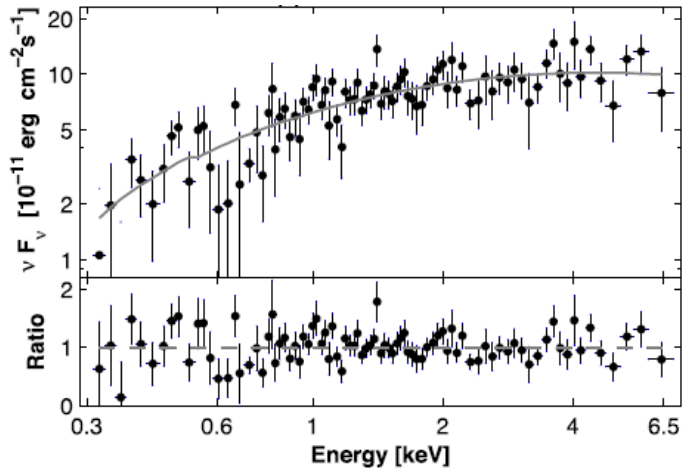
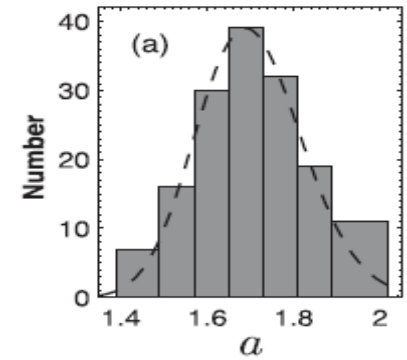
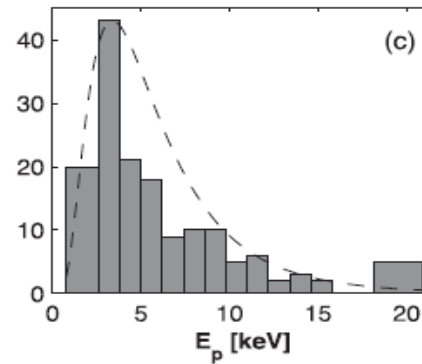
- $E_p - b$ anticorrelation, predicted for stochastic acceleration (Tramacere+2011): **observed** for Mrk 421 in different periods, although weak or very weak





Mrk 501: Very hard X-ray spectra during strong flaring activity in 2014 March – October (Kapanadze+2017a):

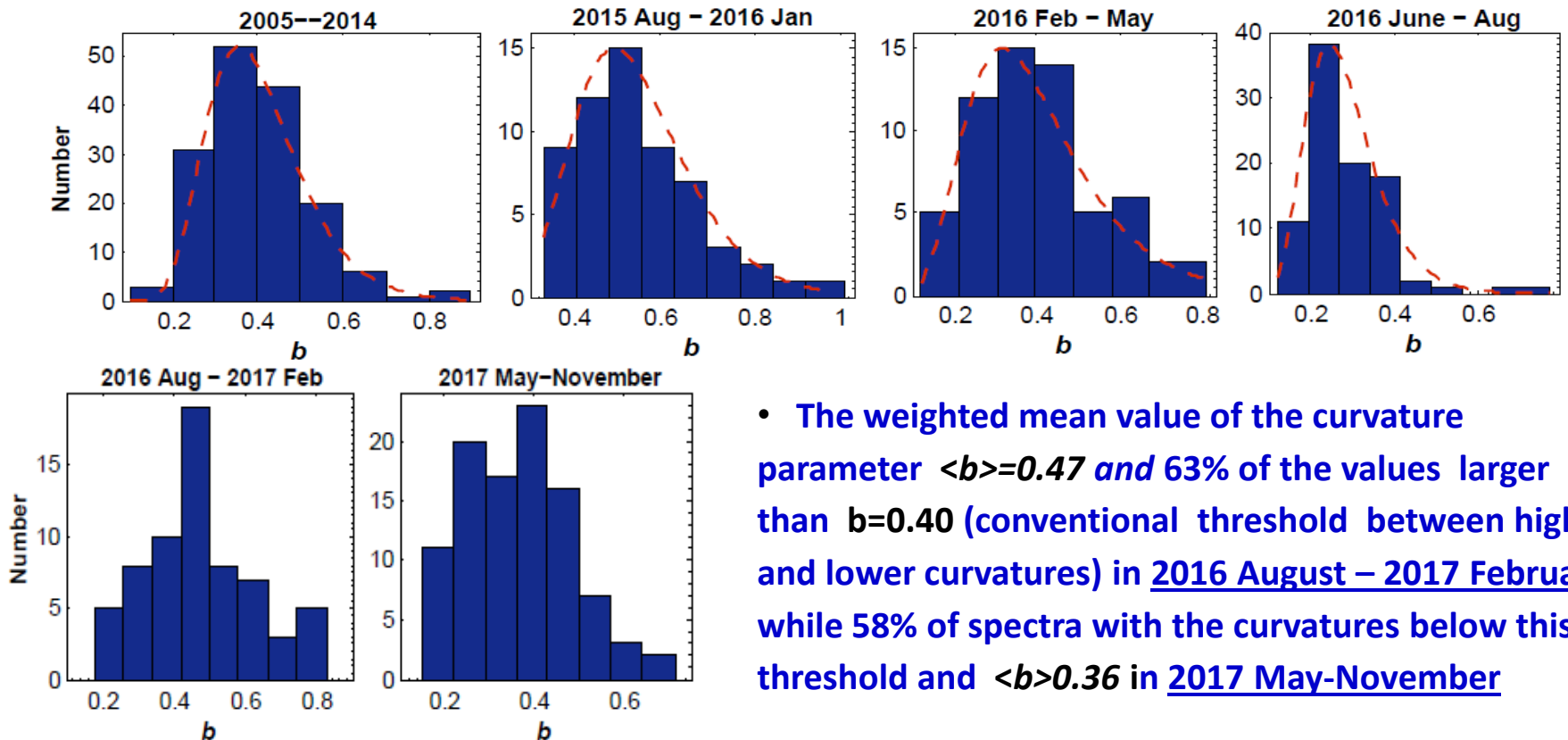
- $E_p > 2$ keV for 95% of LP-spectra
- 18% of LP-spectra harder than $a = 1.60$ (very rare occasion for BLLs!)



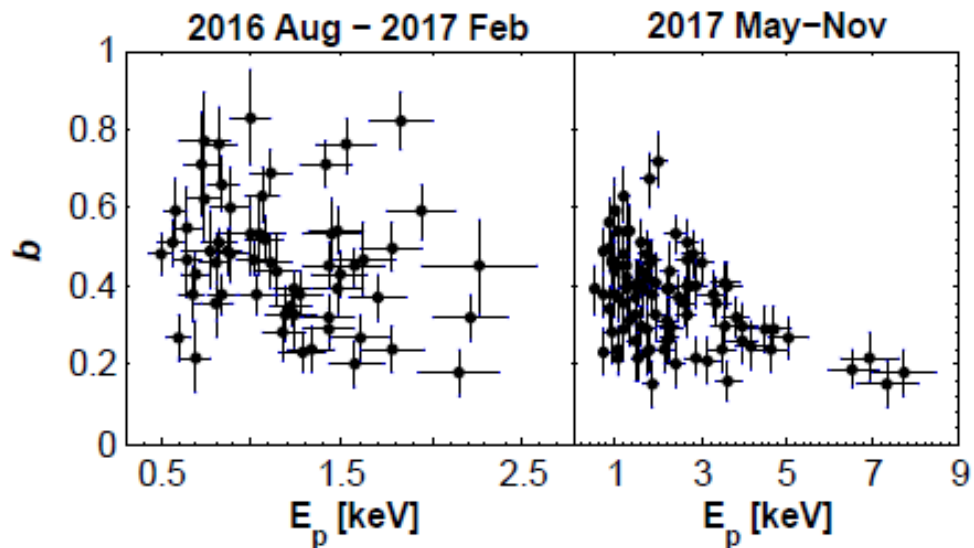
95% of the curvatures with $b \sim 0.3$ or smaller, weak $E_p - b$ correlation

The spectrum corresponding to the first 500-sec segment of the 2014 August 1 XRT observation with $a=1.39 \pm 0.06$, $b = 0.46 \pm 0.13$

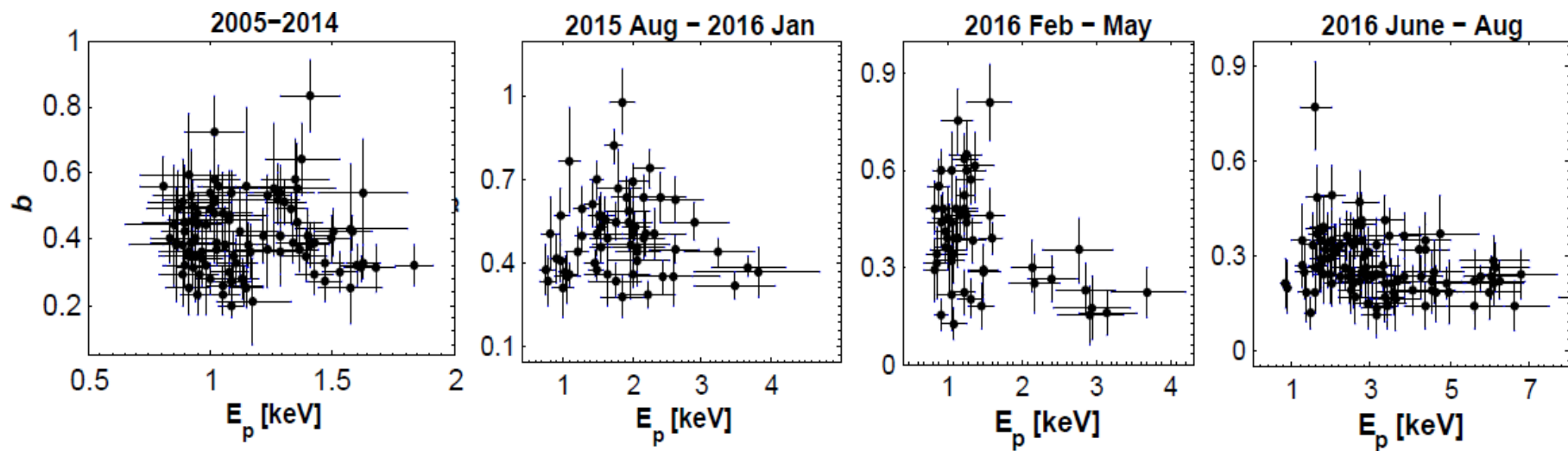
- **1ES 1959+650: TeV-detected BLL making an exclusion from other bright HBLs – mostly larger spectral curvatures: 85% of the spectra with $b > 0.4$, and more than 50% with $b > 0.5$ during 2015 August – 2016 January** → **Less efficient stochastic acceleration in that period?**
- **2016 June – August: 95% of the b values $b \sim 0.3$, expected in the case of efficient stochastic acceleration**
- **Periods 2005–2014 and 2016 February – May: intermediate situations between the aforementioned opposite cases**



- **The weighted mean value of the curvature parameter $\langle b \rangle = 0.47$ and 63% of the values larger than $b = 0.40$ (conventional threshold between higher and lower curvatures) in 2016 August – 2017 February, while 58% of spectra with the curvatures below this threshold and $\langle b \rangle = 0.36$ in 2017 May-November**



- **Weak $E_p - b$ correlation in 2016 February - August and 2017 May - November**
- **No correlation with 99% significance during 2005-2014 and 2015 August - 2016 January**



- **Detection of the correlation**

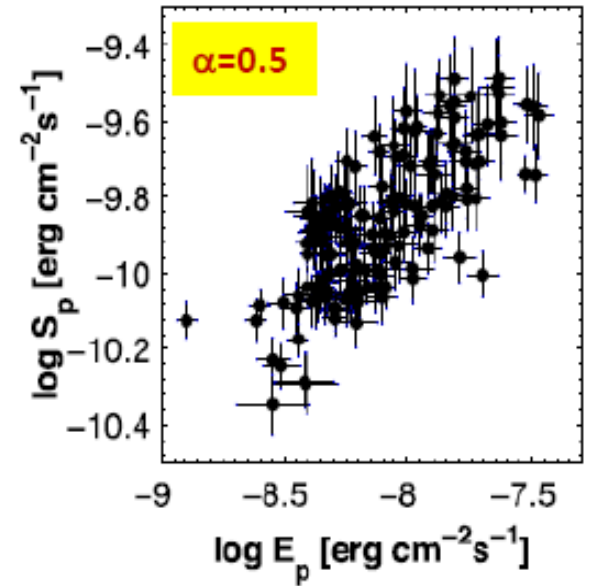
$$S_p \propto E_p^\alpha$$

(S_p - SED peak height): important to draw a conclusion about the **physical factor making the main contribution to the observed spectral variability** depending on the values of the exponent α

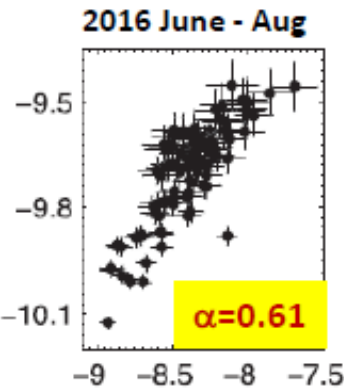
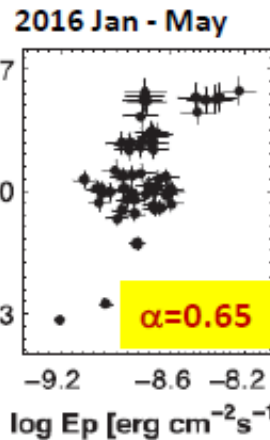
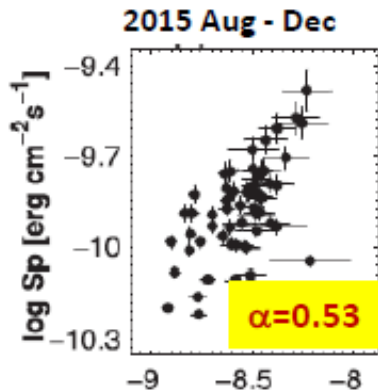
(Tramacere+2011):

- $\alpha=0.6$ - the parameters D_p (*momentum-diffusion coefficient*) and q (*the exponent describing the turbulence spectrum*) variable during the **stochastic acceleration** process: transition from the Kraichnan ($q = 3/2$) into “hard sphere” spectrum ($q = 2$)
- $\alpha=0.6 - 4$: changes in the number and energy of emitting particles, magnetic field, beaming factor)

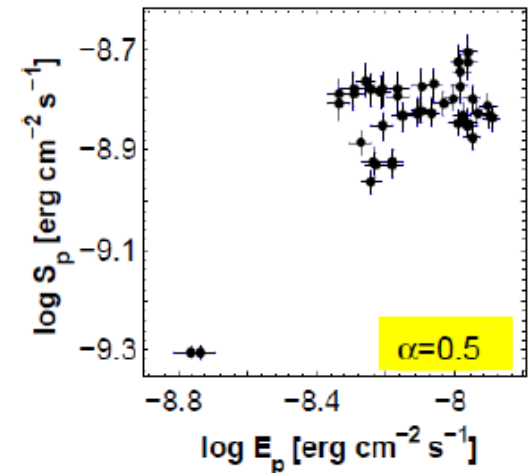
Mrk 501, 2014 March-October



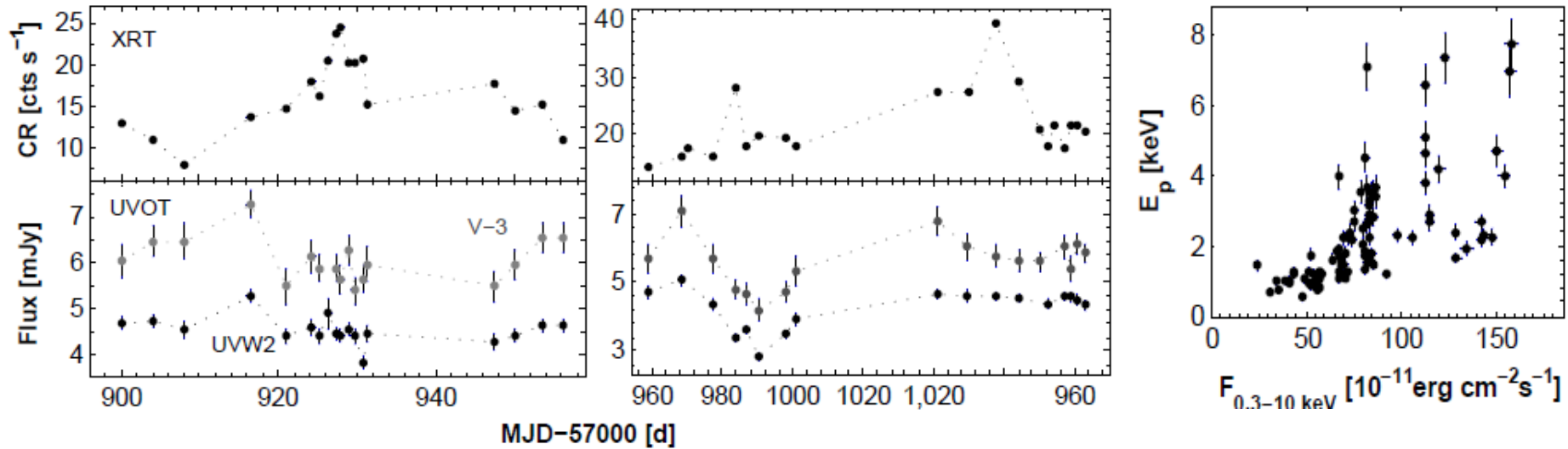
1ES 1959+650



Mrk 421
2008 April - June



1ES 1959+650, 2017 May–September (Kapanadze+2018d)



- **Frequent occurrence of declining optical-UV brightness in the epochs of X-ray flares** (Aleksic +2015; Kapanadze+2016b,2016c,208a etc.)
- **Explanation:** hardening in the electron energy distribution, shifting the entire synchrotron bump to higher energies, leading to a brightness decline at lower frequencies while the X-ray brightness is rising (Aleksic +2015) → **Corroborated by our finding of a positive $E_p-F_{0.3-10\text{ keV}}$ correlation** → **Shift of the synchrotron SED peak toward higher energies with increasing X-ray flux**
- **Underlying physical mechanism:** stochastic acceleration of electrons with a narrow initial energy distribution, having an average energy significantly higher than the equilibrium energy (Katarzynski+2006)

The **acceleration time-scale for stochastic acceleration** is (Rieger, Bosch-Ramon & Duffy 2007)

$$\tau_{\text{FI}} \approx \frac{3}{4} \left(\frac{c}{v_A} \right)^2 \frac{\lambda}{c} \approx \frac{3}{4} \frac{cr_g}{v_A^2}, \quad (4)$$

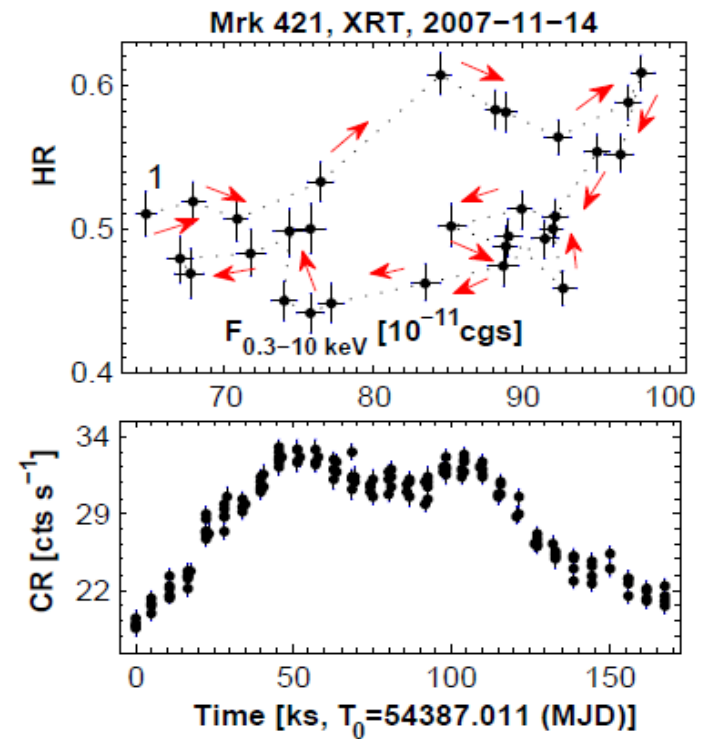
where the **Alfvén speed**, defined by

$$v_A^2 = \frac{(Bc)^2}{4\pi hn + B^2}, \quad (5)$$

depends on the enthalpy, $h = (\rho + P)/n$, with the energy density of the plasma, $\rho = nmc^2$, being a function of the composition and number density, n . The mass m depends on the composition and is $m_{\text{ec}} = 2m_e$ for pure electron-positron plasma, and $m_{\text{ep}} = m_e + m_p$ for ionized hydrogen. The effect of the gas pressure, P , is taken to be negligible.

- **Stochastic acceleration: very slow for**
 - **relatively low magnetic field** and
 - **high matter density**

- **Stochastic mechanism: gradual acceleration of particles versus the fast injection expected within EDAP** (Tammi & Duffy 2009)



- **Counter-clockwise (CCW) spectral evolution in the hardness ratio – flux plane in the case of gradual acceleration** (Cui 2004)

Tammi & Duffy (2009):

The **second-order process will be rapid enough** to occur on a time-scale shorter than the observed flaring time provided that

$$\left(\frac{\gamma}{10^4}\right) \left(\frac{B}{1 \text{ G}}\right)^{-3} \left(\frac{\Gamma}{50}\right)^{-1} \left[a \left(\frac{n}{10^5 \text{ cm}^{-3}}\right) + \left(\frac{B}{1 \text{ G}}\right)^2 \right] < 3.5 \times 10^6 \left(\frac{t_{\text{var}}}{300 \text{ s}}\right), \quad (6)$$

where $a = 2.1$ for an electron–positron plasma and $a = 1.9 \times 10^3$ for the hydrogen case. In a ‘highly magnetized plasma’,

$$a \left(\frac{n}{10^5 \text{ cm}^{-3}}\right) \ll \left(\frac{B}{1 \text{ G}}\right)^2, \quad (7)$$

the above constraint for a rapid stochastic process simplifies to

$$\left(\frac{\gamma}{10^4}\right) \left(\frac{B}{1 \text{ G}}\right)^{-1} \left(\frac{\Gamma}{50}\right)^{-1} < 3.5 \times 10^6 \left(\frac{t_{\text{var}}}{300 \text{ s}}\right). \quad (8)$$

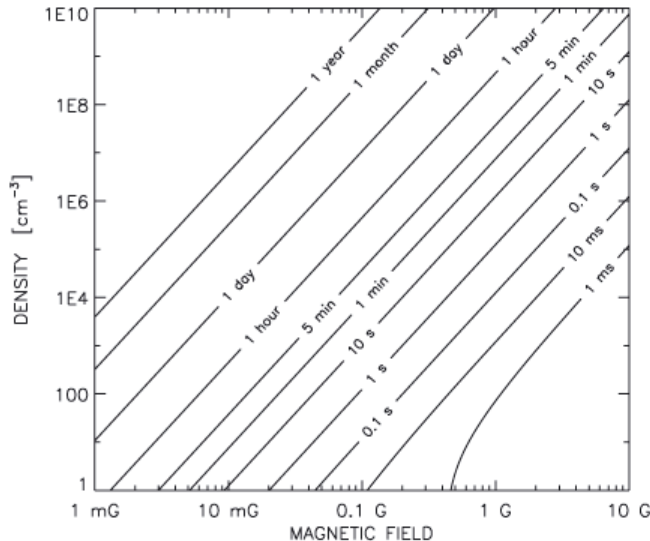


Figure 1. Stochastic-acceleration time-scale τ_{II} (equation 4) for an electron with $\gamma = 10^4$ as a function of plasma number density n and the magnetic-field intensity B in ionized hydrogen plasma. Time-scales vary from years

The stochastic acceleration time-scale as a function of the magnetic field and the number density of the plasma is shown in Fig. 1 for hydrogen plasma and in Fig. 2 for a pair plasma. Although the acceleration is very slow when the magnetic field is relatively low and the density is high, sites such as magnetically dominated active galactic nuclei (AGN) jets with relatively low matter density and compressed magnetic fields could favour fast acceleration with the blob matter density of the order of $\sim 10^3 - 10^6$ particles (protons and electrons) per cm^3 and a magnetic field of the order of 1 G, thus providing acceleration time-scales comparable to the observed minute-scale flickering. **For purely hadron-less pair plasma, the acceleration time-scale is much shorter, and can even be ‘instantaneous’ if the plasma density is not very high. In sources where the plasma is purely or mainly leptonic and has low density, sufficiently high magnetic field can turn the second-order acceleration more rapid than the first-order one. This, however, requires quite ideal turbulence conditions with particle-scattering waves moving in opposite directions over a sufficiently long length-scale.** The

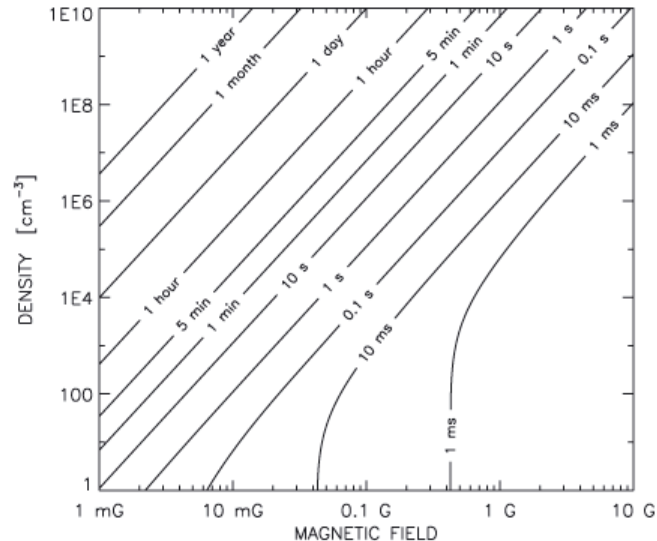
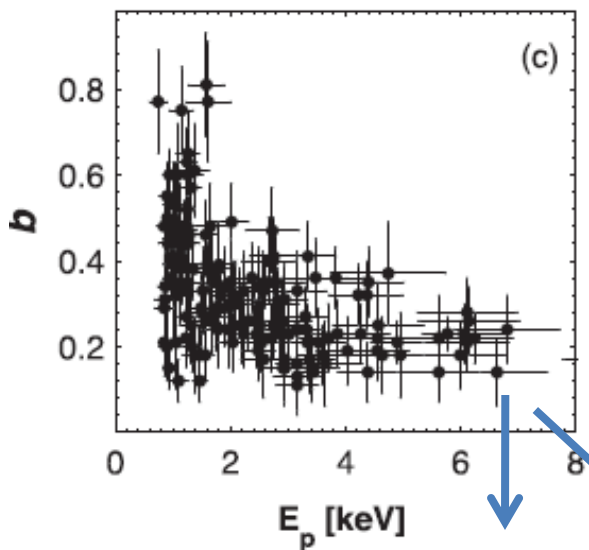


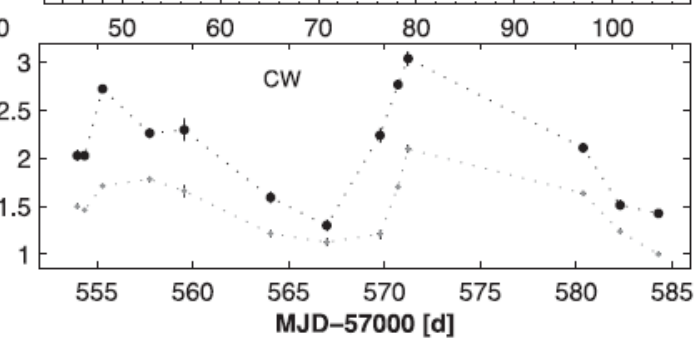
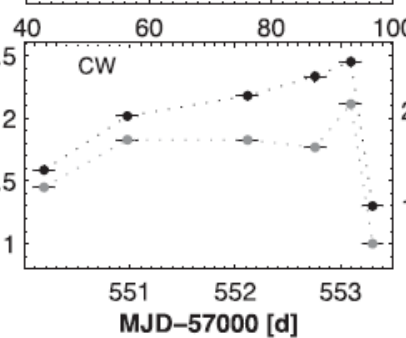
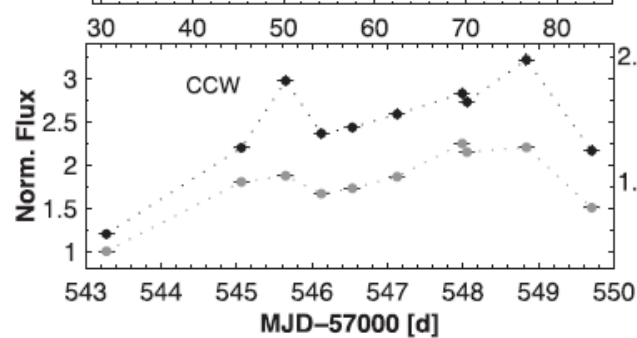
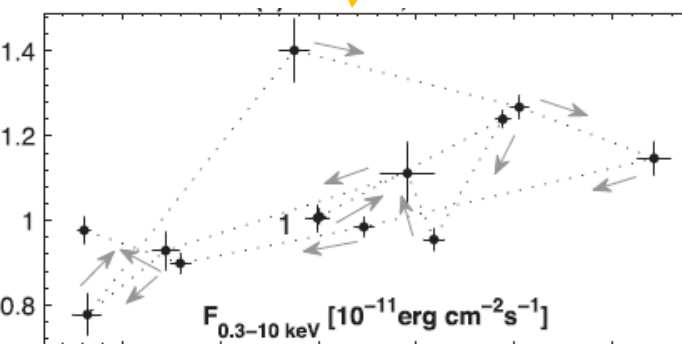
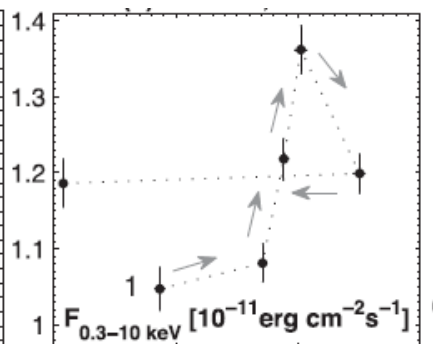
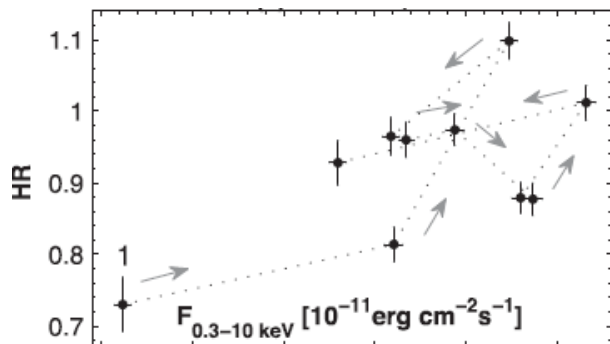
Figure 2. As Fig. 1, but for pure pair plasma.

1ES 1959+650, 2017 May - Nov



• Stochastic acceleration in the jet area with high-density, and/or low magnetic field? (gradual acceleration)

• Stochastic acceleration in the highly-magnetized, low-density jet plasma? (instantaneous injection)



- Frequently observed case for bright HBLs: CCW-loop during some longer-term X-ray flares, although including a CW sub-loop corresponding to the shorter-term, lower amplitude flare superimposed on the long-term variability trend: passage of the shock through jet area with different physical conditions? (e.g. standing shock generated due to different jet instabilities)
- Opposite cases also frequently observed
- Extreme spectral variability in bright HBLs (especially, in Mrk 421): transition from the log-parabolic into a power-law spectrum and vice versa, within 1 ks observational run → Extremely rapid changes of the magnetic field properties in the emission zone: from the state with a decreasing confinement efficiency with increasing gyro-radius (or from the turbulent state, both yielding a log-parabolic spectrum) into that without these properties (power-law spectrum), and vice versa

Summary and Conclusions

- **HBLs – one of the most extreme particle accelerators in the universe**
 - Bright sources in the X-ray band where the injection and radiative evolution of freshly accelerated particles can be tracked (especially in nearby bright HBLs Mrk 421, Mrk 501, 1ES 1959+650 where the flux and spectral variability can be detected within a few hundred seconds)
- **Most plausible acceleration mechanisms:**
 - BZ-mechanism for the jet launching and acceleration of the particles up to ultra-relativistic energies within the hundred Schwarzschild radii
 - Additional acceleration processes in the jets to explain the X-ray and gamma-ray emission to be generated on sub-pc, pc and even on kpc scales on some occasions as well as the observed energetics in the keV – TeV energy range
 - Major additional mechanisms: first and second order Fermi mechanisms, related to the propagation of relativistic shocks and turbulent structures in the jets (corroborated by the observed log-parabolic X-ray and gamma-ray spectra)
 - The features of second-order Fermi (stochastic) acceleration are observed more frequently than those of EDAP (a variety of first-order Fermi mechanism)

- Plausibly, there is “competition” between different acceleration mechanisms in HBL jets (“classical” first-order Fermi acceleration yielding a powerlaw energy spectrum, EDAP, stochastic acceleration etc.) resulting in a weakness or even absence of both $E_p - b$ and $\alpha - b$ correlations
- Observation of the correlation $S_p \propto E_p^\alpha$ with $\alpha \sim 0.6$ in some period for bright HBLs, implying a change in the turbulence spectrum in the jet area producing X-ray emission
- First- and second-order Fermi accelerations in the medium with different matter density, composition and magnetic field may yield as instantaneous, as well as gradual acceleration of the electrons to the energies necessary for producing of X-ray photons, resulted in the both CW and CCW loops in the HR-flux plane
- Optical-UV decline along with X-ray flares, explained by the stochastic acceleration of electrons with a narrow initial energy distribution, having an average energy significantly higher than the equilibrium energy

Thanks for

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